A General Computational Framework to Measure the Expressiveness of Complex Networks using a Tight Upper Bound of Linear Regions

Abstract

The expressiveness of deep neural network (DNN) is a perspective to understand the surprising performance of DNN. The number of linear regions, i.e. pieces that a piece-wise-linear function represented by a DNN, is generally used to measure the expressiveness. And the upper bound of regions number partitioned by a rectifier network, instead of the number itself, is a more practical measurement of expressiveness of a rectifier DNN. In this work, we propose a new and tighter upper bound of regions number. Inspired by the proof of this upper bound and the framework of matrix computation in Hinz & Van de Geer (2019), we propose a general computational approach to compute a tight upper bound of regions number for theoretically any network structures (e.g. DNN with all kind of skip connections and residual structures). Our experiments show our upper bound is tighter than existing ones, and explain why skip connections and residual structures can improve network performance.

References

Peter Hinz and Sara Van de Geer. A framework for the construction of upper bounds on the number of affine linear regions of relu feed-forward neural networks. *IEEE Transactions on Information Theory*, 65(11):7304–7324, 2019.