A Lower Bound of Hash Codes' Performance

Abstract

As a crucial approach for compact representation learning, hashing has achieved
great success in effectiveness and efficiency. Numerous heuristic Hamming space
metric learning objectives are designed to obtain high quality hash codes. Neverthe-
less, a theoretical analysis on criteria of learning good hash codes remains largely
unexploited. In this paper, we prove that inter-class distinctiveness and intra-class
compactness among hash codes determine the lower bound of hash codes' per-
formance. Promoting these two characteristics could lift the bound and improve
hash learning. We propose a surrogate model to fully exploit such objective by
estimating posterior of hash codes and further control it, which results in low-bias
optimization. Extensive experiments reveal effectiveness of the proposed method.
By testing on a series of hashing methods, we obtain performance improvements
among all of them, with an up to 26.5% increase in mean Average Precision and
an up to 20.5% increase in accuracy.

1 Introduction

Explosive increase of multimedia digital contents requires people to develop large-scale processing
techniques for effective and efficient multimedia understanding [30, 44, 47, 8, 26, 22]. Meanwhile,
growing concerns on carbon neutrality initiatives urge researchers to handle above issues with low
power and memory consumption. Fortunately, hashing, as a key role in compact representation
learning, has been demonstrating its efficiency and effectiveness over past few decades towards above
demands [12, 16, 38, 10, 21, 19, 15]. By digesting multimedia contents into short binary codes,
hashing is able to significantly reduce storage, boost speed, and preserve key information.

Recent years, learning to hash shows its advantages to alleviate information loss by performing
data-dependent projection [27, 43, 41, 42, 21, 19, 15, 18]. People design Hamming space metric
learning objectives to optimize hash models, with the purpose of generating meaningful hash codes
for downstream tasks. Specifically, a practical convention is to increase similarities among codes
of same category, otherwise the opposite. Such convention is validated to be effective in many
applications [7, 37, 20, 36, 45, 28], *e.g.*, fast retrieval and hash-based recognition.

Practices in current advances still leave two questions which remains largely unexploited: How
could we determine a hashing model produce *good* codes, and by what means to achieve above
goal? Taking comprehensive study on above questions is essential, since it not only guides us to
develop high performance algorithms in typical hash-based applications, but also provides potential
possibilities to inspire future works in hash learning.

Although some works [43, 1] try to answer above questions by building connections between hash
codes and common descriptors with domain knowledge, we still seeks for theoretical guarantees for
hash codes' performance. Similar situations occur in related works [42, 41, 25], where objectives to
optimize hashing models are designed by intuitions or heuristics.

By formulating correlations among hash codes as inter-class distinctiveness and intra-class compactness, we try to address above issues and provide a lower bound of hash codes' performance. Based on this, we could further adopt them as objectives to lift lower bound. However, estimation on posterior of codes is necessary, since bit-level dependence exists in posterior distribution. If we neglect this, bias will be introduced during hashing model optimization and therefore hinders performance. Summarizing both of them, our contribution is presented below:

1) To the best of our knowledge, we are the first to formulate lower bound of hash codes' performance, which is directly proportional to inter-class distinctiveness and intra-class compactness among codes. Base on this, an objective is further proposed to lift this lower bound and thus enhance performance.

2) To perform low-bias optimization towards above objective, a novel posterior estimation over hash codes is proposed. As a non-linear function, it could be utilized for optimizing hashing models to produce good codes. Such plug-and-play component is able to integrate into current hashing models to boost performance.

3) Extensive experiments on three benchmark datasets reveal effectiveness of proposed method in two typical tasks. Specifically, we obtain an up to 26.5% increase on mean Average Precision for fast retrieval. We also observe an up to 20.5% increase on accuracy for hash-based recognition.

Rest of paper is organized as follows: We first give preliminaries on hash models and applications (Sec. 2). Then, lower bound on hash codes' performance by inter-class distinctiveness and intra-class compactness is proposed (Sec. 3). To fully exploit such guidance, we propose posterior estimation over multivariate Bernoulli distribution for code control (Sec. 4). Discussions (Sec. 5) and experiments (Sec. 6) are conducted in next few sections.

2   Preliminaries

Learning to hash performs data-dependent compact representation learning. Generally, inputs $x \in \mathcal{X} \subseteq \mathbb{R}^d$ are firstly transformed into h-dim real-valued vector ℓ and then binarized to h bits binary code $b \in \mathcal{B} \subseteq \mathbb{H}^h$:

$$x \xrightarrow{\mathcal{F}_\theta} \ell \xrightarrow{\text{bin}} b,$$

where bin is the $\{-1, +1\}$ binary function, and $\mathcal{F}_\theta$ is a projection parameterized by θ to map d-dim input real space to h-dim: $\mathbb{R}^d \xrightarrow{\mathcal{F}_\theta} \mathbb{R}^h$. The posterior distribution of $\mathcal{B}$ given inputs $\mathcal{X}$ is formulated under multivariate Bernoulli distribution: $p(\mathcal{B} \mid \mathcal{X}) \sim \text{MVB}$.

Hash codes' performance. Generally, Hamming space metric learning matters in hash code learning, and neighborhood structure between codes needs to be regarded essentially [27, 28, 24]. We could organize above as a ranking problem: Given an arbitrary query code q, rank list $\mathcal{R}$ with total length r is obtained by sorting distances $d(q, b)$ among all codes of $\mathcal{X}$ in ascending order. For each result in rank list, true positives $tp^i \in TP$ is a sample matched with query while others are false positives $fp^i \in FP$ (i indicates rank). As a criterion, Average Precision (AP) could be adopted to determine hash codes' performance, which encourages true positives to have higher ranks than false positives. Such criterion could cover common hash tasks, *e.g.*, fast retrieval [13, 35, 21, 2], recognition [20, 32, 36, 27], which is explained in supplementary materials.

3   Lower Bound by Inter-Class Distinctiveness and Intra-Class Compactness

To study characteristics of codes under above scenarios, we start at an arbitrary rank list. For simplicity, query and all samples are assumed to be under same distribution, and size of TP and FP is fixed. As a *good* model, we expect true positives are placed near queries in order to be on top of rank list. To easily calculate AP, *mis-rank* is defined to indicate how many false positives take higher places than true positives.

Definition. Mis-rank m indicates how many false positives have higher ranks than a true positive *i.e.*, $tp^{i,m}$ is at rank i, meanwhile $m = |\{d(q, fp) < d(q, tp^{i,m})\}|$, where $|\cdot|$ is number of elements.

Remark. For any true positive $tp^{i,m}$, the rank-i precision $P@i$ is derived to be $(i-m)/i$.

83 Then, average precision could be derived as:

$$AP = \frac{1}{|\boldsymbol{TP}|} \sum_{\forall \boldsymbol{tp}^{i,m} \in \boldsymbol{TP}} P@i = \frac{1}{|\boldsymbol{TP}|} \sum_{\forall \boldsymbol{tp}^{i,m} \in \boldsymbol{TP}} \frac{i-m}{i},$$

84 for all true positives in rank list.

85 **Corollary.** Average precision increases *iff* m decreases.

86 Now we could focus on how to reduce m to thereby increase average precision. Noticed that value of
87 m is highly related to two distances, $d(\boldsymbol{q}, \boldsymbol{fp})$ and $d(\boldsymbol{q}, \boldsymbol{tp})$, following proposition is raised:

Proposition.

$$\overline{m} \propto \frac{\max d(\boldsymbol{q}, \boldsymbol{tp})}{\min d(\boldsymbol{q}, \boldsymbol{fp})} \; \forall \boldsymbol{tp} \in \boldsymbol{TP}, \; \boldsymbol{fp} \in \boldsymbol{FP}$$

88 where $\overline{\cdot}$ denotes upper bound. Correspondingly,

$$\underline{AP} \propto \frac{\min d(\boldsymbol{q}, \boldsymbol{fp})}{\max d(\boldsymbol{q}, \boldsymbol{tp})} \; \forall \boldsymbol{tp} \in \boldsymbol{TP}, \; \boldsymbol{fp} \in \boldsymbol{FP}$$

89 where $\underline{\cdot}$ denotes lower bound.

90 If inputs $\boldsymbol{x}^j$ have associated labels y^j, above studies are able to be generalized. We firstly define
91 class-wise centers to help for introducing inter-class distinctiveness and intra-class compactness.

92 **Definition.** Center $\boldsymbol{c}^c \in \boldsymbol{C}$ is a representative hash code of a specific class c:

$$\boldsymbol{c}^c = \arg\min_{\boldsymbol{b}} \sum_j d(\boldsymbol{b}, \boldsymbol{b}^j), \; \forall \{\boldsymbol{b}^j \mid y^j = c\}$$

93 where $\boldsymbol{b}^j$ is the jth sample in set and y^j is the label of $\boldsymbol{b}^j$, and $|\boldsymbol{C}|$ is number of total classes.

94 **Definition.** *Inter-class distinctiveness* is $\min \mathcal{D}_{inter}$ where $\mathcal{D}_{inter}$ is a set measures distances between
95 all centers over $\boldsymbol{C}$: $\{d(\boldsymbol{c}^j, \boldsymbol{c}^k) \mid \forall \boldsymbol{c}^j, \boldsymbol{c}^k \in \boldsymbol{C}\}$. In contrast, *intra-class compactness* is $1/\max \mathcal{D}_{intra}$
96 where $\mathcal{D}_{intra} = \{d(\boldsymbol{b}^j, \boldsymbol{c}^c) \mid \forall y^j = c\}$ measures distance among samples of same class to their
97 corresponding center.

98 Combining above propositions and definitions, we could reach:

Proposition.

$$\underline{AP} \propto \frac{\min d(\boldsymbol{q}, \boldsymbol{fp})}{\max d(\boldsymbol{q}, \boldsymbol{tp})} \geq const \cdot \frac{\min \mathcal{D}_{inter}}{\max \mathcal{D}_{intra}}.^{[1]} \tag{1}$$

99 This proposition reveals how AP is affected by two factors. Actually, it could cover common scenarios
100 as a criterion of hash codes' performance, *e.g.*, fast retrieval and recognition, which will be explained
101 in supplementary materials. Lifting this lower bound would further enhance performance, which
102 formulates the objective:

$$maximize \; \min \mathcal{D}_{inter}, \tag{2}$$

$$minimize \; \max \mathcal{D}_{intra}, \tag{3}$$

$$s.t. \; \boldsymbol{\mathcal{X}} \subseteq \mathbb{R}^d \xrightarrow{\mathcal{F}_{\boldsymbol{\theta}}, \; \text{bin}} \boldsymbol{\mathcal{B}} \subseteq \mathbb{H}^h.$$

103 Inspired by [48], we give a specific solution to implement Eqs. (2) and (3) under supervised hashing
104 by firstly defining distance maximized class-specific centers and then shrinking hash codes to their
105 corresponding centers. We will validate effectiveness with such solution in experiments. It is worth
106 noting that achieving above goal is an open topic, including unsupervised or other complex scenarios,
107 which we leave for future study. Next, to optimize hashing models with above supervision, we
108 introduce posterior estimation over MVB to model and control codes' distribution.

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[1]Please refer to supplementary materials for detailed proof.

4 Posterior Estimation for Code Control

It is essentially hard to minimize distance between a hash code and a target to achieve Eq. (3) due to two reasons. Firstly, as mentioned before, posterior distribution $p\left(\mathcal{B} \mid \mathcal{X}\right)$ are under multivariate Bernoulli distribution, resulting in biases if we do not take correlations among different variables into account. Secondly, dealing with hash codes is a $\{-1, +1\}$ discrete optimization, which is not trivial to handle. Therefore, we try to organize Eq. (3) as a Maximum Likelihood Estimation (MLE) to tackle above issues. Considering definition of Hamming distance between an arbitrary hash code $\boldsymbol{b}$ and its target $\boldsymbol{t}$:

$$d\left(\boldsymbol{b}, \boldsymbol{t}\right) = \sum_i^h \mathbb{1}\left\{\boldsymbol{b}_i \neq \boldsymbol{t}_i\right\} = \sum_i^h \mathbb{1}\left\{\boldsymbol{\ell}_i \boldsymbol{t}_i > 0\right\}$$

where $\mathbb{1}$ is the characteristic function and $\left(\cdot\right)_i$ indicates value on the ith dimension. Since Hamming distance measures how many bits are different between two codes, probability of $d\left(\boldsymbol{b}, \boldsymbol{t}\right) = \delta$ can be formulated as:

$$p\left(d\left(\boldsymbol{b}, \boldsymbol{t}\right) = \delta\right) = \sum_{\forall i \in \binom{h}{\delta},\, j \notin \binom{h}{\delta}} \left\{p\left(\boldsymbol{b}_i \neq \boldsymbol{t}_i,\ \boldsymbol{b}_j = \boldsymbol{t}_j\right)\right\}.$$

Therefore, $\boldsymbol{b}$ and $\boldsymbol{t}$ have distance δ iff $p\left(d\left(\boldsymbol{b}, \boldsymbol{t}\right) = \delta\right)$ is maximized. However, to precisely calculate it is not trivial, since it involves joint probability of $\boldsymbol{b}$. Therefore, we try to estimate all joint probabilities by adopting a surrogate model $\mathcal{P}_{\boldsymbol{\pi}}$ parameterized by $\boldsymbol{\pi}$ to produce estimation:

$$\boldsymbol{\ell} \xrightarrow{\mathcal{P}_{\boldsymbol{\pi}}} \boldsymbol{o},\ where\ \boldsymbol{o} \in \boldsymbol{O} \subseteq \mathbb{R}^{2^h} \tag{4}$$

where $\boldsymbol{o}$ is probabilities of a Categorical distribution $p\left(\boldsymbol{O} \mid \mathcal{X}\right)$ to estimate $p\left(\mathcal{B} \mid \mathcal{X}\right)$, which contains 2^h entries. Each entry of $\boldsymbol{o}$ estimates one of a specific joint probability by feeding $\boldsymbol{\ell}$ into $\mathcal{P}_{\boldsymbol{\pi}}$:

$$\boldsymbol{o}_i \mathrel{\widehat{=}} p\left(\left(\boldsymbol{b}_k > 0\right)^{\mathbb{1}\{\boldsymbol{\iota}_k = 1\}}, \left(\boldsymbol{b}_k < 0\right)^{\mathbb{1}\{\boldsymbol{\iota}_k = 0\}}\right),\ {\scriptstyle 1 \leq k \leq h,\atop 0 \leq i \leq 2^h - 1}$$

where $\boldsymbol{\iota}$ is the h bits binary representation of i. For example, $\boldsymbol{o}_3 = p\left(\boldsymbol{b} = (\text{-1-1+1+1})\right)$ when $h = 4$. To train $\mathcal{P}_{\boldsymbol{\pi}}$, we perform Maximum Likelihood Estimation (MLE) over $p\left(\boldsymbol{O} \mid \mathcal{X}\right)$:

$$L_{\boldsymbol{\pi}}\left(\mathcal{P}_{\boldsymbol{\pi}}; \boldsymbol{o}\right) = -\log \boldsymbol{o}_i,$$
$$where\ i = \sum_{k=1}^h \mathbb{1}\left\{\boldsymbol{b}_k > 0\right\} \cdot 2^{k-1}. \tag{5}$$

If $\mathcal{P}_{\boldsymbol{\pi}}$ is ready for estimation, we could directly adopt it as a nice non-linear function to maximize $p\left(d\left(\boldsymbol{b}, \boldsymbol{t}\right) = k\right)$. Note that to realize Eq. (3), we want $\boldsymbol{b}$ and its corresponding center $\boldsymbol{c}$ are closest *i.e.* maximize $p\left(d\left(\boldsymbol{b}, \boldsymbol{c}\right) = 0\right)$. So, we could reformulate this maximization as another MLE:

$$L_{\boldsymbol{\theta}}\left(\mathcal{F}_{\boldsymbol{\theta}}; \boldsymbol{b}\right) = -\log p\left(d\left(\boldsymbol{b}, \boldsymbol{c}\right) = 0\right),$$
$$where\ p\left(d\left(\boldsymbol{b}, \boldsymbol{c}\right) = 0\right) = p\left(\boldsymbol{b}_i = \boldsymbol{c}_i, 1 \leq i \leq h\right), \tag{6}$$

which could be optimized by calculating surrogate gradients of Eq. (6) through $\mathcal{P}_{\boldsymbol{\pi}}$:

$$\frac{\partial \hat{L}_{\boldsymbol{\theta}}}{\partial \boldsymbol{\theta}} = \frac{\partial \left\{-\log \boldsymbol{o}_i;\ i = \sum_{k=1}^h \mathbb{1}\left\{\boldsymbol{c}_k > 0\right\} \cdot 2^{k-1}\right\}}{\partial \boldsymbol{\pi}} \frac{\partial \boldsymbol{\pi}}{\partial \boldsymbol{\ell}} \frac{\partial \boldsymbol{\ell}}{\partial \boldsymbol{\theta}} \mathrel{\widehat{=}} \frac{\partial L_{\boldsymbol{\theta}}}{\partial \boldsymbol{\theta}}. \tag{7}$$

However, such joint probability estimation on MVB requires $\mathcal{O}\left(2^h\right)$ complexity. This is not flexible when h is large. By adopting the idea of block code [39], above estimation is able to perform on long hash bits by separating hash codes into a series of blocks. Specifically, any h bits hash codes can be split into m blocks while each block consumes h/m bits. Correspondingly, m independent surrogate networks $\mathcal{P}_{\boldsymbol{\pi}_i}, 1 \leq i \leq m$ are adopted to perform above estimation and back-propagation simultaneously. With such decomposition, the whole problem is transformed into m sub-problems with a reduced complexity $\mathcal{O}\left(2^h\right) \to \mathcal{O}\left(m \cdot 2^{h/m}\right)$.

Summarizing all of things, overall optimization via gradient descent is placed in Alg. 1. We first perform Eq. (2) step by using pre-defined centers and then perform Eq. (3) step by back-propagation via Eq. (7). Two models $\mathcal{F}_{\boldsymbol{\theta}}, \mathcal{P}_{\boldsymbol{\pi}}$ are optimized with learning rate η_1, η_2 respectively.

Algorithm 1 One of implementations under supervised circumstance.

1: **procedure** TRAIN($\mathcal{F}_{\boldsymbol{\theta}}$, $\mathcal{P}_{\boldsymbol{\pi}}$)   ▷ Training procedure of two models $\mathcal{F}_{\boldsymbol{\theta}}$, $\mathcal{P}_{\boldsymbol{\pi}}$.
2:      Generate class-specific centers $\boldsymbol{c} \in \boldsymbol{C}, |\boldsymbol{C}| = $ Class-num;   ▷ (Eq. (2)).
3:      **repeat**   ▷ Main training loop.
4:          Sample $\boldsymbol{x}$ from $\mathcal{X}$ with label y;
5:          $\boldsymbol{\ell} = \mathcal{F}_{\boldsymbol{\theta}}(\boldsymbol{x})$;
6:          $\boldsymbol{o} = \mathcal{P}_{\boldsymbol{\pi}}(\boldsymbol{\ell})$;
7:          $\boldsymbol{\pi} \leftarrow \boldsymbol{\pi} - \eta_1 \frac{\partial L_{\boldsymbol{\pi}}}{\partial \boldsymbol{\pi}}$;   ▷ (Eq. (5)).
8:          $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \eta_2 \frac{\partial L_{\boldsymbol{\theta}}}{\partial \boldsymbol{\theta}}$ with corresponding center $\boldsymbol{c}$;   ▷ Eq. (3), Eq. (7).
9:      **until** Total epoch exceeds;
10:     **return** $\mathcal{F}_{\boldsymbol{\theta}}$;   ▷ Optimized hashing model $\mathcal{F}_{\boldsymbol{\theta}}$
11: **end procedure**
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5 Discussions

In this paper, we provide analysis to guarantee the lower-bound of hashing performance. By maximizing inter-class distinctiveness and intra-class compactness, such lower-bound is lifted. Related works include CSQ [48], which uses centroids and binary cross entropy to achieve above goal, and [36] constrains inter-class relation by auxiliary classification branch. Similar ideas present in a series of works [34, 15, 40, 33], but comprehensive study on effectiveness of them still remains largely unexploited. Our work unifies these ideas by Eq. (1), and try to bridge connection between above intuitions and theory behind it.

To perform discrete optimization, [48] uses binary cross entropy:

$$\sum_{i=1}^{h} \log\left(\sigma\left(\boldsymbol{\ell}_i\right)\right)^{\mathbb{1}\{\boldsymbol{t}_i > 0\}} + \log\left(1 - \sigma\left(\boldsymbol{\ell}_i\right)\right)^{\mathbb{1}\{\boldsymbol{t}_i < 0\}}$$

where $\sigma\left(\cdot\right)$ is sigmoid function. Adopting such objective results in processing hash codes among different dimensions separately, which reveals their assumption that hash codes are under single variate Bernoulli distribution. [3] and similar works [23, 4, 46, 5] rather take approximation of Hamming distance by cosine similarity:

$$d\left(\boldsymbol{b}^j, \boldsymbol{b}^k\right) \approx \frac{h}{2}\left(1 - \frac{\left(\boldsymbol{\ell}^j\right)^{\mathsf{T}} \boldsymbol{\ell}^k}{\|\boldsymbol{\ell}^j\| \cdot \|\boldsymbol{\ell}^k\|}\right)$$

or inner product with $\tanh\left(\boldsymbol{\ell}\right)$. But such practices are validated to be not flexible when code length is large [24]. Both of two approaches will be evaluated in experiments.

6 Experiments

To confirm effectiveness of our proposed method, we conduct extensive experiments on three benchmark datasets. To make fair comparisons, we first provide experiments setup.

6.1 Setup

Our experiments focus on performance evaluation on two typical hash-based tasks, fast retrieval and recognition. The pipeline is to firstly train models on training split, then hash all samples in base split. Then, hashed queries in query split get rank lists from base split for retrieval, while queries' predictions from classifier are used for recognition.

6.1.1 Datasets

Our experiments are conducted on three datasets, varying from scale, variety and perplexity to validate methods' performance in different scenarios. Both single-label and multi-label datasets are included. We follow previous works to generate three splits, which is detailed below:

Figure 1: mAP *w.r.t.* $\frac{\min \mathcal{D}_{inter}}{\max \mathcal{D}_{intra}}$ per epoch on ImageNet 64 bits during training. These two values have a positive relationship.

Figure 2: Convergence curve of three variants on ImageNet 64 bits. Our $\mathcal{P}_{\boldsymbol{\pi}}$ helps for achieving higher performance than other two.

CIFAR-10 [14] is a single-label 10-class dataset. The whole dataset contains $6,000$ images for each class. Following [4], we form the dataset by $500 \mid 5,400 \mid 100$ for each class randomly as train, base, query splits, respectively.

NUS-WIDE [6] consists of 81 labels and images may have one or more labels. We follow previous works [4] to pick the most frequent 21 labels and their associated images ($195,834$) for experiments. Specifically, $193,734$ images are randomly picked to form base split while

Table 1: Total training time with different variants, where we substitute our surrogate model with BCE loss from [48] or Cauchy loss from [3].

Method	Training time per epoch (s)		
	16 bits	32 bits	64 bits
BCE	38.13	38.20	38.52
Cauchy	38.21	38.30	38.42
Ours	38.95	39.78	40.84

remaining $2,100$ images are adopted for queries. $10,500$ images are randomly sampled from base split for training models.

ImageNet [8] is a large-scale dataset consists of $1,000$ classes. To conduct experiments, we follow [4] to pick a subset of 100 classes where all images of these classes in training set / validation set are as base split / query split respectively ($128,503 \mid 4,983$ images). We then randomly sample 100 images per class in base split for training.

6.1.2 Implementation Details

Our method is able to integrate into common hashing models. Due to limitation of computation resources, we choose a series of representative deep hashing methods for comparison, including **HashNet [4]**, **DBDH [49]**, **DSDH [20]**, **DCH [3]**, **GreedHash [36]** and **CSQ [48]** and denote methods with our integration as $(\cdot)_D$. All methods adopted for experiments are from a public benchmark implemented with PyTorch [29].[2] For fair comparisons, we conduct experiments with same backbone (ResNet-50 [11]) and hyper-parameters for all tested methods. We adopt Adam [31] optimizer with default configuration and learning rate of our method $\eta_1 = \eta_2 = 1e^{-3}$ for training. For multi-label datasets, we simply modify Alg. 1 Line 8 with sum of multiple losses. Block number m of $\mathcal{P}$ is set to $^{bits}/_8$. For example, if hash code length is 64, there will be 8 sub models $\mathcal{P}_{\boldsymbol{\pi}_1} \sim \mathcal{P}_{\boldsymbol{\pi}_8}$ trained in parallel. To mitigate randomness, we report average performance for 5 runs on all experiments. Evaluation metric adopted for retrieval is mean Average Precision (mAP@K) where $K = 54,000 \mid 5,000 \mid 1,000$ on CIFAR-10, NUS-WIDE, ImageNet, respectively. For recognition performance, if methods have auxiliary classification branch, we directly use predictions from it. Otherwise, we adopt a kNN classifier built on base split and vote with 100 nearest neighbors.

6.2 Ablation Study

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[2]https://github.com/swuxyj/DeepHash-pytorch

In ablation study, we try to reveal correctness of our proposed objectives Eqs. (2) and (3), and effectiveness of our proposed posterior estimation model $\mathcal{P}_{\pi}$. We answer following questions by conducting experiments on ImageNet to demystify above concerns. We focus on retrieval performance in ablation study, while we observe similar results on recognition.

Is hash model's performance lower-bounded by Eq. (1)? Validating correctness of Eq. (1) is important, but it is essentially hard to conduct experiments to confirm it. Nevertheless, we still reveal correlation between model performance and inter-class distinctiveness / intra-class compactness by tracking mAP *w.r.t.* $\frac{\min \mathcal{D}_{inter}}{\max \mathcal{D}_{intra}}$ during training. To calculate $\mathcal{D}_{inter}$ and $\mathcal{D}_{intra}$, we first hash all samples from base split and calculate centers of them over all classes. The 99.9 percentile of $\min \mathcal{D}_{inter}$ and $\max \mathcal{D}_{intra}$ is picked to avoid outliers. We conduct tracking on CSQ$_D$ with $16, 32, 64$ bits per epoch to draw Fig. 1. As figure shows, lines goes from lower left to upper right with small fluctuations, which indicates linear relationship between mAP and $\frac{\min \mathcal{D}_{inter}}{\max \mathcal{D}_{intra}}$ on all bit-lengths. This may partially confirm our theoretical analysis in Sec. 3.

$$KL\,(Est.,\ Real) = 7.88e^{-3}$$

Method	KL_{real}
Naïve	0.538
Ours	0.008

Figure 3: Multivariate Bernoulli estimation by our proposed $\mathcal{P}_{\pi}$, and KL-divergence comparisons with naïve independent estimation.

Convergence speed and efficiency of $\mathcal{P}_{\pi}$. To verify convergence speed and efficiency of our posterior estimation model $\mathcal{P}_{\pi}$, we test it by substituting with two variants: BCE from [48] and Cauchy loss from [3]. mAP is evaluated per epoch for three variants with three bit-lengths $16, 32, 64$. The convergence curve is plotted in Fig. 2. From the figure, we could see convergence speed of ours and BCE are similar, while Cauchy is slightly slow. Meanwhile, our method has continuously higher performance than other two after ~ 10 epochs for all bit-lengths, which is potentially due to our method could perform low-bias optimization. Note that when training with Cauchy loss, performance is instead lower when codes are longer. This may because inner-product based Cauchy loss could not handle long bits. We also measure averaged training time per epoch of all variants on a single NVIDIA RTX 3090, which is placed in Tab. 1. In this table, our model does not consume significant longer time than others to train. Above observations reveal efficiency of our proposed method. It could be a drop-in replacement without introducing significant overhead. Moreover, convergence speed and efficiency will not drop when bit-length becomes long, which reveals flexibility of the blocked design.

Could $\mathcal{P}_{\pi}$ estimate multivariate Bernoulli distribution? We design an extra toy experiments to validate the ability of estimating multivariate Bernoulli distribution by our surrogate model $\mathcal{P}_{\pi}$. Specifically, 8 bits MVB is generated with randomly 256 joint probabilities. We then take $10,000$ samples from it as inputs to train $\mathcal{P}_{\pi}$. The model will further estimate distribution by feeding another 100 samples and taking mean of all $\boldsymbol{o}$s (Eq. (4)) as result. The estimated distribution is evaluated by Kullback–Leibler divergence [17] (KL) with real distribution, as well as bar plot, demonstrated in Fig. 3. As figure shows, our predicted joint probabilities almost cover real probabilities and KL is low. As a comparison, if we estimate real distribution with product of edge probabilities directly (w/o correlations between variables), KL will be significantly increased (row 1 in table). Such toy experiment reveals our method is better to estimate MVBs than the naïve one.

6.3 Performance Enhancements when Integrating into State-of-the-Art

To evaluate performance gain when integrating our proposed method into hashing models, we conduct experiments on three datasets. Specifically, we first report original performance of tested models and then re-run with our method incorporated to make comparison. Both $16, 32, 64$ bits results are reported to show performance from short codes to long codes.

Retrieval Performance. Retrieval performance comparisons under mAP is shown in Tab. 2. With our method integrated, all of tested hashing methods have performance enhancement. We observe

Table 2: mAP comparisons on three benchmark datasets for $16, 32, 64$ bits codes. $_{\uparrow(\cdot)}$ indicates performance enhancement with our method integrated.

Method	CIFAR-10			NUS-WIDE			ImageNet		
	16 bits	32 bits	64 bits	16 bits	32 bits	64 bits	16 bits	32 bits	64 bits
HashNet	63.0	81.5	84.7	77.8	83.3	85.0	50.6	63.1	68.4
HashNet$_D$	$79.1_{\uparrow16.1}$	$82.3_{\uparrow0.8}$	$85.3_{\uparrow0.6}$	$78.6_{\uparrow0.8}$	$84.0_{\uparrow0.7}$	$86.2_{\uparrow1.2}$	$72.6_{\uparrow22.0}$	$84.3_{\uparrow21.2}$	$87.6_{\uparrow19.2}$
DBDH	83.1	85.0	85.6	82.8	84.6	85.7	55.7	63.8	76.9
DBDH$_D$	$84.6_{\uparrow1.5}$	$86.0_{\uparrow1.0}$	$86.5_{\uparrow0.9}$	$83.8_{\uparrow1.0}$	$85.6_{\uparrow1.0}$	$86.5_{\uparrow0.8}$	$82.2_{\uparrow26.5}$	$85.8_{\uparrow22.0}$	$86.1_{\uparrow9.2}$
DSDH	75.6	83.1	84.5	83.3	84.5	85.6	57.2	72.1	75.3
DSDH$_D$	$84.3_{\uparrow8.7}$	$84.6_{\uparrow1.5}$	$87.3_{\uparrow2.8}$	$83.6_{\uparrow0.3}$	$85.3_{\uparrow0.8}$	$86.4_{\uparrow0.8}$	$81.7_{\uparrow24.5}$	$87.3_{\uparrow15.2}$	$87.9_{\uparrow12.6}$
DCH	83.4	84.4	85.3	80.7	81.7	80.9	85.5	86.2	86.4
DCH$_D$	$83.6_{\uparrow0.2}$	$84.6_{\uparrow0.2}$	$87.1_{\uparrow1.8}$	$82.9_{\uparrow2.2}$	$84.2_{\uparrow2.5}$	$84.9_{\uparrow4.0}$	$86.1_{\uparrow0.6}$	$87.5_{\uparrow1.3}$	$88.1_{\uparrow1.7}$
GreedHash	83.3	84.3	86.9	78.6	80.3	82.0	83.1	85.9	86.4
GreedHash$_D$	$85.2_{\uparrow1.9}$	$85.5_{\uparrow1.2}$	$87.6_{\uparrow0.7}$	$79.4_{\uparrow0.8}$	$83.1_{\uparrow2.8}$	$85.7_{\uparrow3.7}$	$83.8_{\uparrow0.7}$	$86.6_{\uparrow0.7}$	$86.8_{\uparrow0.4}$
CSQ	83.2	83.4	84.7	82.0	83.5	84.6	83.4	86.9	87.9
CSQ$_D$	$88.7_{\uparrow5.5}$	$89.2_{\uparrow5.8}$	$90.3_{\uparrow5.6}$	$83.3_{\uparrow1.3}$	$85.3_{\uparrow1.8}$	$85.8_{\uparrow1.2}$	$88.5_{\uparrow5.1}$	$89.5_{\uparrow2.6}$	$90.2_{\uparrow2.3}$

an up to 26.5% increase and 5.02% in average, which is a significant improvement. Specifically, performance increase on HashNet, DBDH and DSDH is higher than other methods (first 3 rows), especially on ImageNet dataset. A potential reason is all of these three methods use pairwise metric learning objectives and inner-product to approximate Hamming distance, which may not handle optimization well when dataset size and class number is large. Meanwhile, our method works on multi-label datasets where all methods also obtain performance gain, indicating multi-label data may also benefit from our objective. Furthermore, we also observe 3.46% average mAP increase on CSQ. It adopts similar objective but different optimization approach with ours. This increase shows effectiveness of our proposed posterior estimation approach, which could perform better optimization than theirs.

Recognition Performance. Similarly, we report recognition performance of these methods by measuring classification accuracy on ImageNet with $16, 32, 64$ bits codes. Results are reported in Tab. 3. Generally, we observe similar performance gain. When methods have our integration, their accuracy is significantly increased, with an up to 20.5% and 5.3% in average. Specifically, some of them utilize an auxiliary classification branch to produce predictions, *i.e.*, GreedyHash and DSDH. While for others, we adopt a kNN classifier to classify samples by voting with base split. We obtain increase for both approaches. This indicates our approach also helps for hash-based recognition in different scenarios, by leveraging inter-class distinctiveness and intra-class compactness.

From above comparisons, we confirm effectiveness of our proposed method. Overall, our method is validated to be flexible and promising to deploy in above scenarios with various bits.

Table 3: Recognition performance on ImageNet.

Method	ImageNet		
	16 bits	32 bits	64 bits
HashNet	69.1	74.1	82.4
HashNet$_D$	$83.2_{\uparrow14.1}$	$88.1_{\uparrow14.0}$	$89.5_{\uparrow7.1}$
DBDH	80.9	82.9	88.2
DBDH$_D$	$86.8_{\uparrow5.9}$	$89.1_{\uparrow6.2}$	$88.4_{\uparrow0.2}$
DSDH	40.6	49.9	54.1
DSDH$_D$	$61.1_{\uparrow20.5}$	$62.1_{\uparrow12.2}$	$63.2_{\uparrow9.1}$
DCH	88.5	89.2	88.1
DCH$_D$	$88.9_{\uparrow0.4}$	$89.5_{\uparrow0.3}$	$88.7_{\uparrow0.6}$
GreedHash	79.1	86.8	87.0
GreedHash$_D$	$80.2_{\uparrow1.1}$	$87.1_{\uparrow0.3}$	$88.4_{\uparrow1.4}$
CSQ	87.7	89.0	89.4
CSQ$_D$	$87.9_{\uparrow0.2}$	$89.1_{\uparrow0.1}$	$91.0_{\uparrow1.6}$

6.4 Qualitative Comparisons

t-SNE visualization. To further analyze how our objective affects hash codes by adjusting $\mathcal{D}_{inter}$ and $\mathcal{D}_{intra}$, we conduct codes visualization by t-SNE [9] on HashNet and DBDH. Specifically, we randomly sample $5,000$ 64 bits codes from CIFAR-10 base split. t-SNE is then performed on these

(a) HashNet 64 bits, $\frac{\min \mathcal{D}_{inter}}{\max \mathcal{D}_{intra}} = 0.8562$

(b) HashNet$_D$ 64 bits, $\frac{\min \mathcal{D}_{inter}}{\max \mathcal{D}_{intra}} = 0.9334$

(c) DBDH 64 bits, $\frac{\min \mathcal{D}_{inter}}{\max \mathcal{D}_{intra}} = 0.9447$

(d) DBDH$_D$ 64 bits, $\frac{\min \mathcal{D}_{inter}}{\max \mathcal{D}_{intra}} = 0.9761$

Figure 4: t-SNE visualization for HashNet and DBDH $w/$ and w/o our integration.

codes as shown in Fig. 4, where points' color indicates class. From these figures, we give our humble explanation. Firstly, please look at Fig. 4(a), codes extracted from original HashNet are mixed among different classes. While in Fig. 4(c), clusters are loose. These representations result in low $\min \mathcal{D}_{inter}$ and high $\max \mathcal{D}_{intra}$. When they are trained along with our method (Figs. 4(b) and 4(d)), we get more separable and distinct clusters compared to original ones. Meanwhile, codes in a cluster are more compact than original methods. Quantitative results of $\frac{\min \mathcal{D}_{inter}}{\max \mathcal{D}_{intra}}$ placed under figures also reveal this phenomenon.

mAP *w.r.t.* $\frac{\min \mathcal{D}_{inter}}{\max \mathcal{D}_{intra}}$ visualization. To further confirm relationship between hashing performance and $\frac{\min \mathcal{D}_{inter}}{\max \mathcal{D}_{intra}}$, as in Sec. 6.2, we plot mAP *w.r.t.* $\frac{\min \mathcal{D}_{inter}}{\max \mathcal{D}_{intra}}$ for all tested methods on 64 bits ImageNet base split (Fig. 5). As figure shows, most of methods obtain higher mAP with larger $\frac{\min \mathcal{D}_{inter}}{\max \mathcal{D}_{intra}}$ (solid dots to translucent dots indicate original methods to integrated methods). Meanwhile, mAP and $\frac{\min \mathcal{D}_{inter}}{\max \mathcal{D}_{intra}}$ are nearly under linear relationship for different methods. Therefore, we give regression line with 95% confidence interval on plot to confirm our observations.

Figure 5: mAP *w.r.t.* $\frac{\min \mathcal{D}_{inter}}{\max \mathcal{D}_{intra}}$ for different methods. Regression line is also plotted with 95% confidence interval to reveal linear relationship.

7 Conclusion

In this paper, we conduct comprehensive study on characteristics of hash codes. As a result, we point out two key factors that benefit for hash learning. Specifically, by formulating hash performance lower-bound by inter-class distinctiveness and intra-class compactness among hash codes, we are able to further lift it with derived objective. Meanwhile, our proposed surrogate model for posterior estimation are able to model codes' distribution, and fully exploit above guidance to produce good codes. Extensive experiments conducted on three benchmark datasets confirm effectiveness of our proposed method to boost current hashing models' performance with a flexible integration.

Limitation and Broader Impacts

Hashing and related compact representation learning are able to significantly reduce computational requirements while improving memory efficiency when deploying to real scenarios. However, a main challenge for these techniques is information loss which results in performance drop. To tackle this, our work studies on hash codes' performance gives a lower-bound, which is validated in two specific tasks. However, generalizing such lower-bounds to various scenarios, *e.g.* semi-supervised, unsupervised hash learning, are leaved for future study. Meanwhile, our posterior estimation approach is not verified for extremely long bits. Nevertheless, our work may still provide potentials of inspiring researchers to improve not only hash learning, but also broader areas that adopt metric learning. Our study on posterior estimation may also help for precise discrete optimizations.

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