HANDLING UNSTRUCTURED DATA FOR OPERATOR LEARNING USING IMPLICIT NEURAL REPRESENTATIONS

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Paper under double-blind review

ABSTRACT

Operator learning methods are too often constrained by a fixed sampling of both the input and output functions. We propose a novel method to allow current operator learning methods to learn on any sampling. We show that our method can perform inference on unseen samplings, and that it allows returning outputs as continuous functions.

1 INTRODUCTION

An operator is a mapping between two function spaces. An example of operator is the mapping between the initial condition of a Partial Differential Equation (PDE), and the solution of the PDE at a later time. PDEs are ubiquitous in physics, where they model complex systems in many fields (fluid dynamics, electrodynamics...). They can be analytically intractable and require the use of very computationally-expensive numerical solvers. Recently, Deep Learning has emerged as a potential technique for helping to solve PDEs faster.

Recently, two operator learning approaches have been proposed: FNO (Li et al., 2021) and DeepONet (Lu et al., 2021), which have been followed by many applications. However, they come with some limitations over the structure of the input data: DeepONet requires the same sampling across all its input functions, while FNO requires the functions to be sampled on regular grids to be able to perform the Fast Fourier Transform. Such sampling is not always obtainable in real-life, for example in the case of drifting buoys to measure ocean currents. Implicit Neural Representations (INRs) are neural networks that take coordinates as inputs. They allow the representation of different types of signals such as images, 3D shapes via a continuous neural network by using sinusoidal activation functions (Tancik et al. (2020), Sitzmann et al. (2020)), while capturing the high-frequency details of the signal. They have exhibited interesting properties for data compression (Chen et al., 2022), video generation(Yu et al., 2022), and physics-learning(Yin et al., 2023).

In this work, we extend current operator learning algorithms by augmenting the available data to fit the algorithm requirements, using INRs. We show that our approach outperform conventional interpolation methods.

2 METHODOLOGY

Problem Setting We aim to learn an operator $\mathcal{F} : f \mapsto g$, where f and g are two functions defined on spatial domains $\mathcal{D}$ and $\mathcal{D}'$. In practice, we only have access to the values of these functions where they are measured. Thus, our dataset is composed of couples of functions $(f_i, g_i)_{i=1..n}$, evaluated at the respective sets of locations $\mathcal{X}_i = (x_i^j)_{j=1..p_i}$ and $\mathcal{Y}_i = (y_i^j)_{j=1..q_i}$, where $x_i^j \in \mathcal{D}$ and $y_i^j \in \mathcal{D}'$. We can rewrite it as learning a mapping between the sets $(f_i|_{\mathcal{X}_i})_i$ and $(g_i|_{\mathcal{Y}_i})_i$. We note that the number and positions of the sampled points can vary across each considered couple (f_i, g_i). Additionally, note that even if the domains $\mathcal{D}$ and $\mathcal{D}'$ are the same, the sensor locations of the input functions and of the output functions can be different.

Architecture Our method consists of three steps, which are depicted in Fig. 2. • First, we approximate the unknown input and output functions f_i and g_i using Implicit Neural Representations: for

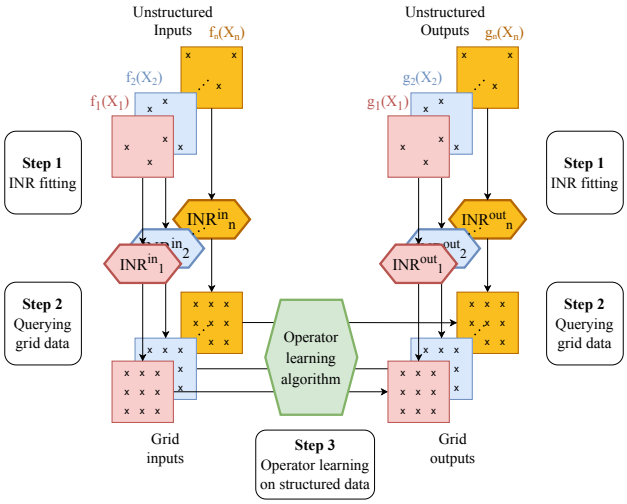

Figure 1: Three-step training architecture

each f_i function, we train an INR INR_i^{in} to fit f_i at the locations of $\mathcal{X}_i$. As in Dupont et al. (2022), we use modulated INRs: the networks share common parameters, which are then modulated differently for each INR, so that each INR also benefits from the common information in the data. The same process is done for the output functions g_i. • In a second time, we use these functions INR_i^{in} and INR_i^{out}, respectively approximating f_i and g_i, to provide values on regular grids $\mathcal{X}_{grid}$ and $\mathcal{Y}_{grid}$ by querying these INRs on the corresponding grids. • Finally, we train our operator learning algorithm on the surrogate regular-grid data: $(INR_i^{in}(\mathcal{X}_{grid}), INR_i^{out}(\mathcal{Y}_{grid}))_i$.

3 Experiments

We demonstrate our method on 2 different datasets: the Shallow-water equation, and the Navier-Stokes equation. We want to learn the operator $\mathcal{F}$ which maps an initial condition $u_0(x) = u(x, t = 0)$ to its state at a later time: $u_1(x) = u(x, t = 1)$. We use FNO as the operator learning algorithm, but our method is applicable to other methods that are constrained by the sampling of its input data.

The input data is measured at different randomly-sampled sensor locations for each initial condition. The sampling rate s is the ratio of the available sensors, compared to a regular grid of reference. We compare our method for handling unstructured data with more traditional interpolation methods: linear and cubic interpolations. We show in Table 1 that our augmentation with INRs outperforms other unbiased interpolation methods.

Table 1: Test relative MSE (↓) on datasets sampled with rates s from 0.05 to 0.9.

	Shallow-water				Navier-Stokes			
	$s = 0.05$	0.15	0.5	0.9	0.05	0.15	0.5	0.9
Linear	2.58e-3	1.41e-3	5.62e-4	1.89e-4	1.47e-2	5.58e-3	1.44e-3	**8.74e-4**
Cubic	2.40e-3	1.13e-3	4.54e-4	1.63e-4	1.00e-2	2.30e-3	1.08e-3	9.45e-4
Ours	**3.09e-4**	**2.40e-4**	**1.57e-4**	**1.27e-4**	**1.17e-3**	**1.28e-3**	**9.81e-4**	1.26e-3

4 Conclusion

We show that our approach can extend operating learning methods for unstructured data using INRs, and shows clear advantages with respect to standard data augmentation techniques.

URM STATEMENT

Please include this URM Statement section at the end of the paper but before the references before. In your anonymized submission, we recommend stating "The authors acknowledge that at least one key author of this work meets the URM criteria of ICLR 2023 Tiny Papers Track." For the camera ready version, we ask authors to identify which author(s) meet the URM criteria, e.g., "Author TFB meets the URM criteria of ICLR 2023 Tiny Papers Track." The authors are also welcome to come up with their own phrases to affirm meeting this criteria.

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A Results visualization

We use the incompressible 2D Navier-Stokes equation dataset from Li et al. (2021), and the 2D Shallow-water dataset from Takamoto et al. (2022)

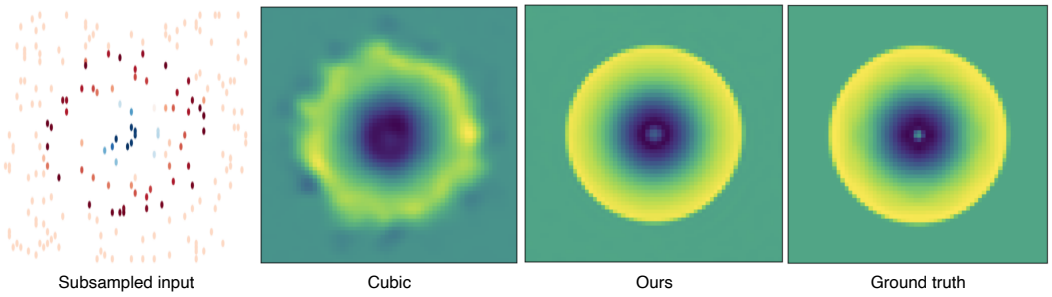

Figure 2: Shallow water comparison with s =0.05

B Implementation details

The model was implemented using PyTorch (Paszke et al. (2019)), as well as the Hydra (Yadan (2019)) and einops (Rogozhnikov (2022)) libraries. The operator learning algorithm was implemented using the FNO algorithm (Li et al., 2021).