Data Leakage in Federated Averaging

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Abstract

Recent attacks have shown that user data can be reconstructed from FedSGD updates, thus breaking privacy. However, these attacks are of limited practical relevance as federated learning typically uses the FedAvg algorithm. It is generally accepted that reconstructing data from FedAvg updates is much harder than FedSGD as: (i) there are unobserved intermediate weight updates, (ii) the order of inputs matters, and (iii) the order of labels changes every epoch. In this work, we propose a new optimization-based attack which successfully attacks FedAvg by addressing the above challenges. First, we solve the optimization problem using automatic differentiation that forces a simulation of the client's update for the reconstructed labels and inputs so as to match the received client update. Second, we address the unknown input order by treating images at different epochs as independent during optimization, while relating them with a permutation invariant prior. Third, we reconstruct the labels by estimating the parameters of existing FedSGD attacks at every FedAvg step. On the popular FEMNIST dataset, we demonstrate that on average we successfully reconstruct >45% of the client's images from realistic FedAvg updates computed on 10 local epochs of 10 batches each with 5 images, compared to only <10% using the baseline. These findings indicate that many real-world federated learning implementations based on FedAvg are vulnerable.

1 Introduction

Federated learning (McMahan et al., 2017) is a general framework for training machine learning models in a fully distributed manner where clients communicate gradient updates to the server, as opposed to their private data. This allows the server to benefit from many sources of data, while preserving client privacy, and to potentially use these massive amounts of data to advance the state of the art in various domains including computer vision, text completion, medicine and others (Kairouz et al., 2021).

Unfortunately, recent works (Zhu et al., 2019; Zhao et al., 2020; Geng et al., 2021; Geiping et al., 2020; Yin et al., 2021) have shown how to compromise the privacy aspect of federated learning. In particular, they demonstrated that an honest-but-curious server can reconstruct clients' private data from the clients' gradient updates only - a phenomenon termed *gradient leakage*. While these attacks can recover data even for complex datasets and large batch sizes, they are still limited in their practical applicability. More specifically, they typically assume that training is performed using FedSGD where clients compute a gradient update on a single local batch of data, and then send it to the server. In contrast, in real-world applications of federated learning, clients often train the model locally for multiple iterations before sending the updated model back to the server, via an algorithm known as federated averaging (FedAvg) (McMahan et al., 2017), thus reducing communication and increasing convergence speed (Kairouz et al., 2021). Under FedAvg, the server only observes the aggregates of the client local updates and therefore, one may expect that this makes the reconstruction of client's private data much more difficult.

This work: Data Leakage in Federated Averaging In this work, we propose a data leakage attack that allows for reconstructing client's data from the FedAvg updates, which constitutes a realistic threat to privacy-preserving federated learning. To this end, we first identify three key challenges that make attacking FedAvg more difficult than FedSGD: (i) during the computation of client updates, the model weights change, resulting in updates computed on weights hidden from the server, (ii) FedAvg updates the model locally using many batches of data, (iii) labels and parameters are simultaneously changing during the update steps.

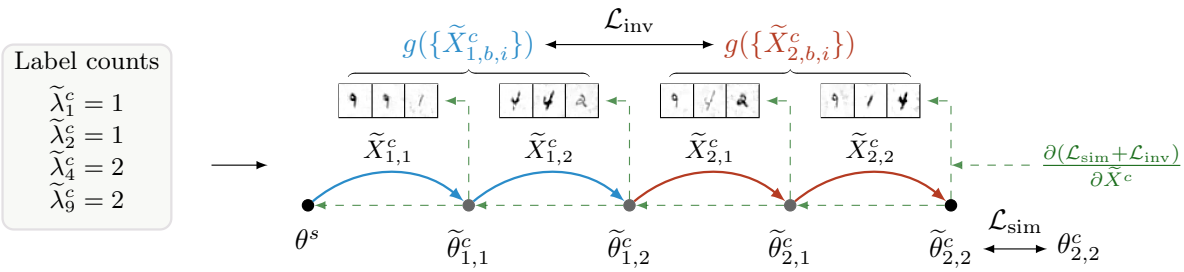

Figure 1: An overview of our attack on FedAvg. Our attack proceeds in two phases: first it estimates the label counts $\widetilde{\lambda}^c$ (left) and then uses these estimates to simulate the FedAvg updates on dummy inputs $\widetilde{X}^c$ (right). We then use automatic differentiation (green) to match the final weight $\widetilde{\theta}_{2,2}$ to the observed weight $\theta_{2,2}$ by minimizing the combination of the reconstruction loss $\mathcal{L}_{\text{sim}}$ and the epoch order-invariant prior $\mathcal{L}_{\text{inv}}$.

These challenges are illustrated in Figure 1, where a client with a total of 6 samples trains the model for 2 epochs, using a batch size of 3 resulting in 2 steps per epoch. Here, the model weights θ change at every step and there are many different batches, with the samples being processed in a different order during the first (blue) and second (red) epoch.

Addressing these challenges, we introduce a new reconstruction attack that targets FedAvg. Our method first runs a FedAvg-specific procedure that approximately recovers the label counts $\widetilde{\lambda}^c$ within the client's data (left in Figure 1) by interpolating the parameters, thus addressing the last challenge. This information is then used to aid the second phase of the attack, which targets the individual inputs. To recover the inputs, our algorithm uses automatic differentiation to optimize a reconstruction loss consisting of two parts. The first part is a FedAvg reconstruction loss $\mathcal{L}_{\text{sim}}$, designed by simulating the client's training procedure, that accounts for the changing parameter values, addressing the first challenge. The second part solves the remaining challenge using an epoch order-invariant prior, $\mathcal{L}_{\text{inv}}$, which exploits the fact that the set of inputs is the same in every epoch, thus mitigating the issue of having a large number of batches during the local updates. To leverage this information, this term uses an aggregating order-invariant function g to compress the sequence of samples in an epoch, and to then compare the results of this operation across different epochs.

We empirically validate the effectiveness of our method at recovering user data when the FedAvg algorithm is used on realistic settings with several local epochs and several batches per epoch on the FEMNIST (Caldas et al., 2018) and CIFAR100 (Krizhevsky et al., 2009) datasets. The results indicate that our method can recover a significant portion of input images, as well as accurate estimates of class frequencies, outperforming previously developed attacks. Further, we perform several ablation studies to understand the impact that individual components of our attack have on overall attack performance.

Main Contributions Our main contributions are:

- Identifying key challenges in applying gradient leakage algorithms to FedAvg updates (Section 4.1).
- Solving these challenges through a novel algorithm based on automatic differentiation through a simulation of the FedAvg client update (Section 4.3) and on an additional term incorporating prior information about the optimization process (Section 4.4), as well as new label reconstruction attack for the setting of FedAvg (Section 4.5).
- An implementation of these insights into a new algorithm, that allows us to create end-to-end attack for realistic FedAvg updates with multiple epochs and multiple batches per epoch.
- A thorough evaluation of our attack on the FEMNIST and CIFAR100 datasets that validates its effectiveness at recovering private client data from FedAvg updates. In some of the settings we consider, our attack is able to accurately recover more than 50% of the clients' images, while prior methods are only able to recover 10%.

2 Related work

In this section we discuss closely related work in the area of data leakage for federated learning.

Federated learning Federated learning (FL) was originally proposed by McMahan et al. (2017) as a framework for training machine learning models with private data from edge devices (or clients), without transferring the original data to a central server and instead communicating only gradient updates. Because of the potential of FL to enable training machine learning models at scale while respecting the sensitive clients' data, a substantial amount of recent research focuses on studying and enhancing the privacy guarantees provided by FL (Abadi et al., 2016; Bonawitz et al., 2017). However, recent work has cast doubt on the extent to which FL is inherently private, by demonstrating that in many cases the information about the model gradients alone is enough to reconstruct the clients' data.

Gradient leakage Melis et al. (2019); Zhu et al. (2019) demonstrated that clients' data can be reconstructed from federated gradient updates. Next, Zhao et al. (2020) improved on the previous attack by showing that reconstructing inputs and labels separately simplifies the optimization process and leads to better reconstruction. Orthogonally, Zhu & Blaschko (2020) formulated gradient leakage attacks as a system of equations and provided an analytical solution under the assumption that the client has computed the update on a single data point. More recently, much work has been done on gradient leakage attacks on specific data domains through the use of input priors (Geiping et al., 2020; Dimitrov et al., 2022; Yin et al., 2021; Li et al., 2022; Deng et al., 2021). What these attacks demonstrate is that it is possible to reconstruct individual inputs from gradients aggregated over a batch of inputs, provided that a strong prior in the form of a regularizer is added to the optimization problem. Our proposed prior in Section 4.4 provides similar benefits. Additionally, following the observations of Zhao et al. (2020), Geng et al. (2021) designed attacks that approximate the frequency counts of labels in a single client's batch of data by looking at the gradients of the last linear layer of the model. We introduce a FedAvg-specific extension of this method in Section 4.5.

While the aforementioned works have achieved substantial progress in making gradient leakage attacks practical, the majority of these works focus on the reconstruction of a single data point or a single batch, given the respective gradient. Even works that consider FedAvg updates, such as Geiping et al. (2020) and Geng et al. (2021) focus on a setting where either small amount of epochs or small number of steps per epoch are executed on the clients' side. In contrast, we develop an end-to-end attack able to recover clients' data points from FedAvg updates computed on several epochs each with multiple intermediate steps – a problem of greater practical relevance given that FL clients usually perform many local iterations before sending their updates to the server to speed up convergence (McMahan et al., 2017).

Defenses against Gradient leakage Several works have studied the task of defending against gradient leakage. There are two main types of defenses – heuristic (e.g Sun et al. (2021) and Scheliga et al. (2022)), which are often broken by stronger attacks (Balunović et al., 2021); and defenses based on adding random noise, that provide differential privacy guarantees (Abadi et al., 2016) but often result in reduced model accuracy. In Appendix B.1, we experiment with networks defended using Gaussian and Laplacian noise, as well as random gradient pruning first suggested by Zhu et al. (2019).

Other threat models In this work, we focus on the honest-but-curious server model for analyzing client's data privacy. That is, we assume that the server follows the standard training algorithm, but can attempt to reconstruct clients data through the received updates. Stronger threat models have also been studied. Works such as Fowl et al. (2022; 2021); Boenisch et al. (2021) demonstrate that the server can achieve much better reconstruction of private data instances by adapting the model weights and/or architecture.

3 Background

In this section, we review the classic components of federated learning, provide an overview of existing gradient leakage methods and their limitations, and introduce the relevant notation.

Algorithm 1 The FedAvg's ClientUpdate Algorithm (Adapted from McMahan et al. (2017))

1: **function** ClientUpdate(X^c, Y^c, f, θ^s, η, m, $\mathcal{E}$)
2: $\quad\mathcal{B}^c \leftarrow \lceil \frac{N^c}{m} \rceil$
3: $\quad\theta^c_{0,\mathcal{B}^c} \leftarrow \theta^s$
4: $\quad$**for** epoch $e \in [\mathcal{E}]$ **do**
5: $\quad\quad\theta^c_{e,0} \leftarrow \theta^c_{e-1,\mathcal{B}^c}$
6: $\quad\quad\{X^c_{e,b}\}, \{Y^c_{e,b}\} \leftarrow$PartitionData($X^c$, Y^c, m)
7: $\quad\quad$**for** batch $b \in [\mathcal{B}^c]$ **do**
8: $\quad\quad\quad\theta^c_{e,b} \leftarrow \theta^c_{e,b-1} - \eta \cdot \nabla_\theta \,\mathrm{CE}(f(X^c_{e,b}, \theta^c_{e,b-1}), Y^c_{e,b})$
9: $\quad\quad$**end for**
10: $\quad$**end for**
11: $\quad$**return** $\theta^c_{\mathcal{E},\mathcal{B}^c}$
12: **end function**

3.1 Federated learning

Throughout the paper, we assume that a server aims to train a neural network f_θ parameterized by a parameter vector θ, with the help of $\mathcal{C}$ clients. We also assume that the neural network has L layers of sizes $n_1, n_2, \ldots, n_L$ respectively, with the dimension of input variables being d and the number of classes equal to K (with $n_L = K$ for notational convenience).

The vast majority of FL methods used in practice are based on the FedAvg algorithm (McMahan et al., 2017). The optimization process consists of T communication rounds. In each round, the server sends the current version of the model θ^s_t to a chosen subset of the clients. Each selected client c then computes an update to the model, based on their own data, and sends an updated version θ^c_t of the model back to the server.

Algorithm 1 outlines the FedAvg's client update algorithm for a selected client c at some communication round. We denote $X^c \in \mathbb{R}^{d \times N^c}$ and $Y^c \in [K]^{N^c}$ the client's private inputs and labels respectively, where N^c signifies the number of private data points of that client and $[K]$ denotes the set $\{1, 2, \ldots, K\}$. At each local client's epoch $e \in \{1, 2, \ldots, \mathcal{E}\}$, the client randomly partitions its private data into sets of $\mathcal{B}^c = \lceil \frac{N^c}{m} \rceil$ batches $\{X^c_{e,b} \mid b \in [\mathcal{B}^c]\}$ and $\{Y^c_{e,b} \mid b \in [\mathcal{B}^c]\}$, each of size m (Line 6). For each batch b and epoch e, a local SGD update (Ruder, 2016) is computed for the cross entropy loss (denoted CE in Algorithm 1) on the respective batch of data. The SGD update is then applied to the network weights, producing the new intermediate weights $\theta^c_{e,b}$ (Line 8). Finally, the client sends the last updated weights $\theta^c = \theta^c_{\mathcal{E},\mathcal{B}^c}$ back to the server.

As a special case of FedAvg, when only one epoch ($\mathcal{E} = 1$) and one batch ($\mathcal{B}^c = 1$) are used locally in all rounds, one obtains the so-called FedSGD algorithm. While easier to attack, FedSGD is rarely used in practice, especially when the number of clients is large, as it leads to a significant increase in communication.

3.2 Gradient leakage attacks

In this section, we review existing attacks based on the honest-but-curious server model. The majority of these reconstruction methods, known as *gradient leakage attacks*, target the FedSGD training procedure (Zhu et al., 2019; Zhao et al., 2020), where the clients communicate a single gradient update to the server. As we attack FedAvg updates in this work, which share weights updates instead of individual gradients, we use the more general term *data leakage attacks* to refer to leakage attacks from both types of updates.

Reconstructions based on a single gradient To describe how existing attacks work, consider the case where the batch size $m = 1$, so that $\frac{\theta^s - \theta^c}{\eta} = \nabla_\theta CE(f(X^c_i, \theta), Y^c_i)$ is a gradient with respect to a single data point (X^c_i, Y^c_i) of the client c. The most common approach for reconstructing (X^c_i, Y^c_i) from the gradient is to search for an input-output pair, whose gradient most closely approximates the one that was sent back to the server. Given a distance measure D in gradient space, this translates to the following optimization problem:

$$\arg\min_{(x,y)} D\left(\nabla_\theta CE(f(x, \theta^s), y), \nabla_\theta CE(f(X^c_i, \theta^s), Y^c_i)\right). \tag{1}$$

The distance measure D is commonly chosen to be the L_2 (Zhu et al., 2019), L_1 (Deng et al., 2021) or cosine distance (Geiping et al., 2020). In the case of FedSGD, it is known that additionally using prior knowledge of the input distribution can improve the attacks (Geiping et al., 2020; Yin et al., 2021; Dimitrov et al., 2022), as it can guide the design of regularization terms that help in navigating the large optimization search space.

Label reconstruction methods Previous work (Zhao et al., 2020; Huang et al., 2021) has shown that in the cases when the labels of the associated data points are known, the optimization problem in Equation 1 is easier to solve for x by standard gradients-based optimization methods. For this reason, a common approach in the gradient leakage literature is to separate the process of recovering the clients' labels from that of the input reconstruction (Geiping et al., 2020; Geng et al., 2021; Yin et al., 2021).

Here we describe the procedure proposed by Geng et al. (2021) to reconstruct a client's label counts from FedSGD updates, which we later on adapt to the case of FedAvg. The paper assumes that the last layer of the network is linear, i.e. $f(X_i^c, \theta) = W_{\text{FC}} \cdot f_{L-1}(X_i^c, \theta) + b_{\text{FC}}$, where $W_{\text{FC}} \in \mathbb{R}^{n_{L-1} \times n_L}$ is a trained weight matrix, $b_{\text{FC}} \in \mathbb{R}^{n_L}$ is an optional bias term and $f_{L-1}(X_i^c, \theta)$ is the activation of the network for input X_i^c at layer $L-1$. In addition to the linearity assumption, Geng et al. (2021) assume that the K network outputs are converted into probabilities p_k by applying the softmax function and are then fed to the cross entropy loss for training, as in Algorithm 1. We note that these assumptions hold for most neural network architectures.

Under these assumptions, Geng et al. (2021) shows that one can precisely compute the label counts λ_k^c from FedSGD updates if the following quantities, computed on θ^s, are known: (i) $\Delta_k W_{\text{FC}}^s$ – the sum of the client's gradients with respect to the CE loss of the weights in W_{FC} that correspond to the kth layer output, (ii) $p_{i,k}^s$ – the softmax probability for class k computed on (X_i^c, Y_i^c) and (iii) O_i^s – the sum of the neuron activations in layer $L-1$ computed on (X_i^c, Y_i^c). While $\Delta_k W_{\text{FC}}^s$ can be computed by the server from the client update, $p_{i,k}^s$ and O_i^s are unknown as they depend on the client's data. Instead, Geng et al. (2021) suggest to use approximations $\widetilde{p}_k^s$ and $\widetilde{O}^s$, computed by feeding the network at parameters θ^s with random dummy data, as opposed to the unknown client's data point. This results in the following approximation to the label counts:

$$\widetilde{\lambda}_k^c = N^c \cdot \widetilde{p}_k^s - \frac{N^c \cdot \Delta_k W_{\text{FC}}^s}{\widetilde{O}^s}. \tag{2}$$

A crucial limitation of these gradient leakage attacks is that they are tailored to FedSGD. As we show in the next section, both the input and the label reconstruction are significantly harder in the context of FedAvg, yielding previously developed attacks inapplicable.

4 Effective Reconstruction Attack on FedAvg

In this section, we first discuss the challenges FedAvg updates pose for data leakage attacks in Section 4.1. We then present an outline of our reconstruction attack in Section 4.2 and discuss how it addresses these challenges. Finally, for the rest of the section we discuss each of the elements of the attack in more details.

4.1 Challenges to Data Leakage in Federated Averaging

We first outline the key challenges for performing gradient leakage when training with FedAvg, making the leakage more difficult than in the case of FedSGD where these problems do not exist.

The most discernible challenge is that in FedAvg clients perform multiple local updates, each computed on intermediate model parameters $\theta_{e,b}^c$ that are unavailable to the server. If the number of local updates is large, the change in parameter values is likely to be large, making the data leakage problem substantially more difficult. We address this issue in Section 4.3.

The second important challenge is that FedAvg updates the model locally using many different batches of data. More concretely, the client updates the model for several local epochs, in each epoch performing several local steps, and in each step using a different batch of data. As the server only sees the final parameters, it does not know which batches were selected at each step. This is a more difficult case than FedSGD which performs its update using only a single batch of data. We address this issue in Section 4.4.

Algorithm 2 Overview of our attack

1: **function** ATTACK(f, θ^s, θ^c, η, m, $\mathcal{E}$)
2: $\nabla\bar{\theta}^c \leftarrow \frac{1}{\eta \cdot U^c}(\theta^s - \theta^c)$
3: $\{\widetilde{\lambda}_k^c\} \leftarrow$ RECLABELS(θ^s, θ^c, U^c)
4: $\widetilde{Y}^c \leftarrow$ RANDORDER($\{\widetilde{\lambda}_k^c\}$)
5: $\widetilde{X}^c \leftarrow$ RANDINIT(m, $\mathcal{E}$)
6: **while** not converged **do**
7: $\widetilde{\theta}^c \leftarrow$ SIMUPDATE($\widetilde{X}^c$, $\widetilde{Y}^c$, f, θ^s, η, m, $\mathcal{E}$)
8: $\nabla\widetilde{\theta}^c \leftarrow \frac{1}{\eta \cdot U^c}(\theta^s - \widetilde{\theta}^c)$
9: $\ell \leftarrow \mathcal{L}_{\text{sim}}(\nabla\bar{\theta}^c, \nabla\widetilde{\theta}^c) + \lambda_{\text{inv}} \cdot \mathcal{L}_{\text{inv}}(\widetilde{X}^c)$
10: $\widetilde{X}^c \leftarrow \widetilde{X}^c - \eta_{\text{rec}} \cdot \frac{\partial \ell}{\partial \widetilde{X}^c}$
11: **end while**
12: **return** MATCHEPOCH($\widetilde{X}^c$), $\widetilde{Y}^c$
13: **end function**

Finally, the third challenge is that labels and parameters are simultaneously changing during local updates. In practice, this makes it difficult to reconstruct label counts (necessary to reconstruct the inputs) via exiting FedSGD methods. In Section 4.5 we introduce our method for label reconstruction which is more robust w.r.t to changing batch sizes and number of epochs.

4.2 Federated Averaging Attack

We now present our attack, which aims to address the aforementioned challenges by reconstructing the clients' data from the original weights θ^s, the client weights θ^c and client parameters η, m and $\mathcal{E}$. We provide an overview of our method in Algorithm 2. First, we invoke a label reconstruction procedure to generate the reconstructed labels $\widetilde{Y}^c$ (Line 3–4) and then use $\widetilde{Y}^c$ as an input to an optimization problem that minimizes a reconstruction loss ℓ (Line 9) with respect to a randomly initialized (via RANDINIT) input set $\widetilde{X}^c$ (Line 5–11).

To address the challenge with label reconstruction, Algorithm 2 relies on our algorithm RECLABELS. This algorithm is an improved, more robust version of the method of Geng et al. (2021) discussed in Section 3.2 and tailored to FedAvg. The algorithm no longer directly estimates network statistics $\widetilde{p}_k^s$ and $\widetilde{O}^s$ from θ^s (as introduced in Section 3.2), but instead estimates the statistics for each $\theta_{e,b}^c$ by interpolating between the statistics at θ^s and the statistics at θ^c.

For the input reconstruction, Algorithm 2 uses our novel FedAvg-specific reconstruction loss ℓ (Line 9), which consists of two components. The first component is the gradient similarity loss $\mathcal{L}_{\text{sim}}(\nabla\bar{\theta}^c, \nabla\widetilde{\theta}^c)$, which uses a server-side simulation of the client's updates via the SIMUPDATE method to obtain the approximated averaged update $\nabla\widetilde{\theta}^c$ (Line 8) and measures its similarity to the true averaged update $\nabla\bar{\theta}^c = \frac{1}{\eta \cdot U^c}(\theta^s - \theta^c)$, where $U^c = \mathcal{E} \cdot \mathcal{B}^c$ denotes the number of local update steps of the weights. This component aims to address the issue of having multiple hidden local steps by simulating the client's updates end-to-end. The second part is our epoch order-invariant prior $\mathcal{L}_{\text{inv}}(\widetilde{X}^c)$, which takes advantage of the fact that, even though the number of local steps $\mathcal{U}^c$ is large in realistic FedAvg settings, each input is part of exactly one batch per epoch. Finally, we reconcile our predictions by invoking MATCHEPOCH (discussed later in the section).

For simplicity, we present the optimization step in Algorithm 2 as an SGD update with a fixed learning rate η_{rec}. In practice, we instead rely on Adam (Kingma & Ba, 2014), as it was shown to perform better for solving gradient leakage problems (Zhao et al., 2020). The gradients $\frac{\partial \ell}{\partial \widetilde{X}^c}$, required by SGD and Adam, are calculated via an automatic differentiation tool – in our case JAX (Bradbury et al., 2018).

Over the next few sections we formally describe the components of Algorithm 2. In particular, we first discuss the reconstruction of inputs $\widetilde{X}^c$ from given label counts λ_k^c, as the inputs are typically of primary interest to the attacker. To this end, we describe our server simulation method SIMUPDATE in Section 4.3 and our epoch order-invariant prior $\mathcal{L}_{\text{inv}}(\widetilde{X}^c)$ in Section 4.4. Finally, in Section 4.5 we explain how to compute the estimated label counts $\widetilde{\lambda}_k^c$, thus creating a complete end-to-end attack.

4.3 Input Reconstruction through Simulation

As described in Section 4.1, a FedAvg client computes its gradient steps on the intermediate model parameters $\theta^c_{e,b}$ unknown to the server. These parameters can substantially differ from both θ^s and θ^c when the number of local steps U^c is high. To address this issue, our attack relies on the method SimUpdate to simulate the client's updates from Algorithm 1 and thus generate approximations of the intermediate $\theta^c_{e,b}$'s.

An important consideration for our simulation is how to split the variables $\widetilde{X}^c$ that Algorithm 2 optimizes over, as well as the label counts λ^c_k (or their reconstructions $\widetilde{\lambda}^c_k$ in the case of an end-to-end attack), into batches, as the client's original split is not available to the attacker. We fix the split randomly into $\mathcal{B}^c$ batches $\widetilde{Y}^c = \{\widetilde{Y}^c_b \mid b \in [\mathcal{B}^c]\}$ (via RandOrder) before the optimization in Algorithm 2 begins and keep the same split for all $\mathcal{E}$ epochs throughout the reconstruction. This provides a stable input order throughout the reconstruction and making the optimization simpler. We do this for two reasons: (i) the original client split of the labels into batches is hard to reconstruct, and (ii) we experimentally found that most batch splits produce similar weights at the end of each epoch. For the optimization variables $\widetilde{X}^c$, we choose to split it into $U^c = \mathcal{E} \cdot \mathcal{B}^c$ separate variables $\widetilde{X}^c = \{\widetilde{X}^c_{e,b} \mid e \in [\mathcal{E}], b \in [\mathcal{B}^c]\}$, one for each batch b and epoch e.

With these batch splits, SimUpdate executes a modified version of Algorithm 1. The two modification are: (i) The original partition of the data (Line 6) is replaced by our partitioning as described above, and (ii) we replace all occurrences of $X^c_{e,b}$ and $Y^c_{e,b}$ in Line 8 with $\widetilde{X}^c_{e,b}$ and $\widetilde{Y}^c_b$. The final reconstruction error is then calculated as the distance $\mathcal{L}_{\text{sim}}$ between the true average weight update $\nabla\bar{\theta}^c$ and the simulated update $\nabla\widetilde{\theta}^c$. In this paper, we rely on the cosine distance for $\mathcal{L}_{\text{sim}}$, as Geiping et al. (2020) showed it reconstructs data well in a wide range of challenging settings.

In addition to the approach described above, we also experimented with other variants. In particular, we investigated a version that uses $\mathcal{B}^c$ input optimization variables $\widetilde{X}^c = \{\widetilde{X}^c_b \mid b \in [\mathcal{B}^c]\}$ that are shared across all $\mathcal{E}$ epochs, as in Geiping et al. (2020). Opposed to our algorithm (described above), the optimization problem now becomes more complicated, since the optimization variables enter into the computation of multiple local steps. Further, we considered a simulation where $\mathcal{B}^c = 1$, which is relatively precise when the number of batches that the client uses per epoch $\mathcal{B}^c$ is small, but significantly diverges for larger $\mathcal{B}^c$, as typically used by FedAvg. We compare to these variants in Section 5 and find that they perform worse, confirming the effectiveness of our method.

4.4 Epoch Order-Invariant Prior

The use of multiple local batches of data for several epochs in the FedAvg client update makes the optimization problem solved by Algorithm 2 harder compared to FedSGD. In particular, several reconstructions of each input have to be generated and combined, as the inputs are repeatedly fed to the network. In this section, we propose to solve this issue by using the prior knowledge that each input is processed exactly once per epoch. This implies that the value of any order-invariant function g on the inputs in an epoch remains the same across epochs. During simulation, our prior encourages this property for every pair of epochs (e_1, e_2), by penalizing a chosen distance function D_{inv} between the values of g on the optimization variables at epoch e_1 and e_2. Let $\widetilde{X}^c_{e,b,i}$ denote the ith element in batch b and epoch e in $\widetilde{X}^c$. This results in the loss term:

$$\mathcal{L}_{\text{inv}} = \frac{1}{\mathcal{E}^2} \sum_{e_1=1}^{\mathcal{E}} \sum_{e_2=1}^{\mathcal{E}} D_{\text{inv}} \left(g(\{\widetilde{X}^c_{e_1,b,i} \mid b \in [\mathcal{B}^c], i \in [m]\}), g(\{\widetilde{X}^c_{e_2,b,i} \mid b \in [\mathcal{B}^c], i \in [m]\}) \right). \tag{3}$$

In Section 5.4, we experiment with several different order-invariant functions g and show that the best choice is dataset-dependent. Further, we show that the ℓ_2 distance is a good choice for D_{inv}. In Line 9 of Algorithm 2, the prior is balanced with the rest of the reconstruction loss using the hyperparameter λ_{inv}.

Once we have reconstructed the individual inputs at different epochs, we need to combine them into our final input reconstructions $\{\widetilde{X}^c_i \mid i \in [N^c]\}$. We do this in MatchEpoch (Line 12 in Algorithm 2) by optimally matching the images at different epochs based on a similarity measure and averaging them. The full details of MatchEpoch are presented in Appendix C. In Section 5.5, we experimentally show that our matching and

averaging procedure improves the final reconstruction, compared to using the epoch-specific reconstructions, which aligns with the observations from previous work (Yin et al., 2021).

4.5 Label Count Reconstruction

As we point out in Section 2, state-of-the-art FedSGD attack methods aim to recover the client's label counts λ_k^c before moving on to reconstruct the inputs X^c. This is commonly done by computing statistics on the weights and the gradients of f (Yin et al., 2021; Geng et al., 2021). As demonstrated by our experiments in Section 5.2, existing FedSGD methods struggle to consistently and robustly reconstruct the label counts λ_k^c. We postulate that this is due to the simultaneous changes across local steps of both the labels inside the processed client's batch and the network parameters. To address the challenge of recovering label counts robustly (to different batch and epoch sizes), we make the observation that the model statistics used by these methods often change smoothly throughout the client training process.

As the label counts reconstruction mechanism demonstrated in Geng et al. (2021) is the current FedSGD state-of-the-art, we chose to focus on adapting it to the FedAvg setting using the observation above. Recall from Section 3.2 that the method depends on the estimates $\widetilde{p}_k^s$ and $\widetilde{O}^s$ of the average softmax probabilities p_k^s and of average sum of neuron activation at layer $L-1$. For FedSGD, both estimates are computed at θ^s and with respect to a dummy client dataset, since the true data of the client is unknown to the server. However, in the case of FedAvg, multiple local steps are taken by the client, with each batch using different labels and changing the intermediate model parameters (both labels and parameters remain unobserved by the server). To account for these changes, we compute approximate counts $\widetilde{\lambda}_{k,i}^c$ for each local step $i \in [U^c]$ and aggregate these counts to obtain the approximate label counts $\widetilde{\lambda}_k^c$ for the full dataset of the client.

We proceed as follows. First, we compute approximations $\widetilde{p}_k^s$ and $\widetilde{O}^s$ at θ^s (using a client dummy dataset) and approximations $\widetilde{p}_k^c$ and $\widetilde{O}^c$ at θ^c (using the same client dummy dataset), as described in Section 3.2. Then, we linearly interpolate these estimates between $\widetilde{p}_k^s$ and $\widetilde{O}^s$ and $\widetilde{p}_k^c$ and $\widetilde{O}^c$ for the U^c steps of local training, that is, we set $\widetilde{p}_{k,i}^c = \frac{i}{U^c}\widetilde{p}_k^s + \frac{U^c-i}{U^c}\widetilde{p}_k^c$ and $\widetilde{O}_i^c = \frac{i}{U^c}\widetilde{O}^s + \frac{U^c-i}{U^c}\widetilde{O}^c$ for each $i \in [U^c]$ and $k \in [K]$. Next, we use these approximations to obtain approximate label counts $\widetilde{\lambda}_{k,i}^c$ for every training step $i \in [U^c]$, using Equation 2. To compute the final approximate counts, we set $\widetilde{\lambda}_k^c = \frac{1}{\mathcal{E}}\sum_{i=1}^{U^c}\widetilde{\lambda}_{k,i}^c$. Note that we may need to adjust $\widetilde{\lambda}_k^c$ in order to enforce the invariant that $\sum_{k=1}^{K}\widetilde{\lambda}_k^c = N^c$. We remark that this interpolation of the local statistics is general and can be applied to other label reconstruction methods.

5 Experimental Evaluation

In this section we present an experimental evaluation of our proposed attack and various baselines.

Experimental setup We conduct our experiments on two image classification datasets. One is FEMNIST, part of the commonly used federated learning framework LEAF (Caldas et al., 2018). FEMNIST consists of 28×28 grayscale images of handwritten digits and letters partitioned into 62 classes. We evaluate with 100 random clients from the training set and select $N^c = 50$ data points from each. The other dataset is CIFAR100 (Krizhevsky et al., 2009), which consists of 32×32 images partitioned into 100 classes. Here, we simply sample 100 batches of size $N^c = 50$ from the training dataset to form the individual clients' data.

We ran all of our experiments on a single NVIDIA RTX 2080 Ti GPU. Each experiment takes between 5 and 30 hours to complete, with the exact runtime depending on the attack method. We apply our reconstruction methods on undefended CNN networks at initialization time. We provide additional experiments on defended and trained networks in Appendix B. Throughout the section, FEMNIST attacks that use our epoch prior set the order-invariant function g to the mean of the images in an epoch, while CIFAR100 attacks set it to the pixelwise maximum of the randomly convolved images. We investigate different choices for g in Section 5.4. We provide the exact network architectures and hyperparameters used in our experiments in Appendix A.

The rest of the section is organized as follows. First, in Section 5.1, we focus on image reconstruction with known label counts. In this setting, we compare our attack to using non-simulation-based reconstruction

Table 1: The effectiveness of different methods for reconstructing images with known labels. We measure the percentage of successfully reconstructed images and the average PSNR of the reconstructions.

					Ours (prior)		Ours (no prior)		Shared		FedSGD-Epoch		FedSGD	
Dataset	$\mathcal{E}$	m	N^c	U^c	Rec (%)	PSNR	Rec (%)	PSNR	Rec (%)	PSNR	Rec (%)	PSNR	Rec (%)	PSNR
FEMNIST	1	5	50	10	**42.3**	**19.7**	**42.3**	**19.7**	**42.3**	**19.7**	37.7	19.5	37.7	19.5
	5	5	50	50	**63.1**	**21.2**	56.2	20.7	59.0	20.6	56.1	20.4	13.7	17.4
	10	10	50	50	**63.8**	**21.3**	58.5	20.9	62.0	20.7	60.2	20.6	11.8	17.2
	10	5	50	100	**65.5**	**21.5**	50.6	20.3	59.9	20.3	56.0	19.6	8.3	16.7
	10	1	50	500	**49.5**	**20.1**	21.5	16.7	14.9	16.1	1.6	4.5	5.2	16.6
CIFAR100	1	5	50	10	**9.1**	**16.2**	**9.1**	**16.2**	**9.1**	**16.2**	7.7	16.1	7.7	16.1
	5	5	50	50	**55.2**	**19.2**	53.0	19.1	10.8	16.4	10.9	16.3	1.2	14.2
	10	10	50	50	**62.7**	**20.3**	60.1	20.0	23.8	17.2	26.2	17.4	0.6	13.8
	10	5	50	100	**58.8**	**19.8**	56.4	19.6	15.7	16.4	8.5	15.0	0.4	13.2
	10	1	50	500	**12.2**	**15.4**	11.0	15.2	0.4	12.5	0.0	8.9	1.3	12.8

methods and justify our algorithm design choices by showing that our method compares favorably to a selection of baselines. Next, in Section 5.2, we experiment with label counts recovery and show that compared to Geng et al. (2021) our method is more robust while achieving similar or better label reconstruction results. In Section 5.3, we evaluate our end-to-end attack that reconstructs both the inputs and label counts. Finally, in Section 5.4 and Section 5.5 we justify our choice for g for the different datasets and demonstrate the importance of averaging individual reconstructions across epochs for the quality of our results, respectively.

5.1 Input Reconstruction Experiments

In this section, we compare the image reconstruction quality, assuming the label counts per epoch are known (but not to which batches they belong), with methods from prior work (Geiping et al., 2020; Geng et al., 2021) as well as variants of our approach: (i) **Ours (prior)**, our full input-reconstruction method, including the order-invariant prior $\mathcal{L}_{\text{inv}}$, as described in Section 4.4, as well as the simulation-based reconstruction error $\mathcal{L}_{\text{sim}}$ that uses separate optimization variables for the inputs at different epochs, (ii) **Ours (no prior)**, same as **Ours (prior)**, but not using the epoch prior $\mathcal{L}_{\text{inv}}$, (iii) **Shared** (Geiping et al., 2020), the approach proposed by Geiping et al. (2020) which assumes same order of batches in different epochs in SimUpdate, as described in Section 4.3 (allows sharing of optimization variables without using the prior), (iv) **FedSGD-Epoch**, this variant simulates the FedAvg update with a single batch per epoch ($\mathcal{B}^c = 1$) and thus no explicit regularization is needed, (v) **FedSGD** (Geng et al., 2021), the approach proposed by Geng et al. (2021), disregards the simulation, and instead reconstructs the inputs from the average update $\nabla\bar{\theta}^c$ like in FedSGD.

To deal with the unknown label batch split at the client, for all attack methods we pick the same random batch split in Line 4 in Algorithm 2. We report the results for different values of the number of epochs $\mathcal{E}$ and batch sizes m in Table 1. For each experiment we report two measures: (i) the percentage of images that were successfully reconstructed by the attack, and (ii) the average peak signal-to-noise ratio (PSNR) (Hore & Ziou, 2010), which is a standard measure of image reconstruction quality, between the client and the reconstructed images. We note that PSNR is on a logarithmic scale, so even small differences in PSNR correspond to large losses in image quality. For the methods which produce multiple reconstructions of the same image (the two **Ours** methods and **FedSGD-Epoch**), we use matching and averaging approach, described in Section 4.4, to generate a single final reconstruction. We use a linear assignment problem similar to the one in Appendix C to match these final reconstructions to the client's original images before computing the reported PSNR values. We consider a reconstruction on FEMNIST (CIFAR100) successful if the corresponding PSNR value is > 20 (> 19). All results are averaged across all images and users. In Appendix B.3, we further experiment with the setting where both the label counts and their batch assignments are known.

Evaluating reconstructions From Table 1 we make the following observations. First, the full version of our method provides the best reconstructions in essentially all cases. Second, we find that our epoch prior $\mathcal{L}_{\text{inv}}$ improves the results in almost all cases except for the case of a single client epoch ($\mathcal{E} = 1$). We point out that in this case the three methods **Ours (prior)**, **Ours (no prior)** and **Shared**, as well as the two methods **FedSGD-Epoch** and **FedSGD**, are equivalent in implementation. We observe that the use of the prior results in a bigger reconstruction gap on the FEMNIST dataset compared to CIFAR100. Our hypothesis

Table 2: The effectiveness of our method and the baselines for the task of label counts reconstruction.

Dataset	$\mathcal{E}$	m	N^c	U^c	Ours	Geng et al. θ^s	Geng et al. θ^c
FEMNIST	1	5	50	10	3.4 ± 2.01	$\mathbf{3.2} \pm 1.84$	3.6 ± 2.07
	5	5	50	50	3.4 ± 2.10	7.8 ± 6.77	$\mathbf{3.2} \pm 1.92$
	10	10	50	50	$\mathbf{3.2} \pm 1.94$	8.2 ± 7.06	$\mathbf{3.2} \pm 1.96$
	10	5	50	100	$\mathbf{5.2} \pm 2.98$	9.2 ± 4.35	$\mathbf{5.2} \pm 3.55$
	10	1	50	500	14.0 ± 6.28	15.4 ± 4.73	$\mathbf{13.3} \pm 6.35$
CIFAR100	1	5	50	10	4.2 ± 1.41	$\mathbf{3.9} \pm 1.30$	4.6 ± 1.55
	5	5	50	50	3.2 ± 1.24	7.6 ± 1.83	$\mathbf{2.5} \pm 1.09$
	10	10	50	50	2.8 ± 1.22	8.1 ± 1.84	$\mathbf{2.0} \pm 0.86$
	10	5	50	100	$\mathbf{4.9} \pm 1.57$	12.2 ± 1.96	5.2 ± 1.45
	10	1	50	500	$\mathbf{8.1} \pm 1.77$	10.2 ± 1.95	15.7 ± 3.48

is that the reason is the added complexity of the CIFAR100 dataset. In Section 5.4, we further demonstrate that more complex order-invariant functions g can help close this gap. Additionally, we observe that using separate optimization variables in SimUpdate sometimes performs worse than having shared variables when no epoch prior is used. However, this is not the case on the harder CIFAR100 dataset or when the prior is used, justifying our choice of using the separate variables. Third, while the **FedSGD** method performs worse when the number of epochs $\mathcal{E}$ is large, all other methods, perhaps counter-intuitively, benefit from the additional epochs. We think this is due to the client images being used multiple times during the generation of the client update resulting in easier reconstructions. Finally, our experiments show that **FedSGD-Epoch** performs well when the number of batches per epoch $\mathcal{B}^c$ is small, but its performance degrades as more updates are made per epoch, to the point where it becomes much worse than FedSGD on batch size $m = 1$.

5.2 Label Count Reconstruction Experiments

We now experiment with the quality of our label count reconstruction method and compare it to prior work (Geng et al., 2021). We consider the following methods: (i) **Ours**, our label count reconstruction algorithm RecLabels described in Section 4.5, (ii) **Geng et al.** θ^s, the label count reconstruction algorithm, as described in Geng et al. (2021), (iii) **Geng et al.** θ^c, same as **Geng et al.** θ^s, but with the parameters $\widetilde{O}^c$ and $\widetilde{p}_k^c$ estimated on the weights θ^c returned by the client.

In Table 2, we report the numbers of incorrectly reconstructed labels and their standard deviation for different values of the number of epochs $\mathcal{E}$, batch sizes m and different methods averaged across users. Unlike image and text reconstructions where prior knowledge about the structure of inputs can be used to judge their quality, judging the quality of label count reconstructions is hard for an attacker as they do not have access to the client counts. To this end, a key property we require from label count reconstruction algorithms is that they are *robust* w.r.t. FedAvg parameters such as datasets, number of epochs $\mathcal{E}$ and batch sizes m, as choosing them on the fly is not possible without additional knowledge about the client label distribution. Results in Table 2 show that the algorithm in Geng et al. (2021), as originally proposed, performs well when the number of local FedAvg steps U^c is small. Conversely, when approximating the parameters $\widetilde{O}^c$ and $\widetilde{p}_k^c$ at the client weights θ^c, the algorithm works well predominantly when U^c is large, except on CIFAR100 and $U^c = 500$. This makes it difficult to decide which variant of Geng et al. (2021) to apply in practice. In contrast, the results in Table 2 show that our label count reconstruction algorithm performs well across different settings. Further, it achieves the best reconstruction in the most challenging setting of CIFAR100 with small batch sizes ($m = 1, 5$) and its predictions often vary less than the alternatives. Therefore, we propose our method as the default choice for the FedAvg label count reconstruction algorithm due to its robustness.

5.3 End-to-End Attack

In this section, we show the results of our end-to-end attack, that combines our methods for label count and input reconstruction. We evaluate it under the same settings and compare it to the same methods as in Section 5.1. For all methods under consideration, we use our label count reconstruction method since, as we showed in Section 5.2, it performs well under most settings. We present the results in Table 3. Compared to the results in Table 1 (where we assumed label counts are known), the performance of our end-to-end attack remains strong even though we are now reconstructing the labels as well, showing the effectiveness of

Table 3: The effectiveness of our method and the baselines for the image and label counts reconstruction.

					Ours (prior)		Ours (no prior)		Shared		FedSGD-Epoch		FedSGD	
Dataset	$\mathcal{E}$	m	N^c	U^c	Rec(%)	PSNR	Rec(%)	PSNR	Rec(%)	PSNR	Rec(%)	PSNR	Rec(%)	PSNR
FEMNIST	1	5	50	10	**37.2**	**19.3**	**37.2**	**19.3**	**37.2**	**19.3**	35.7	19.2	35.7	19.2
	5	5	50	50	**51.9**	**20.3**	47.5	20.1	40.8	19.5	47.0	19.8	12.5	17.3
	10	10	50	50	**55.0**	**20.5**	50.0	20.2	45.7	19.7	50.0	19.9	10.6	17.0
	10	5	50	100	**48.5**	**20.2**	43.0	19.8	43.6	19.3	45.8	19.1	7.2	16.6
	10	1	50	500	**21.4**	**18.3**	14.5	16.9	7.5	15.2	1.1	3.6	4.7	16.5
CIFAR100	1	5	50	10	**5.0**	**15.7**	**5.0**	**15.7**	**5.0**	**15.7**	4.8	**15.7**	4.8	**15.7**
	5	5	50	50	**46.7**	**18.5**	45.0	18.3	7.0	15.8	7.7	15.8	0.8	14.1
	10	10	50	50	**54.3**	**19.4**	52.0	19.1	15.1	16.5	18.8	16.7	0.5	13.7
	10	5	50	100	**46.0**	**18.5**	43.2	18.2	8.7	15.7	8.1	14.9	0.6	13.1
	10	1	50	500	**6.7**	**14.7**	5.7	14.6	0.4	11.9	0.1	9.0	1.4	12.7

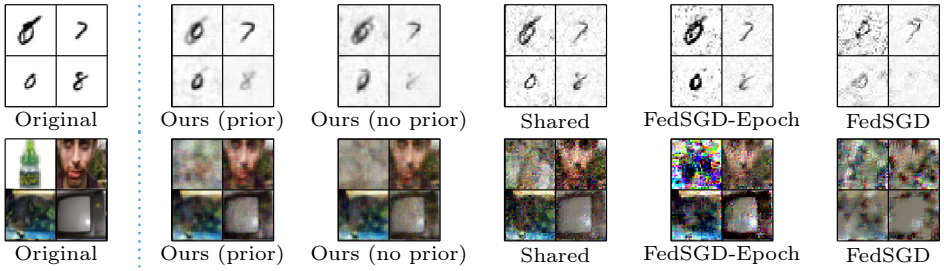

Figure 2: Reconstructions of 4 images from the $10 \times 5 \times 50$ FEMNIST and CIFAR end-to-end experiments.

our label reconstruction. In particular, most results remain within 10% of the original reconstruction rate (resulting in $> 45\%$ success rate of our attack in most settings) except for the case of $U^c = 500$, where due to the higher error rate of the label reconstruction we observe more significant drop in performance. Note that even in this setting our attack performs the best and is able to reconstruct 20% of the FEMNIST images successfully. Furthermore, we observe that the **Shared** method is less robust to errors in the label counts compared to the other methods, as with known label counts it performs better than the **Ours (no prior)** method on FEMNIST, but the trend reverses when the label counts are imperfect. We think the reason is the sensitivity to label count errors of the shared variables optimization procedure which further justifies our use of separate optimization variables.

Reconstruction Visualizations In addition to Table 3, we also provide visualizations of the reconstructions from the different attacks for $\mathcal{E} = 10$, $m = 5$, and $N^c = 50$ in Figure 2. For both FEMNIST and CIFAR, we show the reconstructions of 4 images from the first user's batch. We provide the full batch reconstructions in Appendix E.1. We observe that reconstructions obtained using our method are the least noisy. As we will see in Section 5.5, this is due to the matching and averaging effect. Further, we see that the prior has positive effect on the reconstruction as it can sharpen some of the edges in both the FEMNIST and CIFAR reconstructions. Finally, we also observe that the reconstructions of the **FedSGD** method are very poor.

5.4 Epoch Order-Invariant Priors

In this section, we investigate the effectiveness of various order-invariant functions g and distance measures D_{inv}. We experiment with the ℓ_1 and ℓ_2 distances for D_{inv}, as they are the most natural choices for a distance functions. For g, inspired by PointNet (Qi et al., 2017) that also relies on the choice of an order-invariant function, we experiment with the mean and max functions. Further, following Li et al. (2022) that show image features produced by randomly initialized image networks can serve as a good image reconstruction priors, we also consider versions of g that apply the mean and max functions on the result of applying 1 layer random convolution with 96 output channels, kernel size 3 and no stride on the images (denoted with $\text{conv} + \text{mean}$ and $\text{conv} + \text{max}$ in Table 4). As our performance increased by adding more output convolution channels, we chose the largest number of channels that fits in GPU memory. For both FEMNIST and CIFAR, we ran our experiments on $\mathcal{E} = 10$, $m = 5$, and $N^c = 50$ with known label counts. We chose λ_{inv} by exponential search on the $D_{\text{inv}} = \ell_2$ and $g = \text{mean}$ ($D_{\text{inv}} = \ell_1$ and $g = \text{conv} + \text{max}$) combination on FEMNIST (CIFAR100). Due to

Table 4: The effectiveness of our method with different priors on $10 \times 5 \times 50$ FEMNIST and CIFAR100.

		FEMNIST		CIFAR100	
g	D_{inv}	Rec(%)	PSNR	Rec(%)	PSNR
mean	ℓ_1	61.6	21.2	55.1	19.5
conv + mean	ℓ_1	61.5	21.2	56.0	19.6
max	ℓ_1	36.8	19.2	56.9	19.6
conv + max	ℓ_1	56.0	20.7	58.0	**19.8**
mean	ℓ_2	**65.5**	**21.5**	56.0	19.5
conv + mean	ℓ_2	64.4	21.4	56.3	19.6
max	ℓ_2	39.6	19.4	56.0	19.6
conv + max	ℓ_2	63.6	21.2	**58.8**	**19.8**

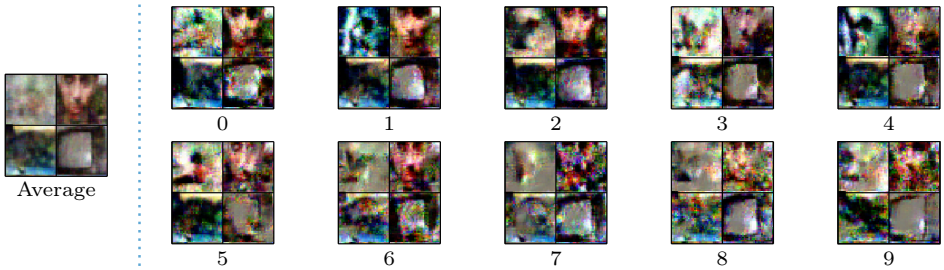

Figure 3: An end-to-end reconstruction of 4 images at individual epochs from the $10 \times 5 \times 50$ CIFAR100.

the computational costs, we adapted these values of λ_{inv} to the other combinations in a way that the value of $\lambda_{\text{inv}} \cdot \mathcal{L}_{\text{inv}}(\widetilde{X}^c)$ matches on average at the first iteration of the optimization process. Comparison is performed on the same 100 clients as before, as the attacker can select the prior which obtains the highest quality of the reconstruction. The results are shown in Table 4. We observe that $D_{\text{inv}} = \ell_2$ performs consistently better in all experiments and thus we chose to use it. Furthermore, on FEMNIST g does not benefit from the additional complexity of the convolution, likely due to being simpler dataset and that the max is a very poor choice for this dataset. To this end, we choose g to be simply the mean. In contrast, on CIFAR we observe that the random convolution is always helpful to the reconstruction. Also, we see that the combination of the convolution and the max is particularly effective. To this end, we choose to use this combination on CIFAR. We believe more complex feature extracting functions such as ones that use stacks of convolutions or more output channels will have even greater benefit on this dataset. We leave that as future work.

5.5 Importance of averaging across epochs

In this section we visualize the effect of the matching and averaging, described in Section 4.4, used to generate the final reconstructions for our method. In Figure 3, we show the reconstructions at different epochs for our end-to-end attack computed on the same setup as Section 5.3 with $\mathcal{E} = 10$, $m = 5$, and $N^c = 50$. Further, we provide full batch reconstructions with known and unknown label counts in Appendix E.2. We make two observations. First, the reconstructions at later epochs are of worse quality. We think this is due to the imperfect label reconstruction making the simulation worse at later epochs, since in Appendix E.2 we show this effect is not seen when the label counts are known. Second, while the reconstructions at individual epochs are noisy, the average reconstructs the original images significantly better than any individual epoch reconstruction. This confirms that matching and averaging substantially improves the reconstruction results.

6 Conclusion

In this work, we presented a new data leakage attack for the commonly used federated averaging learning algorithm. The core ingredients of our attack were a novel simulation-based reconstruction loss combined with an epoch order-invariant prior, and our FedAvg-specific extension of existing label reconstruction algorithms. We experimentally showed that our method can effectively attack FedAvg updates computed on combinations of large number of epochs and batches per epoch, thus for the first time demonstrating that realistic FedAvg updates are vulnerable to data leakage attacks. We believe that our results indicate a need for a more thorough investigation of data leakage in FedAvg and refer to our Broader Impact Statement in Appendix D, that further discusses the practical implications of our work.

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