Variance Reduced ProxSkip: Algorithm, Theory and Application to Federated Learning

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Abstract

We study distributed optimization methods based on the *local training (LT)* paradigm: achieving communication efficiency by performing richer local gradient-based training on the clients before parameter averaging. Looking back at the progress of the field, we *identify 5 generations of LT methods*: 1) heuristic, 2) homogeneous, 3) sublinear, 4) linear, and 5) accelerated. The 5th generation, initiated by the ProxSkip method of Mishchenko et al. [2022] and its analysis, is characterized by the first theoretical confirmation that LT is a communication acceleration mechanism. Inspired by this recent progress, we contribute to the 5th generation of LT methods by showing that it is possible to enhance them further using *variance reduction*. While all previous theoretical results for LT methods ignore the cost of local work altogether, and are framed purely in terms of the number of communication rounds, we show that our methods can be substantially faster in terms of the *total training cost* than the state-of-the-art method ProxSkip in theory and practice in the regime when local computation is sufficiently expensive. We characterize this threshold theoretically, and confirm our theoretical predictions with empirical results.

1 Introduction

Announced in April 2017 in a Google AI blog [McMahan and Ramage, 2017], and citing four foundational papers [Konečný et al., 2016b,a, McMahan et al., 2017, Bonawitz et al., 2017] of what was to become a new and rapidly growing interdisciplinary field, *federated learning* (FL) constitutes a novel paradigm for training supervised machine learning models. The key idea is the acknowledgement that increasing amounts of data are being captured and stored on edge devices, such as mobile phones, sensors and hospital workstations, and that moving the data to a datacenter for centralized processing may be infeasible or undesirable for various reasons, including high energy costs and data privacy concerns [Kairouz et al., 2019, Li et al., 2020a]. FL faces a multitude of challenges which are being actively addressed by the research community.

1.1 Formalism

We study the standard optimization formulation of federated learning [Konečný et al., 2016a, McMahan et al., 2017, Kairouz et al., 2019, Wang et al., 2021] given by

$$\min_{x\in\mathbb{R}^{d'}} \phi(x), \qquad f(x) := \sum_{i=1}^{M} \frac{n_i}{n}\phi_i(x), \qquad \phi_i(x) := \frac{1}{n_i}\sum_{j=1}^{n_i} \phi_{ij}(x), \tag{1}$$

where M is the number of clients (devices, machines, workers), n_i is the number of training data points on client $i \in \{1, 2, \ldots, M\}$, and $n := \sum_{i=1}^{M} n_i$ is the total number of training data points

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collectively owned by this federation of M clients. Note that ϕ is the empirical risk over the federated dataset. Perhaps conceptually the simplest method for solving (1) is *gradient descent (GD)*,

$$x_{t+1} = x_t - \gamma \nabla \phi(x_t) = x_t - \gamma \sum_{i=1}^{M} \frac{n_i}{n} \nabla \phi_i(x_t) = \sum_{i=1}^{M} \frac{n_i}{n} \left(x_t - \gamma \nabla \phi_i(x_t) \right), \tag{2}$$

where $\gamma > 0$ is the stepsize. It will be useful to describe how GD would be implemented in a federated environment. First, all clients $i \in \{1, \ldots, M\}$ in parallel perform a single local gradient step starting from the current global model x_t, arriving at the local models $x_{it} \coloneqq x_t - \gamma \nabla \phi_i(x_t)$, $i \in \{1, \ldots, M\}$. These local models are then communicated to the *orchestrating server*, which aggregates them via weighted averaging, arriving at the new global model $x_{t+1} = \sum_{i=1}^{M} \frac{n_i}{n} x_{it}$. This new model is then broadcast back to all clients, and the process is repeated until a model of sufficient quality is found.

1.2 Federated averaging

Proposed by Povey et al. [2015], Moritz et al. [2016], McMahan et al. [2017], federated averaging (FedAvg) is arguably the most popular method for solving the standard FL formulation (1). Motivated by the specific constraints of federated environments, FedAvg can be seen as a practical enhancement of GD via the simultaneous application of three techniques: a) data sampling (DS), b) client sampling (CS), and c) local training (LT). That is,

FedAvg = GD + (DS + CS + LT).

We will now briefly describe each of these three GD-enhancing techniques separately.

a) GD + Data Sampling. In situations when the local datasets are so large that the computation of the exact local gradients becomes a bottleneck, it makes sense to approximate them via data sampling. That is, instead of passing through all local data to compute the local gradient $\nabla \phi_i(x_t)$, each client i computes the gradients $\nabla \phi_{ij}(x_t)$ for $j \in \mathcal{D}_{it}$ only, where $\mathcal{D}_{it}$ is a suitably chosen small-enough subset of the local dataset $\{1, \ldots, n_i\}$. These gradients are then used to form gradient estimators $g_i(x_t) \approx \nabla \phi_i(x_t)$ which are used to perform a local SGD step on all clients. The rest of the procedure is the same as in the case of GD. That is, the local models obtained in this way are sent to the orchestrating server, the server aggregates them via weighted averaging and broadcasts the resulting model back to all clients. Combination of GD and DS can be seen as a particular version of SGD, where the stochastic gradient estimator is formed from the gradients $\nabla \phi_{ij}(x_t)$ associated with the datapoints (i, j) where $j \in \cup_{i=1}^{M} \mathcal{D}_{it}$. While DS is still an active area of research, it has been studied for a long time, and is in general well understood [Takáč et al., 2013, Li et al., 2014, Csiba and Richtárik, 2018, Gower et al., 2019a, Horváth and Richtárik, 2019, Khaled and Richtárik, 2020].

b) GD + Client Sampling. In practical federated environments, and especially in cross-device FL [Kairouz et al., 2019], the number of clients is enormous, they are not all available at all times, and the orchestrating server has limited compute and memory capacity. For these and other reasons, practical FL methods need to work in an environment in which a small subset $\mathcal{S}_t \subseteq \{1, \ldots, M\}$ of the clients is sampled ("participates") in each communication/aggregation/training round only. Since only the participating clients $i \in \mathcal{S}_t$ perform a local GD step and communicate the resulting local model to the orchestrating server for aggregation, this induces an error compared to GD, which has an adverse effect on the convergence rate. Combination of GD and CS can be seen as a particular version of SGD, where the stochastic gradient estimator is formed from the gradients $\nabla \phi_{ij}(x_t)$ associated with the datapoints (i, j) where $j \in \cup_{i=1}^{\mathcal{S}_t} \{1, \ldots, n_i\}$. While CS is still an active area of research, since CS is a special type of DS, much was known about CS long before FedAvg was proposed [Gower et al., 2019a, Horváth and Richtárik, 2019]. Still, CS poses new challenges tackled by the community Eichner et al. [2019], Chen et al. [2020], Gower et al. [2019a], Cho et al. [2020], Charles et al. [2021].

c) GD + Local Training. In federated learning, the cost of communication between the clients and the orchestrating server forms the key bottleneck. Indeed, in their FedAvg paper, which introduced LT to the world of federated learning, McMahan et al. [2017] wrote:

> *"In contrast[1], in federated optimization communication costs dominate".*

LT is a conceptually simple and surprisingly powerful communication-acceleration technique. The basic idea behind LT is for the clients to perform *multiple* local GD steps instead of a single step

[1] to datacenter optimization

Table 1: Five generations of local training (LT) methods summarizing the progress made by the ML/FL community over the span of 7+ years in the understanding of the *communication acceleration properties of LT*.

Generation[(a)]	Theory	Assumptions	Comm. Complexity[(b)]	Selected Key References
1. Heuristic	✗	—	empirical results only	LocalSGD [Povey et al., 2015]
	✗	—	empirical results only	SparkNet [Moritz et al., 2016]
	✗	—	empirical results only	FedAvg [McMahan et al., 2017]
2. Homogeneous	✓	bounded gradients	sublinear	FedAvg [Li et al., 2020b]
	✓	bounded grad. diversity[(c)]	linear but worse than GD	LFGD [Haddadpour and Mahdavi, 2019]
3. Sublinear	✓	standard[(d)]	sublinear	LGD [Khaled et al., 2019]
	✓	standard	sublinear	LSGD [Khaled et al., 2020]
4. Linear	✓	standard	linear but worse than GD	Scaffold [Karimireddy et al., 2020]
	✓	standard	linear but worse than GD	S-Local-GD [Gorbunov et al., 2020a]
	✓	standard	linear but worse than GD	FedLin [Mitra et al., 2021]
5. Accelerated	✓	standard	linear & better than GD	ProxSkip/Scaffnew [Mishchenko et al., 2022]
	✓	standard	linear & better than GD	ProxSkip-VR **[THIS WORK]**

[(a)] Since client sampling (CS) and data sampling (DS) can only *worsen* theoretical communication complexity, our historical breakdown of the literature into 5 generations of LT methods focuses on the full client participation (i.e., no CS) and exact local gradient (i.e., no DS) setting. While some of the referenced methods incorporate CS and DS techniques, these are irrelevant for our purposes. Indeed, from the viewpoint of communication complexity, all these algorithms enjoy best theoretical performance in the no-CS and no-DS regime.

[(b)] For the purposes of this table, we consider problem (1) in the *smooth* and *strongly convex* regime only. This is because the literature on LT methods struggles to understand even in this simplest (from the point of view of optimization) regime.

[(c)] *Bounded gradient diversity* is a uniform bound on a specific notion of gradient variance depending on client sampling probabilities. However, this assumption (as all homogeneity assumptions) is very restrictive. For example, it is not satisfied the standard class of smooth and strongly convex functions.

[(d)] The notorious FL challenge of handling non-i.i.d. data by LT methods was solved by Khaled et al. [2019] (from the viewpoint of *optimization*). From generation 3 onwards, there was no need to invoke any data/gradient homogeneity assumptions. Handling non-i.i.d. data remains a challenge from the point of view of *generalization*, typically by considering *personalized* FL models.

(which is how GD operates) before communication and aggregation takes place. The intuitive reasoning used in virtually all papers on this topic is: performing multiple local GD steps results in "richer" and ultimately more useful local training in the sense that fewer communication rounds will *hopefully* suffice to finish the training. McMahan et al. [2017] supported this intuition with ample empirical evidence, and credited LT as the critical component behind the success of FedAvg:

> *"Thus, our goal is to use additional computation in order to decrease the number of rounds of communication needed to train a model. . . " "Communication costs are the principal constraint, and we show a reduction in required communication rounds by 10–100× as compared to synchronized stochastic gradient descent." ". . . the speedups we achieve are due primarily to adding more computation on each client".*

2 Five Generations of Local Training Methods

We now offer several historical comments on the most important developments related to the *theoretical* understanding of LT. To this end, we have identified 5 distinct generations of LT methods, each with its unique challenges and characteristics. To make the narrative simple, and since we focus on this regime in our paper, we limit our overview to loss functions ϕ_i that are μ-strongly convex and L-smooth. This is arguably the most studied class of functions in continuous optimization [Nesterov, 2004], and for this reason, it presents a valuable litmus test for any theory of LT.

Generation 1: Heuristic Age. While LT ideas were used in several machine learning domains before [Povey et al., 2015, Moritz et al., 2016], LT truly rose to prominence as a practically potent communication acceleration technique due to the seminal paper of McMahan et al. [2017] which introduced the FedAvg algorithm. However, no theory was provided in their work, nor in any prior work. LT-based heuristics, i.e., methods without any theoretical guarantees, dominated the initial development of the field up to, and including, the FedAvg paper.

Generation 2: Homogeneous Age. The first theoretical results for LT methods offering explicit convergence rates relied on various data/gradient *homogeneity*[2] assumptions. The intuitive rationale behind such assumptions comes from the following thought process. In the extreme case when all the local functions ϕ_i are *identical* (this is often referred to as the *homogeneous* or *i.i.d. data* regime), there is a very simple approach to making GD communication-efficient: push the idea of LT to its extreme by running GD on all clients, independently and in parallel, without any

[2] We use the term *homogeneity* to refer to various related assumptions used in the literature, including *bounded gradient norms* [Li et al., 2020b], *bounded gradient variance* [Li et al., 2019, Yu et al., 2019] and *bounded gradient diversity* [Haddadpour and Mahdavi, 2019].

communication/synchronization/averaging whatsoever. Extrapolating from this, it is reasonable to assume that as we increase heterogeneity, taking multiple local steps should still be beneficial as long as we do not take too many steps. Several authors analyzed various LT methods under such assumptions, and obtained rates [Haddadpour and Mahdavi, 2019, Yu et al., 2019, Li et al., 2019, 2020b]. However, bounded dissimilarity assumptions are highly problematic. First, they do not seem to be satisfied even for some of the simplest function classes, such as strongly convex quadratics [Khaled et al., 2019, 2020], and moreover, it is well known that practical FL datasets are highly heterogeneous/non-i.i.d. McMahan et al. [2017], Kairouz et al. [2019]. So, analyses relying on such strong assumptions are both mathematically questionable, and practically irrelevant.

Generation 3: Sublinear Age. The third generation of LT methods is characterized by the successful removal of the bounded dissimilarity assumptions from the convergence theory. Khaled et al. [2019] first achieved this breakthrough by studying the simplest LT method: local gradient descent (LGD) (i.e., a simple combination of GD and LT). While works belonging to this generation elevated LT to the same theoretical footing as GD in terms of the assumptions, which marked an important milestone in our understanding of LT, unfortunately, the obtained communication complexity theory of LGD is pessimistic when compared to vanilla GD. Indeed, the inclusion of LT did *not* lead to an improvement upon the communication complexity of vanilla GD. Moreover, while GD enjoys a linear communication complexity (in the smooth and strongly convex regime), the communication complexity of LGD is *sublinear*. In a follow-up work, Khaled et al. [2020] later analyzed LGD in combination with DS as well. Woodworth et al. [2020] and Glasgow et al. [2022] provided lower bounds for LGD with DS showing that it is not better than minibatch SGD in heterogeneous setting. See the work of Malinovsky et al. [2020] for a fixed-point theory viewpoint.

Generation 4: Linear Age. The fourth generation of LT methods is characterized by the effort to design *linearly* converging variants of LT algorithms. In order to achieve this, it was important to tame the adverse effect of the so-called *client drift* [Karimireddy et al., 2020], which was identified as the culprit of the worse-than-GD theoretical performance of the previous generation of LT methods. The first LT-based method that successfully tamed client drift, and as a result obtained a linear convergence rate, was Scaffold [Karimireddy et al., 2020]. Several alternative approaches to obtaining the same effect were later proposed by Gorbunov et al. [2020a] and Mitra et al. [2021] . While obtaining a linear rate for LT methods under standard assumptions was a major achievement, the communication complexity of these methods is still somewhat worse[3] than that of vanilla GD, and is at best equal to that of GD.

Generation 5: Accelerated Age. Finally, the fifth generation of LT methods was initiated recently by Mishchenko et al. [2022] with their ProxSkip method which enjoys *accelerated communication complexity*. Acceleration comes from the LT steps coupled with a new client drift reduction technique and a probabilistic approach to deciding whether communication takes place or not. Mishchenko et al. [2022] first reformulate (1) using into the equivalent consensus form

$$\min_{x\in\mathbb{R}^d} f(x) + r(x), \tag{3}$$

where $d = Md'$, $x = (x_1, \ldots, x_M) \in \mathbb{R}^d$, and

$$f(x) := \sum_{i=1}^{M} \tfrac{n}{n_i}\phi_i(x_i), \qquad r(x) = \begin{cases} 0 & \text{if } x_1 = \cdots = x_M, \\ +\infty & \text{otherwise.} \end{cases} \tag{4}$$

The ProxSkip method is a randomized variant of proximal gradient descent (ProxGD) [Nesterov, 2013, Beck, 2017] for solving (3), with the proximity operator of r, given by $\operatorname{prox}_r(x) := \arg\min_y \left(r(y) + \frac{1}{2}\|y - x\|^2\right)$, being evaluated in each iteration with probability $p \in (0, 1]$ only. Remarkably, Mishchenko et al. [2022] showed that it is possible to choose p as low as $1/\sqrt{\kappa}$, where κ is the condition number of f, without this worsening the rate of its parent method ProxGD. In summary, ProxSkip lets the M clients perform $\sqrt{\kappa}$ local gradient steps in expectation, followed by the evaluation of the prox of r, which in the case of the consensus reformulation of (1) means averaging across all M nodes, i.e., communication.

[3] Both GD, and LT methods such as Scaffold [Karimireddy et al., 2020], S-Local-GD [Gorbunov et al., 2020a] and FedLin [Mitra et al., 2021] enjoy the linear rate $\mathcal{O}(\kappa \log \frac{1}{\varepsilon})$, where κ is a condition number. However, this condition number is in general slightly worse for the LT methods.

Algorithm 1 ProxSkip-VR

Parameters: stepsize $\gamma > 0$, probability $p \in (0, 1]$, initial iterate $x_0 \in \mathbb{R}^d$, initial control vector $y_0 \in \mathbb{R}^d$, initial gradient shift $h_0 \in \mathbb{R}^d$, number of iterations $T \geq 1$
for $t = 0, 1, \ldots, T-1$ **do**
  $g_t = g(x_t, y_t, \xi_t)$ ◇ Sample ξ_t and construct an unbiased estimator of $\nabla f(x_t)$
  $\hat{x}_{t+1} = x_t - \gamma(g_t - h_t)$ ◇ Take a gradient-type step adjusted via the shift h_t
  Construct new control vector y_{t+1}
  Flip a coin $\theta_t \in \{0, 1\}$ where $\mathrm{Prob}(\theta_t = 1) = p$ ◇ Decides whether to skip the prox or not
  if $\theta_t = 1$ **then**
    $x_{t+1} = \mathrm{prox}_{\frac{\gamma}{p} r}\left(\hat{x}_{t+1} - \frac{\gamma}{p} h_t\right)$ ◇ Apply prox, but only with probability p
  else
    $x_{t+1} = \hat{x}_{t+1}$ ◇ Skip the prox!
  end if
  $h_{t+1} = h_t + \frac{p}{\gamma}(x_{t+1} - \hat{x}_{t+1})$ ◇ Update the shift h_t
end for

3 ProxSkip-VR: A General Variance Reduction Framework for ProxSkip

In this work we contribute to the fifth generation of LT methods by extending the work of Mishchenko et al. [2022] to allow for a very large family of gradient estimators, including variance reduced (VR) ones [Johnson and Zhang, 2013, Defazio et al., 2014, Kovalev et al., 2020a, Mishchenko et al., 2019]. Like ProxSkip, our method ProxSkip-VR (Algorithm 1) is aimed to solve the composite problem (3) in a more general setting (see Assumptions 1–3), with the special structure (4) coming from the consensus reformulation being a special case only. Our method differs from ProxSkip in that we replace the gradient $\nabla f(x_t)$ by an unbiased estimator $g_t = g(x_t, y_t, \xi_t)$, where ξ_t is the source of randomness controlling unbiasedness and y_t is a control vector whose role is to progressively reduce the variance of the estimator, so that $\mathbb{E}\left[g_t \mid x_t, y_t\right] = \nabla f(x_t)$.

There are several motivations behind this endeavor. First, it is a-priori not clear whether the novel proof technique employed by Mishchenko et al. [2022] can be combined with the proof techniques used in the analysis of VR methods, and hence it is scientifically significant to investigate the possibility of such a merger of two strands of the literature. We show in Section 3.1 that this is possible. Second, marrying VR estimators with ProxSkip can lead to novel system architectures which are more elaborate than the simplistic client-server architecture (see Section 4). Lastly, while researchers contributing to generations 1–4 of LT methods were preoccupied with trying to close the gap on GD in terms of communication efficiency, they *ignored* the number of the local steps appearing in their algorithms, and reported their bounds primarily in terms of the number of communication rounds. Bounds reported this way make complete sense in the scenario when the cost of local work (e.g., one SGD step w.r.t. a single data point), say δ, is negligible compared to the cost of communication, which can w.l.o.g. assume to be 1, and when the number of local steps is small. With the advent of the fifth generation of LT methods, we can (to a large degree) stop worrying about communication efficiency, and can now ask more refined questions, such as: *Are there gradient estimators which, when combined with* ProxSkip*, lead to faster algorithms in terms of the total cost, which includes the communication cost as well as the cost of local training?* We give an affirmative answer to the question in Sections 4 and 5.

3.1 Framework for expressing VR estimators and our main result

We assume throughout that f is differentiable, and let $D_f(x, y) \coloneqq f(x) - f(y) - \langle \nabla f(y), x - y \rangle$ denote the Bregman divergence of f. Throughout the work we make the following assumptions:

Assumption 1 (L-smoothness)**.** *There exists $L > 0$ s.t. $2D_f(x, y) \leq L \left\|x - y\right\|^2$ for all $x, y \in \mathbb{R}^d$.*

Assumption 2 (μ-convexity)**.** *There exists $\mu > 0$ s.t. $\mu \left\|x - y\right\|^2 \leq 2D_f(x, y)$ for all $x, y \in \mathbb{R}^d$.*

Assumption 3. *The regularizer $r : \mathbb{R}^d \to \mathbb{R} \cup \{+\infty\}$ is proper, closed and convex.*

Under the above assumptions, (3) has a unique minimizer $x_\star$. Let $h_\star \coloneqq \nabla f(x_\star)$.

Table 2: Special cases of ProxSkip-VR, depending on the choice of the gradient estimator g_t.

Estimator of ∇f	Communication Complexity of ProxSkip-VR	Iteration Complexity of ProxSkip-VR	Corollaries of Theorem 5
GD [(b)]	$\mathcal{O}\left(\sqrt{L/\mu}\log 1/\varepsilon\right)$	$\mathcal{O}\left(L/\mu \log 1/\varepsilon\right)$	Theorem 6
SGD [(c)]	$\mathcal{O}\left(\left(\sqrt{A/\mu}+\sqrt{2C/\varepsilon\mu^2}\right)\log 1/\varepsilon\right)$	$\mathcal{O}\left(\left(A/\mu + 2C/\varepsilon\mu^2\right)\log 1/\varepsilon\right)$	Theorem 8
HUB **[NEW]**	$\mathcal{O}\left(\sqrt{L_{\max}/\mu\,(1+\omega/\tau)}\log 1/\varepsilon\right)$	$\mathcal{O}\left(L_{\max}/\mu\,(1+\omega/\tau)\log 1/\varepsilon\right)$	Theorem 9
LSVRG **[NEW]**	$\mathcal{O}\left(\sqrt{L\tau/\mu}\log 1/\varepsilon\right)$	$\mathcal{O}\left(L\tau/\mu \log 1/\varepsilon\right)$	Corollary 1
Q **[NEW]**	$\mathcal{O}\left(\sqrt{L_{\max}/\mu\,(1+\omega/M)}\log 1/\varepsilon\right)$	$\mathcal{O}\left(L_{\max}/\mu\,(1+\omega/M)\log 1/\varepsilon\right)$	Corollary 2

[(a)] Any estimator satisfying Assumption 4
[(b)] ProxSkip-VR with the GD estimator reduces to the ProxSkip method of Mishchenko et al. [2022]
[(c)] ProxSkip-VR with the SGD estimator satisfying Assumption 7 reduces to the SProxSkip method of Mishchenko et al. [2022]
[(d)] $L_\tau = \left(\frac{m-\tau}{\tau(m-1)}\max_i L_i + \frac{m(\tau-1)}{\tau(m-1)}L\right)$, where τ is the mini-batch size and m is the number of clients belonging to one hub

Our next assumption, initially introduced by Gorbunov et al. [2020b], postulates several parametric inequalities characterizing the behavior and ultimately the quality of a gradient estimator. Similar assumptions appeared later in [Gorbunov et al., 2020a,c].

Assumption 4. *Let $\{x_t\}$ be iterates produced by ProxSkip-VR. First, we assume that the stochastic gradients $g_t = g(x_t, y_t, \xi_t)$ are unbiased for all $t \geq 0$, namely $\mathbb{E}\left[g_t \mid x_t, y_t\right] = \nabla f(x_t)$. Second, we assume that there exist non-negative constants $A, B, C, \tilde{A}, \tilde{B}, \tilde{C}$, with $\tilde{B} < 1$, and a nonnegative mapping $y_t \mapsto \sigma(y_t) := \sigma_t$ such that the following two relations hold for all $t \geq 0$,*

$$\mathbb{E}\left[\|g_t - \nabla f(x_*)\|^2 \mid x_t, y_t\right] \leq 2AD_f(x_t, x_*) + B\sigma_t + C, \tag{5}$$

$$\mathbb{E}\left[\sigma_{t+1} \mid x_t, y_t\right] \leq 2\tilde{A}D_f(x_t, x_*) + \tilde{B}\sigma_t + \tilde{C}. \tag{6}$$

Assumption 4 covers a very large collection of gradient estimators, including an infinite variety of subsampling/minibatch estimators, gradient sparsification and quantization estimators, and their combinations; see [Gorbunov et al., 2020b] for examples. VR estimators are characterized by $C = \tilde{C} = 0$; most non-VR estimators by $\tilde{A} = \tilde{B} = \tilde{C} = B = 0$ and $C > 0$ [Gower et al., 2019b]. Now we are ready to formulate our main result.

Theorem 5. *Let Assumptions 2 and 3 hold, and let g_t be a gradient estimator satisfying Assumption 4. If $B > 0$, choose any $W > B/(1-\tilde{B})$ and $\beta = (B + W\tilde{B})/W$. If $B = 0$, let $W = 0$ and $\beta = \tilde{B}$. Choose stepsize $0 < \gamma \leq \min\left\{1/\mu, 1/(A + W\tilde{A})\right\}$. Then the iterates of ProxSkip-VR for any $p \in (0, 1]$ satisfy*

$$\mathbb{E}\left[\Psi_T\right] \leq \max\left\{(1-\gamma\mu)^T, \beta^T, (1-p^2)^T\right\}\Psi_0 + \frac{(C + W\tilde{C})\gamma^2}{\min\{\gamma\mu, p^2, 1-\beta\}}, \tag{7}$$

where the Lyapunov function is defined by $\Psi_t := \|x_t - x_\star\|^2 + \frac{\gamma^2}{p^2}\|h_t - h_\star\|^2 + \gamma^2 W\sigma_t$.

3.2 Two examples of estimators

Here we give two illustrating examples of estimators satisfying Assumption 4.

Theorem 6 (GD estimator)**.** *Let Assumption 1, 2 and 3 hold. Then for the trivial estimator $g_t = \nabla f(x_t)$, Assumption 4 holds with the following parameters:*

$$A = L, \quad B = 0, \quad C = 0, \quad \tilde{A} = 0, \quad \tilde{B} = 0, \quad \tilde{C} = 0, \quad \sigma_t \equiv 0.$$

Choose a stepsize satisfying $0 < \gamma \leq 1/L$. Then the iterates of ProxSkip-VR for any $p \in (0, 1]$ satisfy

$$\mathbb{E}\left[\Psi_T\right] \leq \max\left\{(1-\gamma\mu)^T, (1-p^2)^T\right\}\Psi_0, \tag{8}$$

where $\Psi_t := \|x_t - x_\star\|^2 + \frac{\gamma^2}{p^2}\|h_t - h_\star\|^2$. Let $\gamma = 1/L$ and $p = \sqrt{\mu/L}$ then the communication complexity of ProxSkip-VR is $\mathcal{O}\left(\sqrt{L/\mu}\log 1/\varepsilon\right)$ and iteration complexity is $\mathcal{O}\left(L/\mu \log 1/\varepsilon\right)$

This recovers the result obtained by Mishchenko et al. [2022] for their ProxSkip method. The next assumption holds for virtually all (non-VR) estimators based on subsampling [Gower et al., 2019b].

Assumption 7 (Expected smoothness)**.** *We say that an unbiased estimator $g(x;\xi) : \mathbb{R}^d \to \mathbb{R}^d$ of the gradient $\nabla f(x)$ satisfies the expected smoothness inequality if there exists $A'' > 0$ such that*

$$\mathbb{E}\left[\|g(x;\xi) - g(x_\star;\xi)\|^2\right] \leq 2A'' D_f(x, x_\star), \quad \forall x \in \mathbb{R}^d.$$

Theorem 8. *Let $g(x,\xi)$ satisfy Assumption 7 and define $g_t := g(x_t, \xi_t)$, where ξ_t is chosen independently at time t. Then Assumption 4 holds with the following parameters:*

$$A = A'', \quad B = 0, \quad C = \mathrm{Var}(g(x_\star,\xi)), \quad \tilde{A} = 0, \quad \tilde{B} = 0, \quad \tilde{C} = 0, \quad \sigma_t \equiv 0.$$

Moreover, assume that Assumption 2 holds. Choose stepsize $0 < \gamma \leq \min\left\{1/\mu, 1/A\right\}$. Then the iterates of ProxSkip *for any probability $p \in (0,1]$ satisfy*

$$\mathbb{E}\left[\Psi_T\right] \leq \max\left\{(1-\gamma\mu)^T, (1-p^2)^T\right\}\Psi_0 + \gamma^2 \tfrac{\mathrm{Var}(g(x_\star,\xi))}{\min\{\gamma\mu, p^2\}}, \tag{9}$$

where the Lyapunov function is defined by $\Psi_t := \|x_t - x_\star\|^2 + \frac{\gamma^2}{p^2}\|h_t - h_\star\|^2$. If we choose $\gamma = \min\left\{1/A, \varepsilon\mu/2C\right\}$ and $p = \sqrt{\gamma\mu}$ then the communication complexity is $\mathcal{O}((\sqrt{A/\mu} + \sqrt{2C/\varepsilon\mu^2})\log(1/\varepsilon))$ and the iteration complexity is $\mathcal{O}((A/\mu + 2C/\varepsilon\mu^2)\log(1/\varepsilon))$.

This recovers the result obtained by Mishchenko et al. [2022] for their SProxSkip method.

4 New FL Architecture: Regional Hubs Connecting the Clients to the Server

We now illustrate the versatility of our ProxSkip-VR framework by designing a new "FL architecture" and proposing an algorithm that can efficiently operate in this setting. In particular, we consider the situation where the clients are clustered (e.g., based on region), and where *a hub* is placed in between each cluster and the central server. Clients communicate with their regional hub only, which can communicate with the central server (see Figure 1). There are M hubs, hub i handles n_i clients, and client j associated with hub i owns loss function ϕ_{ij}. Mathematically, this can be modeled by problem (1). In this situation, we care about two sources of communication cost: the server and the hubs, and between the hubs and the clients. We propose to handle this via *local training (LT)* between the server and the hubs, and via *client sampling (CS)* and *compressed communication (CC)* between the hubs and the clients. Algorithmically, from the server-hubs perspective, we are applying a particular variant of ProxSkip-VR to (3)–(4), where ϕ_i is the aggregate loss handles by hub i. This takes care of communication efficiency between the server and the hub. Note also that we need not worry about partial participation of hubs, as these are designed to be always available. However, in this situation, it is costly for hub i to compute the gradient of ϕ_i as this involves communication with all the clients it handles. In order to alleviate this burden, we propose a combination of CS and CC. However, we need to be very careful about how to do this. Indeed, both CS and CC, even when applied in isolation, and without ProxSkip in the mix, can lead to a substantial slowdown in convergence. For example, one will typically lose linear convergence in the strongly convex regime. However, techniques for preserving linear convergence in the presence of CS and CC exist: this is what variance reduction strategies are designed to do. For example, L-SVRG [Hofmann et al., 2015, Kovalev et al., 2020a] is a VR technique for reducing the variance due to CS, and DIANA [Mishchenko et al., 2019] is a VR technique for reducing the variance due to CC. However, we are not aware of any VR method that combines CS (applied first) and CC (applied second).

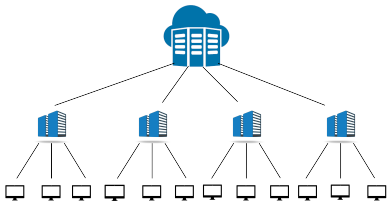


Figure 1: Server-hubs-clients FL architecture with 4 hubs and 12 clients.

We now propose such a technique. In iteration t, every hub $i \in \{1, 2, \ldots, M\}$ selects a random subset $\mathcal{S}_t^i \subseteq \{1, 2, \ldots, n_i\}$ of the clients it handles of cardinality τ_i, chosen uniformly at random, and estimates the hub gradient via

$$\nabla\phi_i(x_t) \approx g_t^i := \tfrac{1}{|\mathcal{S}_t^i|} \sum_{j \in \mathcal{S}_t^i} \mathcal{Q}_t^{ij}\left(\nabla\phi_{ij}(x_t) - \nabla\phi_{ij}(y_t)\right) + \nabla\phi_i(y_t), \tag{10}$$

where $\mathcal{Q}_t^{ij} : \mathbb{R}^{d'} \to \mathbb{R}^{d'}$ is a randomized compression (e.g., sparsification or quantization) operator [Alistarh et al., 2017, Khirirat et al., 2018, Horváth et al., 2019b,a, Philippenko and Dieuleveut,

2020], i.e., a mapping satisfying

$$\mathbb{E}\left[Q_t^{ij}(x)\right] = x, \quad \mathbb{E}\left[\|Q_t^{ij}(x) - x\|^2\right] \leq \omega \|x\|^2, \quad \forall x \in \mathbb{R}^{d'},$$

and the control vector y_t is updated probabilistically as follows:

$$y_{t+1} = \begin{cases} x_t & \text{with probability} \quad q \\ y_t & \text{with probability} \quad 1-q \end{cases}. \tag{11}$$

The global gradient (a vector in $\mathbb{R}^{Md'}$) is then a concatenation of the above hub estimators:

$$\nabla f(x_t) := (\tfrac{n_i}{n} \nabla \phi_i(x_t))_{i=1}^M \approx g_t := g(x_t, y_t, \xi_t) := (\tfrac{n_i}{n} g_t^i)_{i=1}^M, \tag{12}$$

where ξ_t represents the combined randomness from the compressors $\{\mathcal{Q}_t^{ij}\}$ and random sets $\{\mathcal{S}_t^i\}$.

In order to analyze ProxSkip-VR in the consensus form, we assume that $n_i = m = n/M$ and $\tau_i = \tau$ for all i, and rely on a slightly different, more general reformulation:

$$\min_{x \in \mathbb{R}^d} \frac{1}{m} \sum_{j=1}^m \widetilde{\phi}_j(x) + r(x), \quad \widetilde{\phi}_j(x) := \frac{1}{M} \sum_{i=1}^M \phi_{ij}(x_i), \quad r(x) := \begin{cases} 0 & \text{if } x_1 = \cdots = x_M, \\ +\infty & \text{otherwise.} \end{cases}$$

Our proposed method is thus ProxSkip-VR combined with the novel estimator (12). The following result first claims that the above estimator satisfies Assumption 4 with $C = \tilde{C} = 0$ (i.e., it is variance reduced), and the rest of the claim follows by application of our general theorem Theorem 5.

Theorem 9. *Assume that $\nabla \widetilde{\phi}_j$ is L_j-smooth for all j and let Assumptions 2 and 3 hold. Then for the gradient estimator (12), Assumption 4 holds with the following constants:*

$$A = 4\left(L_\tau + \tfrac{\omega}{\tau} L_{\max}\right), \quad B = 4\left(1 + \tfrac{\omega}{\tau}\right), \quad C = 0, \quad \tilde{A} = qL_{\max}, \quad \tilde{B} = 1 - q, \quad \tilde{C} = 0,$$

and $\sigma_t := \sigma(y_t)$, $\sigma(y) := \frac{1}{m}\sum_{j=1}^m \|\nabla \widetilde{\phi}_j(y) - \nabla \widetilde{\phi}_j(x_\star)\|^2$, $L_{\max} := \max_j L_j$. Set $W = 2B/(1-\tilde{B})$ and $0 < \gamma \leq \min\left\{1/\mu, 1/(A + W\tilde{A})\right\}$. Then the iterates of ProxSkip-VR for any $p \in (0,1]$ satisfy

$$\mathbb{E}\left[\Psi_T\right] \leq \max\left\{(1-\gamma\mu)^T, (1-p^2)^T, (1-q/2)^T\right\} \Psi_0,$$

where the Lyapunov function is defined by $\Psi_t := \|x_t - x_\star\|^2 + \frac{\gamma^2}{p^2}\|h_t - h_\star\|^2 + \gamma^2 \frac{8}{q}\left(1 + \frac{\omega}{\tau}\right)\sigma_t$.

Corollary 1. *If we do not use compression ($\omega = 0$), then the iteration complexity is $\mathcal{O}\left(L_{\max}/\mu \log 1/\varepsilon\right)$ and communication complexity is $\mathcal{O}\left(\sqrt{L_{\max}/\mu} \log 1/\varepsilon\right)$. However, if we use the estimator (10) in Theorem 5 directly, then the iteration complexity is $\mathcal{O}\left(L_\tau/\mu \log 1/\varepsilon\right)$ and the communication complexity is $\mathcal{O}\left(\sqrt{L_\tau/\mu} \log 1/\varepsilon\right)$.*

Corollary 2. *If we do not use client sampling (i.e., $\tau = m$), then the iteration complexity is $\mathcal{O}\left(L_{\max}/\mu\,(1 + \omega/M) \log 1/\varepsilon\right)$ and the communication complexity is $\mathcal{O}\left(\sqrt{L_{\max}/\mu\,(1+\omega/M)} \log 1/\varepsilon\right)$.*

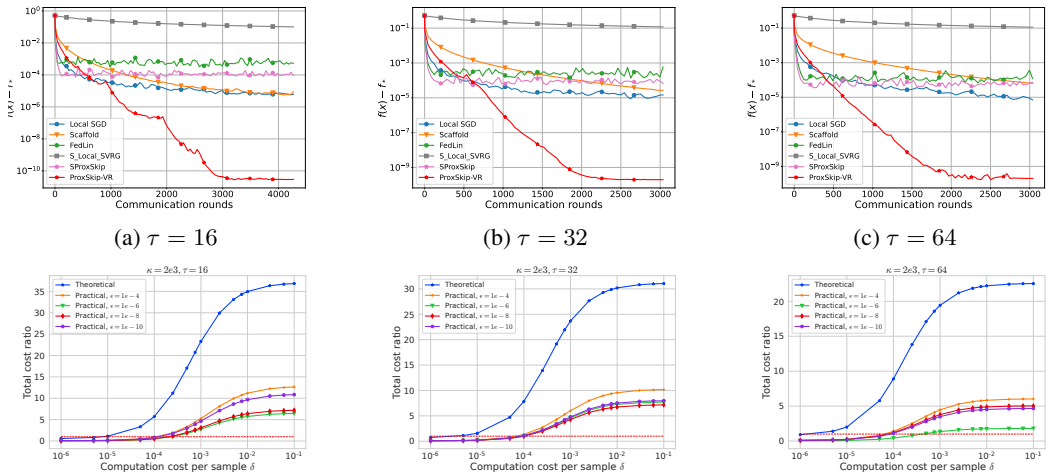
Assume $r(x) \equiv 0$. If $\mathcal{Q}_t^{ij}(x) \equiv x$, then $\omega = 0$, and we restore the well-known LSVRG method [Hofmann et al., 2015, Kovalev et al., 2020b], assuming that the functions ϕ_{ij} have the same smoothness constant. We can recover the same rate exactly as well, but with a slightly more refined analysis, one in which we do not need to work with compressors (L-SVRG does not involve any), which makes for a tighter analysis. On the other hand, if $\tau = m$, we restore the rate of the well-known DIANA [Mishchenko et al., 2019, Horváth et al., 2019b] method and its sibling Rand-DIANA [Shulgin and Richtárik, 2021].

5 Experiments

To illustrate the predictive power of our theory, it suffices to consider L_2-regularized logistic regression in the distributed setting (1), with

$$\phi_i(x) = \frac{1}{n_i} \sum_{j=1}^{n_i} \log\left(1 + \exp\left(-b_{ij} a_{ij}^\top x\right)\right) + \frac{\lambda}{2}\|x\|^2,$$

where n_i is the number of data points per worker $a_{ij} \in \mathbb{R}^{d'}$ and $b_{ij} \in \{-1, +1\}$ are the data samples and labels. We choose $n_i = n/M$ for all i. We set the regularization parameter $\lambda = 5 \cdot 10^{-4} L$ by default, where L is the smooth constant of f. We conduct a bunch of experiments on the `w8a` dataset from LibSVM library [Chang and Lin, 2011]. In Figure 2 (first row), we show convergence of various baselines which utilize local steps and client drift correction for different mini-batch sizes. As we can see, ProxSkip-VR outperforms all other methods significantly because of its accelerated nature.



(a) $\tau = 16$ (b) $\tau = 32$ (c) $\tau = 64$

Figure 2: The top row shows the convergence results compared with baselines and the second row is the total cost ratio of ProxSkip over our ProxSkip-VR.

Next, we derive the total cost, which includes communication cost (assumed to be 1, for normalization purposes), and computational cost (assumed to be δ; and equal to the cost of performing one SGD step with a single data point). Let us consider the total cost of ProxSkip-VR in case of LSVRG estimator: in each iteration we compute 1 stochastic gradient, and with probability q we compute the exact gradient. We do not need to compute a second stochastic gradient since we can use memory and the relation $y_{t+1} = x_t$. The total cost for ProxSkip-VR is equal to

$$\text{Cost}(\text{ProxSkip-VR}) := T_{\text{comm.}}(\text{ProxSkip-VR}) + \delta\left(qm + (1-q)\tau + \tau\right) T_{\text{iter}}(\text{ProxSkip-VR}).$$

ProxSkip requires full/exact gradient computation at each iteration, so the total cost of ProxSkip is

$$\text{Cost}(\text{ProxSkip}) := T_{\text{comm.}}(\text{ProxSkip}) + \delta m T_{\text{iter}}(\text{ProxSkip}).$$

Using Theorems 6 and 9 and the value of the expected smoothness constant for sampling with minibatch size τ,

$$L_\tau = \tfrac{m-\tau}{\tau(m-1)} L_{\max} + \tfrac{m(\tau-1)}{\tau(m-1)} L,$$

we get the following expression for the cost ratio, expressed as a function of δ:

$$\text{Cost ratio}(\delta) := \frac{\text{Cost}(\text{ProxSkip})}{\text{Cost}(\text{ProxSkip-VR})} = \frac{\sqrt{\mu L} + mL\delta}{\sqrt{L_\tau \mu} + (2m\mu + (2L_\tau - 2\mu)\tau)\delta}. \tag{13}$$

We can easily calculate the limits of this expression:

$$\text{Cost ratio}(\delta = 0) = \sqrt{\frac{L}{L_\tau}}, \qquad \text{Cost ratio}(\delta \to \infty) = \frac{mL}{2m\mu + (2A'' - 2\mu)\tau}.$$

In Figure 2 (second row), we depict the theoretical cost ratio according to (13) and the corresponding experimental ratio obtained by an actual run of both methods to achieve ε-accuracy, with $\varepsilon = 10^{-6}$ and $\varepsilon = 10^{-8}$. Remarkably, the experimental results follows the same pattern as our theoretical prediction. The empirical curves appear lower because we use approximations for $L_{\max}$, L and μ. As we can see, starting from $\delta = 10^{-4}$, ProxSkip-VR starts to outperform ProxSkip. As δ grows, the advantage of variance reduction embedded in ProxSkip-VR over vanilla ProxSkip grows. These results suggest that variance reduction is of practical utility in terms of the total cost, especially for large values of δ.

References

D. Alistarh, D. Grubic, J. Li, R. Tomioka, and M. Vojnovic. QSGD: Communication-efficient SGD via gradient quantization and encoding. In *Advances in Neural Information Processing Systems*, pages 1709–1720, 2017.

A. Beck. *First order methods in optimization*. MOS-SIAM Series on Optimization, 2017.

K. Bonawitz, V. Ivanov, B. Kreuter, A. Marcedone, H. B. McMahan, S. Patel, D. Ramage, A. Segal, and K. Seth. Practical secure aggregation for privacy-preserving machine learning. In *Proceedings of the 2017 ACM SIGSAC Conference on Computer and Communications Security*, pages 1175–1191, 2017.

C.-C. Chang and C.-J. Lin. LibSVM: A library for support vector machines. *ACM Transactions on Intelligent Systems and Technology (TIST)*, 2(3):27, 2011.

Z. Charles, Z. Garrett, Z. Huo, S. Shmulyian, and V. Smith. On large-cohort training for federated learning. *arXiv preprint arXiv:2106.07820*, 2021.

W. Chen, S. Horváth, and P. Richtárik. Optimal client sampling for federated learning. *Privacy Preserving Machine Learning (NeurIPS 2020 Workshop)*, 2020.

Y. J. Cho, J. Wang, and G. Joshi. Client selection in federated learning: Convergence analysis and power-of-choice selection strategies. *arXiv preprint arXiv:2010.01243*, 2020.

D. Csiba and P. Richtárik. Importance sampling for minibatches. *Journal of Machine Learning Research*, 19(27):1–21, 2018.

A. Defazio, F. Bach, and S. Lacoste-Julien. SAGA: A fast incremental gradient method with support for non-strongly convex composite objectives. In *Advances in Neural Information Processing Systems 27*, 2014.

Y. Deng, M. M. Kamani, and M. Mahdavi. Adaptive personalized federated learning. *arXiv preprint arXiv:2003.13461*, 2020.

H. Eichner, T. Koren, H. B. McMahan, N. Srebro, and K. Talwar. Semi-cyclic stochastic gradient descent. In *International Conference on Machine Learning*, 2019.

A. Fallah, A. Mokhtari, and A. Ozdaglar. Personalized federated learning with theoretical guarantees: A model-agnostic meta-learning approach. In *34th Conference on Neural Information Processing Systems*, 2020.

E. Gasanov, A. Khaled, S. Horváth, and P. Richtárik. Flix: A simple and communication-efficient alternative to local methods in federated learning. In *24th International Conference on Artificial Intelligence and Statistics (AISTATS 2022)*, 2022.

M. R. Glasgow, H. Yuan, and T. Ma. Sharp bounds for federated averaging (local sgd) and continuous perspective. In *International Conference on Artificial Intelligence and Statistics*, pages 9050–9090. PMLR, 2022.

E. Gorbunov, F. Hanzely, and P. Richtárik. Local SGD: unified theory and new efficient methods. In *NeurIPS*, 2020a.

E. Gorbunov, F. Hanzely, and P. Richtárik. A unified theory of sgd: Variance reduction, sampling, quantization and coordinate descent. In *International Conference on Artificial Intelligence and Statistics*, pages 680–690. PMLR, 2020b.

E. Gorbunov, D. Kovalev, D. Makarenko, and P. Richtárik. Linearly converging error compensated sgd. *Advances in Neural Information Processing Systems*, 33:20889–20900, 2020c.

E. Gorbunov, K. Burlachenko, Z. Li, and P. Richtárik. MARINA: Faster non-convex distributed learning with compression. In *38th International Conference on Machine Learning*, 2021.

R. M. Gower, N. Loizou, X. Qian, A. Sailanbayev, E. Shulgin, and P. Richtárik. SGD: General analysis and improved rates. In K. Chaudhuri and R. Salakhutdinov, editors, *Proceedings of the 36th International Conference on Machine Learning*, volume 97 of *Proceedings of Machine Learning Research*, pages 5200–5209, Long Beach, California, USA, 09–15 Jun 2019a. PMLR. URL http://proceedings.mlr.press/v97/qian19b.html.

R. M. Gower, N. Loizou, X. Qian, A. Sailanbayev, E. Shulgin, and P. Richtárik. Sgd: General analysis and improved rates. In *International Conference on Machine Learning*, pages 5200–5209. PMLR, 2019b.

F. Haddadpour and M. Mahdavi. On the convergence of local descent methods infederated learning. *arXiv preprint arXiv:1910.14425*, 2019.

F. Hanzely and P. Richtárik. Federated learning of a mixture of global and local models. *arXiv:2002.05516*, 2020.

F. Hanzely, S. Hanzely, S. Horváth, and P. Richtárik. Lower bounds and optimal algorithms for personalized federated learning. In *NeurIPS*, 2020.

T. Hofmann, A. Lucchi, S. Lacoste-Julien, and B. McWilliams. Variance reduced stochastic gradient descent with neighbors. *Advances in Neural Information Processing Systems*, 28, 2015.

S. Horváth and P. Richtárik. Nonconvex variance reduced optimization with arbitrary sampling. In K. Chaudhuri and R. Salakhutdinov, editors, *Proceedings of the 36th International Conference on Machine Learning*, volume 97 of *Proceedings of Machine Learning Research*, pages 2781–2789, Long Beach, California, USA, 09–15 Jun 2019. PMLR. URL http://proceedings.mlr.press/v97/horvath19a.html.

S. Horváth, C.-Y. Ho, Ľudovít Horváth, A. N. Sahu, M. Canini, and P. Richtárik. Natural compression for distributed deep learning. *arXiv preprint arXiv:1905.10988*, 2019a.

S. Horváth, D. Kovalev, K. Mishchenko, S. Stich, and P. Richtárik. Stochastic distributed learning with gradient quantization and variance reduction. *arXiv preprint arXiv:1904.05115*, 2019b.

S. Horváth, M. Sanjabi, L. Xiao, P. Richtárik, and M. Rabbat. Fedshuffle: Recipes for better use of local work in federated learning. *arXiv preprint arXiv:2204.13169*, 2022.

Y. Jiang, J. Konečný, K. Rush, and S. Kannan. Improving federated learning personalization via model agnostic meta learning. In *NeurIPS Workshop on Federated Learning for Data Privacy and Confidentiality*, 2019.

R. Johnson and T. Zhang. Accelerating stochastic gradient descent using predictive variance reduction. In *Advances in Neural Information Processing Systems 26*, pages 315–323, 2013.

P. Kairouz, H. B. McMahan, B. Avent, A. Bellet, M. Bennis, A. N. Bhagoji, K. Bonawitz, Z. Charles, G. Cormode, R. Cummings, R. G. D'Oliveira, H. Eichner, S. E. Rouayheb, D. Evans, J. Gardner, Z. Garrett, A. Gascón, B. Ghazi, P. B. Gibbons, M. Gruteser, Z. Harchaoui, C. He, L. He, Z. Huo, B. Hutchinson, J. Hsu, M. Jaggi, T. Javidi, G. Joshi, M. Khodak, J. Konečný, A. Korolova, F. Koushanfar, S. Koyejo, T. Lepoint, Y. Liu, P. Mittal, M. Mohri, R. Nock, A. Özgür, R. Pagh, M. Raykova, H. Qi, D. Ramage, R. Raskar, D. Song, W. Song, S. U. Stich, Z. Sun, A. T. Suresh, F. Tramèr, P. Vepakomma, J. Wang, L. Xiong, Z. Xu, Q. Yang, F. X. Yu, H. Yu, and S. Zhao. Advances and open problems in federated learning. *Foundations and Trends®in Machine Learning*, 14(1–2):1–210, 2019.

S. Karimireddy, S. Kale, M. Mohri, S. Reddi, S. Stich, and A. Suresh. SCAFFOLD: Stochastic controlled averaging for on-device federated learning. In *ICML*, 2020.

A. Khaled and P. Richtárik. Better theory for SGD in the nonconvex world. *arXiv Preprint arXiv:2002.03329*, 2020.

A. Khaled, K. Mishchenko, and P. Richtárik. First analysis of local GD on heterogeneous data. In *NeurIPS Workshop on Federated Learning for Data Privacy and Confidentiality*, pages 1–11, 2019.

A. Khaled, K. Mishchenko, and P. Richtárik. Tighter theory for local SGD on identical and heterogeneous data. In *The 23rd International Conference on Artificial Intelligence and Statistics (AISTATS 2020)*, 2020.

S. Khirirat, H. R. Feyzmahdavian, and M. Johansson. Distributed learning with compressed gradients. *arXiv preprint arXiv:1806.06573*, 2018.

J. Konečný, H. B. McMahan, D. Ramage, and P. Richtárik. Federated optimization: distributed machine learning for on-device intelligence. *arXiv:1610.02527*, 2016a.

J. Konečný, H. B. McMahan, F. Yu, P. Richtárik, A. T. Suresh, and D. Bacon. Federated learning: strategies for improving communication efficiency. In *NIPS Private Multi-Party Machine Learning Workshop*, 2016b.

D. Kovalev, S. Horváth, and P. Richtárik. Don't jump through hoops and remove those loops: SVRG and Katyusha are better without the outer loop. In *Proceedings of the 31st International Conference on Algorithmic Learning Theory*, 2020a.

D. Kovalev, S. Horváth, and P. Richtárik. Don't jump through hoops and remove those loops: Svrg and katyusha are better without the outer loop. In *Algorithmic Learning Theory*, pages 451–467. PMLR, 2020b.

M. Li, T. Zhang, Y. Chen, and A. J. Smola. Efficient mini-batch training for stochastic optimization. In *Proceedings of the 20th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, KDD '14, pages 661–670, New York, NY, USA, 2014. ACM. ISBN 978-1-4503-2956-9. doi: 10.1145/2623330.2623612. URL http://doi.acm.org/10.1145/2623330.2623612.

T. Li, A. K. Sahu, A. Talwalkar, and V. Smith. Federated learning: challenges, methods, and future directions. *IEEE Signal Processing Magazine*, 37(3):50–60, 2020a. doi: 10.1109/MSP.2020.2975749.

X. Li, W. Yang, S. Wang, and Z. Zhang. Communication-efficient local decentralized SGD methods. *arXiv preprint arXiv:1910.09126*, 2019.

X. Li, K. Huang, W. Yang, S. Wang, and Z. Zhang. On the conver-gence of FedAvg on non-IID data. In *International Conference on Learning Representations*, 2020b.

Z. Li, D. Kovalev, X. Qian, and P. Richtárik. Acceleration for compressed gradient descent in distributed and federated optimization. In *International Conference on Machine Learning*, 2020c.

Y. Lin, S. Han, H. Mao, Y. Wang, and B. Dally. Deep gradient compression: Reducing the communication bandwidth for distributed training. In *International Conference on Learning Representations*, 2018.

C. Ma, J. Konečný, M. Jaggi, V. Smith, M. I. Jordan, P. Richtárik, and M. Takáč. Distributed optimization with arbitrary local solvers. *Optimization Methods and Software*, 32(4):813–848, 2017.

G. Malinovsky, D. Kovalev, E. Gasanov, L. Condat, and P. Richtárik. From local SGD to local fixed point methods for federated learning. In *International Conference on Machine Learning*, 2020.

G. Malinovsky, K. Mishchenko, and P. Richtárik. Server-side stepsizes and sampling without replacement provably help in federated optimization. *arXiv preprint arXiv:2201.11066*, 2021.

B. McMahan and D. Ramage. Federated learning: Collaborative machine learning without centralized training data. GoogleAIBlog, Apr. 2017.

H. B. McMahan, E. Moore, D. Ramage, S. Hampson, and B. Agüera y Arcas. Communication-efficient learning of deep networks from decentralized data. In *Proceedings of the 20th International Conference on Artificial Intelligence and Statistics (AISTATS)*, 2017.

K. Mishchenko, E. Gorbunov, M. Takáč, and P. Richtárik. Distributed learning with compressed gradient differences. *arXiv preprint arXiv:1901.09269*, 2019.

K. Mishchenko, G. Malinovsky, S. Stich, and P. Richtárik. Proxskip: A simple and provably effective communication-acceleration technique for federated learning. *arXiv preprint arXiv:2202.09357*, 2022.

A. Mitra, R. Jaafar, G. Pappas, and H. Hassani. Linear convergence in federated learning: Tackling client heterogeneity and sparse gradients. In *Advances in Neural Information Processing Systems 34*, 2021.

P. Moritz, R. Nishihara, I. Stoica, and M. I. Jordan. SparkNet: Training deep networks in Spark. In *International Conference on Learning Representations (ICLR)*, 2016.

Y. Nesterov. *Introductory lectures on convex optimization: a basic course (Applied Optimization)*. Kluwer Academic Publishers, 2004.

Y. Nesterov. Gradient methods for minimizing composite functions. *Mathematical Programming*, 140(1):125–161, 2013.

C. Philippenko and A. Dieuleveut. Bidirectional compression in heterogeneous settings for distributed or federated learning with partial participation: tight convergence guarantees. *arXiv preprint arXiv:2006.14591*, 2020.

D. Povey, X. Zhang, and S. Khudanpur. Parallel training of DNNs with natural gradient and parameter averaging. In *ICLR Workshop*, 2015.

X. Qian, R. Islamov, M. Safaryan, and P. Richtárik. Basis matters: better communication-efficient second order methods for federated learning. In *International Conference on Artificial Intelligence and Statistics (AISTATS)*, 2022.

P. Richtárik, I. Sokolov, and I. Fatkhullin. EF21: A new, simpler, theoretically better, and practically faster error feedback. In *Advances in Neural Information Processing Systems 34*, 2021.

P. Richtárik, I. Sokolov, I. Fatkhullin, E. Gasanov, Z. Li, and E. Gorbunov. 3pc: Three point compressors for communication-efficient distributed training and a better theory for lazy aggregation. *arXiv preprint arXiv:2202.00998*, 2022.

M. Safaryan, R. Islamov, X. Qian, and P. Richtárik. FedNL: Making Newton-type methods applicable to federated learning. *arXiv preprint arXiv:2106.02969*, 2021.

E. Shulgin and P. Richtárik. Shifted compression framework: Generalizations and improvements. In *OPT2021: 13th Annual Workshop on Optimization for Machine Learning*, 2021.

M. Takáč, A. Bijral, P. Richtárik, and N. Srebro. Mini-batch primal and dual methods for SVMs. In *30th International Conference on Machine Learning*, pages 537–552, 2013.

O. Thakkar, G. Andrew, and H. B. McMahan. Differentially private learning with adaptive clipping. *arXiv preprint arXiv:1905.03871*, 2019.

P. Vepakomma, T. Swedish, R. Raskar, O. Gupta, and A. Dubey. No peek: A survey of private distributed deep learning. *arXiv preprint arXiv:1812.03288*, 2018.

J. Wang, Z. Charles, Z. Xu, G. Joshi, H. B. McMahan, B. A. y Arcas, M. Al-Shedivat, G. Andrew, S. Avestimehr, K. Daly, D. Data, S. Diggavi, H. Eichner, A. Gadhikar, Z. Garrett, A. M. Girgis, F. Hanzely, A. Hard, C. He, S. Horvath, Z. Huo, A. Ingerman, M. Jaggi, T. Javidi, P. Kairouz, S. Kale, S. P. Karimireddy, J. Konecny, S. Koyejo, T. Li, L. Liu, M. Mohri, H. Qi, S. J. Reddi, P. Richtarik, K. Singhal, V. Smith, M. Soltanolkotabi, W. Song, A. T. Suresh, S. U. Stich, A. Talwalkar, H. Wang, B. worth, S. Wu, F. X. Yu, H. Yuan, M. Zaheer, M. Zhang, T. Zhang, C. Zheng, C. Zhu, and W. Zhu. A field guide to federated optimization. *arXiv preprint arXiv:2107.06917*, 2021.

B. E. Woodworth, K. K. Patel, and N. Srebro. Minibatch vs local sgd for heterogeneous distributed learning. *Advances in Neural Information Processing Systems*, 33:6281–6292, 2020.

H. Yu, R. Jin, and S. Yang. On the linear speedup analysis of communication efficient momentum SGD for distributed non-convex optimization. In *International Conference on Machine Learning (ICML)*, 2019.

Checklist

1. For all authors...
   (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [Yes]
   (b) Did you describe the limitations of your work? [Yes] In the main paper
   (c) Did you discuss any potential negative societal impacts of your work? [N/A] Our paper is mostly theoretical and it cannot cause negative societal impacts.
   (d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [Yes]
2. If you are including theoretical results...
   (a) Did you state the full set of assumptions of all theoretical results? [Yes] We state all assumptions for all theorems in this paper.
   (b) Did you include complete proofs of all theoretical results? [Yes] We provide all proofs in supplementary materials.
3. If you ran experiments...
   (a) Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? [Yes] We provide it in supplementary materials.
   (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? [Yes]
   (c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? [No] Our methods have low level of variance, so there is no need to include error bars.
   (d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? [Yes]
4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...
   (a) If your work uses existing assets, did you cite the creators? [Yes]
   (b) Did you mention the license of the assets? [N/A] We use open source assets.
   (c) Did you include any new assets either in the supplemental material or as a URL? [N/A] We do not use any new assets.
   (d) Did you discuss whether and how consent was obtained from people whose data you're using/curating? [N/A] We do not use personal data.
   (e) Did you discuss whether the data you are using/curating contains personally identifiable information or offensive content? [N/A] We do not use personal data.
5. If you used crowdsourcing or conducted research with human subjects...
   (a) Did you include the full text of instructions given to participants and screenshots, if applicable? [N/A] We do not use crowdsourcing or conducted research with human subjects.
   (b) Did you describe any potential participant risks, with links to Institutional Review Board (IRB) approvals, if applicable? [N/A] We do not use crowdsourcing or conducted research with human subjects.
   (c) Did you include the estimated hourly wage paid to participants and the total amount spent on participant compensation? [N/A] We do not use crowdsourcing or conducted research with human subjects.