# Learning What To Do by Simulating the Past

## Abstract

Since reward functions are hard to specify, recent work has focused on learning policies from human feedback. However, such approaches are impeded by the expense of acquiring such feedback. Recent work proposed that agents have access to a source of information that is effectively free: in any environment that humans have acted in, the state will *already* be optimized for human preferences, and thus an agent can extract information about what humans want from the state. Such learning is possible in principle, but requires simulating all possible past trajectories that could have led to the observed state. This is feasible in grid worlds, but how do we scale it to complex tasks? In this work, we show that by combining a learned feature encoder with learned inverse models, we can enable agents to simulate human actions backwards in time to infer what they must have done. The resulting algorithm is able to reproduce a specific skill in MuJoCo environments given a *single state* sampled from the optimal policy for that skill.

## 1 Introduction

As deep learning has become popular, many parts of AI systems that were previously coded by hand have been replaced with learned components. Neural architecture search has automated architecture design (Zoph & Le, 2017; Elsken et al., 2019), population-based training has automated hyperparameter tuning (Jaderberg et al., 2017), and self-supervised learning has led to impressive results in language modeling (Devlin et al., 2019; Radford et al., 2019; Clark et al., 2020) and reduced the need for labels in image classification (Oord et al., 2018; He et al., 2020; Chen et al., 2020). However, in reinforcement learning, one component continues to be designed by humans: the task specification. Handcoded reward functions are notoriously difficult to specify (Clark & Amodei, 2016; Krakovna, 2018), and learning from demonstrations (Ng et al., 2000; Fu et al., 2018) or preferences (Wirth et al., 2017; Christiano et al., 2017) requires a lot of human input. Is there a way that we can automate even the specification of what must be done?

It turns out that we can learn what the user wants *simply by looking at the state of the environment*: after all, the user will already have optimized the state towards their own preferences (Shah et al., 2019). For example, when a robot is deployed in a room containing an intact vase, it can reason that if its user wanted the vase to be broken, it would already have been broken; thus she probably wants the vase to remain intact.

However, we must ensure that the agent distinguishes between aspects of the state that the user *couldn't control* from aspects that the user *deliberately designed*. This requires us to simulate what the user *must have done* to lead to the observed state: anything that the user put effort into in the past is probably something the agent should do as well. As illustrated in Figure 1, if we observe a Cheetah balancing on its front leg, we can infer how it must have launched itself into that position. Unfortunately, it is unclear how to simulate these past trajectories that lead to the observed state. So far, this has only been done in gridworlds, where all possible trajectories can be considered using dynamic programming (Shah et al., 2019).

Our key insight is that we can sample such trajectories by starting at the observed state and simulating *backwards in time*. To enable this, we derive a gradient that is amenable to estimation through backwards simulation, and learn an inverse policy and inverse dynamics model using supervised learning to perform the backwards rollouts. The only remaining challenge is finding a reward representation that can be meaningfully updated from a single state observation. To that end, rather than defining the reward directly on the raw input space, we represent it as a linear combination of

Figure 1: Suppose we observe a Cheetah balancing on its front leg (left). Given a simulator for the environment, Deep RLSP is able to infer how the cheetah must have acted to end up in this position. It can then imitate these actions in order to recreate this skill. Note that the state contains joint *velocities* in addition to positions, which makes the task more tractable than this picture might suggest.

features learned through self-supervised representation learning techniques. Putting it all together gives our *Deep Reward Learning by Simulating the Past* (Deep RLSP) algorithm.

We evaluate Deep RLSP on MuJoCo environments and show that it can recover fairly good performance on the task reward given access to a few states sampled from a policy optimized for that reward. We also use Deep RLSP to imitate skills generated using a skill discovery algorithm (Sharma et al., 2020), in some cases given just a *single state* sampled from the policy for that skill.

Information from the environment state cannot completely replace reward supervision. For example, it would be hard to infer how clean Bob would ideally want his room to be, if the room is currently messy because Bob is too busy to clean it. Nonetheless, we are optimistic that information from the environment state can be used to significantly reduce the burden of human supervision required to train useful, capable agents.

## 2 METHOD

Here we show how to produce an algorithm that can learn a reward function for high dimensional environments given access only to a simulator and the observed state $s_0$.

**Notation.** A finite-horizon Markov Decision Process (MDP) $\mathcal{M} = \langle \mathcal{S}, \mathcal{A}, \mathcal{T}, r, \mathcal{P}, T \rangle$ contains a set of states $\mathcal{S}$ and a set of actions $\mathcal{A}$. Given a state and an action, the transition function $\mathcal{T} : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \mapsto [0, 1]$ determines the distribution over next states. The reward function $r : \mathcal{S} \mapsto \mathbb{R}$ determines the agent's objective. $\mathcal{P}$ is the prior over initial states. $T \in \mathbb{Z}_+$ is the finite planning horizon. The goal is to find a *policy* $\pi : S \times A \mapsto [0, 1]$ that maximizes the expected reward $\mathbb{E}_\tau \left[ \sum_{t=1}^{T} r(s_t) \right]$, where $\tau$ (a trajectory) is a sequence of state-action pairs where the first state is sampled from $\mathcal{P}$, and every subsequent action is sampled from $\pi$ and every subsequent state is sampled from $\mathcal{T}$.

We use $\phi : \mathcal{S} \to \mathbb{R}^n$ to denote a feature function (whether handcoded or learned) that produces a feature vector of length $n$ for every state. The reward function $r$ is linear over $\phi$ if it can be expressed in the form $r(s) = \theta^T \phi(s)$ for some $\theta \in \mathbb{R}^n$.

We assume that some *past* trajectory $\tau_{-T:0} = s_{-T} a_{-T} \dots a_{-1} s_0$ produced the observed state $s_0$.

### 2.1 IDEALIZED ALGORITHM

We first explain what we would ideally do, if we had a handcoded a feature function $\phi$ and an enumerable (small) state space $\mathcal{S}$ that affords dynamic programming. This is a recap of *Reward Learning by Simulating the Past* (RLSP) (Shah et al., 2019).

We assume that the human is acting according to the Boltzmann-rational policy $\pi_t(a \mid s, \theta) \propto \exp(Q_t(s, a; \theta))$, where the $Q$ values are computed using soft value iteration. This defines a distribution over the observed state $p(s_0 \mid \theta)$ by marginalizing over past trajectories: $p(s_0 \mid \theta) = \sum_{s_{-T} \dots a_{-1}} p(\tau_0 \mid \theta)$. We compute the MLE, $\operatorname{argmax}_\theta \ln p(s_0 \mid \theta)$, via gradient descent, by reducing the gradient of the *observed state* to a weighted combination of gradients of *consistent trajectories*:

$$\nabla_\theta \ln p(s_0 \mid \theta) = \mathbb{E}_{\tau_{-T:-1} \sim p(\tau_{-T:-1} \mid s_0, \theta)} [\nabla_\theta \ln p(\tau \mid \theta)] \tag{1}$$

$\nabla_\theta \ln p(\tau \mid \theta)$ is a gradient for inverse reinforcement learning. Given our assumption of a Boltzmann-rational human, this is the gradient for Maximum Causal Entropy Inverse Reinforcement Learning (Ziebart et al., 2010). However, there is still an intractable expectation over all trajectories that end in $s_0$. Shah et al. (2019) use dynamic programming to compute this gradient in tabular settings.

## 2.2 GRADIENT AS BACKWARDS-FORWARDS CONSISTENCY

**Approximating the expectation.** For higher-dimensional environments, we must approximate the expectation over past trajectories $p(\tau_{-T:-1} \mid s_0, \theta)$. We would like to sample from the distribution, but it is not clear how to sample the past conditioned on the present. Our key idea is that just as we can sample the future by rolling out forwards in time, *we should be able to sample the past by rolling out backwards in time*. Note that by the Markov property we have:

$$p(\tau_{-T:-1} \mid s_0, \theta) = \prod_{t=-1}^{-T} p(a_t \mid s_{t+1}, a_{t+1}, \ldots s_0, \theta) p(s_t \mid a_t, s_{t+1}, \ldots s_0, \theta)$$

$$= \prod_{t=-1}^{-T} p(a_t \mid s_{t+1}, \theta) p(s_t \mid a_t, s_{t+1}, \theta)$$

Thus, given the *inverse policy* $\pi_t^{-1}(a_t \mid s_{t+1}, \theta)$ and the *inverse dynamics* $\mathcal{T}_t^{-1}(s_t \mid a_t, s_{t+1}, \theta)$, as well as the observed state $s_0$, we can sample a past trajectory $\tau_{-T:-1} \sim \langle s_0, \pi^{-1}, \mathcal{T}^{-1} \rangle$. Thus, our gradient can be written as $\mathbb{E}_{\tau_{-T:-1} \sim \langle s_0, \pi^{-1}, \mathcal{T}^{-1} \rangle} [\nabla_\theta \ln p(\tau \mid \theta)]$.

**Learning $\pi$, $\pi^{-1}$ and $\mathcal{T}^{-1}$.** In order to learn $\pi^{-1}$, we must first have $\pi$. We assumed that the human was Boltzmann-rational, which corresponds to the *maximum entropy* reinforcement learning objective (Levine, 2018). We use Soft Actor-Critic (SAC) (Haarnoja et al., 2018) to estimate the policy $\pi(a \mid s, \theta)$, since it explicitly optimizes the maximum entropy RL objective.

Given the forward policy $\pi(a \mid s, \theta)$ and simulator $\mathcal{T}$, we can construct a dataset of sampled forward trajectories, and learn the inverse policy $\pi^{-1}$ and the inverse dynamics $\mathcal{T}^{-1}$ using supervised learning. Given these, we can then sample $\tau_{-T:-1}$, allowing us to approximate the expectation in the gradient. In general, both $\pi^{-1}$ and $\mathcal{T}^{-1}$ could be stochastic and time-dependent.

**Estimating the gradient for a trajectory.** We now turn to the term within the expectation. Assuming that the user is Boltzmann-rational, this is the Maximum Causal Entropy Inverse Reinforcement Learning gradient (Ziebart et al., 2010), which can be written as (Shah et al., 2019, Appendix A):

$$\nabla_\theta \ln p(\tau \mid \theta) = \left( \sum_{t=-T}^{0} \phi(s_t) \right) - \mathcal{F}_{-T}(s_{-T}) + \sum_{t=-T}^{-1} \left( \mathbb{E}_{s'_{t+1} \sim \mathcal{T}(\cdot \mid s_t, a_t)} \left[ \mathcal{F}_{t+1}(s'_{t+1}) \right] - \mathcal{F}_{t+1}(s_{t+1}) \right) \tag{2}$$

$\mathcal{F}$ is the expected feature count under $\pi$, that is, $\mathcal{F}_t(s_t) \triangleq \mathbb{E}_{\tau_{t:0} \sim \langle s_t, \pi, \mathcal{T} \rangle} \left[ \sum_{t'=t}^{0} \phi(s_{t'}) \right]$.

The last term is essentially a correction for the observed dynamics: if we see that $s_t, a_t$ led to $s_{t+1}$, it corrects for the fact that we "could have" seen some other state $s'_{t+1}$. Since this correction is zero in expectation (and expensive to compute), we drop it for our estimator.

**Gradient estimator.** After dropping the last term in Equation 2, expanding the definition of $\mathcal{F}$, and substituting in to Equation 1, our final gradient estimator is:

$$\nabla_\theta \ln p(s_0 \mid \theta) = \mathbb{E}_{\tau_{-T:-1} \sim \langle s_0, \pi^{-1}, \mathcal{T}^{-1} \rangle} \left[ \left( \sum_{t=-T}^{0} \phi(s_t) \right) - \mathbb{E}_{\tau' \sim \langle s_{-T}, \pi, \mathcal{T} \rangle} \left[ \left( \sum_{t=-T}^{0} \phi(s'_t) \right) \right] \right] \tag{3}$$

Thus, given $s_0, \theta, \pi, \mathcal{T}, \pi^{-1}$, and $\mathcal{T}^{-1}$, computing the gradient consists of three steps:

1. Simulate backwards from $s_0$, and compute the feature counts of the resulting trajectories.

2. Simulate forwards from $s_{-T}$ of these trajectories, and compute their feature counts.

3. Take the difference between these two quantities.

Since $r(s) = \theta^T \phi(s)$, the gradient increases the reward of backward features, moving it towards "what the human must have done", and decreases the reward of forward features, moving it away from "what the human would do if they had this reward". Thus, the gradient incentivizes *consistency* between the backwards and forwards trajectories, by changing $\theta$ so that the forwards trajectories become closer to the backwards trajectories. The gradient becomes zero when they are identical.

It may seem like the backwards and forwards trajectories should always be consistent with each other, since $\pi^{-1}$ and $\mathcal{T}^{-1}$ are inverses of $\pi$ and $\mathcal{T}$. The key difference is that $s_0$ imposes constraints on the backwards trajectories, but not on the forward trajectories. For example, suppose we observe $s_0$ in which a vase is unbroken, and our current hypothesis is that the user wants to break the vase. When we simulate backwards, our trajectory will contain an unbroken vase, but when we simulate forwards from $s_{-T}$, $\pi$ will break the vase. The gradient would then reduce the reward for a broken vase and increase the reward for an unbroken vase.

## 2.3 LEARNING A LATENT MDP

Our gradient still relies on a feature function $\phi$, with the reward parameterized as $r(s) = \theta^T \phi(s)$. A natural way to remove this assumption would be to instead allow $\theta$ to parameterize a neural network, which can then learn whatever features are relevant to the reward from the RLSP gradient.

However, this approach will not work. The information contained in the RLSP gradient is insufficient to identify the appropriate features to construct: after all, it is derived from a single state. If we were to learn a single unified reward using the same gradient, the resulting reward would likely be degenerate: for example, it may simply identify the observed state, that is $R(s) = \mathbb{1}[s = s_0]$.

Thus, we continue to assume that the reward is linear in features, and instead *learn the feature function* using self-supervised learning (Oord et al., 2018; He et al., 2020). A recurrent state space model (RSSM) (Karl et al., 2017; Doerr et al., 2018; Hafner et al., 2019; 2020; Buesing et al., 2018; Kurutach et al., 2018) would let us handle partial observability: such methods aim to learn a *latent MDP* in which the states satisfy the Markov property. They do so by allowing the states to be computed by a recurrent model over the observations, thus allowing the states to encode history. When using such a model, we can imagine that the underlying POMDP has been converted into a latent MDP whose feature function $\phi$ is the identity. We can then compute gradients directly in this latent MDP.

Unfortunately, we found that existing RSSM methods were too brittle for our experiments, and so we used a variational autoencoder (VAE) (Kingma & Welling, 2014) instead. Since our experiments do not involve partial observability, this was not a problem.

## 2.4 DEEP RLSP

Putting these components together gives us the Deep RLSP algorithm (Algorithm 1). We update $\theta$ using Equation 3, and continually train $\pi$, $\pi^{-1}$, and $\mathcal{T}^{-1}$ alongside $\theta$ to keep them up to date. The full algorithm also adds a few bells and whistles that we describe below.

**Initial state distribution $\mathcal{P}$.** The attentive reader may wonder why our gradient appears to be independent of $\mathcal{P}$. This is actually not the case: while $\pi$ and $\mathcal{T}$ are independent of $\mathcal{P}$, $\pi^{-1}$ and $\mathcal{T}^{-1}$ *do* depend on it. For example, if we observe Alice exiting the San Francisco airport, the corresponding $\pi^{-1}$ should hypothesize different flights if she started from New York than if she started from Tokyo.

However, in order to actually produce such explanations, we must train $\pi^{-1}$ and $\mathcal{T}^{-1}$ solely on trajectories of length $T$ starting from $s_{-T} \sim \mathcal{P}$. We instead train $\pi^{-1}$ and $\mathcal{T}^{-1}$ on a variety of trajectory data, which loses the useful information in $\mathcal{P}$, but leads to several benefits. First, we can train the models on exactly the distributions that they will be used on, allowing us to avoid failures due to distribution shift. Second, the horizon $T$ is no longer critical: previously, $T$ encoded the separation in time between $s_{-T}$ and $s_0$, and as a result misspecification of $T$ could cause bad results. Since we now only have information about $s_0$, it doesn't matter much what we set $T$ to, and as a result we can use it to set a curriculum (discussed next). Finally, this allows Deep RLSP to be used in domains where an initial state distribution is not available.

Since we are no longer able to use the information from $\mathcal{P}$ through $\pi^{-1}$ and $\mathcal{T}^{-1}$, we add in a heuristic to incorporate the information elsewhere. Specifically, we weight every backwards trajectory, cosine-similarity between the final state $s_{-T}$, and a sample $\hat{s}_{-T} \sim \mathcal{P}$.

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**Algorithm 1** The DEEP RLSP algorithm. The initial dataset of environment interactions $D$ can be constructed in many different ways: random rollouts, human play data, curiosity-driven exploration, etc. The specific method will determine the quality of the learned features.

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**procedure** DEEP RLSP($\{s_0\}, \mathcal{T}$)

    $D \leftarrow$ dataset of environment interactions

    Initialize $\phi_e, \phi_d, \pi, \pi^{-1}, \mathcal{T}^{-1}, \theta$ randomly.

    $\phi_e, \phi_d \leftarrow$ SelfSupervisedLearning($D$)            ▷ Train encoder and decoder for latent MDP

    $\mathcal{T}^{-1} \leftarrow$ SupervisedLearning($D$)                        ▷ Train inverse dynamics

    $T \leftarrow 1$                                                   ▷ Start horizon at 1

    **for** $i$ in [1..num_epochs] **do**

        $\pi \leftarrow$ SAC($\theta$)                                    ▷ Train policy

        $\{\tau\} \leftarrow$ Rollout($\pi, \mathcal{T}$)               ▷ Collect dataset by rolling out $\pi$

        $\pi^{-1} \leftarrow$ SupervisedLearning($\phi_e, \{\tau\}$)            ▷ Train inverse policy

        $\theta \leftarrow \theta + \alpha \times$ ComputeGrad($\{s_0\}, \pi, \mathcal{T}, \pi^{-1}, \mathcal{T}^{-1}, T, \phi_e$)     ▷ Update $\theta$

        **if** gradient magnitudes are sufficiently low **then**

            $T \leftarrow T + 1$                                 ▷ Advance horizon

    **return** $\theta, \phi_e$

**procedure** COMPUTEGRAD($\{s_0\}, \pi, \mathcal{T}, \pi^{-1}, \mathcal{T}^{-1}, T, \phi_e$)

    $\{\tau_{\text{backward}}\} \leftarrow$ Rollout($\{s_0\}, \pi^{-1}, \mathcal{T}^{-1}, T$)           ▷ Simulate backwards from $s_0$

    $\phi_{\text{backward}} \leftarrow$ AverageFeatureCounts($\phi_e, \{\tau_{\text{backward}}\}$)    ▷ Compute backward feature counts

    $\{s_{-T}\} \leftarrow$ FinalStates($\{\tau_{\text{backward}}\}$)

    $\{\tau_{\text{forward}}\} \leftarrow$ Rollout($\{s_{-T}\}, \pi, \mathcal{T}, T$)              ▷ Simulate forwards from $s_{-T}$

    $\phi_{\text{forward}} \leftarrow$ AverageFeatureCounts($\phi_e, \{\tau_{\text{forward}}\}$)       ▷ Compute forward feature counts

    **return** $\phi_{\text{backward}} - \phi_{\text{forward}}$

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**Curriculum.** Since the horizon $T$ is no longer crucial, we can use it to provide a curriculum. We initially calculate gradients with low values of $T$, to prevent compounding errors in our learned models, and making it easier to enforce backwards-forwards consistency, and then slowly grow $T$, making the problem harder. We found this to be a significant improvement in practice.

**Multiple input states.** If we get multiple independent $s_0$ as input, we average their gradients.

**Replay buffer.** We also maintain a replay buffer that persists across policy training steps.

## 3 EXPERIMENTS

### 3.1 GRIDWORLD ENVIRONMENTS

As a first sanity check, we consider the gridworld environments in Shah et al. (2019). In these stylized gridworlds, self-supervised learning should not be expected to learn the necessary features. For example, in the room with vase environment, the two door features are just some locations, with no patterns for self-supervised learning to identify these locations as important. So, we run Algorithm 1 without the feature learning and instead use the pre-defined feature function of the environments. With this setup we are able to use Deep RLSP to recover the desired behavior from a single state in all environments in which the exact RLSP algorithm is able to recover it. In Appendix A, we provide more details on the environments and how our experiments compare to the ones by Shah et al. (2019).

### 3.2 SETUP

To demonstrate that Deep RLSP can be scaled to complex, continuous, high-dimensional environments, we use the MuJoCo physics simulator (Todorov et al., 2012). We consider the *Inverted Pendulum*, *Half-Cheetah* and *Hopper* environments implemented in Open AI Gym (Brockman et al., 2016). The hyperparameters of our experiments are described in detail in Appendix B[1].

**Baselines.** To our knowledge, this is the first work to train policies using a single state as input. Due to lack of alternatives, we compare against GAIL (Ho & Ermon, 2016) using the implementation

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[1]We will provide source code for our experiments after the review phase.

| Environment | SAC | # states | Deep RLSP | AverageFeatures | Waypoints | GAIL |
|---|---|---|---|---|---|---|
| Inverted Pendulum | 1000 | 1 | 262 (246) | 6 (2) | N/A | **1000 (0)** |
| | | 10 | 605 (242) | 3 (1) | 4 (1) | **1000 (0)** |
| | | 50 | 258 (198) | 6 (4) | 3.7 (0.3) | **1000 (0)** |
| Cheetah (forward) | 13236 | 1 | **4833 (2975)** | **6466 (3343)** | N/A | -288 (55) |
| | | 10 | **6299 (559)** | **6245 (2352)** | -10 (23) | -296 (172) |
| | | 50 | **7657 (177)** | 4504 (2970) | -126 (38) | -54 (295) |
| Cheetah (backward) | 13361 | 1 | 5694 (2513) | **12443 (645)** | N/A | -335 (46) |
| | | 10 | 8102 (624) | **12829 (651)** | -80 (388) | -283 (45) |
| | | 50 | 7795 (551) | **11616 (178)** | -509 (87) | 2113 (1015) |
| Hopper (terminate) | 3274 | 1 | 80 (21) | 99 (45) | N/A | **991 (9)** |
| | | 10 | 168 (58) | 159 (126) | 58 (7) | **813 (200)** |
| | | 50 | 130 (12) | 65 (36) | 14 (4) | **501 (227)** |
| Hopper (penalty) | 3363 | 1 | **1964 (545)** | **2537 (363)** | N/A | 990 (9) |
| | | 10 | 564 (75) | **3103 (64)** | 709 (133) | 784 (229) |
| | | 50 | **2130 (744)** | **2078 (581)** | 1612 (785) | 508 (259) |

Table 1: Average returns achieved by the policies learned through various methods, for different numbers of input states. The states are sampled from a policy trained using SAC on the true reward function; the return of that policy is given as a comparison. Besides the SAC policy return, all values are averaged over 3 seeds and the standard error is given in parentheses. We don't report Waypoints on 1 state as it is identical to AverageFeatures on 1 state.

from the `imitation` library (Wang et al., 2020). For each state we provide to Deep RLSP, we provide a transition $(s, a, s')$ to GAIL. Note that although GAIL does get more information than Deep RLSP, it is not designed to handle settings with such sparse data.

**Ablations.** In Section 2.2, we derived a gradient for Deep RLSP that enforces consistency between the backwards and forwards trajectories. However, we could also ignore the temporal information altogether. If an optimal policy led to the observed state $s_0$, then it is probably a good bet that $s_0$ is high reward, and that the agent should try to keep the state similar to $s_0$. Thus, we can simply set $\theta = \frac{\phi(s_0)}{||\phi(s_0)||}$, and not deal with $\pi^{-1}$ and $\mathcal{T}^{-1}$ at all.

How should we handle multiple states $s_0^1, \ldots, s_0^N$? Given that these are all sampled i.i.d. from rollouts of an optimal policy, a natural choice is to simply average the feature vectors of all of the states, which we call *AverageFeatures*. Alternatively, we could view each of the observed states as a potential *waypoint* of the optimal policy, and reward an agent for being near any one of them. We implement this *Waypoints* method as $R(s) = \max_i \frac{\phi(s_0^i)}{||\phi(s_0^i)||} \cdot \phi(s)$.

Note that both of these ablations still require us to learn the feature function $\phi$.

**Feature learning dataset.** By default, we use random rollouts to generate the initial dataset that is used to train the features $\phi$ and the inverse model $\mathcal{T}^{-1}$. (This is $D$ in Algorithm 1.) However, in the inverted pendulum environment, the pendulum falls very quickly in random rollouts, and $\mathcal{T}^{-1}$ never learns what a balanced pendulum looks like. So, for this environment only, we combine random rollouts with rollouts from an expert policy that balances the pendulum.

### 3.3 SOLVING THE ENVIRONMENTS WITHOUT ACCESS TO THE REWARD FUNCTION

First we look at the typical target behavior in each environment: balancing the inverted pendulum, and making the half-cheetah and the hopper move forwards. Additionally we consider the goal of making the cheetah run backwards (that is, the negative of its usual reward function). We aim to use Deep RLSP to learn these behaviors *without having access to the reward function*.

We train a policy using soft actor critic (SAC) (Haarnoja et al., 2018) to optimize for the true reward function, and sample either 1, 10 or 50 states from rollouts of this policy to use as input. We then

infer a reward, train a policy by running SAC on the inferred reward function, and evaluate the policy on the true reward function (which Deep RLSP does not have access to).

Results are shown in Table 1. In Hopper, we noticed that videos of the policies learned by Deep RLSP looked okay, but the quantitative evaluation said otherwise. It turns out that the policies learned by Deep RLSP do jump, as we might want, but they often fall down, terminating the episode; in contrast GAIL policies stand still or fall over slowly, leading to later termination and explaining their better quantitative performance. We wanted to also evaluate the policies without this termination bias, and so we evaluate the same policies in an environment that does not terminate the episode, but provides a negative reward instead; in this evaluation both Deep RLSP and AverageFeatures perform much better. We also provide videos of the learned policies at `https://sites.google.com/view/deep-rlsp`, which show that the policies learned by Deep RLSP do exhibit hopping behavior (though with a strong tendency to fall forward).

GAIL is only able to learn a truly good policy for the (very simple) inverted pendulum. Deep RLSP on the other hand achieves reasonable behavior (though clearly not expert behavior) in all of the environments. Surprisingly, the AverageFeatures method also performs quite well, even beating the full algorithm on some tasks, though failing quite badly on Pendulum. It seems that the task of running forward or backward is very well specified by a single state, since it can be inferred even without any information about the dynamics (except that which is encoded in the features learned from the initial dataset).

## 3.4 LEARNING SKILLS FROM A SINGLE STATE

We investigate to what extent Deep RLSP can learn other skills where the reward is not clear. Evaluation on these tasks is much harder, because there is no ground truth reward. Therefore we evaluate qualitatively how similar the policies learned by Deep RLSP are to the original skill.

We consider skills learned by running Dynamics-Aware Unsupervised Discovery of Skills (DADS) (Sharma et al., 2020). Since we are not interested in navigation, we remove the "x-y prior" used to get directional skills in DADS. We run DADS on the half-cheetah environment and select all skills that are not some form of running. This resulted in two skills: one in which the cheetah is moving forward making big leaps (*"jumping"*) and one in which it is slowly moving forward on one leg (*"balancing"*). As before we roll out these policies and sample individual states from the trajectories to provide as an input for Deep RLSP. We then evaluate the policy learned by Deep RLSP. Since the best evaluation here is to simply watch what the learned policy does, we provide videos of the learned policies at `https://sites.google.com/view/deep-rlsp`. We also provide visualizations in Appendix C.

The first thing to notice is that only Deep RLSP is even close to imitating the skill. None of the other policies resemble the original skills at all. While AverageFeatures could perform well on simple tasks such as running, the full algorithm is crucial to imitate more complex behavior.

Deep RLSP can learn the balancing skill fairly well from a single state, which we visualize in Figure 2 (though we emphasize that the videos are much clearer). Like the original skill, the learned policy balances on one leg and slowly moves forward by jumping, though with slightly more erratic behavior. However, the learned policy sometimes drops back to its feet or falls over on its back. We suspect this is an artifact of the short horizon ($T \leq 30$) used for simulating the past in our algorithm. A small horizon is necessary to avoid compounding errors in the learned inverse dynamics model, but can cause the resulting behavior to be more unstable on time-scales greater than $T$.

However, the jumping behavior is harder to learn, especially from a single state. We speculate that it is less informative than the balancing state. In the balancing state, the low joint velocities tell us that the cheetah is not performing a flip, suggesting that we had optimized for this specific balancing state. On the other hand, with the jumping behavior, we only get a single state of the cheetah in the air with high velocity, which is likely not sufficient to determine what the jump looked like exactly.

In line with this observation we see a significant improvement of the learned jumping policy if we provide Deep RLSP with 10 or 50 input states instead of a single state. Given a single state it learns a policy that makes little jumps that bear some resemblance to the original skill; with 10 states as input the jumps are already bigger; and given 50 states the learned policy closely mimics the original skill.

Figure 2: We sample a few states from a policy performing a specific skill to provide as input. Here, Deep RLSP learns to balance the cheetah on the front leg from a *single state*. We provide videos of the original skills and learned policies at: `https://sites.google.com/view/deep-rlsp`.

## 4 RELATED WORK

**Learning from human feedback.** Many algorithms aim to learn good policies from human demonstrations, including ones in imitation learning (Ho & Ermon, 2016) and inverse reinforcement learning (IRL) (Ng et al., 2000; Abbeel & Ng, 2004; Fu et al., 2018). Useful policies can also be learned from other types of feedback, such as preferences (Christiano et al., 2017), corrections (Bajcsy et al., 2017), instructions (Bahdanau et al., 2019), or combinations of feedback modalities (Ibarz et al., 2018).

While these methods require expensive human feedback, Deep RLSP instead *simulates* the trajectories that must have happened. This is reflected in the algorithm: in Equation 1, the inner gradient corresponds to an inverse reinforcement learning problem. While we used the MCEIRL formulation (Ziebart et al., 2010), other IRL algorithms could be used instead (Fu et al., 2018).

**Learning from observations.** For many tasks, we have demonstrations *without action labels*, e.g. YouTube videos. Learning from Observations (LfO) (Torabi et al., 2018; Gandhi et al., 2019) aims to recover a policy from such demonstrations. Similarly to LfO, we do not have access to action labels, but our setting is further restricted to observing only a small number of states.

## 5 LIMITATIONS AND FUTURE WORK

**Summary.** Learning useful policies with neural networks requires significant human effort, whether it is done by writing down a reward function by hand, or by learning from explicit human feedback such as preferences or demonstrations. We showed that it is possible to reduce this burden by extracting "free" information present in the current state of the environment. This enables us to imitate policies in MuJoCo environments with access to just a few states sampled from those policies. We hope that Deep RLSP will help us train agents that are better aligned with human preferences.

**Learned models.** The Deep RLSP gradient depends on having access to a good model of $\pi$, $\mathcal{T}$, $\pi^{-1}$, and $\mathcal{T}^{-1}$. In practice, it was quite hard to train sufficiently good versions of the inverse models. This could be a significant barrier to practical implementations of Deep RLSP. It can also be taken as a sign for optimism: self-supervised representation learning through deep learning is fairly recent and is advancing rapidly; such advances will likely translate directly into improvements in Deep RLSP.

**Computational cost.** Imitation learning with full demonstrations can already be quite computationally expensive. Deep RLSP learns several distinct deep neural net models, and then *simulates* potential demonstrations, and finally imitates them. Unsurprisingly, this leads to increased computational cost.

**Multiagent settings.** In any realistic environment, there is not just a single "user" who is influencing the environment: many people act simultaneously, and the state is a result of joint optimization by all of them. However, our model assumes that the environment state resulted from optimization by a single agent, which will not take into account the fact that each agent will have constraints imposed upon them by other agents. We will likely require new algorithms for such a setting.

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## A GRIDWORLD ENVIRONMENTS

Here we go into more detail on the experiments in Section 3.1, in which we ran Deep RLSP on the environment suite constructed in the original RLSP paper Shah et al. (2019).

In this test suite, each environment comes equipped with an *observed state* $s_0$, an *initial state* $s_{-T}$, a specified reward $R_{\mathrm{spec}}$, and a true reward $R_{\mathrm{true}}$. A given algorithm should be run on $s_0$ and optionally also $s_{-T}$ and produce an inferred reward $R_{\mathrm{inferred}}$. This is then added to the specified reward to produce $R_{\mathrm{final}} = R_{\mathrm{spec}} + \lambda R_{\mathrm{inferred}}$, where $\lambda$ is a hyperparameter that determines the weighting between the two. An optimal policy for $R_{\mathrm{final}}$ is then found using value iteration, and the resulting policy is evaluated according to $R_{\mathrm{true}}$.

There is no clear way to set $\lambda$: it depends on the scales of the rewards. We leverage the fact that $R_{\mathrm{spec}}$ is deliberately chosen to incentivize bad behavior, such that we know $\lambda = 0$ will always give incorrect behavior. So, we normalize $R_{\mathrm{inferred}}$, and then increase $\lambda$ from 0 until the behavior displayed by the final policy changes.

Figure 3: Reproduction of part of Figure 2 in Shah et al. (2019) illustrating the gridworld environments that we test on.

From left to right, the environments are:

1. Room with vase: We observe a state in which the vase is unbroken. This suggests that the human deliberately avoided breaking the vase, and so the inferred reward should put a negative weight on broken vases. Deep RLSP indeed does this and so avoids breaking the vase.

2. Toy train: In this environment, we observe a state in which an operational train is moving around a track. Once again, the inferred reward should put a negative weight on broken trains. Deep RLSP does this and avoids breaking the train.

3. Batteries: We observe a state in which the human has put a battery in the train to keep it operational. Algorithms should infer that it is good to put batteries in the train to keep it operational, even though this irreversibly uses up the battery. Deep RLSP correctly does this.

4. Apples: We observe a state in which the human has collected some apples and placed them in a basket. Algorithms should infer that it is good for there to be apples in the basket. Deep RLSP does this, learning a policy that continues to collect apples and place them in the basket.

5. Room with far away vase: This is an environment that aims to show what *can't* be learned: in this case, the breakable vase is so far away, that it is not much evidence that the human has not broken it so far. As a result, algorithms should *not* learn anything significant about whether or not to break vases. This is indeed the case for Deep RLSP.

Overall, Deep RLSP has the same behavior on these environments as RLSP.

## B  ARCHITECTURE AND HYPERPARAMETER CHOICES

In this section we describe the architecture choices for all models used in our algorithm and the hyperparameter choices in our experiments. All models are implemented using the TensorFlow framework.

### B.1  FEATURE FUNCTION

We use a variational autoencoder (VAE) (Kingma & Welling, 2014) to learn the feature function. The encoder and decoder consist of 3 feed-forward layers of size 512. The latent space has dimension 30. The model is trained for 100 epochs on 100 rollouts of a random policy in the environment. During training we use a batch size of 500 and learning rate $10^{-5}$. We use the standard VAE loss function, but weight the KL-divergence term with a factor $c = 0.001$, which reduces the regularization and empirically improved the reconstruction of the model significantly in our experiments. We hypothesize that the standard VAE regularizes too much in our setting, because the latent space has a higher dimension than the input space, which is not the case in typical dimensionality reduction settings.

### B.2  INVERSE DYNAMICS MODEL

Our inverse dynamics model is a feed-forward neural network with 5 layers of size 1024 with ReLU activations. We train it on 1000 rollouts of a random policy in the environment for 100 epochs, with a batch size 500 and learning rate $10^{-5}$.

Note that the model predicts the previous observation given the current observation and action; it does not use the feature representation. We found the model to perform better if it predicts the residual $o_{t-1} - o_t$ given $o_t$ and $a_t$ instead of directly predicting $o_{t-1}$.

We normalize all inputs to the model to have zero mean and unit variance. To increase robustness, we also add zero-mean Gaussian noise with standard deviation 0.001 to the inputs and labels during training and clip the outputs of the model to the range of values observed during training.

### B.3  POLICY

For learning the policy we use the *stable-baselines* implementation of Soft Actor-Critic (SAC) with its default parameters for the MuJoCo environments (Haarnoja et al., 2018; Hill et al., 2018). Each policy update during Deep RLSP uses $10^4$ total timesteps and we evaluate the final reward function using $2 \times 10^6$ timesteps.

### B.4  INVERSE POLICY

Because the inverse policy is not a deterministic function, we represent it with a *mixture density network*, a feed-forward neural network that outputs a mixture of Gaussian distributions (Bishop, 1994).

The network has 3 layers of size 512 with ReLU activations and outputs a mixture of 5 Gaussians with a fixed diagonal covariance matrix $0.05 \cdot I$.

During Deep RLSP, we maintain an experience replay that is initialized with random rollouts. In every iteration all states encountered during the algorithm are added to the experience replay.

To update the inverse policy we sample batches with batch size 500 from the experience replay, apply the forward policy and the forward transition model on the states to label the data. We then train the model with a learning rate of $10^{-4}$.

### B.5  DEEP RLSP HYPERPARAMETERS

We run Deep RLSP with a learning rate of 0.01, and use 200 forward and backward trajectories to estimate the gradients. Starting with $T = 1$ we increment the horizon when the gradient norm drops below 2 or after 10 steps, whichever comes first. We run the algorithm until $T = 30$.

## C  VISUALIZATION OF LEARNED SKILLS

Here we provide larger visualizations of the skills learned in the experiments discussed in Section 3.4 of the main paper. For each experiment we show the original policy, the states sampled from this policy and given as an input to Deep RLSP, the policy learned by the AverageFeatures ablation, and the policy learned by Deep RLSP in figs. 4 to 9 (on future pages). We don't show the Waypoints ablation and the policy learned by GAIL, because they are not close to the desired skill. Again, we emphasize that the visual comparison is easier with videos of the policies which we provide at `https://sites.google.com/view/deep-rlsp` (including Waypoints and GAIL).

## D  THINGS WE TRIED THAT DID NOT WORK

Here we list a few variations of the Deep RLSP algorithm that we tested on the MuJoCo environments that failed to provide good results.

- As mentioned in the main paper, we tried to learn a latent state-space jointly with a dynamics model in the latent space using a recurrent state-space model (RSSM). However, we found the model too brittle to reliably learn a good dynamics model. In particular, we found the reward function and policy learned by Deep RLSP worked in the RSSM but did not generalize to the actual environment.

- We also tried learning a supervised dynamics model from the initial set of rollouts, similarly to how we learn an inverse dynamics model, rather than relying on the simulator $\mathcal{T}$. However, we found this to cause a similar issue as the RSSM: the reward function and policy learned by Deep RLSP did not generalize to the actual environment. However, we hope that progress in model-based RL will allow us to implement Deep RLSP using only learned dynamics models in the future.

- Using an MDN instead of an MLP to model the inverse dynamics did not improve the performance of the algorithm. We suspect this to be because in the MuJoCo simulator the dynamics and the inverse dynamics are "almost deterministic".

- Updating the inverse dynamics model and the feature function during Deep RLSP by training it on data from the experience replay did not improve performance and in some cases significantly decreased performance. The decrease in performance seems to have been caused by the feature function changing too much and the training of the other models suffering from catastrophic forgetting as a result.

Figure 4: Deep RLSP learning the balancing skill from a single state. The first row shows the original policy from DADS, the second row shows the sampled state from this policy, the third row is the AverageFeatures algorithm that fails to learn the skill, and the last row shows the policy learned by Deep RLSP.

Figure 5: Deep RLSP learning the balancing skill from 10 states. The first row shows the original policy from DADS, the second row shows the sampled states from this policy, the third row is the AverageFeatures algorithm that fails to learn the skill, and the final row shows the policy learned by Deep RLSP.

Figure 6: Deep RLSP learning the balancing skill from 50 states. The first row shows the original policy from DADS, the next five rows show the sampled states from this policy, the second to last row is the baseline algorithm, and the last row shows the policy learned by Deep RLSP. In this case both approaches fail to learn the desired behavior. The AverageFeatures policy falls over immediately, while the Deep RLSP policy balances slightly and then falls over too.

Figure 7: Deep RLSP learning the jumping skill from a single state. The first row shows the original policy from DADS, the second row shows the sampled state from this policy, the third row is the AverageFeatures algorithm that fails to learn the skill, and the last row shows the policy learned by Deep RLSP. The AverageFeatures policy jumps in place, but falls over immediately.

Figure 8: Deep RLSP learning the jumping skill from 10 states. The first row shows the original policy from DADS, the second row shows the sampled states from this policy, the third row is the AverageFeatures algorithm that fails to learn the skill, and the final row shows the policy learned by Deep RLSP. The AverageFeatures policy falls over backward when jumping.

Figure 9: Deep RLSP learning the jumping skill from 50 states. The first row shows the original policy from this policy, the second to last row is the AverageFeatures algorithm, and the last row shows the policy learned by Deep RLSP. Here the AverageFeatures policy just stands still.