Graph Theory-Based Deep Graph Similarity Learning: A Unified Survey of Pipeline, Techniques, and Challenges

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Abstract

Graph similarity computation, which measures the resemblance between graphs, is a crucial operation in fields such as graph search. Recent advances in graph neural networks have enabled the embedding of graphs into low-dimensional vector spaces, where the similarity or distance between graphs can be efficiently quantified. However, these methods are often tailored to specific tasks and function as black boxes, limiting both generalization and interpretability. To address these challenges, there is growing interest in incorporating domain-agnostic and interpretable concepts from graph theory—such as subgraph isomorphism, maximum common subgraph, and graph edit distance—into graph similarity learning as training objectives. This survey presents a comprehensive review of recent advancements in deep graph similarity learning, focusing on models that integrate these graph theory concepts. Despite the different training objectives of these approaches, they share significant commonalities in the training pipeline, techniques, and challenges. We analyze them within a unified lens referred to as graph theory-based deep similarity learning (GTDGSL) methods. We systematically compare existing GTDGSL methods alongside their common training pipeline, highlighting the technique trend and discussing key challenges, applications, and future research directions in this domain.

1 Introduction

Entities can be represented as nodes within a graph, with edges depicting their relationships. This graph-based representation effectively captures the complex interactions present in real-world systems, ranging from social networks to biochemical molecule structures. This representation, in turn, facilitates the identification of similar interaction patterns, i.e., structural similarities between graphs. These structural similarities often correlate with functional similarities, highlighting the importance of graph similarity learning in exploring these relationships.

Such similarity measurement serves as a fundamental operation in various downstream tasks, including graph classification (Mohamed et al., 2019), clustering (Liu et al., 2024), and search (Zheng et al., 2014; He & Singh, 2006; Wang et al., 2010; Zhu et al., 2012; Zheng et al., 2013). For instance, in bioinformatics, identifying similar structures can lead to the discovery of new drugs or treatments (Schadt et al., 2009). In social network analysis, graph similarity is crucial for detecting communities, identifying influencers, and understanding the spread of information (Narayan & Kumar, 2016). Similarly, in recommendation systems and fraud detection, graph similarity plays a key role in matching user preferences and identifying anomalous patterns (Kim et al., 2022).

As graph similarity learning has been studied for several decades, many techniques have been proposed, such as kernel-based methods (Borgwardt & Kriegel, 2005; Costa & Grave, 2010; Shervashidze et al., 2011; Kriege, 2022; Nikolentzos et al., 2022) and graph spectral-based methods (ElGhawalby & Hancock, 2008; Crawford et al., 2017). However, these methods often rely on predefined patterns or structures to represent graphs. Recent advances in graph neural networks (GNNs) have empowered models to learn abstract representations that capture the most relevant substructures across diverse graphs according to the downstream

task. This shift reduces dependence on hand-crafted features, enhances the generalizability of the methods, and broadens the design space for model architectures and training paradigms.

Based on deep learning techniques such as graph neural networks (GNNs) (Kipf & Welling, 2017; Xu et al., 2019; Veličković et al., 2018; Brody et al., 2022), deep graph similarity learning (DGSL) methods project graphs into low-dimensional vector spaces, where the distances between graph pairs effectively capture and reflect their structural similarities and differences. More detailed surveys on general DGSL can be found in the work of Ma et al. (2019). However, the distance measurements produced by these models are often task-specific and can be difficult to interpret, complicating the understanding of the models' behaviors. In contrast, classical graph theory provides well-defined and domain-agnostic concepts of graph structural similarity through various problems. This survey focuses on three of them that are widely applied in graph search, which range from the stringent subgraph isomorphism (SI)—which is NP-complete (Garey & Johnson, 1979) and involves determining whether one graph is equivalent to a subgraph of another—to more flexible approaches that are robust to noise in real-world scenarios, such as the maximum common subgraph (MCS) and graph edit distance (GED). While both MCS and GED are NP-hard problems (Hjorth, 2005), they provide useful frameworks for evaluating similarity: MCS identifies the largest subgraph common to both graphs, while GED quantifies the minimum cost required to transform one graph into another through a sequence of edits.

Although these three problems are theoretically significant and form a basis for understanding and quantifying graph similarities, their inherent computational complexity makes them notoriously difficult to solve. Computing these concepts typically requires establishing a bijective mapping between elements, such as nodes and edges, across graph pairs that satisfy certain criteria. For example, in graph edit distance (GED), the total edit cost to transform the source graph into the target graph must be optimized, necessitating exhaustive combinatorial enumeration. Existing efforts have proposed strategies for ordering the search process and pruning unpromising search branches to reduce the search space (Abu-Aisheh et al., 2015; Chang et al., 2020; McCreesh et al., 2017; Carletti et al., 2018). However, these methods operate in an on-the-fly manner, which means each computation relies on a specific graph pair, rendering intermediate results from one search inapplicable to another, impacting the efficiency of these methods.

Recently, several end-to-end learning-based methods (Bai et al., 2019; Ying et al., 2020; Doan et al., 2021) have proposed leveraging distances in embedding spaces to reflect the similarity between graph pairs based on the aforementioned graph theory concepts. In addition, some methods operate in learn-to-search scenarios, enhancing the searching process of conventional algorithms with pretrained deep learning models (Yang & Zou, 2021; Wang et al., 2021; Bai et al., 2021; Wang et al., 2022; He et al., 2022). These approaches not only offer clearer insights into the structural relationships between graphs but also significantly accelerate the computation of these complex graph theory problems. In recent years, various works have followed this trend and further refined existing methods, leading to notable advancements in both accuracy and efficiency (Zhuo & Tan, 2022; Liu et al., 2023b; Roy et al., 2022a). Although these methods are tailored to different problems, this paper analyzes them through a unified lens, systematically presenting their theoretical connections, technical similarities, and differences. Moreover, this unified view enables a more comprehensive discussion of common challenges, such as preserving graph characteristics and ground-truth acquisition problems, etc. Finally, this perspective facilitates the identification of future research directions and broadening the field.

Scope and contributions. Unlike previous surveys on general deep graph similarity learning or deep similarity learning, such as Ma et al. (2019); Yang et al. (2024), which primarily focus on model taxonomies or data-specific discussions and provide limited insights into technical developments and trends, this survey distinguishes itself in two key aspects. First, it specifically focuses on deep graph similarity learning approaches that approximate three selected graph theory concepts for quantifying graph similarity: subgraph isomorphism, maximum common subgraph, and graph edit distance, collectively referred to as Graph Theory-based Deep Graph Similarity Learning (GTDGSL). By examining these approaches under a unified lens, this survey highlights their commonalities and differences in both theoretical foundations and technical implementations. Second, it provides an in-depth review of techniques, emphasizing technical developments and trends that have been underexplored in prior works. To the best of our knowledge, this is the first survey

dedicated specifically to the GTDGSL problem. The main contribution of this survey can be summarized as follows.

- We briefly summarize three graph theory concepts—subgraph isomorphism, maximum common subgraph, and graph edit distance—and analyze their theoretical connections, along with the technical similarities and differences among conventional algorithms for these problems.
- We categorize existing GTDGSL methods alongside their training pipeline, establishing a unified framework for systematic comparison.
- We analyze and review GTDGSL methods at each step of the training pipeline to align their inputs and outputs, highlight technical trends at each step, and elucidate their design space to provide technical insights.
- We provide a detailed discussion of dataset generation, evaluation metrics, and their downstream applications.
- We identify key challenges and opportunities for future research in this domain.

Organization. This paper is organized as follows. In Section 2, we provide the necessary background on the GTDGSL problem, covering key graph theory concepts, traditional techniques, as well as a formal problem formulation of GTDGSL. In Section 3, we categorize and review the current GTDGSL methods approximating SI, MCS, and GED, alongside their common training pipeline, revealing the design space and technique trends of GTDGSL methods. In Section 4, we introduce the dataset generation techniques in existing work. In Section 5, we present the metrics evaluating GTDGSL models. In Section 6, we discuss the applications of GTDGSL methods. Finally, in Section 7, we identify the key challenges shared among GTDGSL methods and highlight the future directions. The survey is briefly concluded in Section 8.

2 Background

In this section, we begin with the notations that will be used throughout this survey. Next, we provide formal definitions for the three selected graph theory concepts—subgraph isomorphism, maximum common subgraph, and graph edit distance—along with a brief overview of their conventional solutions. Following this, we explore the connection between graph neural networks (GNNs) and the computation of these concepts, laying the foundation for graph theory-based deep graph similarity learning.

Notation. This survey focuses on node-labeled undirected graphs. For a given graph $G = (\mathcal{V}_G, \mathcal{E}_G)$, where $\mathcal{V}_G$ represents the set of nodes and $\mathcal{E}_G$ denotes the collection of edges $e(v, u)$ for $v, u \in \mathcal{V}_G$, each node $v \in \mathcal{V}_G$ is associated with a feature vector $\mathbf{x}^0 \in \mathbf{X}$, with $\mathbf{X}$ being the collection of node features. This notation can be easily extended to unlabeled graphs by assigning identical features to each node. Since G is undirected, for every edge $e(v, u) \in \mathcal{E}_G$, there exists a counterpart $e(u, v)$. The cardinalities of the node and edge sets are denoted by $|\mathcal{V}_G|$ and $|\mathcal{E}_G|$, respectively. The sparse edge set $\mathcal{E}_G$ can be represented as a dense adjacency matrix $\mathbf{A}^{|\mathcal{V}_G|\times|\mathcal{V}_G|}$, where $\mathbf{A}(v, u) = 1$ if $e(v, u) \in \mathcal{E}_G$ and $\mathbf{A}(v, u) = 0$ otherwise, with $\mathbf{A}(v, u)$ representing the entry at row v and column u. A connected subgraph $G' = (\mathcal{V}_{G'}, \mathcal{E}_{G'})$ of G consists of subsets of $\mathcal{V}_G$ and $\mathcal{E}_G$ such that for each edge $e(v, u) \in \mathcal{E}_{G'}$, both v and u belong to $\mathcal{V}_{G'}$, and all vertices in G' are reachable from one another by traversing edges within $\mathcal{E}_{G'}$.

2.1 Graph Similarity Concepts in Graph Theory

In this survey, we focus on three fundamental graph theory concepts—subgraph isomorphism (SI), maximum common subgraph (MCS), and graph edit distance (GED)—as the foundation for graph theory-based deep graph similarity learning (GTDGSL). These concepts are closely related, sharing common theoretical foundations, techniques, and applications, while also facing similar challenges. Despite these similarities, each concept offers distinct and valuable insights into the structural relationships between graphs. In the following, we will present formal definitions of these concepts and discuss their conventional solutions, emphasizing both their similarities and differences to deepen the understanding of their interconnectedness.

2.1.1 Subgraph Isomorphism (SI)

Definition. Given a query graph Q and a data graph D, if Q is isomorphic to a subgraph D' of D, there exists at least one bijective function f between Q and D', such that $f : \mathcal{V}_Q \mapsto \mathcal{V}_{D'}$ satisfying (1) for all $v \in \mathcal{V}_Q$ and $v' \in \mathcal{V}_{D'}$, there exists $f(v) = v'$ and $\mathbf{x}_v^0 = \mathbf{x}_{v'}^0$. (2) For all $e(v, u) \in \mathcal{E}_Q$, there exists $e(f(v), f(u)) \in \mathcal{E}_{D'}$. We denote (Q, D) as a subgraph isomorphism. In this context, graph isomorphism is a special case of subgraph isomorphism where $D' = D$. Another related variant of subgraph isomorphism is subgraph edit distance (SED), which depicts the minimum edit cost for transforming a graph into a subgraph of another. The SED for matched pair (Q, D) equals zero; otherwise, it would be a non-negative value.

According to downstream scenarios, conventional solutions for subgraph isomorphism can be divided into two groups: subgraph enumeration (also known as subgraph matching) and subgraph indexing. Subgraph enumeration methods aim to exhaustively enumerate all bijective mappings (isomorphisms) from the query graph to subgraphs within the data graphs (He & Singh, 2008; Shang et al., 2008; Carletti et al., 2018; Archibald et al., 2019; Han et al., 2013; 2019; Sun et al., 2020). State-of-the-art methods in this group typically follow a preprocessing-ordering-enumeration paradigm, they first apply heuristic rules to filter out unpromising vertices and edges based on the graph pairs, then define an optimal search order to guide the enumeration process, which is often executed using a backtracking search, with optimizations to further reduce the search space. For a detailed review of these methods, we refer readers to Sun & Luo (2020); Zhang et al. (2024).

In contrast to these on-the-fly enumeration methods, subgraph indexing methods rely on predefined subgraph patterns (i.e., features) to construct an index for the data graphs (Zhang et al., 2009; Zhao & Han, 2010; Klein et al., 2011; Xie & Yu, 2011; Giugno et al., 2013; He et al., 2024). These methods typically count the occurrences of specific subgraph patterns within each graph and use these counts to build an index, which accelerates subsequent queries. Unlike enumeration methods, which exhaustively search each graph pair during query processing, the constructed vector index in indexing methods can be stored for further related computations. Intuitively, if (Q, D) is a subgraph isomorphism, the index of D on any feature should contain higher or equal counts compared with Q's. Given a query graph and a collection of data graphs, indexing methods aim to filter out unpromising data graphs by comparing the indices of graph pairs. Additionally, once the promising data graphs have been identified, subgraph enumeration algorithms can be applied to further enumerate the matches if necessary.

2.1.2 Maximum Common Subgraph (MCS)

Definition. The graph M is the maximum common connected graph between Q and D, if M satisfying (1) (M, Q) and (M, D) are subgraph isomorphisms, (2) M has the most nodes compared with other common subgraphs exist Q and D.

Conventional Maximum Common Subgraph (MCS) algorithms generally share a similar search process with subgraph enumeration algorithms (McGregor, 1981; Raymond & Willett, 2002; Ehrlich & Rarey, 2010; Ndiaye & Solnon, 2011; McCreesh et al., 2016; 2017). These algorithms first define a search order and, guided by this order, perform a backtracking search to gradually expand the partial matching between the query graph Q and the data graph D until they find the partial mappings with the most nodes. Similar to subgraph enumeration methods, the search order is crucial for effectively solving the MCS problem. Existing MCS algorithms often incorporate heuristic techniques to prioritize the most promising nodes for exploration, thereby reducing the overall computational effort. Additionally, state-of-the-art algorithms address the MCS problem using a branch and bound optimization approach (McCreesh et al., 2017). This methodology systematically explores the search space by dividing it into smaller subproblems, allowing for effective pruning of unpromising branches based on a bounding function. The bounding function estimates the maximum possible size of the common subgraph that can be formed from the current partial mappings, enabling the algorithm to discard branches that cannot lead to an optimal solution.

2.1.3 Graph Edit Distance (GED)

Definition. The graph edit distance $\text{GED}(Q, D)$ is the minimum cost of the edit operation sequence that transforms Q into a graph isomorphic to D. The viable edit operations are node/edge insertion, deletion, and label substitution. Each operation can be assigned with a distinct non-negative cost, and different cost settings can produce different GEDs, making GEDs a flexible tool for various downstream tasks. Especially, GED is a metric under a uniform cost setting, thus is symmetric and exists $\text{GED}(Q, D) = \text{GED}(D, Q)$ (note that they may still have different edit operation sequences), otherwise, it is a measure, $\text{GED}(Q, D)$ can be different from $\text{GED}(D, Q)$.

Conventionally, the exact GED problem is formulated as a path-finding problem, while the approximate GED problem is regarded as a quadratic assignment problem or bipartite graph matching problem (Bunke, 1997; Zeng et al., 2009; Riesen & Bunke, 2009; Riesen et al., 2013; Abu-Aisheh et al., 2015; Bougleux et al., 2017; Stauffer et al., 2017; Blumenthal & Gamper, 2020; Chang et al., 2020). These formulations typically involve creating a cost matrix that quantifies the differences between each cross-graph node (or edge) pair. The matrix reflects the costs of various edit operations (insertions, deletions, substitutions), with each operation assigned a customizable edit cost to enable flexible usage of GED for diverse downstream tasks. Based on the cost matrix, the exact ones gradually expand the search path till each node in Q is matched to a node in D or a dummy node, while the approximate ones directly construct node mapping from Q to D to minimize the total cost. Similar to the above concepts, a well-defined search order is crucial for exact GED algorithms to identify optimal or near-optimal edit paths. Numerous heuristic techniques have been proposed to determine this search order, such as prioritizing nodes with higher degrees or selecting node pairs with lower costs.

2.1.4 Commonalities and Differences Between Graph Similarity Concepts

All three problems—Subgraph Isomorphism (SI), Maximum Common Subgraph (MCS), and Graph Edit Distance (GED)—are NP problems and can be regarded as attempts to establish (partial) node mappings across pairs of graphs. SI imposes strict constraints on the typologies and properties of these graph pairs, requiring an exact structural match between the query graph and a subgraph in the data graph. This rigidity can pose significant challenges, especially in the context of real-world graph datasets that are often noisy and may contain irregularities, incomplete information, or variations in node and edge attributes.

In cases where a query graph does not exist in a data graph, MCS and GED emerge as valuable tools for quantifying similarities between graphs. MCS identifies the largest common substructure shared by two graphs, providing a way to compare graphs even when they do not perfectly align. This approach allows for flexibility in matching, accommodating variations in the structure while still capturing the essence of the graphs being compared. Similarly, GED quantifies the dissimilarity between graphs by measuring the minimum cost of operations required to transform one graph into another. This distance enables a more sophisticated comparison, considering both similarities and differences.

Despite all three concepts—SI, MCS, and GED—sharing similar underlying processes such as heuristic filtering, pruning, bounding, and backtracking search, they exhibit distinct computational characteristics. Specifically, GED algorithms prune unpromising branches dynamically based on transformation costs exceeding a bound established during the search, enabling them to adjust strategies according to estimated edit costs. In contrast, SI and MCS algorithms typically perform a preprocessing step to narrow the candidate space of query nodes by eliminating infeasible matches. During the search, they adjust exploration based on the structural compatibility of potential mappings, reducing the search space and improving efficiency.

Another notable distinction lies in the handling of editing costs. Since different node pair matches can incur varying costs in GED, this method can leverage those costs to establish search orders. This cost-sensitive approach facilitates the identification of the most promising paths for exploration. In comparison, such an ordering mechanism is relatively rare in SI and MCS algorithms, which typically rely on graph properties like vertex degrees, label frequency, or connectivity rather than explicit edit costs.

In summary, while SI, MCS, and GED problems share similarities in their objective and methodologies, they are different in constraints, preprocessing strategies, search dynamics, and cost handling. SI's strict structural requirements can hinder performance in noisy datasets, whereas MCS and GED provide more flexible

frameworks for similarity assessment. GED dynamically prunes branches based on estimated transformation costs, while SI and MCS improve efficiency by filtering out unpromising nodes before the search and pruning branches during the search process. Furthermore, GED's ability to leverage varying edit costs for search ordering distinguishes it from SI and MCS, with the latter primarily relying on graph properties. These differences give each method a unique approach to graph similarity, making them well-suited for various applications with different structural constraints.

2.2 Graph Theory-Based Deep Graph Similarity Learning (GTDGSL)

Deep graph similarity learning, empowered by deep learning techniques, aims to construct a similarity function that assigns meaningful similarity scores to graph pairs. Graph Neural Networks (GNNs) naturally emerge as a key tool in this context, as they effectively generate node, edge, and graph representations while considering the connectivity patterns of graphs.

2.2.1 Graph Neural Networks

Message-passing Graph Neural Networks (MPNNs), commonly referred to as GNNs, generate node embeddings for each graph using an aggregation-update paradigm. For a node $v \in \mathcal{V}$ with an initial feature vector $\mathbf{x}_v^0$, the process involves aggregating features from its neighboring nodes according to the adjacency matrix of the graph, and then updating the feature of v, this process at l-th layer can be described as follows:

$$\mathbf{x}_v^l = \text{Update}(\mathbf{x}_v^{l-1}, \text{Aggr}(\mathbf{x}_u^{l-1} : u \in \mathcal{N}_v)),$$

where $\mathbf{x}_v^l$ is the embedding of node v at the l-th layer, and $\mathcal{N}_v$ denotes the set of neighboring nodes of v. The function $\text{Aggr}(\cdot)$ aggregates the features of the neighboring nodes u, and the $\text{Update}(\cdot,\cdot)$ function updates the feature of the node v based on its previous feature and the aggregated information. The outputs of a k-layer GNNs for each node can be further summarized as the graph representation with a pooling function, such as a dimensional-wise sum pooling. We refer to Wu et al. (2021) for more details on GNNs.

2.2.2 Problem Formulation

Given any graph pair (G_i, G_j), $D(\cdot,\cdot)$ is a learnable pairwise similarity function, such that $D(\phi(G_i), \phi(G_j)) \mapsto d_{ij} \in \mathbb{R}$, where ϕ is a projection function, such as GNNs, that transforms graphs into low-dimensional embeddings. Unlike general deep graph similarity learning, GTDGSL requires d_{ij} to approximate specific graph theory targets, such as those defined above.

In the GTDGSL framework, subgraph isomorphism prediction at the graph level can be framed as a binary classification task, where the model outputs 1 if (Q, D) is a subgraph isomorphism and 0 otherwise. Certain methods also propose to predict such subgraph isomorphism at the node level, they extract the k-hop subgraph $\mathcal{N}^k$ induced by a given central node, predicting whether $(\mathcal{N}^k(v), \mathcal{N}^k(u))$ is a subgraph isomorphism, where $v \in Q$ and $u \in D$. Compared with conventional subgraph isomorphism algorithms, the prediction of subgraph isomorphism is more akin to subgraph indexing methods rather than enumeration algorithms. Subgraph indexing involves counting predefined subgraph patterns and using these counts as indices. Since this process requires enumerating smaller subgraphs in both the query and data graphs, it is NP-complete. In contrast, GTDGSL methods bypass this computationally expensive process by learning to capture abstract graph features, which improves generalization and enhances efficiency, though at the cost of interpretability.

The predictions of MCS and GED are often formulated as a regression task. For MCS problem, the training objective of a GTDGSL model is the number of nodes within M (ranging from 0 to infinity) or the normalized value $\frac{|\mathcal{V}_M|}{(|\mathcal{V}_Q|+|\mathcal{V}_D|)/2}$ (ranging from $[0, 1]$). Similarly, for GED problem, the target value is the non-negative GED value or the normalized GED value $e^{-\frac{GED(Q,D)}{(|\mathcal{V}_Q|+|\mathcal{V}_D|)/2}}$ to ensure a range of $(0, 1]$.

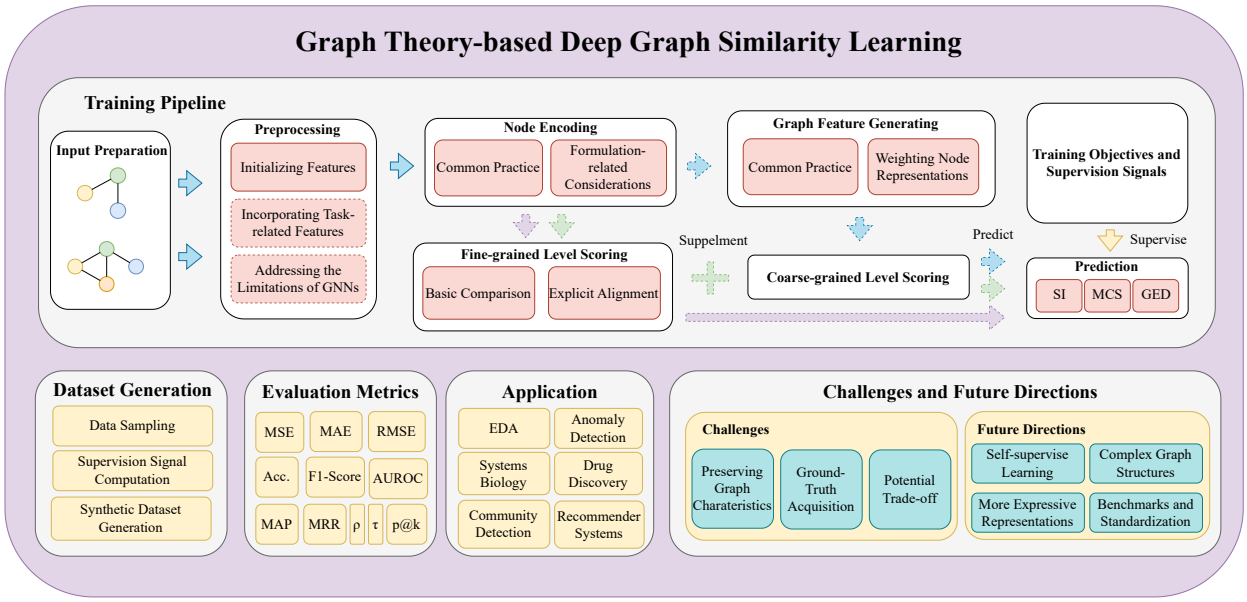

Figure 1: The visualization illustrates the organization of the survey and the training pipeline of GTDGSL methods. In the training pipeline, solid blocks or arrows represent indispensable steps or operations, while dashed ones indicate optional steps. For purple, green, or blue dashed arrows, one option must be selected based on their corresponding color.

3 Design Space of GTDGSL Methods

3.1 Overview

In this survey, we explore the design space of GTDGSL methods alongside the training pipeline, enabling alignment between model inputs and outputs and offering a clearer understanding of their operational flow. Given an *input graph pair*, GTDGSL methods typically employ deep learning techniques, such as graph neural networks, to *extract neighborhood information* for each node. This step is crucial, as it provides the basic information required for subsequent similarity computations between graph pairs. Since similarity is inherently a pairwise relationship, incorporating interactions between graphs within each pair is a natural approach to *computing similarity scores*, whether at a fine-grained or/and coarse-grained level. Building on this, the training pipeline for GTDGSL methods typically encompasses several key steps: input preparation, preprocessing, node encoding, fine-grained level scoring, graph feature generation, coarse-grained level scoring, and defining training objectives and supervision signals, as illustrated in Figure 1. Each of these steps offers opportunities for divergence in model design, contributing to the expansive and diverse design space of GTDGSL methods.

In the following sections, we closely examine each step of the training pipeline shared by many GTDGSL models. We discuss common practices at each step and highlight specific optimizations introduced by various models. This structure not only highlights the evolution of GTDGSL methods but also establishes a unified framework for comparison, enabling a more systematic evaluation of their design choices and identifying key trends in the field. The categorization alongside the training pipeline, including their targeted problems and model outcomes, is summarized in Table 1.

3.2 Input Preparation

Common Practice. GTDGSL methods generally take the entire original graph pair as input and treat similarity computation as a regression/classification task. By circumventing the combinatorial search process, they enable end-to-end similarity computation, expediting the process significantly. These models aim to

Table 1: Categorization of methods alongside the training pipeline.

Methods	Targeted Problem			Input Graphs		Scoring Level		Model Outcomes	
	SI	MCS	GED	Original Graphs	Subgraphs	Fine-grained	Coarse-grained	Similarity	Mapping
SimGNN (Bai et al., 2019)			✓	✓		✓	✓	✓	
GMN-emb (Li et al., 2019)			✓	✓			✓	✓	
GMN-match (Li et al., 2019)			✓	✓		✓	✓	✓	
NeuroMatch (Ying et al., 2020)	✓			✓			✓	✓	
GraphSim (Bai et al., 2020)			✓	✓		✓		✓	
Noah (Yang & Zou, 2021)			✓		✓	✓		✓	✓
GOTSim (Doan et al., 2021)		✓	✓	✓		✓		✓	
TAGSim (Bai & Zhao, 2021)			✓	✓			✓	✓	
GENNA* (Wang et al., 2021)			✓		✓	✓		✓	✓
GLSearch (Bai et al., 2021)		✓			✓	✓			✓
H2MN (Zhang et al., 2021)			✓	✓		✓		✓	
EGSC (Qin et al., 2021)			✓	✓			✓	✓	
IsoNet (Roy et al., 2022b)	✓			✓		✓		✓	
RLQVO (Wang et al., 2022)	✓				✓	✓			✓
Eric (Zhuo & Tan, 2022)			✓	✓		✓	✓	✓	
MCSNet (Roy et al., 2022a)		✓		✓		✓		✓	
Prune4Sed (Liu et al., 2022)	✓			✓		✓		✓	
FAST (He et al., 2022)	✓				✓		✓	✓	✓
Greed (Ranjan et al., 2022)	✓			✓			✓	✓	
D2Match (Liu et al., 2023b)	✓			✓		✓	✓	✓	
GEDGNN (Piao et al., 2023)			✓	✓		✓		✓	✓
MATA* (Liu et al., 2023a)			✓	✓		✓		✓	✓
GED-CDA (Jia et al., 2023)			✓	✓		✓	✓	✓	
AEDNet (Lan et al., 2023)	✓			✓		✓		✓	

directly predict similarity scores for each graph pair, making them particularly advantageous in scenarios involving a large number of graphs, where fast inference and approximate similarity scores are prioritized over detailed node mappings.

Operate in Learn-to-search Scenarios. Certain methods treat the regression/classification task as a subcomponent, using pretrained models to evaluate partial solutions and guide the search process of conventional algorithms (Yang & Zou, 2021; Bai et al., 2021; He et al., 2022). These models process subgraphs from the original graph pair to estimate costs or scores for unprocessed parts. While subgraph inputs introduce variations in the inference framework, the training pipeline remains unchanged, as the pretrained models are designed to assess similarities for both processed (i.e., subgraphs within partial solutions) and unprocessed graph parts. This survey focuses on their evaluation models, with their training objectives and supervision signals detailed in Section 3.8. For more information on the overall frameworks of learn-to-search-oriented methods, we refer readers to Yang et al. (2024).

In the following, we intentionally distinguish between the terms *substructure* and *subgraph* to avoid ambiguity. The former term refers to local structures within the input graphs, while the latter represents instances in learn-to-search scenarios where the input graphs are derived subgraphs from original graph pairs. We refer to both the original graph pair and the subgraphs derived from them as *input graphs* for simplicity.

3.3 Preprocessing

In existing GTDGSL methods, the primary goal of the preprocessing step is to incorporate the necessary information for the subsequent node encoding step. This involves initializing features, incorporating task-related features, and addressing the limitations of GNN.

Initializing Features. Before feeding input graphs into GNNs, existing GTDGSL methods generally initialize node features with either node labels (Bai et al., 2019; Li et al., 2019; Ying et al., 2020; Yang & Zou, 2021; Doan et al., 2021; Wang et al., 2021; Qin et al., 2021; Zhuo & Tan, 2022; Liu et al., 2022; He et al., 2022; Ranjan et al., 2022; Liu et al., 2023b; Piao et al., 2023) or node degree information (Wang et al., 2021; Liu et al., 2023b;a). Similarly, edge features can be initialized with edge labels (Li et al., 2019). Notably, IsoNet (Roy et al., 2022b) and MCSNet (Roy et al., 2022a), which specifically tackle feature-agnostic

subgraph matching and MCS problems, initialize all node features to an identical value, ignoring the original attributes of the nodes.

Incorporating Task-related Features. Following conventional algorithms, some GTDGSL models incorporate task-specific knowledge through heuristics as additional node features to enhance the understanding of the problem. One notable approach is RL-QVO (Wang et al., 2022), which is designed to optimize the search order for conventional subgraph matching algorithms. RL-QVO incorporates scaled node degrees, integer labels, and query node IDs to initialize query node features, facilitating the differentiation of the input node order for the graph neural network. Furthermore, it also introduces two precomputed heuristics: (1) the frequency of vertices in the data graph D with a higher degree than the query vertex u, and (2) the frequency of vertices in D sharing the same label as u. These heuristics provide an estimation of the matching difficulty for each node in the query graph, helping to anticipate the potential solution space that will emerge as the matching progresses. Similarly, GLSearch (Bai et al., 2021), which operates under a branch-and-bound search framework to address the Maximum Common Subgraph (MCS) problem, leverages heuristics such as local degree profiles.

Addressing the Limitations of GNNs. GNNs are effective at generating graph representations, making them powerful tools for a range of graph-based tasks. However, they face limitations when dealing with complex graph structures.

- *Limited Expressiveness*: GNNs based on the message-passing mechanism focus on the local structures of nodes, producing similar representations for nodes with identical local structures. Consequently, GNNs struggle to distinguish between nodes, such as in cycle or regular graphs, where each node has the same local structure. To address this limitation, D2Match (Liu et al., 2023b) tackles the subgraph matching problem by transforming it into a subtree matching problem. It enhances performance by replacing chordless cycles shorter than length l in graph pairs with supernodes, thereby avoiding ambiguities in these structures. To increase GNN expressiveness on regular graphs, NeuroMatch (Ying et al., 2020) builds on the concept introduced in (You et al., 2021). It incorporates identity information indicating whether a node is an anchor node into the features of each node-anchored subgraph, making them identity-aware and better equipped to distinguish nodes.

- *Loss of Overall Structure*: GNNs generally treat graphs as unordered sets of nodes to ensure permutation invariance, but this approach can overlook global structural information, which is crucial for targeted problems. To capture such information, MATA* (Liu et al., 2023a) perturbs graphs by randomly adding or removing edges, calculating random-walk probabilities for each node in both the original and perturbed graphs, and using these probabilities as relative position encodings. GED-CDA (Jia et al., 2023) generates spectral encodings by calculating the eigenvalues and eigenvectors, which embed the global structural properties of the graph. H2MN (Zhang et al., 2021) proposes transforming input graphs into hypergraphs to capture richer substructure information, where each hyperedge can connect multiple nodes. Specifically, H2MN performs random-walk or k-hop subgraph extractions around designated center nodes, viewing each extracted subgraph as a hyperedge that connects all nodes within that subgraph.

- *Lack of Scalability*: Operating GNNs on large-scale graphs requires substantial memory to store the entire adjacency matrix and node embeddings, and the burden increases with graph size and model depth. To mitigate this issue, NeuroMatch and Greed (Ranjan et al., 2022) partition large input graphs into smaller, overlapping, node-anchored substructures. By breaking down the SI problem into smaller subproblems, each substructure can be processed independently, enabling scalable parallel computations and reducing the memory and resource load. This partitioning approach helps GNNs handle large graphs more efficiently, as the localized subgraphs preserve important structural details while reducing computational demands.

3.4 Node Encoding

Given the input graph pairs and their features, the node encoding step involves extracting the neighboring information for each node using deep learning techniques such as GNNs.

Common Practice. Popular GNNs such as Graph Convolutional Network (GCN) (Bai et al., 2019; 2020; Doan et al., 2021), Graph Isomorphism Network (GIN) (Ying et al., 2020; Zhuo & Tan, 2022; Yang & Zou, 2021), Graph Attention Network (Bai et al., 2021; Liu et al., 2022; Ye et al., 2024) are commonly applied as the backbone of models. The choice of backbone in GNNs depends on how each method considers the contributions of neighboring nodes to the central nodes' representation. GCN assumes that a node's influence on its neighbors should be weighted by their degrees. It aggregates and normalizes neighborhood information using the degrees of both the central node and its neighbors, updating the central node's representation by averaging the normalized features. In contrast, GAT dynamically adjusts the contributions of neighboring nodes, assigning weights based on the similarity between the central node and its neighbors. GIN treats all neighboring nodes equally, considering them as elements of a multiset and summing their information to update the central node's representation. Another well-known GNN model, GraphSAGE, employs a sampling-based approach to aggregate information from neighbors. While this method enhances scalability, GraphSAGE is less frequently used as a backbone in GTDGSL methods. Additionally, the choice of backbone also depends on the data source; for instance, GENN-A* (Wang et al., 2021) utilizes SplineCNN (Fey et al., 2018) for graphs derived from 2D images. To address the potential over-smoothing issue in GNNs—where, as the depth of aggregation increases, all nodes within a graph can share a similar receptive field, thus their node representations become indistinguishable—methods, NeuroMatch (Ying et al., 2020) and Greed (Ranjan et al., 2022) introduce skip layers, which allow information from earlier layers to be directly passed to later layers and concatenate the outputs from different layers as output, to mitigate this effect.

Formulation-related Considerations. The choice of backbones is also related to problem formulation. To mimic the edit operations such as node/edge substitution, insertion, and deletion. TaGSim (Bai & Zhao, 2021) proposes generating type-aware graph embedding. To this end, it solely leverages the message-passing mechanism of GNN to aggregate k-hop node/edge-label multiset. Since H2MN (Zhang et al., 2021) transforms input graphs into hypergraphs, it encodes them with Hypergraph Convolutional Networks (HGCN), which aggregate information based on the incidence matrix of hypergraphs to generate node representations. It further devises a hyperedge pooling operation according to the Personalized PageRank (PPR) values to keep the top-ranked hyperedges based on their importance. To consider edge features, GMN (Li et al., 2019) follows the practice in Li et al. (2015) and proposes concatenating the edge information and the node information at the ends of the edge to update the nodes, which can be expressed as follows.

$$\mathbf{m}_{ji} = \mathrm{MLP}(\mathbf{x}_i^l, \mathbf{x}_j^l, \mathbf{e}_{ij})$$
$$\mathbf{x}_i^{l+1} = \mathrm{RNN}(\mathbf{x}_i^l, \sum_{j \in \mathcal{N}(i)} \mathbf{m}_{ji})$$

Where MLP is a Multilayer Perceptron, and RNN is a recurrent neural network, and it can be replaced by its variants such as GRU and LSTM. Such a backbone is further adopted in (Roy et al., 2022b;a).

3.5 Fine-grained Level Scoring

The graph similarity of input graphs can be evaluated at fine- or coarse-grained levels. Coarse-grained scoring assesses similarity by comparing embeddings representing entire input graphs, which will be discussed later. In contrast, fine-grained scoring focuses on capturing detailed structural similarities and differences at the level of nodes, edges, or substructures, and can be performed either during or after the node encoding step, with the resulting scores either supplementing or replacing the coarse-grained similarity score.

Since graph structures can differ widely in node and edge counts, these methods need to handle structural irregularities. To address this and leverage GPU efficiency, they often pad the smaller graph in each pair with dummy elements. This padding ensures both graphs are represented as tensors of equal size, enabling

streamlined batch processing on GPUs. The dummy elements, set to neutral values like zeros, are designed to avoid influencing similarity calculations.

Based on how they use produced scores, fine-grained scoring methods can be further categorized as follows:

- *Basic Comparisons:* These approaches rely on direct comparisons to capture the overall similarity distribution. This distribution can supplement or replace graph-level representations to predict similarity scores, as demonstrated in SimGNN (Bai et al., 2019) and GraphSim (Bai et al., 2020).
- *Explicit Alignment:* These methods establish one-to-one correspondences between nodes or edges across graphs and calculate similarity based on these mappings to minimize the overall transport cost, as exemplified by GOTSim (Doan et al., 2021).

In the following sections, we examine the fine-grained scoring techniques, highlighting their commonalities and differences to reveal emerging trends in the field.

3.5.1 Basic Comparison-based Models

The most representative basic comparison-based models are SimGNN (Bai et al., 2019) and GraphSim (Bai et al., 2020). Both models compare cross-graph node pairs after all or one step of node encoding and then use the pairwise inner product as similarity scores between node pairs as follows $sim(\mathbf{X}_1, \mathbf{X}_2) = \sigma(\mathbf{X}_1\mathbf{X}_2^\top)$. To address the absence of a natural ordering among nodes within graphs, SimGNN computes a permutation-invariant but non-differentiable histogram of pairwise similarity scores, represented as $\text{hist}(s) \in \mathbb{R}^B$, where B is the number of bins. This histogram serves as a supplementary feature for graph-level scores, offering insights into the scale and overall similarity distribution of the graphs. In contrast, GraphSim permutes the similarity matrix using a breadth-first search (BFS) node-ordering scheme, ensuring that nearby nodes are placed close together to capture their connections. Additionally, GraphSim treats the similarity matrices from each layer as images and employs convolutional neural networks (CNNs) to extract information from each matrix, capturing the local similarity distribution among graph pairs.

Unlike the two methods mentioned above, which compare representations after one step of node encoding, GMN-match (Li et al., 2019) scores local similarities during the node encoding process. It updates node representations in one graph based on similarity-weighted influences from nodes in the other graph. Specifically, GMN-match alternates between updating node representations within one graph and incorporating information from its counterpart. For a node i in G_1, the model first updates its representation based on G_1's local structure using the adjacency matrix $\mathbf{A}_{G_1}$. It then calculates the similarity between node i and all nodes i' in G_2. These similarities are used to compute attention weights $\alpha_{ii'}$, which indicate the influence of each node in G_2 on the updated representation of node i. The final representation of node i is obtained by subtracting the weighted sum of representations from nodes in G_2, thereby integrating cross-graph information into node i's feature vector. This process is formally expressed as follows.

$$\alpha_{ii'} = \frac{\exp\left(sim(\mathbf{x}_i^l, \mathbf{x}_{i'}^l)\right)}{\sum_{i' \in G_2} \exp\left(sim(\mathbf{x}_i^l, \mathbf{x}_{i'}^l)\right)}$$
$$\mathbf{x}_i^{l+1} = \mathbf{x}_i^l - \sum_{i' \in G_2} \alpha_{ii'} \mathbf{x}_{i'}^l$$

This approach combines both cross-graph node similarity and local graph structure, and has significantly influenced subsequent methods.

H2MN (Zhang et al., 2021) extends this practice to hypergraphs. Specifically, for each hyperedge e_i in a graph, it first measures its cosine similarity score with all hyperedges $e_{i'}$ in the other graph to compute cross-graph attention coefficients $\alpha_{i,i'}$ then aggregates the relevant information in the other graph based on such coefficients to compute contextual hyperedge representations. Furthermore, it compares the original hyperedge representation $\mathbf{e}_i$ with its contextual one $\tilde{\mathbf{e}}_i$ as follows $\mathbf{m}_i = \text{cosine}(\mathbf{e}_i \odot \mathbf{W}, \tilde{\mathbf{e}}_i \odot \mathbf{W})$, to compute matching vectors, which further serves as inputs for the next layers. It then concatenates the readout outputs of each graph from each layer to form the final matching representations and then predicts scores with an

MLP. Prune4Sed (Liu et al., 2022) learns representations of the data graph conditioned on the query graph, generating data node embeddings that capture their potential relevance to the query. The model iteratively calculates a keep probability for each data node, pruning nodes based on these probabilities. After pruning, it computes the distance between the pruned data graph and the query graph to estimate the subgraph edit distance (SED).

Although node-, edge-, and substructure-level interactions capture fine-grained similarity between graph pairs, they have at least quadratic complexity. Eric (Zhuo & Tan, 2022) introduces an Alignment Regularization (AReg), similar to (Hassani & Khasahmadi, 2020), to reduce this complexity and avoid the need for explicit node-to-node matching. During training, AReg implicitly aligns nodes by maximizing the mutual information between node representations and the graph representations of both their own and another graph in an unsupervised manner. During inference, the learned graph-level representations are directly used to compute similarity scores, bypassing AReg to reduce the inference time.

3.5.2 Explicit Alignment-based Methods

To improve the interpretability of the similarity computation process, GOTSim (Doan et al., 2021) first formulates GED computation as a graph optimal transport problem whose goal is to minimize the transport cost from one graph to another. To do so, it first proposes to explicitly establish the node correspondences between graph pairs with a linear assignment solver. Considering the characteristics of GED computation, it further proposes augmenting the cost matrix to reflect the costs of the node-deletion and the node-insertion operations, which accordingly increase the complexity of pair-wise node comparison to $\mathcal{O}(\max(|\mathcal{V}_{G_1}|, |\mathcal{V}_{G_2}|)^{2.5})$. The final graph similarity score is determined by the minimum transport cost.

IsoNet (Roy et al., 2022b) further proposes to align edges within two graphs to predict SI. Different from GED computation, SI is a partial order relationship (Ying et al., 2020), thus an edge e in a query graph matches another edge e' in a data graph, meaning the subgraph induced by e is contained by the counterpart of e'. To reflect such containment constraint, the distance of two edge representations is computed as $D(e_{ij}, e_{i'j'}) = \max(0, \mathbf{e}_{ij} - \mathbf{e}_{i'j'})$, following (Ying et al., 2020). It further applied the Gumbel Sinkhorn Network to solve the optimal assignment problem, which entailed a complexity of $\mathcal{O}(k \cdot \max(|\mathcal{V}_{G_1}|, |\mathcal{V}_{G_2}|)^2)$.

Extending the practice in IsoNet, MCSNet (Roy et al., 2022a) further proposes late and early interaction variants to tackle the MCS computation problem. Using the Gumbel Sinkhorn Network, the former first computes the node embeddings of graph pairs, the latter aligns nodes during the node embedding computation step following GMN-match (Li et al., 2019). It further proposes a gossip protocol to iteratively find the largest connected component in a graph. To generate edit paths, GEDGNN (Piao et al., 2023) and MATA* (Liu et al., 2023a) adopt a similar strategy, they both propose to train a model that computes the edit paths and GED value. To this end, GEDGNN computes a matching matrix and a cost matrix of node pairs and then predicts GED based on the results of two matrices. Similarly, MATA* proposes to generate a similarity matrix of node pairs but predicts the GED based on the representations of graph pairs. Furthermore, given the k-best matching of the matching matrix, They both propose to compute the edit paths using conventional GED algorithms.

Orient toward SI, AEDNet (Lan et al., 2023) addresses subgraph matching by adaptively removing unnecessary edges from the data graph to better align it with the query. It follows GMN-Match's alternative propagation strategy with a unidirectional cross-propagation mechanism, which transfers information from data to query nodes to align representations and approximate the ground truth matching matrix. Additionally, the model introduces a sample-wise adaptive mechanism that generates a query-specific vector, assigning minimal or zero weights to irrelevant edges. Finally, AEDNet predicts and supervises the likelihood of each node and edge in the data graph matching the query. D2Match (Liu et al., 2023b) computes an indicator matrix for graph pairs, where each entry reflects whether the subtrees rooted at nodes from the query and target graphs are subgraph isomorphic, determining SI by finding a perfect matching on a bipartite graph composed of query-data nodes.

3.6 Graph Feature Generation

In this step, GTDGSL methods summarize the graph representation of graph pairs using node representations, preparing for the coarse-grained level scoring. This step is not necessary for methods that solely rely on fine-grained scores.

Common Practice. Existing GTDGSL methods typically generate graph-level representations using pooling techniques such as Sum (Ying et al., 2020; Bai & Zhao, 2021; Zhuo & Tan, 2022; He et al., 2022; Ranjan et al., 2022) and Max (Bai et al., 2021) pooling. Sum pooling aggregates node embeddings by performing an element-wise summation, while Max pooling selects the maximum value for each dimension across all node embeddings. These methods operate under the assumption that all nodes within the graph contribute equally to the final graph representation, without assigning different levels of importance or weights to individual nodes.

Weighting Node Representations. Certain methods suggest that nodes can contribute differently to the graph representation. To consider this aspect, some methods proposed weighting node representations based on the node distribution of the own graph (Bai et al., 2019; Li et al., 2019). To consider the contribution of each node within a graph, SimGNN (Bai et al., 2019) proposed an attentional graph representation generation based on a global context, i.e., the average of weighted node embeddings, which can be written as follows.

$$\mathbf{c} = \tanh(\frac{1}{|\mathcal{V}|}\mathbf{W}\sum_{i=1}^{|\mathcal{V}|}\mathbf{x}_i)$$

Where $\mathbf{W} \in \mathbb{R}^{d\times d}$ is a learnable weight matrix, $\tanh(\cdot)$ is a non-linear function. Then, it considers the inner product between each node and the global context $\mathbf{c}$ to ensure greater attention for nodes similar to c, which can be written as follows.

$$\mathbf{G} = \sum_{i=1}^{|\mathcal{V}|}\sigma(\mathbf{x}_i^\top \mathbf{c})$$

This practice captures the compactness of node representations within graphs and focuses on nodes that are more similar to the global context. This approach is further followed by Wang et al. (2021); Zhang et al. (2021); Qin et al. (2021); Jia et al. (2023).

Similarly, GMN-emb (Li et al., 2019), the graph embedding variant of GMN-match, computes the graph representation with a gate-weighted sum of node representation:

$$\mathbf{G} = \mathrm{MLP}\left(\sum_{i\in G}\sigma(\mathrm{MLP}_{gate}(\mathbf{x}_i^k)) \odot \mathrm{MLP}(\mathbf{x}_i^k)\right)$$

To handle potentially unconnected graphs, Noah (Yang & Zou, 2021) introduces a hypernode that connects all nodes for each subgraph and generates graph-level embedding through a weighted sum of node-level embeddings based on the embedding of this hypernode as follows, considering the similarity between individual node with the hypernode.

$$\mathbf{G} = \sum_{i\in\mathcal{V}_G}\sigma(\mathrm{sim}(\mathbf{x}_i^\top, \mathbf{x}_{hyper})) \cdot \mathbf{x}_i$$

3.7 Coarse-grained Level Scoring

In this step, GTDGSL methods compute similarity scores by comparing representations of graph pairs. In GED and MCS computations, the scores are often calculated using either the Euclidean distance between graph-level representations (Li et al., 2019) or fully connected predictors (Bai et al., 2019; Zhang et al., 2021). Another widely used approach for scoring is the Neural Tensor Network (NTN) (Socher et al., 2013), applied in methods such as (Bai et al., 2019; Wang et al., 2021; Zhuo & Tan, 2022; Bai & Zhao, 2021; Yang & Zou, 2021; Jia et al., 2023). The NTN leverages a bilinear tensor layer to model complex interactions

between graph embeddings, producing similarity scores that capture the joint influence of each embedding in the pair. Mathematically, the NTN takes two input vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ and models their relationship through a bilinear tensor product:

$$sim(\mathbf{G}_1, \mathbf{G}_2) = \mathbf{G}_1^T \mathbf{W}[\mathbf{G}_2] + \mathbf{V}\left(\mathbf{G}_1 \oplus \mathbf{G}_2\right) + \mathbf{b}$$

Here, $\mathbf{W}$ is a tensor that captures the pairwise interactions in different spaces, while $\mathbf{V}$ and $\mathbf{b}$ capture linear combinations and biases. Most approaches use NTN to directly compute the similarity scores of graph pairs. In contrast, TaGSim (Bai & Zhao, 2021) considers the impact of different operations on graph structures, generating operation-specific graph representations based on node or edge representations from multiple layers. It then predicts the cost of each operation type using NTN. Supplementing this, SimGNN combines these scores with histogram features derived from pairwise node comparisons, while Eric (Zhuo & Tan, 2022) introduces a multi-scale GED discriminator that leverages NTN for interaction scoring and integrates it with the Euclidean distance between graph representations at each layer to predict.

EGSC (Qin et al., 2021) devises an attentional embedding fusion process at each layer to capture joint embeddings.

$$\mathbf{h}_{ij} = \text{CONCAT}(\mathbf{h}_i, \mathbf{h}_j)$$
$$\mathbf{h}_{ij}^* = \text{MLP}(\sigma(\mathbf{W}_U \text{ReLU}(\mathbf{W}_D \mathbf{h}_{ij})) \cdot \mathbf{h}_{ij} + \mathbf{h}_{ij})$$

The joint embeddings $\mathbf{h}_{ij}^*$ from each layer are then concatenated across layers and passed through an MLP, producing a single fused embedding to predict. To enable offline storage, EGSC introduces a method to decompose the joint embeddings into individual embeddings. This is achieved by training a student model. Specifically, the teacher model first generates self-embeddings by having each graph interact with itself, then computes pseudo-individual embeddings by subtracting these self-embeddings from the joint embeddings. The student model is trained to approximate these pseudo-individual embeddings, learning to efficiently replicate the decomposed embeddings for storage.

Unlike GED and MCS, the binary SI relationship is challenging to represent with Euclidean distance due to potential size differences between query and data graphs. To solve this issue, NeuroMatch (Ying et al., 2020) models this relationship as a partial order, predicting whether a query graph is contained by a data graph within the embedding space. This model introduces an order embedding constraint (Vendrov et al., 2016), which enforces that each dimension of the query graph's representation does not exceed the corresponding dimension in the supergraph. Violations of this dimensional ordering indicate a partial violation of the containment constraint. Greed (Ranjan et al., 2022) further extends this approach to predict both subgraph edit distance (SED). The extent of constraint violation can be computed as follows.

$$dist(\mathbf{G}_1, \mathbf{G}_2) = \sum_{i \in d} ||\max(0, \mathbf{G}_2 - \mathbf{G}_1)_i|| \tag{1}$$

Greed uses this score to approximate SED, while NeuroMatch feeds the violation score into a linear layer to predict SI. D2Match employs an NTN to compute the graph-level similarity score and combines it with an indicator matrix, which represents the subtree matching of nodes, to make predictions.

3.8 Training Objectives and Supervision Signals

Given the scores computed at fine-grained and/or coarse-grained levels, the training objective of GTDGSL methods typically focuses on minimizing the divergence between predicted scores and ground-truth labels. However, the way divergences are assessed and the choice of ground-truth labels, i.e., supervision signals, can vary across models.

Methods for end-to-end GED and MCS predictions, such as SimGNN, and MCSNet (Bai et al., 2019; Roy et al., 2022a), typically use Mean Squared Error (MSE) loss to quantify the error between predicted and ground-truth values, which can be either normalized or unnormalized. EGSC (Qin et al., 2021) adopts a teacher-student pipeline for training its model on GED prediction: the teacher model minimizes GED prediction errors using MSE loss, while the student model minimizes reconstruction loss with Huber loss on pseudo-individual embeddings derived from the teacher model. To ensure the predicted similarity score falls

between 0 and 1, SimGNN normalizes the score predicted by the MLP layer using a sigmoid function. In contrast, GOTSim scales the predicted distance by dividing it by the average node count of the graph pairs, accounting for varying graph sizes.

TAGSim (Bai & Zhao, 2021) introduces a slightly different training objective by predicting Graph Edit Vectors (GEV), where each entry corresponds to the cost of a specific graph edit operation. This objective provides more detailed predictions of GED but requires more sophisticated supervision using the true cost of each operation type. GEDGNN (Piao et al., 2023) supervises its model using MSE loss for the GED prediction and Binary Cross-Entropy (BCE) loss for the matching matrix, requiring the supervision of the ground-truth GED and matching matrix. Similarly, AEDNet (Lan et al., 2023), designed for SI, applies contrastive loss on the matching matrix and adaptive edge deletion. The goal is to ensure the embeddings of matched nodes are significantly more similar than unmatched nodes while simultaneously aligning the local adjacency structures by removing non-relevant edges.

In contrast, methods like NeuroMatch (Ying et al., 2020), D2Match (Liu et al., 2023b), and Greed (Ranjan et al., 2022), which address SI, make predictions based on the violation scores, which quantify the extent to which the SI relationship is violated, as outlined in Equation 1. NeuroMatch uses BCE loss to supervise the binary SI classification, along with the violation scores. It trains the model with positive examples (subgraphs of the anchor graph) and negative examples (non-subgraphs), applying a max-margin loss to ensure a clear separation between the violation scores of positive and negative pairs. D2Match, while also handling binary classification, adopts a different strategy. It uses Mean Absolute Error (MAE) loss to supervise the indicator matrix, enforcing the outputs to be either 0 (for non-matching) or 1 (for matching). Additionally, D2Match employs MSE loss to train the graph-level similarity score. On the other hand, Greed computes lower and upper bounds for SED using an approximate algorithm. The model is then trained based on these bounds using the following loss function:

$$\mathcal{L} = \max(0, \text{lb} - dist)^2 + \max(0, dist - \text{ub})^2$$

Where lb is the lower bound, ub is the upper bound, *dist* is computed with Equation 1.

As discussed at the beginning of Section 3, approaches operating in learn-to-search scenarios propose interacting with processed and unprocessed subgraphs to capture changes introduced by the sequential decision process. Although they share a similar training pipeline with end-to-end methods, their supervision signals can differ slightly. For instance, Noah (Yang & Zou, 2021) trains its evaluation model using MSE loss, supervised by the distance between unprocessed subgraphs. GENNA* (Wang et al., 2021) supervises its model with the cost of the optimal solution under MSE loss and then fine-tunes it to predict the similarity between unprocessed graphs. GLSearch (Bai et al., 2021) uses GNNs as a DQN component to evaluate partial solutions (two subgraphs) and potential node matches (node pairs). It trains the model to predict the remaining size of the largest common subgraph starting from the current partial solutions, also using MSE loss. In contrast, RLQVO (Wang et al., 2022) adopts a reinforcement learning paradigm. It uses use the reduced number of enumeration compared with order produced by existing subgraph matching algorithm, as part of the reward signal to improve its GCN and MLP-based policy network.

4 Dataset Generation

Since all three graph theory concepts are domain-agnostic, existing studies often evaluate the proposed methods on graphs extracted from real-world datasets in various domains, such as small molecules (AIDS, COX2, PTC), program dependency graphs (Linux), social networks (IMDB-MULTI), bioinformatics (Enzymes, DD, PPI), and computer vision (MSRC_21, FIRSTMM_DB) from (Morris et al., 2020), etc.

Data Sampling. It is worth noting that since there are no specific datasets for GTDGSL tasks, existing methods often extract or sample graphs from the original datasets. For instance, SimGNN and GraphSim provide the pairwise GED value of graphs collected from AIDS, Linux, and IMDB-MULTI datasets[1]. They choose graphs with 10 or fewer nodes from AIDS and Linux to evaluate the efficiency, and use the full

[1] https://pytorch-geometric.readthedocs.io/en/latest/generated/torch_geometric.datasets.GEDDataset.html

IMDB-MULTI dataset without any selection to test the scalability. NeuroMatch randomly chose an original graph G_o from the dataset according to the graph scale within the dataset. Based on the chosen graph, it samples the anchor graph by randomly choosing a central node u and performing a random breadth-first traversal (BFS) of the graph, extracting the traversed substructure, and sampling its positive examples by performing the same process on the anchor graph starting from the same central node. Then, it proposes to sample negative examples from G_o starting from a node other than u or perturb the sampled positive examples to make it no longer a subgraph of the anchor graph.

Supervision Signal Computation. To generate ground-truth supervision signals, conventional exact or approximate algorithms are often used. For example, exact GED can be computed with the A* algorithm (Abu-Aisheh et al., 2015)[2] and more advanced A*LSa (Chang et al., 2020)[3], though A*LSa does not support customizable edit costs. Alternatively, the smallest distance among Beam, Hungarian, and VJ algorithms can serve as an approximation[4]. GEDLIB (Blumenthal et al., 2019)[5] offers another approximate method that computes lower and upper bounds for GED and SED, as used in (Ranjan et al., 2022). The exact MCS value can be computed with MCSplit (McCreesh et al., 2017) [6]. The algorithm for the exact subgraph isomorphism computation[7] [8], including VF3 (Carletti et al., 2018) and other algorithms, as examined in (Sun & Luo, 2020). RLQVO evaluates its ordering planning for subgraph matching using datasets from previous studies (Sun & Luo, 2020).

Synthetic Dataset Generation. Since computing ground-truth solutions for this graph theory problem is intractable, synthetic GED datasets have become a promising alternative. In this context, GMN generates training data by sampling random binomial graphs with a specified number of nodes and edge probability. From each synthetic graph, it creates positive and negative examples by randomly substituting edges, ensuring a greater number of substitutions for the negative examples than for the positive ones. As exact GED methods provide only an overall GED score for a graph pair without tracking fine-grained values for each graph edit type in GEV, TaGSim proposes generating synthetic graph pairs that adhere to a specified GEV. Meanwhile, GLSearch validates its performance on large graphs by sampling a connected subgraph twice to produce two overlapping subgraphs. NeuroMatch, on the other hand, generates Erdos-Rényi (ER) random graphs and extended Barabasi (BA) graphs as base graphs and then applies a BFS strategy to generate positive and negative examples. To further investigate model generalization, NeuroMatch suggests sampling unseen queries from various distributions, including random BFS, degree-weighted sampling, and random walk sampling.

5 Evaluation Metrics

Since GED and MCS similarity computations are generally formulated as regression tasks. The GTDGSL models for GED and MCS are often evaluated by Mean Squared Error (MSE), Mean Absolute Error (MAE), or Rooted Mean Squared Error (RMSE) to quantify the gap between the predicted value and the ground truth. Towards scenarios such as graph search for a database, which requires returning the top-k similar data graph for a given query graph, the ranking metrics such as Spearman's Rank Correlation Coefficient (ρ), Kendall's Rank Correlation Coefficient (τ) and Precision at k ($p@k$) are also applied.

In addition to the above metrics, learn-to-search-oriented models, such as Noah, also propose the use of accuracy and feasibility. The former measures the accuracy of the computed GEDs compared to the ground-truth GEDs, the latter measures the ratio that the computed GEDs are feasible (i.e., they are equal to or smaller than the ground-truth GEDs). In contrast, metrics such as Area Under the Receiver Operating Characteristic Curve (AUROC), Accuracy, and F1-score are adopted in evaluating subgraph isomorphism

[2] https://networkx.org/documentation/stable/reference/algorithms/generated/networkx.algorithms.similarity.graph_edit_distance.html
[3] https://github.com/LijunChang/Graph_Edit_Distance
[4] https://github.com/dzambon/graph-matching-toolkit
[5] https://dbblumenthal.github.io/gedlib/
[6] https://github.com/jamestrimble/ijcai2017-partitioning-common-subgraph/tree/master
[7] https://github.com/RapidsAtHKUST/SubgraphMatching
[8] https://github.com/MiviaLab/vf3lib

prediction, and Mean Average Precision (MAP) and Mean Reciprocal Rank (MRR) can be further adopted to evaluate the ranking ability, as in (Roy et al., 2022b).

6 Applications

Graphs are ubiquitous in numerous fields, serving as essential structures for representing complex relationships and interactions. Before the advent of GTDGSL methods, MCS, GED, and SI computations were already extensively applied across various domains to measure graph similarity, including graph search (Chang et al., 2023), drug discovery (Mohamed et al., 2019), community detection (Sangkaran et al., 2020), and social network analysis (Guo et al., 2022). Compared with conventional algorithms solving these problems, GTDGSL methods can provide inexact answers within a much shorter response time, making them suitable for real-time applications, such as in dynamic graphs where structures may change rapidly, necessitating frequent updates of similarity assessments, which enhances efficiency and facilitates the retrieval and comparison of graph similarities across large datasets. Furthermore, unlike these conventional algorithms that operate in an on-the-fly manner, some GTDGSL methods can produce reusable representations to construct graph vector databases. Additionally, GTDGSL approaches provide more interpretable measures of graph similarity compared with general deep graph similarity methods. This interpretability is crucial for understanding and justifying decision-making processes, especially in applications such as graph anomaly detection. Practitioners can gain insights into fraudulent activities and identify potential false predictions for further improvements in the algorithms.

GTDGSL methods can be applied in various domains, such as in Electronic Design Automation (EDA). For instance, GTDGSL methods facilitate the comparison of different circuit designs, enabling designers to identify and evaluate modifications aimed at optimizing performance, power consumption, or area constraints (Lu & Pingali, 2018; Shrestha & Savidis, 2024; Li et al., 2024). In the field of drug discovery, GTDGSL methods are instrumental in comparing chemical compounds represented as graphs to identify potential drug candidates. By analyzing the structural similarity of molecules, researchers can predict biological activity and optimize lead compounds (Mohamed et al., 2019; Jayaraj et al., 2016; Shiokawa et al., 2024; Naoi & Shiokawa, 2023; Ranu & Singh, 2012; Schadt et al., 2009).

In systems biology, GTDGSL methods can analyze protein interaction networks to identify potential interactions based on graph similarities, thereby aiding in the understanding of cellular processes and disease mechanisms (Koch et al., 1996; Peng & Tsay, 2010; Shen et al., 2012; Ibragimov et al., 2013; 2014). Moreover, GTDGSL methods are valuable in social network analysis (Guo et al., 2022), such as community detection, where they can identify clusters within social networks by examining the structural similarities between different user profiles or groups. This capability allows for the detection of anomalous patterns that may indicate fraudulent activities in social networks or online platforms (Xiang et al., 2009; Cui et al., 2014; Rong et al., 2018; Sangkaran et al., 2020). In e-commerce and streaming services, GTDGSL methods can enhance item recommender systems by analyzing user-item interaction graphs and the similarity between items (Lalithsena et al., 2016; Wang et al., 2023).

7 Challenges and Future Directions

7.1 Challenges in GTDGSL

Given the detailed analysis of existing models, in this section, we discuss the general challenges or open problems encountered by GTDGSL methods.

7.1.1 Preserving Graph Charateristics

Scale Information. Scale information is crucial in similarity assessment because it impacts how graph differences are interpreted, especially when comparing graphs of different sizes. In MCS and GED computations, the raw measures tend to be biased towards larger graphs, as they naturally have more nodes and edges, making them more likely to have a higher commonality (in MCS) or a larger number of required edits (in GED). This bias can distort the similarity score, making larger graphs seem more similar to or differ-

ent from each other than they actually are. Therefore, considering the scale information helps to compare graphs of varying sizes in a fair and proportional way. Scale information is also essential in SI, which aims to determine whether a substructure in one graph exactly matches another graph. This inherently involves considering the relative sizes of the graphs. Despite the importance of scale information for MCS, GED, and SI, each of them encounters challenges related to handling scale in different ways.

- *Loss of Scale Information.* In the context of graph similarity, GNNs primarily focus on local structural similarity, often overlooking the sizes of graphs. They project nodes with varying local neighborhood sizes onto a single point within the embedding space, potentially losing scale information. SimGNN (Bai et al., 2019) addresses this by summing unnormalized weighted node representations to reflect graph size. Yet, since node representations are a single point, it is unclear whether this method fully captures scale information. SimGNN also generates histogram features based on pairwise node similarity. Intuitively, the height and distribution of bins can depict the scale information and overall similarity. However, SimGNN proposes to normalize the histogram, potentially causing bins with different counts to have the same proportion. This normalization can obscure the differences in graph size, diminishing the role of scale in the final similarity assessment. Moreover, the histogram feature is non-differentiable, meaning it cannot be optimized through backpropagation. Similarly, GOTSim (Doan et al., 2021) proposes normalizing the total transport cost by the average node size of graph pairs. This may limit the model's ability to learn from the data and embody the inductive biases that designers intend to introduce.

- *Scale Difference.* Unlike MCS and GED, SI is a binary classification task rather than a numerical similarity measure. The scale difference between the query and data graphs introduces a unique challenge, as it is often difficult for GNNs to ensure that node representations for matchable pairs are similar when their neighborhoods differ greatly in size. This scale disparity can make it hard to correctly identify subgraph matches. Different methods approach this challenge in various ways. NeuroMatch (Ying et al., 2020) leverages the size difference between the query and data graphs as a filtering mechanism, using order embeddings to ensure that the representation of a data graph contains that of its subgraphs. However, because larger graphs inherently have more complex structures and a broader receptive field during GNN's message-passing process, their representations can inadvertently encompass smaller graphs', making it difficult to maintain strict containment constraints. AEDNet (Lan et al., 2023) proposes a different strategy, pruning irrelevant edges from the data graphs to reduce the representation difference between matched node pairs. However, this method may be unstable, as it heavily relies on the correct identification and removal of redundant edges. The model's ability to capture the precise subgraph structure may be compromised if crucial edges are mistakenly removed.

Structural Information. Capturing Structural Information that depicts the node connections within graphs is crucial for MCS, GED, and SI computations, as these problems focus not just on the local structure around nodes but also on the nodes' positions relative to each other. Since graphs have varying sizes and, within which nodes do not have a natural order. Thus, nodes within graphs generally are treated as a bag of elements during the message-passing and pooling process to emphasize the permutation-invariant nature of graphs. This practice, although shown to be effective in most applications, can cause the loss of structural information, leading to inaccurate predictions. Furthermore, within the embedding space, nodes that share similar local substructures can end up with similar embeddings. Based on this inductive bias, two different nodes in a graph that share isomorphic local substructures can have an identical representation and, thus, cannot be distinguished, which is termed automorphism in (Chamberlain et al., 2023), further confusing the matching process. Despite approaches that operate in learn-to-search scenarios employing a sequential searching process, the loss of structural information can still impede cost estimation for unprocessed subgraphs, leading to a diminished boost in search efficiency.

- *Capture Structural Relationships.* To capture the structural relationships between nodes, methods such as Laplacian matrix-based spectral encoding (Jia et al., 2023) and BFS-based ordering schemes on cost matrices (Bai et al., 2020) have been proposed. The spectral encoding utilizes the eigenvalues

and eigenvectors of the Laplacian matrix to offer insights into node connections and overall graph structure. In contrast, the BFS-based ordering arranges the cost matrix based on a breadth-first search traversal, aiming to align similar structures between graphs more effectively during comparison. However, both methods have notable limitations. Spectral encoding often struggles to differentiate between automorphic nodes, as it can yield similar spectral embeddings for such nodes, hindering its effectiveness in accurately distinguishing them. On the other hand, the BFS ordering captures only 2-hop local connectivity when encountering complex graphs. Additionally, both methods are computationally demanding, with complexities of $\mathcal{O}(|\mathcal{V}|^3)$ for spectral encoding and $\mathcal{O}(|\mathcal{V}|^2)$ for BFS ordering scheme in their worst cases.

- *Break Automorphism.* To break automorphism, identities can be assigned to each node within the graph as augmented features, such as unique numbers (Wang et al., 2022) or random-walk-based features (Liu et al., 2023a). However, while unique numbers are deterministic, they do not generalize well across different graphs and may lead to a loss of permutation invariance. Conversely, random-walk-based features can be computationally expensive, especially in large graphs, as they often require multiple walks or sampling processes. Additionally, being probabilistic, random-walk-based features can produce slightly different representations for the same graph across different runs, potentially affecting their consistency. To enhance the stability of random-walk-based features, MATA* (Liu et al., 2023a) suggests augmenting graphs by randomly adding or removing edges and conducting random walks on both the original and augmented graphs, which further increases computational costs.

7.1.2 Ground-Truth Acquisition

The problems of maximum common subgraph (MCS), graph edit distance (GED), and SI are notoriously challenging due to their combinatorial nature, which complicates the acquisition of ground truth. Recent research indicates that computing the exact GED is particularly difficult, even for graphs with as few as 16 nodes (Blumenthal & Gamper, 2020). Popular GED datasets, such as AIDS and LINUX, comprise graphs with fewer than 10 nodes. In contrast, larger GED datasets, like IMDB-MULTI, contain graphs with a maximum node count of 89 and a maximum graph diameter of 2.

The GED values in IMDB-MULTI are calculated as the smallest distance determined by three well-known approximate algorithms: Beam, Hungarian, and VJ. However, studies (Bai et al., 2019; 2020; Doan et al., 2021; Zhuo & Tan, 2022) have shown that these conventional algorithms often perform significantly worse than current neural network methods when evaluated on the AIDS and LINUX datasets. Since learning-based methods rely on accurate GED values for training and supervision, the performance of these inexact algorithms on the AIDS and LINUX datasets raises concerns about the reliability of the GED values in IMDB-MULTI. Consequently, models trained on such datasets may inherit biases and struggle to learn how to compute GED accurately.

Compared with GED computation, MCS algorithms can handle somewhat larger graphs, but their scalability remains limited. The most advanced MCS algorithm, MCSplit (McCreesh et al., 2017), can solve 2,000 out of 4,110 unlabelled, undirected instances with up to 50 nodes per graph, given a time limit of 0.5 seconds per instance, as shown in the original paper. In contrast, subgraph matching algorithms can process significantly larger graphs. State-of-the-art subgraph matching frameworks, such as RapidMatch (Sun et al., 2020), can enumerate the first 10^5 mappings for queries with 32 nodes in large-scale graphs, such as YouTube, which contains 1,134,890 nodes and 2,987,624 edges, within 100 seconds.

However, it is important to note that subgraph matching algorithms aim to enumerate mappings for a given query, whereas learning-based SI predictors focus on predicting the existence of a SI relationship. In cases of positive instances, where a SI exists, subgraph matching algorithms can terminate early upon finding a valid mapping. Conversely, for negative instances, where no SI exists, these algorithms may have to exhaustively search the solution space to confirm the absence of any matching subgraph. This exhaustive search can be more time-consuming than identifying a positive match, particularly as the size and complexity of the graph increase. While numerous large-scale datasets are available for subgraph matching, there is a notable lack of datasets specifically designed for SI prediction.

Although some methods (Li et al., 2019; Bai & Zhao, 2021; Piao et al., 2023; Ying et al., 2020) propose training and testing models on generated datasets for MCS, GED, and SI, these datasets are often created by modifying original graphs to meet target prediction values. Such modifications require careful design, or the target prediction values given during the generation process may not represent the optimal solution for the graph pairs. This is because modifications made to a subgraph extracted from a larger graph may inadvertently make it resemble other parts of the larger graph.

Recent research toward search-to-learn scenarios has shown that conventional algorithms empowered by deep learning techniques can deliver better solutions on larger datasets in less time compared with the original ones (Liu et al., 2023a; Bai et al., 2021). However, these methods cannot guarantee optimal solutions. Due to the approximate nature of neural network-based approaches, they may overlook critical information, potentially missing correct solutions and compromising the approach's overall reliability (He et al., 2024).

7.1.3 Potential Trade-off Between Performance and Scalability

The trade-off between performance and scalability poses significant challenges in the GTDGSL problem. Notably, these aspects are rarely explored in existing studies, with the exception of Piao et al. (2023), which evaluates the performance of methods such as Noah (Yang & Zou, 2021), a model that predicts costs based on graph representations, and GEDGNN (Piao et al., 2023), which leverages both a cost matrix and a matching matrix for cost prediction. The results indicate that the performance of both methods declines as graph size increases, particularly for Noah. This suggests that while methods predicting using graph representations are generally more efficient than those that implicitly or explicitly establish cross-graph node mappings, they may generalize worse on larger graphs. Although explicit alignment-based methods such as GEDGNN may perform better on large graphs, they typically have at least quadratic complexity, which may hinder the ability to handle large graphs.

7.2 Future Directions

Based on the above limitations, several promising directions for future research can be identified:

Self-Supervised Learning. The integration of self-supervised learning offers a promising solution to the challenges of data acquisition in GTDGSL methods. By leveraging abundant real-world graph pairs without ground-truth labels, self-supervised techniques can reduce reliance on computationally expensive datasets for GED, MCS, and SI. Exploring diverse self-supervised objectives, such as contrastive learning or masked graph prediction, could enhance the adaptability and performance of GTDGSL methods, making them more suitable for real-world applications.

Handling Complex Graph Structures. Currently, most GTDGSL methods are limited to undirected graphs without edge labels, which restricts their utility across broader graph types. Extending these models to support directed graphs with labeled edges would enable them to better capture the nuanced, directional relationships that often exist in real-world scenarios such as knowledge graphs. Furthermore, expanding GTDGSL methods to handle heterogeneous graphs—those with multiple types of nodes and edges—presents an important research direction. Heterogeneous graphs require specialized representations and similarity metrics that account for diverse entities and relationships. Developing these capabilities could significantly enhance the applicability of GTDGSL methods in areas such as social networks, biomedical research, and recommender systems, where accurate similarity assessments are essential in multi-relational and data-rich environments.

More Expressive Representations. Another key direction is advancing graph representations to enhance GTDGSL methods. This involves overcoming the limitations of the 1-WL test, increasing sensitivity to global graph structures, addressing computational constraints, and mitigating over-smoothing in GNNs. These improvements could enable more effective capture of fine-grained differences between graphs.

Benchmarks and Standardization. Establishing benchmarks and standardizing evaluations are essential for advancing GTDGSL research. The lack of standardized datasets that cover diverse graph sizes and

are tailored to GTDGSL challenges hinders consistent assessment and comparison of methods. Developing benchmark datasets and tasks that reflect real-world applications would improve reproducibility, foster collaboration, and enable the identification of best practices, ultimately driving progress in graph similarity learning.

8 Conclusion

This survey presents a comprehensive overview of Graph Theory-based Deep Graph Similarity Learning (GTDGSL). To the best of our knowledge, it is the first work to examine graph similarity learning methods based on graph theory concepts, including subgraph isomorphism, maximum common subgraph, and graph edit distance. We review existing GTDGSL methods, analyzing their training pipelines and techniques to identify commonalities and distinctions. Through this analysis, we highlight current technical trends, applications, and key challenges of GTDGSL methods. Despite promising results in achieving interpretable graph similarity with high efficiency and accuracy, GTDGSL methods face key limitations, such as challenges in preserving critical graph characteristics and obtaining ground-truth supervision signals. These limitations highlight future directions, including the development of more expressive representations and the adoption of self-supervised learning approaches.

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