# Supervised Q-Learning for Continuous Control

**Anonymous Author(s)**
Affiliation
Address
email

## Abstract

Policy gradient (PG) algorithms have been widely used in reinforcement learning (RL). However, PG algorithms rely on exploiting the value function being learned with the first-order update locally, which results in limited sample efficiency. In this work, we propose an alternative method called Zeroth-Order Supervised Policy Improvement (ZOSPI). ZOSPI exploits the estimated value function $Q$ globally while preserving the local exploitation of the PG methods based on zeroth-order policy optimization. This learning paradigm follows Q-learning but overcomes the difficulty of efficiently operating argmax in continuous action space. It finds max-valued action within a small number of samples. The policy learning of ZOSPI has two steps: First, it samples actions and evaluates those actions with a learned value estimator, and then it learns to perform the action with the highest value through supervised learning. We further demonstrate such a supervised learning framework can learn multi-modal policies. Experiments show that ZOSPI achieves competitive results on the continuous control benchmarks with a remarkable sample efficiency.[1]

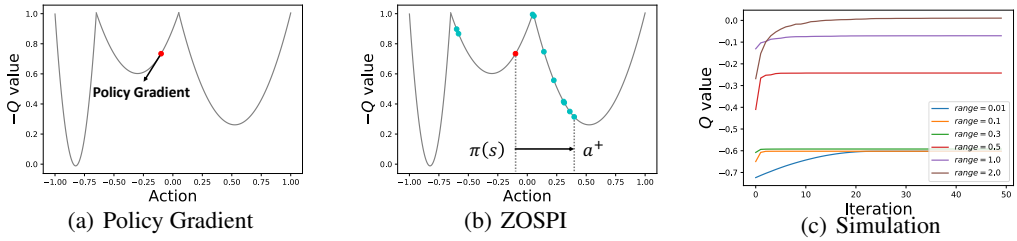
## 1 Introduction

Model-free Reinforcement Learning achieves great successes in many challenging tasks [1–3], however one hurdle for its applicability to real-world control problems is the low sample efficiency. To improve the sample efficiency, off-policy methods [4–8] reuse the experiences generated by previous policies to optimize the current policy, therefore can obtain a much higher sample efficiency than the on-policy methods [9, 10]. Alternatively, SAC [11] improves sample efficiency by conducting more active exploration with maximum entropy regularizer [12] to the off-policy actor critic [4, 13].OAC [14] further improves SAC by combining it with the Upper Confidence Bound heuristics [15] to incentivize more informed exploration. Nonetheless, all of those previous methods rely on Gaussian-parameterized policies and conducts local exploration strategies that simply add noises on the action space. Consequentially, those methods can converge to sub-optimal solutions, as has been shown in [16].

In this work, we explore an alternative approach to the conventional policy gradient paradigm for continuous control. We introduce a zeroth-order method that can be optimized through supervised learning. The proposed method carries out non-local exploration through global value-function exploitation to achieve higher sample efficiency for continuous control tasks. To better exploit the learned value function $Q$, we propose to search the action space globally for a better target action, which is in contrast to previous policy gradient methods that only utilize the local information of the learned value functions (e.g. the Jacobian matrix [17]). The key insight is to improve sample efficiency at expense of additional computation, which is acceptable as samples are much more expensive than computations in real-world applications such as Healthcare and Robotics.

[1] Code is made open-sourced at https://anonymous.4open.science/r/Submission7213-FBF5

Submitted to 36th Conference on Neural Information Processing Systems (NeurIPS 2022). Do not distribute.




(a) Policy Gradient (b) ZOSPI (c) Simulation

Figure 1: (a) Landscape of $Q$ value for a 1-dim continuous control task. Policy gradient methods optimize the policy according to the local information of $Q$. (b) For the same task, ZOSPI directly updates the predicted actions to the sampled action with the largest $Q$ value. (c) Simulation results, where in each optimization iteration 10 actions are uniformly sampled under different ranges. The reported results are averaged over 100 random seeds. It can be seen that a larger random sample range improves the chance of finding global optima. Similar phenomenon also exist in practice as shown in Appendix A.

The idea behind our work is related to the value-based policy gradient methods [7, 8], where the policy gradient optimization step takes the role of finding a well-performing action given a learned state-action value function. In previous work, the policy gradient step tackles the curse of dimensionality for deep Q-learning since it is intractable to directly search for the maximal value in the continuous action space [1]. Differently, we circumvent searching for the action with the maximal value in the continuous action space by finding the max-valued action within a small set of sampled actions.

**A Motivating Example** To perform a global exploitation of the learned value function $Q$, we propose to apply a zeroth-order optimization scheme and update the target policy through supervised learning, which is inspired by works of evolution strategies [18–20] that adopt zeroth-order optimizations in the parameter space. Figure 1 shows a motivating example that demonstrates the difference between the zeroth-order and first-order optimizations. Different from the standard policy gradient, combining the zeroth-order optimization with supervised learning forms a new way of policy update. Such an update avoids the local improvement of policy gradient: when policy gradient is applied, the target policy uses policy gradient to adjust its predictions according to the deterministic policy gradient theorem [17], but such adjustments can only lead to local improvements and may induce sub-optimal policies due to the non-convexity of the policy function [16]; on the contrary, our integrated approach of zeroth-order optimization and supervised learning can greatly improve the non-convex policy optimization and more likely escape the potential local minimum.

**Contributions** of this paper can be summarized as follows:

1. We introduce a simple yet effective policy optimization method called Zeroth-Order Supervised Policy Improvement (ZOSPI), as an alternative to the policy gradient approaches for continuous control. The policy from ZOSPI exploits global information of the learned value function $Q$ and updates itself through sample-based supervised learning.
2. Practically, we point out the flexibility of the proposed supervised learning paradigm and show ZOSPI is capable of working with multi-modal policy class. And introduce a hybrid method that leverages both zeroth-order and first-order gradients for policy learning.
3. Empirically, we demonstrate the improved performance of ZOSPI over policy gradient methods on the continuous control benchmarks, in terms of both higher sample efficiency and asymptotic performance.

# 2 Related Work

## 2.1 Policy Gradient Methods

The policy gradient methods solve an MDP by directly optimizing the policy to maximize the cumulative reward [21, 22]. While the prominent on-policy policy gradient methods like TRPO [9]

and PPO [10] improve the learning stability of vanilla policy gradient [21] via trust region updates, the off-policy methods such as [4, 7, 6] employ an experience replay mechanism to achieve higher sample efficiency. The work of TD3 [8] further addresses the function approximation error and boosts the stability of DDPG with several improvements. Another line of works is the combination of policy gradient methods and the max-entropy regularizer, which leads to better exploration and stable asymptotic performances [12, 11]. All of these approaches adopt function approximators [23] for state or state-action value estimation and take directionally-uninformed Gaussian as the policy parameterization, which lead to a local exploration behavior [14, 16].

### 2.2 RL by Supervised Learning

Iterative supervised learning and self-imitate learning are becoming an alternative approach for model-free RL. Instead of applying policy gradient for policy improvement, methods based on supervised methods update policies by minimizing the mean square error between target actions and current actions predicted by a policy network [24], or alternatively by maximizing the likelihood for a stochastic policy class [25]. While those previous works focus on the Goal-Conditioned tasks in RL, in this work we aim to tackle more general RL tasks. Some other works use supervised learning to optimize the policy towards manually selected policies to achieve better training stability under both offline [26] and online settings [27–29]. Differently, in our work the policy learning is based on a much simpler formulation than the previous attempts [28, 30, 31], and it does not rely on any expert data to achieve competitive performance.

### 2.3 Zeroth-Order Methods

Zeroth-order optimization methods, also called gradient-free methods, are widely used when it is difficult to compute the gradients. They approximate the local gradient with random samples around the current estimate. The works in [32, 33] show that a local zeroth-order optimization method has a convergence rate that depends logarithmically on the ambient dimension of the problem under some sparsity assumptions. It can also efficiently escape saddle points in non-convex optimizations [34, 35]. In RL, many studies have verified an improved sample efficiency of zeroth-order optimization [36, 20, 18]. In this work we provide a novel way of combining the local sampling and the global sampling to ensure that our algorithm can approximate the gradient descent locally and also find a better global region.

## 3 Preliminaries

### 3.1 Markov Decision Processes

We consider the deterministic Markov Decision Process (MDP) with continuous state and action spaces in the discounted infinite-horizon setting. Such MDPs can be denoted as $\mathcal{M} = (\mathcal{S}, \mathcal{A}, \mathcal{T}, r, \gamma)$, where the state space $\mathcal{S}$ and the action space $\mathcal{A}$ are continuous, and the unknown state transition probability representing the transition dynamics is denoted by $\mathcal{T} : \mathcal{S} \times \mathcal{A} \mapsto \mathcal{S}$. $r : \mathcal{S} \times \mathcal{A} \mapsto [0, 1]$ is the reward function and $\gamma \in [0, 1]$ is the discount factor. An MDP $\mathcal{M}$ and a learning algorithm operating on $\mathcal{M}$ with an arbitrary initial state $s_0 \in \mathcal{S}$ constitute a stochastic process described sequentially by the state $s_t$ visited at time step $t$, the action $a_t$ chosen by the algorithm at step $t$, the reward $r_t = r(s_t, a_t)$ and the next state $s_{t+1} = \mathcal{T}(s_t, a_t)$ for any $t = 0, \ldots, T$. Let $H_t = \{s_0, a_0, r_0, \ldots, s_t, a_t, r_t\}$ be the trajectory up to time $t$. Our algorithm finds the policy that maximizes the discounted cumulative rewards $\sum_{t=0}^{T} \gamma^t r_t$. Our work follows the general Actor-Critic framework [37, 38, 4, 6], which learns in an unknown environment using a value network denoted by $Q_{w_t} : \mathcal{S} \times \mathcal{A} \mapsto \mathbb{R}$ for estimating $Q$ values and a policy network for learning the behavior policy $\pi_{\theta_t} : \mathcal{S} \mapsto \mathcal{A}$. Here $w_t$ and $\theta_t$ are the parameters of these two networks at step $t$, respectively.

### 3.2 Mixture Density Networks

The optimal policies in many environments tend to have multi-modal property. Mixture Density Networks (MDN), previously proposed in [39] to solve the stochastic prediction problem, is a potential multi-modal policy function to be introduced in this work. One challenge here is that previous policy gradient methods are not compatible with multi-modal policies based on MDN.

**Algorithm 1** Policy Update with Zeroth-Order and First-Order Optimization

**Require**
Objective function $Q_s$, domain $\mathcal{A}$, current policy network $\pi_\theta$, perturbation network $\pi_\phi$ current point $a_0 = \pi_\theta(s)$, number of global samples $n_1$, number of local samples $n_2$
**Global sampling**
Sample $n_1$ points uniformly in the entire space by

$$a_i \sim \mathcal{U}_\mathcal{A}, \text{ for } i = 1, \ldots, n_1,$$

where $\mathcal{U}_\mathcal{A}$ is the uniform distribution over $\mathcal{A}$.
**Local sampling**
Sample $n_2$ points locally from a Gaussian centered at $a_0$ with a covariance matrix $\sigma^2 I$:

$$a_{i+n_1} \sim \mathcal{N}(a_0, \sigma^2 I), i = 1, \ldots, n_2.$$

Set $a^+ = \arg\max_{a \in \{a_0, \ldots, a_{n_1+n_2}\}} Q_s(a)$.
Update policy $\pi_\theta$ according to Eq.(2)
**Local perturbation**
Update perturbation network according to Eq. (4).

While in normal regression tasks, the learning objective is to maximize the log-likelihood of observed data given the current parameters of a predictive model, MDN uses the mixture of Gaussian with multiple parameters as $\mathcal{D} = \{X_i, Y_i\}_\mathbb{N}$. Then the likelihood is given by

$$\mathcal{L} = \sum_{i=1}^{i=\mathbb{N}} \mathcal{L}(Y_i|X_i, \theta) = \sum_{i=1}^{i=\mathbb{N}} \sum_{k=1}^{K} w_k(X_i, \theta)\phi(Y_i|\mu_k(X_i, \theta), \sigma_k(X_i, \theta)), \tag{1}$$

where $\theta$ denotes the parameters of neural networks with three branches of outputs $\{w_i, \mu_i, \sigma_i\}_K$, $\phi$ is the probability density function of normal distribution and $\sum_{i=1}^{K} w_i = 1$.

# 4 Zeroth-Order Supervised Continuous Control

Q-learning in the tabular setting finds the best discrete action given the current state, which can be difficult in the continuous action space due to the non-convexity of $Q$. A policy network is thus trained to approximate the optimal action. However, gradient-based algorithm may be trapped by local minima or saddle point depending on the initialization. In this section, we introduce a hybrid method with a global policy and a perturbation policy. The global policy predicts a coarse action by supervised learning on uniformly sampled actions and the perturbation network iterates based on the coarse prediction. In the following section, we give a motivating example to demonstrate the benefits of the hybrid framework.

Figure 1 shows a motivating example to demonstrate the benefits of applying zeroth-order optimization to policy updates. Consider we have a learned $Q$ function with multiple local optima. Here we assume the conventional estimation of $Q$ function is sufficient for a global exploitation [8, 11] and we will discuss an improved estimation method in the Appendix. Our deterministic policy selects a certain action at this state, denoted as the red dot in Figure 1(a). In deterministic policy gradient methods [17, 7], the policy gradient is conducted according to the chain rule to update the policy parameter $\theta$ with regard to $Q$-value by timing up the Jacobian matrix $\nabla_\theta \pi_\theta(s)$ and the derivative of $Q$, *i.e.*, $\nabla_a Q(s, a)$. Consequently, the policy gradient can only guarantee to find a local minima, and similar local improvement behaviors are also observed in stochastic policy gradient methods like PPO and SAC [10, 11, 16, 14]. Instead, if we can sample sufficient *random* actions in a broader range of the action space, denoted as blue dots in Figure 1(b), and then evaluate their values respectively through the learned $Q$ estimator, it is possible to find a better initialization, from which the policy gradient can more likely find the global minima. Figure 1(c) shows the simulation result using different sample ranges for the sample-based optimization starting from the red point. It is clear that a larger sample range improves the chance of finding the global optima. Utilizing such a global exploitation on the learned value function is the key insight of this work.

### 4.1 Zeroth-Order Supervised Policy Improvement

Based on the above motivation, we propose a framework called ZOSPI (Zeroth-Order Supervised Policy Improvement). ZOSPI consists of two policy networks: the global policy network that gives us a coarse estimate on the optimal action and a perturbation network that iterates based on the output of the global policy network using policy gradient.

**Supervised learning for global policy network.** We denote the global policy network by $\pi_\theta$. At any step $t$, we sample a set of actions uniformly over the entire action space as well as a set of actions sampled from Gaussian distribution centered at current prediction $\pi_{\theta_t}$. We denote by $a_t^+$, the action that gives the highest $Q$ value with respect to current state $s_t$. Then we apply the supervised policy improvement that minimizes the $L_2$ distance between $a_t^+$ and $\pi_{\theta_t}(s_t)$, which gives the descent direction:

$$\nabla_\theta \frac{1}{2}(a_t^+ - \pi_{\theta_t}(s_t))^2 = (a_t^+ - \pi_{\theta_t}(s_t))\nabla_\theta \pi_{\theta_t}(s_t). \tag{2}$$

The global samples can help finding the regions that are better in the whole space. The local samples from Gaussian distribution accelerate later-stage training when the prediction is accurate enough and most global samples are not as good as the current prediction. The implementation detail is shown in Algorithm 1.

**Policy gradient for perturbation network.** We introduce another perturbation network, denoted by $\pi_{\phi_t}$, as [40]. Such a perturbation network is parameterized by $\phi_t$ that performs fine-grained control on top of the global policy network $\pi_{\theta_t}$. Different from the $\pi_{\theta_t}$, the perturbation network $\pi_{\phi_t}$ takes both the current state $s_t$ and the predicted action $\pi_{\theta_t}(s_t)$ as inputs, thus the final executed action is as follows:

$$a_t = \pi_{\theta_t}(s_t) + \pi_{\phi_t}(s_t, \pi_{\theta_t}(s_t)). \tag{3}$$

The range of the outputs for the perturbation network is limited to $0.05$ times the value of maximal action. Therefore, it is only able to *perturb* the action provided by the policy network $\pi_{\theta_t}$.

The perturbation network $\pi_{\phi_t}$ is trained with the policy gradient:

$$\nabla_\phi J = \mathbb{E}[\nabla_{a'} Q_w(s_t, a')|_{a'=\pi_{\theta_t}(s_t)+\pi_{\phi_t}(s_t)} \nabla_\phi \pi_{\phi_t}(s_t)]. \tag{4}$$

The intuition behind such an empirical design can be drawn from the example of Figure 1: although the global sampling step as well as the zeroth-order method helps policy optimization escape sub-optimal regions of the non-convex value function, the first order method can help to optimize the decision afterwards inside the locally convex region more efficiently. Eq. (4) shows that the updates are accessed only through the gradient, $\nabla_a Q_w$.

### 4.2 Analyses on the Benefits of Global Sampling

In this section, we give some analyses on the benefits of global sampling in terms of the sampling efficiency. Our analyses does not consider the improvement from the local sampling set, with which the quality of supervised learning can only be even better.

**Error rate of the $a_t^+$.** The performance of the supervised policy improvement heavily depends on the goodness of $a_t^+$, the best action in the sampling set. By assuming the continuity of the estimated $Q$ function, we give an upper bound on the error rate $\|a_t^+ - a^*(s)\|$, where $a^*(s)$ is the true optimal action given the current state $s$.

**Lemma 1.** Assume the estimated $Q$ function $Q_w(s_t, \cdot)$ given any $s_t$ is $L$-Lipschitz with respect to action inputs. Let the action space $\mathcal{A} \subset \mathbb{R}^d$ for some positive integer $d$ and assume that any action $a \in \mathcal{A}$, $\|a\|_2 \leq k$ for $k > 0$. Then with a probability at least $1 - \delta$, we have

$$\|a_t^+ - a^*(s)\|_2 \leq 2k\sqrt{d}L \left(\frac{\log^2(n/\delta)}{n}\right)^{1/d}.$$

Though the dependence of $n$ is $1/n^{1/d}$, the estimation of $a_t^+$ is sufficient for a coarse prediction, because our prediction prediction $\pi_\theta$ aggregates all the historical information, which further reduces the error and we only need $\pi_\theta$ in a local convex region of the global optima. This rate can be much lower when we have a fairly better prediction and the best actions are given by the local sampling set instead.

**Algorithm 2** Zeroth-Order Supervised Policy Improvement (ZOSPI)

**Require**
  Number of epochs $M$, size of mini-batch $N$, momentum $\tau > 0$.
  Random initialized policy network $\pi_\theta$, target policy network $\pi_{\theta'}$, perturbation network $\pi_\phi$, $\theta' \leftarrow \theta$.
  Two random initialized $Q$ networks, and corresponding target networks, parameterized by $w_1, w_2, w_1', w_2'$. $w_i' \leftarrow w_i$.
  Empty experience replay buffer $\mathcal{D} = \{\}$.
**for** iteration $= 1, 2, ...$ **do**
  **for** t $= 1, 2, ..., T$ **do**
    # Interaction
    Rollout with $a_t = \pi_{\theta_t}(s_t) + \pi_{\phi_t}(s_t, \pi_{\theta_t}(s_t))$, store transition $(s_t, a_t, s_{t+1}, r_t)$ in $\mathcal{D}$.
    **for** epoch $= 1, 2, ..., M$ **do**
      Sample a mini-batch of transition tuples $\mathcal{D}' = \{(s_{t_j}, a_{t_j}, s_{t_j+1}, r_{t_j})\}_{j=1}^N$.
      # Update $Q$
      Calculate target $Q$ value $y_j = r_{t_j} + \min_{i=1,2} Q_{w_i'}(s_{t_j+1}, \pi_{\theta'}(s_{t_j}))$.
      Update $w_i$ with one step gradient descent on the loss $\sum_j (y_j - Q_{w_i}(s_{t_j}, a_{t_j}))^2$, $i = 1, 2$.
      # Update $\pi$
      Call Algorithm 1 for policy optimization to update $\theta$ and $\phi$.
    **end for**
    $\theta' \leftarrow \tau\theta + (1-\tau)\theta'$; $w_i' \leftarrow \tau w_i + (1-\tau) w_i'$
  **end for**
**end for**

**Quality of $\pi_\theta$.** It is hard to directly evaluate the error rate of $\pi_\theta(s_t)$ because of the online stochastic gradient descent applied on a non-stationary distribution of states and a Q-function that is continuously being updated. Thus we take a step back and consider a simpler scenario. We assume a fixed $Q$ function and a fixed state distribution. We assume that the policy network is a linear model with $\theta \in \mathbb{R}^p$. We consider a global empirical risk minimizer, so called ERM, since it is easy to achieve for linear models. Let our dataset with $T$ samples be $\{(s_t, a_t^+)\}_{t=1}^T$. Our ERM estimate is defined by

$$\hat{\theta} = \arg\min_{\theta \in \Theta} \sum_{t=1}^{T} (a_t^+ - \pi_\theta(s_t))^2.$$

**Theorem 1.** If assumptions in Lemma 1 hold, with a high probability we have

$$\mathbb{E}\|\pi_{\hat{\theta}}(s) - a^*(s)\|_2^2 = \tilde{\mathcal{O}}\left(\frac{k\sqrt{d}pL}{n^{1/d}T} + \min_\theta \mathbb{E}\|\pi_\theta(s) - a^*(s)\|_2\right).$$

where $\tilde{\mathcal{O}}$ hides all the constant and logarithmic terms.

Theorem 1 implies that though the error of $a_t^*$ at one step can be high, the error of our policy network can be further reduced by aggregating information from multiple steps. However, when $\pi_\theta$ is nonlinear functions, the quality may be worse than the case in our analyses. That is why we need some local perturbation through policy gradient to further improve our policy. Missing proofs in this section are given by Appendix B.

### 4.3 Multi-Modal Continuous Control with ZOSPI

Different from standard policy gradient methods, the policy optimization step in ZOSPI can be considered as sampling-based supervised learning. Such a design enables many extensions of the proposed learning paradigm. In this section, we introduce the combination of ZOSPI and first-order method for stabilized training and the combination of ZOSPI and MDN for multi-modal policy learning.

**Learning Multi-Modal Policies with Mixture Density Networks** In the context of RL, there might be multiple optimal actions for some certain states, e.g., stepping up-ward and then right-ward may lead to the same state as stepping right and then upward, therefore both choices result in identical return in Figure 2. However, normal deterministic policy gradient methods can not capture such multi-modality due to the limitation that the DPG theorem is not applicable to a stochastic policy class.

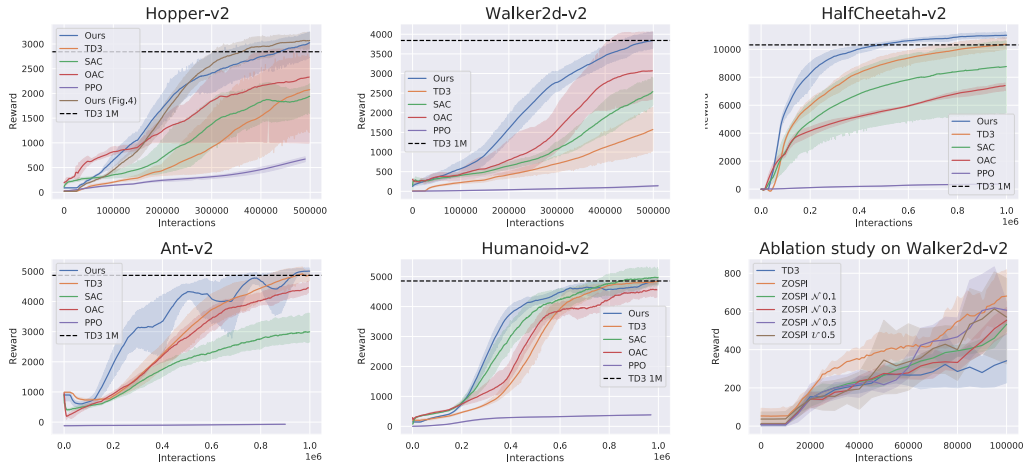



Figure 3: Experimental results on the MuJoCo locomotion tasks. The shaded region represents standard deviation. The dashed lines indicate asymptotic performance of TD3 after 1M interactions. ZOSPI is able to reach on-par performance within much less interactions. In all main experiments the results reported are collected from 10 random seeds and in ablations studies we use 5 random seeds. Curves are smoothed uniformly for visual clarity.

In this section, we further integrate ZOSPI with the Mixture Density Networks (MDNs) [39], which was introduced for multi-modal regression. Applying MDNs to ZOSPI leads to a more flexible and interpretable multi-modal policy class [16]: different from normal Dirac policy parameterization used in TD3, ZOSPI with MDN predicts a mixture of Dirac policies, $i.e.$, the policy $\pi_{\text{MDN}}(s)$ predicts $K$ choices for action $\vec{a} = \{a_1, ..., a_K\} \in \mathcal{A}^K$, with their corresponding probability $\vec{p} = \{p_1, ..., p_K\} \in \mathbb{R}^K$, and $\sum_i^K p_i = 1$. Then the action is sampled by

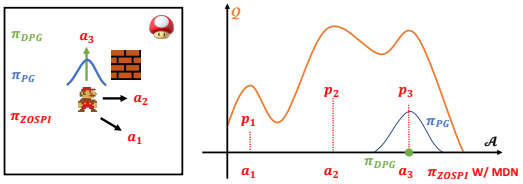



Figure 2: Illustration of how ZOSPI with MDN works when multiple optimal actions exist for a certain state: (left) Supermario is going to collect the mushroom, both $a_2$ and $a_3$ are the optimal actions for the current location. While deterministic policy gradient methods (green color) are only able to learn one of those optimal actions, ZOSPI with MDN is able to learn both. Policy gradient methods normally learn a Gaussian policy class.

$$\pi_{\text{MDN}}(s) = a_i, \quad \text{w.p. } p_i, \text{ for } i = 1, ..., K \tag{5}$$

Thereby, instead of learning the mean value of multiple optimal actions, ZOSPI with MDN is able to learn multiple optimal actions. Different from previous stochastic multi-modal policies discussed in [11], the learning of multi-modal policy does not aim to fit the entire $Q$ function distribution. Rather, ZOSPI with MDN focuses on learning the multi-modality in the best choices of actions, thus addresses the difficulty in generating multi-modal policies [12, 16]. Figure 2 illustrates the difference between ZOSPI with MDN and the previous policy gradient methods.

## 5 Experiments

In this section, we conduct experiments on five MuJoCo locomotion benchmarks to demonstrate the effectiveness of the proposed method. Specifically, we validate the following statements:

1. If we use ZOSPI with locally sampled actions, the performance of ZOSPI should be the same as its policy gradient counterpart like TD3; if we increase the sampling range, ZOSPI can better exploit the $Q$ function thus find better solution than the policy gradient methods.
2. If we continuously increase the sampling range, it will result in an uniform sampling, and the $Q$ function can be maximally exploited.
3. The perturbation network can help to improve the sample efficiency of the primal ZOSPI frame work purely based on zeroth-order optimization.

Table 1: Quantitative results. ZOSPI achieves comparable performance with only half the number of interactions with the environment, and achieves superior performance for 1M interactions.

| METHOD/TASK | HOPPER-V2 | WALKER2D-V2 | HALFCHEETAH-V2 | ANT-V2 | HUMANOID-V2 |
|---|---|---|---|---|---|
| TD3 | $2843 \pm 197$ | $3842 \pm 239$ | $10314 \pm 93$ | $4868 \pm 388$ | $4855 \pm 263$ |
| SAC | $2158 \pm 388$ | $\mathbf{4154 \pm 333}$ | $8735 \pm 170$ | $3051 \pm 469$ | $\mathbf{5012 \pm 478}$ |
| OAC | $2983 \pm 317$ | $3075 \pm 183$ | $4497 \pm 296$ | $4497 \pm 297$ | $4624 \pm 351$ |
| OURS | $\mathbf{3268 \pm 234}$ | $4027 \pm 28$ | $\mathbf{10992 \pm 126}$ | $\mathbf{5006 \pm 135}$ | $4881 \pm 164$ |
| IMPROV. OVER TD3/SAC | $\uparrow 15\% / \uparrow 51\%$ | $\uparrow 5\% / \downarrow 3\%$ | $\uparrow 7\% / \uparrow 26\%$ | $\uparrow 3\% / \uparrow 64\%$ | $\uparrow 1\% / \downarrow 3\%$ |

| METHOD/TASK | HOPPER-V2 | WALKER2D-V2 | HALFCHEETAH-V2 | ANT-V2 | HUMANOID-V2 |
|---|---|---|---|---|---|
| TD3-0.5M | $2234 \pm 231$ | $1648 \pm 204$ | $8801 \pm 176$ | $3232 \pm 268$ | $3036 \pm 567$ |
| SAC-0.5M | $1926 \pm 410$ | $2640 \pm 529$ | $7255 \pm 180$ | $2167 \pm 399$ | $4068 \pm 610$ |
| OAC-0.5M | $2395 \pm 166$ | $3074 \pm 182$ | $7405 \pm 115$ | $2903 \pm 322$ | $3610 \pm 376$ |
| OURS-0.5M | $\mathbf{3048 \pm 307}$ | $\mathbf{3932 \pm 125}$ | $\mathbf{10290 \pm 129}$ | $\mathbf{4304 \pm 260}$ | $\mathbf{4175 \pm 302}$ |
| IMPROV. OVER TD3/SAC | $\uparrow 58\% / \uparrow 36\%$ | $\uparrow 139\% / \uparrow 49\%$ | $\uparrow 17\% / \uparrow 42\%$ | $\uparrow 33\% / \uparrow 99\%$ | $\uparrow 38\% / \uparrow 3\%$ |

4. The supervised learning policy update paradigm of ZOSPI permits it to be flexibly work with multi-modal policy class.

### 5.1 ZOSPI on the MuJoCo Locomotion Tasks.

We evaluate ZOSPI on the Gym locomotion tasks based on the MuJoCo engine [41, 42]. The five locomotion tasks are Hopper-v2, Walker2d-v2, HalfCheetah-v2, Ant-v2, and Humanoid-v2. We compare our method with TD3 and SAC, the deterministic and the stochastic SOTA policy gradient methods. We also include PPO and OAC [14] to better show the learning efficiency of ZOSPI. We compare different methods within $0.5$M environment interactions in the two easy tasks to demonstrate the high learning efficiency of ZOSPI, and present results during $1$M interactions for the other tasks. The results of TD3 and OAC are obtained by running the code released by the authors, the results of PPO are obtained through the high quality open-source implementation [43], and the results of SAC are extracted from the training logs of [11].

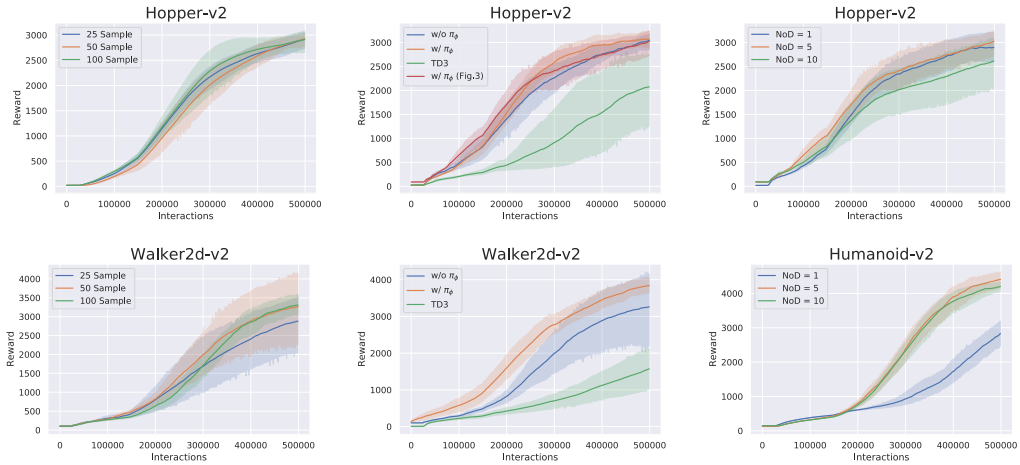
The learning curves of those tasks are shown in Figure 3. It is worth noting that in all of the results we reported, only $50$ actions are sampled and it is sufficient to learn well-performing policies. With a high sampling efficiency, ZOSPI works well in the challenging environments that have high-dimensional action spaces such as Ant-v2 and Humanoid-v2. In all the tasks the sample efficiency is consistently improved over TD3, which is the DPG counterpart of ZOSPI. While a total of $50$ sampled actions should be very sparse in the high dimensional space, we attribute the success of ZOSPI to the generality of the policy network as well as the sparsity of meaningful actions. For example, even in the tasks that have high dimensional action spaces, only limited dimensions of the action are crucial.

We provide quantitative comparison in Table 1. We report both the averaged score and the standard deviation of different methods after $1$M step of interactions with the environment. Moreover, we provide the quantitative results of ZOSPI with $0.5$M interactions during training to emphasize on its high sample efficiency. In $4$ out of the $5$ environments (except Humanoid), ZOSPI is able to achieve on-par performance compared to TD3 and SAC, with only half number of interactions. The last line of Table 1 reports the improvement of ZOSPI over TD3 and SAC respectively. All results are collected by experimenting with $10$ random seeds.

The last plot in Figure 3 shows the ablation study on the sampling range in ZOSPI, where a sampling method based on a zero-mean Gaussian is applied and we gradually increase its variance from $0.1$ to $0.5$. We also evaluate the uniform sampling method with radius of $0.5$, which is denoted as $\mathcal{U}$ $0.5$ in the Figure. The results suggest that zeroth-order optimization with local sampling performs similarly to the policy gradient method, and increasing the sampling range can effectively improve the performance.

### 5.2 Ablation Study

In this section, we conduct a series of experiments as the ablation study to evaluate different components of ZOSPI. Specifically, we investigate the performance of ZOSPI under different number of samples, with or without perturbation networks, and Number of Diracs (NoD) used in the policy network.




(a) Number of Samples (b) Perturbation Network (c) Number of Gaussians

Figure 4: Ablation study. The first column (a) shows the performance of our proposed method with different number of samples; the second column (b) shows the performance difference when ZOSPI is work with or without the perturbation network; the last column (c) shows the performance difference between using different number of Diracs when combining MDNs with ZOSPI.

**Number of Samples Used in ZOSPI** The first column of Figure 4 shows the performance of ZOSPI when using different number of samples. In both environments, more samples lead to better performance, but in the easier task of Hopper, there are only tiny differences, while in the task of Walker2d, using more than $50$ samples noticeably improves the performance. Trading off the improvements and the computational costs when using more samples, we choose to use $50$ samples for the experiments we reported in the previous section.

**Perturbation Network** The second column of Figure 4 shows the ablation studies on the perturbation network. While in the simpler control task of Hopper, the perturbation network helps improve the learning efficiency only a little bit, in more complex task the perturbation network is crucial for efficient learning. This result demonstrate the effectiveness of the combination of global and local information in exploiting of the learned value function. In all of our main experiments, we use a perturbation network with a perturbation range of $0.05$.

**The Application of MDNs** Our ablation studies on the usage of MDNs are shown in the last column of Figure 4. In the easier task of Hopper, using a larger number of NoD in ZOSPI with MDN only improves a little but introduce several times of extra computation expense. However, such multi-modal policies clearly improves the performance in the complex tasks like Humanoid. In our experiments, we find larger NoD benefits the learning for the Walker2d and Humanoid tasks, and do not improve performance in the other three environments. More implementation details of ZOSPI with MDN can be found in Appendix D.

## 6 Conclusion

In this work, we propose the method of Zeroth-Order Supervised Policy Improvement (ZOSPI) as an alternative approach of policy gradient algorithms for continuous control. ZOSPI improves the learning efficiency of previous policy gradient learning by exploiting the learned $Q$ functions globally and locally through sampling the action space. Different from previous policy gradient methods, the policy optimization of ZOSPI is based on supervised learning so that the learning of actor can be implemented with regression. Such a property enables potential extensions such as multi-modal policies, which can be seamlessly cooperated with ZOSPI. We evaluate ZOSPI on five locomotion tasks, where it remarkably improves the performance in terms of both sample efficiency and asymptotic performance compared to previous policy gradient methods.

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