MEASURING, EVALUATING AND IMPROVING LOGICAL CONSISTENCY IN LARGE LANGUAGE MODELS

Anonymous authors
Paper under double-blind review

ABSTRACT

Recent research in Large Language Models (LLMs) has shown promising progress related to LLM alignment with human preferences. LLM-empowered decision-making systems are expected to be predictable, reliable and trustworthy, which implies being free from paradoxes or contradictions that could undermine their credibility and validity. However, LLMs still exhibit inconsistent and biased behaviour when making decisions or judgements. In this work, we focus on studying *logical consistency of LLMs* as a prerequisite for more reliable and trustworthy systems. Logical consistency ensures that decisions are based on a stable and coherent understanding of the problem, reducing the risk of erratic or contradictory outputs. We first propose a universal framework to quantify the logical consistency via three fundamental proxies: *transitivity*, *commutativity* and *negation invariance*. We then evaluate logical consistency, using the defined measures, of a wide range of LLMs, demonstrating that it can serve as a strong proxy for overall robustness. Additionally, we introduce a data refinement and augmentation technique that enhances the logical consistency of LLMs without sacrificing alignment to human preferences. It augments noisy and sparse pairwise-comparison annotations by estimating a partially or totally ordered preference rankings using rank aggregation methods. Finally, we show that logical consistency impacts the performance of LLM-based logic-dependent algorithms, where LLMs serve as logical operators.

1 INTRODUCTION

Recent research in Large Language Models (LLMs; Brown et al. 2020; OpenAI 2023; Anil et al. 2023a;b) has achieved substantial progress concerning their instruction-following capabilities and generating responses aligned with human preferences. Consequently, these advancements have broadened their application to an even wider range of complex natural language tasks across diverse domains, often with minimum or even no supervised data (Kojima et al., 2022). The instruction-following abilities of LLMs are primarily achieved through supervised training with instruction-following data (Wei et al., 2022a; Chung et al., 2022) and reinforcement learning from human feedback (RLHF; Christiano et al. 2017; Stiennon et al. 2020; Ouyang et al. 2022b). During the alignment phases, LLMs are enhanced to align with human values, enabling them to generate responses that better support their decision-making and problem-solving (Dai et al., 2024).

Despite these advancements, key challenges still exist regarding the reliability and trustworthiness of LLMs. Issues such as hallucination (Zhang et al., 2023), bias (Gallegos et al., 2024), and inconsistencies in reasoning (Huang & Chang, 2023) continue to affect their credibility. These limitations hinder the full practical deployment of LLMs, particularly in professional and high-stakes applications where *trustworthiness* and *reliability* are crucial.[1] The foundation of a reliable and trustworthy system is the **consistency** of its predictions. A consistent system produces explainable and tractable decisions, enhancing its dependability and reliability. In this work, we focus on a key form of consistency in LLMs: **logical consistency**. This is especially critical for applications requiring structured reasoning and coherent decision-making (Creswell et al., 2023). Logical inconsistencies can lead to unreliable conclusions (Restall, 2002) and even paradoxes (Hyde, 2011), posing significant

[1]For instance, in the fields of behavioral economics and psychology systems are traditionally evaluated on two key dimensions: *validity* and *reliability* (Schmidt et al., 2000; Guion, 2004; Miller et al., 2023).

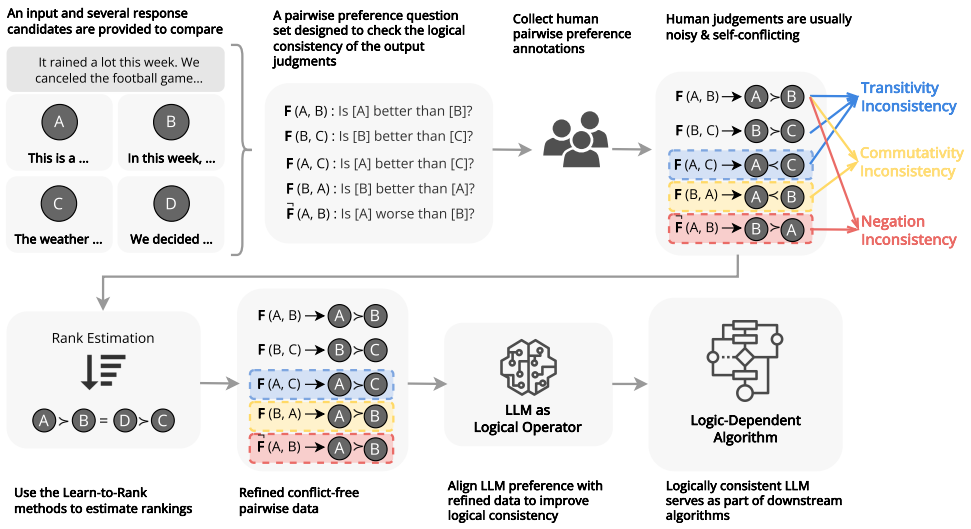

Figure 1: Three types of logical inconsistencies are observed in real-world pairwise annotations (top row): Transitivity, Commutativity, and Negation Invariance. By refining the data for self-consistency using rank estimation, we can train LLMs with enhanced logical consistency, improving their performance in logic-dependent algorithms (bottom row).

risks in domains that demand rigorous logical judgment, such as temporal or spatial reasoning (Mostafazadeh et al., 2016b), optimization (Guo et al., 2024), and automated decision systems.

In this paper, we focus on three fundamental aspects of logical consistency: **transitivity**, **commutativity**, and **negation invariance**. These properties are crucial for a coherent understanding of relationships between items, and our goal is to measure, evaluate, and enhance them in LLMs. We propose a universal framework for quantifying these consistency properties, applicable to any number of items, supporting both partial and complete comparisons, and adaptable across various domains. Our empirical investigation spans a diverse set of LLMs, including Llama 3 (Dubey et al., 2024), Gemma (Rivière et al., 2024), and Phi-3 (Abdin et al., 2024), among others. Through extensive evaluations across three representative domains, we find a strong correlation between logical consistency and the internal robustness and reliability of LLMs. This suggests that logical consistency could be a valuable (proxy) indicator of overall model reliability.

To enhance the logical consistency of LLM judgements, we propose a data refinement and augmentation framework that improves noisy and sparse pairwise comparison annotations by estimating preference rankings using rank aggregation methods. The framework then augments the data by 'logically extrapolating' additional pairwise comparisons from these rankings. Our results show that LLMs trained with this approach achieve improved internal logical consistency without compromising alignment with human preferences. Furthermore, we demonstrate that, when applied to logic-dependent downstream applications, LLMs with better logical consistency outperform less consistent models. They show better performance and reduced computational requirements when employed as logical operators in a sorting-based preference ranking aggregation algorithm (Liu et al., 2024) that relies on logical coherence.

In sum, our contributions are as follows: **1)** We argue that logical consistency, in addition to human alignment, should be taken into account as an essential factor when aligning LLMs' preferences. We present a general mathematical formulation for quantifying/measuring three key properties of logical consistency: transitivity, commutativity and negation invariance. **2)** We then conduct extensive experiments to evaluate logical consistency across a range of state-of-the-art LLMs and analyze the correlations between consistency and models' reliability. **3)** We propose a data refinement and augmentation method for instruction-tuning that enhances logical consistency while maintaining alignment with human preferences and **4)** we show that LLMs with improved logical consistency yield improved performance in logic-dependent algorithms where LLMs are employed as logical operators. We will release our source code and refined dataset at `ANONYMOUS` to facilitate future work.

2 MEASURING LOGICAL CONSISTENCY

We evaluate the logical consistency of LLMs by assessing their ability to predict logically consistent relations among a *set* of *items*. These items could represent diverse *entities* or *events* with a uniform *relation* between them; such a relation might be (i) comparing the preference among response candidates to a query or (ii) determining the causal order of shuffled events, among other possibilities. This evaluation is grounded in relational algebra and order theory (Abiteboul et al., 1995; Imieliński & Lipski Jr, 1984), with the goal of assessing whether the model maintains coherent and self-consistent judgments across its predictions.

To formalize this concept, we define the *logical consistency evaluation process* by treating an LLM as an operator function that compares pairs of items and outputs a decision on the relation between the items. Let $X = \{x_1, x_2, \ldots, x_N\}$ represent a set of items, and we define a relation (e.g., comparison) function $F : X \times X \rightarrow R$, which compares two items, such as (x_i, x_j), and assigns a relational decision $F(x_i, x_j) = r$, where $r \in R$ denotes the directional relation between x_i and x_j. For simplicity, we consider R to be a binary relation set, $R = \{r_{ij}, r_{ji}\}$, where r_{ij} represents a preferential relation $x_i \succ x_j$ (i.e., item x_i is preferred over item x_j), and r_{ji} indicates the reverse preference $x_j \succ x_i$.

In evaluating logical consistency, we focus on whether the function F adheres to the following key properties over the item set X: *1) transitivity*, *2) commutativity*, and *3) negation invariance*, as demonstrated in Figure 1. These properties are foundational for logically consistent operations and are critical for determining the reliability and reasoning capability of an LLM (Hamon et al., 2020).

Transitivity ensures that the LLM's predictions and judgements are internally coherent and do not suffer from logical contradictions within a given context. Commutativity tests whether the model's decisions are invariant to the order in which items are compared. The model should produce consistent judgments regardless of whether the pair (x_i, x_j) or (x_j, x_i) is presented first. The negation invariance checks whether the model maintains consistency when dealing with relational negations. Inconsistencies here would indicate a failure to correctly understand and apply the complementary nature of relations. By systematically applying these tests, we are able to assess the extent to which the model's outputs conform to logically consistent behavior, providing a quantitative proxy measure of its decision reliability.

2.1 MEASURING TRANSITIVITY

Grounded in our problem setup and definitions above, respecting transitivity implies the following: if a model predicts $A \succ B$ and $B \succ C$, it must also predict $A \succ C$. The ability to make transitive predictions is critical to forming a robust, global understanding of relationships within a set of items. Without transitivity, the model's reasoning would be prone to contradictions, which could lead to unreliable outcomes in tasks requiring consistent decision-making or ranking (Liu et al., 2024; Li et al., 2019; Qin et al., 2024).

We define that the function F is fully transitive if it does not predict any intransitive relation within the set X. This means that if $F(x_i, x_j) = r_{ij}$ and $F(x_j, x_k) = r_{jk}$, then $F(x_i, x_k) = r_{ik}$ must hold for all $i, j, k \in X$. This can be visualized in Figure 2, where we represent the pairwise relations by F as a relation graph. If F is transitive over X, the corresponding relation graph should be a Directed Acyclic Graph (DAG). A DAG implies that *no cycles* exist in the relation graph, denoting that there are no contradictions in the model's relational judgments. Consequently, to determine if F is fully transitive over the item set X, we only have to verify whether the predicted relation graph contains any cycle or not. We show how to construct the relation graph from the judgements of the LLM operator function F, and the algorithm to check whether a graph contains cycles in Appendix §A.

We introduce a metric, $s_{tran}(K)$, designed to quantify transitivity over an arbitrary number of items. LLMs often struggle to maintain perfect/full transitivity, especially as the number of items increases. The proposed metric $s_{tran}(K)$ captures the *degree of transitivity* across subsets of K sampled items, where $3 \leq K \leq |X|$. The metric is defined as:

$$s_{tran}(K) = \frac{1}{M} \sum_{i=1}^{M} \mathbb{1}_{\text{acyclic}}(S_i^K). \tag{1}$$

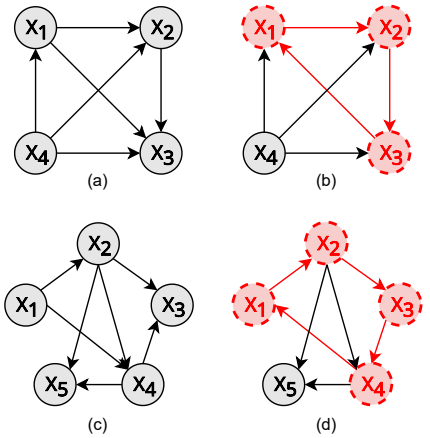

Figure 2: Example of relation graphs illustrating transitivity, where items are represented as nodes, and directed edges indicate pairwise preferential relations. Red dashed cycles in the graph highlight violations of transitivity. The cycle in (d), spanning 4 items, cannot be captured by $s_{tran}(3)$. The s_{tran} metric can be applied to partial relation graphs, as shown in (c) and (d).

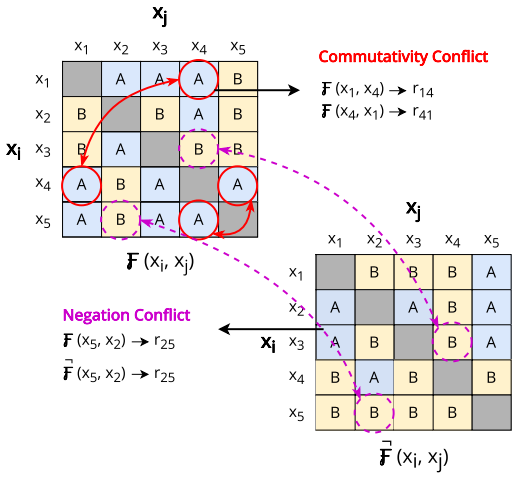

Figure 3: Examples illustrating violations of commutativity and negation invariance. Each entry of the two preference matrices represents predicted judgments of $x_i \succ x_j$ and $x_i \prec x_j$, labelled with A and B respectively. The top-left matrix is based on the original relation, while the bottom-right matrix reflects the negated relation. Linked red cycles highlight non-commutative pairs, and linked dashed purple cycles indicate negation inconsistencies.

Here, S_i^K represents a randomly sampled sub-graph of size K, and the indicator function $\mathbb{1}_{acyclic}$ returns 1 if the sub-graph S_i^K contains no cycles (i.e., is transitive), and 0 if otherwise. M denotes the total number of the sampled sub-graphs. As the size of the item set increases, the number of possible sub-graph combinations grows exponentially. To manage this complexity, we cap the number of samples to 1,000 sub-graphs to estimate transitivity for larger sets. Therefore, the metric ranges from 0 to 1, where 1 represents perfect transitivity.

Maintaining transitivity becomes increasingly difficult as the size of the subset grows. For any set of K items, there are 2^K possible combinations of pairwise relationships, but only $K!$ of these can form a transitive ranking. As a result, the degree of transitivity tends to decrease with larger sub-graph sizes. This means that for the same LLM, $s_{tran}(K)$ typically decreases as K increases, reflecting that preserving consistent rankings over larger sets of items is increasingly challenging. Since $s_{tran}(K)$ measures transitivity for a fixed subset size, it allows for fair transitivity comparisons between item sets of different sizes.

2.2 Measuring Commutativity

Commutativity refers to the logical property that ensures the model's judgments remain consistent when the order of comparison between two items is reversed. Prior studies have shown that LLMs are susceptible to permutation bias, also referred to as positional bias (Wang et al., 2024b; Liusie et al., 2024a). This bias can result in inconsistent outputs when the order of items in a comparison is altered. To measure the degree of commutativity, we propose a metric s_{comm}, which evaluates whether the model's judgment changes when the order of the items is swapped in the prompt. Specifically, it is defined as follows:

$$s_{comm} = \frac{2}{|X| \cdot (|X| - 1)} \sum_{0<i<j\leq|X|} \mathbb{1}(F(x_i, x_j) = F(x_j, x_i)). \tag{2}$$

Here, $F(x_i, x_j)$ represents the model's judgment when comparing items x_i and x_j. The indicator function $\mathbb{1}$ returns 1 if the model's judgment remains consistent when the order of the items is reversed, i.e., $F(x_i, x_j) = F(x_j, x_i)$, and 0 otherwise. We visualize this comparison in Figure 3. The normalization term ensures that s_{comm} is averaged across all pairwise combinations of the items in set X. As a result, the metric ranges from 0 to 1, with 1 indicating perfect commutativity, meaning that the model is completely robust to the order of item comparisons.

2.3 MEASURING NEGATION INVARIANCE

Negation invariance tests whether the model maintains consistency when confronted with the negation or inversion of a relational statement. Inconsistencies here could indicate a failure to correctly understand and apply the complementary nature of relations. Previous work suggested that LLMs struggle to automatically infer appropriate inverse relationships when acquiring knowledge (Allen-Zhu & Li, 2023; Berglund et al., 2024). To quantify negation invariance, we propose the metric s_{neg}, which examines if the model can correctly reverse its judgement when prompted with a negated relationship between items. The metric is defined as below:

$$s_{neg} = \frac{1}{|X| \cdot (|X| - 1)} \sum_{0<i,j\leq|X|, i\neq j} \mathbb{1}(\overset{\neg}{F}(x_i, x_j) = \neg F(x_i, x_j)). \tag{3}$$

In this formulation, $\neg F(x_i, x_j)$ represents the negation of the original relation(e.g., reversing a preference or relational direction). The comparison function $\overset{\neg}{F}(x_i, x_j)$ refers to the model's judgment when explicitly prompted with the negated relation. The indicator function returns 1 if the model's response to the negated relation matches the expected negated judgment (i.e., $\overset{\neg}{F}(x_i, x_j) = \neg F(x_i, x_j)$), and 0 otherwise. We also visualize this comparison in Figure 3. The normalization factor averages across all pairwise permutations in the set X, ensuring that s_{neg} ranges from 0 to 1. The maximum score of 1 indicates perfect negation invariance, where the model consistently handles negated relations.

3 EVALUATING LOGICAL CONSISTENCY OF LARGE LANGUAGE MODELS

After defining the measures quantifying the three aspects of logical consistency in Section 2, we now proceed to evaluate LLMs' judgements from the consistency angle on three representative tasks, each reflecting different levels of subjectivity.

3.1 EVALUATION SETUP

Tasks and Datasets. We employ three representative tasks to evaluate LLMs' logical consistency. The first task, *abstractive summarization evaluation*, uses the SummEval dataset (Fabbri et al., 2021) and focuses on the model's ability to make preference judgments between summaries, particularly regarding the coherence aspect. The second task, *document reranking*, leverages the NovelEval dataset (Sun et al., 2023), where LLMs assess the relevance of retrieved documents in response to queries. The final task, *temporal event ordering*, uses the CaTeRS dataset (Mostafazadeh et al., 2016b) and tests the model's capability to reason about temporal and causal relationships between events, critical for maintaining consistency in narrative understanding. Further task and dataset details are available in Appendix §B.

Metrics and Reliability Measurement. In Appendix §F, we show metric computation details and the prompt templates used. We compute the logical consistency metrics at the instance level and report averages across the test sets for each task. In addition, we report the human agreement rate (abbreviated as H.) by calculating the pairwise judgement accuracy between judgements made by LLMs and the provided human annotations. It serves as a reference for how closely the model's judgements align with human judgements. For the measurement of LLMs' reliability, we perform Monte Carlo Sampling of Chain-of-Thought (Wei et al., 2022b) reasoning outputs, using a temperature of 0.7. Self-agreement is defined as the percentage of outputs that agree with the majority judgment across multiple samples. This measurement ranges from 0.5 to 1, with higher values indicating greater stability in the model's reasoning processes.

3.2 RESULTS AND ANALYSIS

Performance of Different (Families of) Models. As shown in Table 1, recent LLMs like Gemma2-9B and Phi-3-medium demonstrate stronger overall consistency compared to earlier models. In particular, models such as Deepseek-chat, Phi-3-medium, and Gemma-2-9B perform well across all three evaluated consistency dimensions. However, it is important to note that strong performance in one aspect of consistency does not guarantee similar performance in others. For example, while Mistral-7B excels in transitivity, its performance in other consistency aspects is weaker.

Table 1: Logical consistency evaluation results. We report human accuracy (H.), transitivity over 5 items ($s_{tran}(5)$), commutativity (s_{comm}) and negation invariance (s_{neg}), all measured in accuracy.

Models	SummEval (Coh)				NovelEval				CaTeRS			
	H.	$s_{tran}(5)$	s_{comm}	s_{neg}	H.	$s_{tran}(5)$	s_{comm}	s_{neg}	H.	$s_{tran}(5)$	s_{comm}	s_{neg}
Direct Judgements												
Llama-2-7B	57.5	88.3	57.5	66.2	57.5	68.1	57.5	78.1	61.9	88.4	56.0	49.7
Llama-2-13B	58.3	86.6	59.3	84.0	58.3	88.2	62.9	76.9	65.6	95.3	70.8	54.5
Llama-3-8B	67.8	91.0	76.1	48.9	60.6	73.0	73.3	79.1	73.1	88.2	79.9	63.3
Mistral-7B	63.6	95.1	59.9	51.2	60.1	90.5	68.0	82.1	70.2	95.9	73.9	76.9
Zephyr-7B-beta	61.3	87.8	52.8	74.1	60.5	91.5	63.8	86.5	70.6	93.5	77.8	80.8
Phi-3-mini	65.6	92.8	66.9	75.1	59.7	92.5	55.5	39.4	60.2	**97.2**	85.5	73.3
Phi-3-medium	68.8	**96.2**	71.0	78.1	62.7	93.7	76.3	77.6	67.2	93.4	87.9	84.1
Gemma-2-9B	**73.8**	94.8	**78.1**	78.2	63.6	89.9	85.3	88.2	72.4	96.0	86.5	66.9
GPT-3.5-0125	66.3	82.5	67.5	65.8	61.2	89.8	71.6	83.4	69.3	86.2	66.7	81.7
Deepseek-chat	72.7	93.1	72.8	**84.9**	**65.7**	**95.1**	**86.7**	**89.2**	**73.5**	93.8	**91.9**	**91.1**
CoT Prompting												
Llama-2-7B	55.0	42.6	45.7	68.9	56.5	50.8	66.7	60.1	59.5	40.5	56.7	56.1
Llama-2-13B	66.6	55.8	63.2	40.3	59.2	60.8	71.0	56.4	60.6	61.1	63.5	74.3
Llama-3-8B	67.2	65.0	64.8	61.2	60.2	69.3	76.8	72.4	67.9	62.2	46.1	78.6
Mistral-7B	61.6	64.7	65.3	58.0	58.5	64.9	73.2	77.7	60.3	66.1	69.0	79.1
Zephyr-7B-beta	58.4	46.4	51.7	74.0	60.0	64.5	70.6	71.2	65.7	69.3	78.4	77.3
Phi-3-mini	65.1	67.9	61.9	43.0	56.5	61.2	47.8	80.5	58.4	81.2	82.6	78,7
Phi-3-medium	69.1	**82.3**	72.5	58.9	**63.1**	89.0	87.3	79.3	54.5	81.4	87.4	85.9
Gemma-2-9B	73.5	74.0	71.9	81.3	62.1	89.9	85.9	82.3	63.5	**83.8**	87.1	69.9
GPT-3.5-0125	63.9	63.3	64.8	68.4	59.3	79.3	73.4	67.5	62.8	68.6	70.5	74.9
Deepseek-chat	**74.9**	79.4	**76.1**	**82.8**	61.5	**94.2**	**90.4**	**83.0**	**69.1**	81.6	**88.2**	**88.4**

The Phi-3 family consistently shows high logical consistency, which we hypothesize is due to its reliance on synthesized data which is cleaner and less self-contradictory. This type of data likely reduces internal conflicts, resulting in more consistent logical behavior. However, no strong correlation is observed between logical consistency and human agreement accuracy. For instance, while Gemma-2-9B and Llama-3-8B achieve similar levels of human agreement, Gemma-2-9B shows higher consistency in its outputs. This is further confirmed by the low correlations (below 0.05) between transitivity consistency and human preference accuracy across all datasets.

Another observation is that LLMs tend to perform more consistently on the CaTeRS dataset. We attribute this to the dataset's focus on temporal and causal relations. The more objective and logical preferential relations may make it easier for models to maintain consistency.

Impact of CoT Prompting. We also investigated the effect of CoT prompting on logical consistency. Surprisingly, CoT reasoning did not generally improve consistency, and in some cases, it led to a decrease in transitivity performance. We hypothesize that this decline may be due to the additional reasoning tokens generated during CoT prompting, which could introduce variations in the judgement standards used for different comparisons. When a model's understanding of the preferential relations is not uniform, it may produce inconsistent (non-transitive) outcomes. This suggests that CoT prompting, while beneficial for complex reasoning, might introduce unintended inconsistencies in certain logical judgement tasks.

3.3 Consistency and Reliability

Transitivity as a Proxy for Self-Agreement. We next investigated the relationship between transitivity and self-agreement across different LLMs. As shown in Figure 4, there are strong correlations between transitivity and self-agreement across all three datasets, regardless of the task's level of subjectiveness. Self-agreement indicates the model's internal consistency and robustness, as a reliable model should not fluctuate significantly in its responses. Higher self-agreement suggests that the model exhibits a stable understanding of the underlying relations in the input. Given that transitivity captures this self-consistency, we argue that transitivity serves as a useful proxy for evaluating the local robustness of LLMs.

Commutativity Correlates Strongly with Human Preferences. For each task, we paraphrased the comparison prompt template 10 times using `GPT-4-turbo` to explore the sensitivity of LLMs to

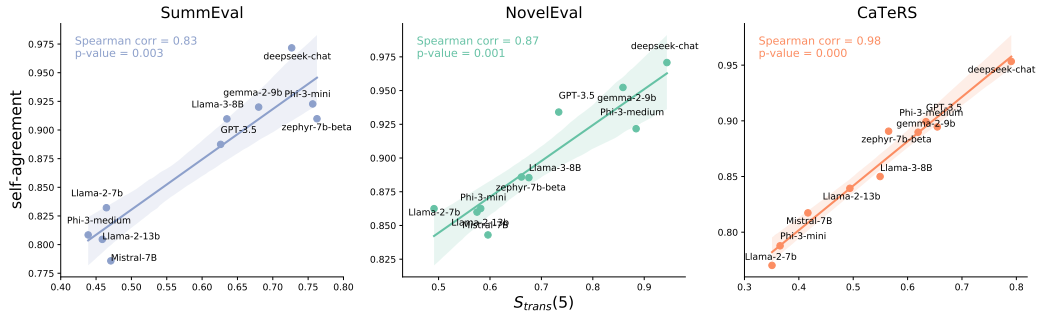

Figure 4: *Transitivity shows strong correlations with self-agreement* across all three datasets. Self-agreement is measured as the percentage of majority choices from 10 CoT inferences, generated with a temperature of 0.7.

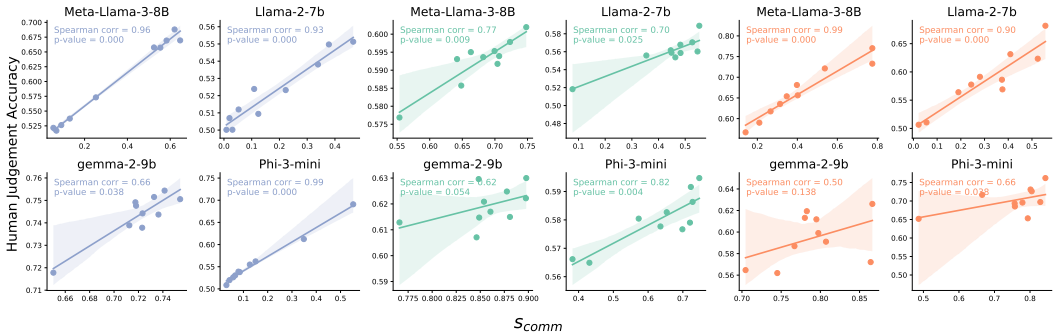

Figure 5: *Commutativity shows a generally strong correlation with human preference agreement rates* across various LLMs and all three datasets.

input variations. Despite these paraphrases being semantically equivalent, they produced different performance outcomes, as shown in Figure 5.

We found strong correlations between commutativity and human agreement rates across nearly all datasets and models. This finding is not surprising, as a lack of commutativity often indicates a strong positional bias. These results align with previous research (Zhou et al., 2024), which demonstrated that positional bias can significantly impact alignment with human judgments.

We also show that the strength of this correlation varies between models. For instance, Llama-3-8B exhibits a higher correlation with human preferences compared to Gemma-2-9B. We hypothesize that this difference is due to Gemma-2-9B's training, which was designed to be more robust to input prompts, while other models, like Llama-3-8B, are more fine-tuned to specific prompting styles.

4 A Data Refinement and Augmentation Approach to Improve Logical Consistency in LLMs

Our analysis so far has demonstrated that many LLMs exhibit varying degrees of logical (in)consistency when making judgements. To address this challenge, we propose a data refinement and augmentation framework that aims to mitigate this phenomenon. The idea is to filter out self-conflicting information from noisy preference data and generate additional conflict-free pairwise comparisons. This approach aims to enhance the logical consistency of LLMs while maintaining alignment with human preferences, enabling them to function more reliably as logical operators.

Motivation. Previous work has proven the benefits of using pairwise comparisons derived from preference rankings to train LLMs for a better understanding of preference orderings (Song et al., 2024). Additionally, Asai & Hajishirzi (2020) showed that incorporating logically derived counterparts of

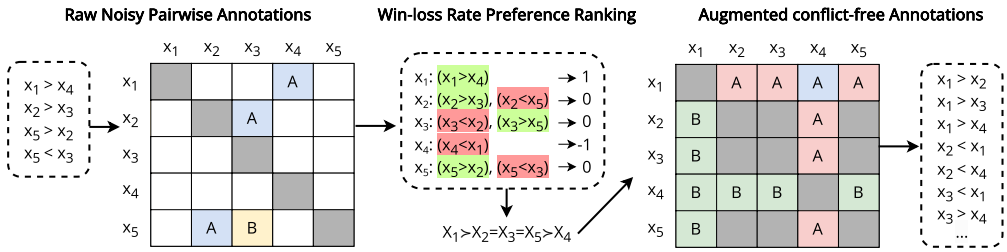

Figure 6: *Illustration of data refinement and augmentation* based on preference rank estimation using win-loss rates. The original annotations (left) are sparse and exhibit inconsistencies (non-transitivity), while the augmented annotations are consistent and include *more* comparisons. For partially ordered rankings, items are not compared when their preference relationship is unknown.

Table 2: Data statistics at each augmentation stages. # Data refers to data size and AvgComp/Inst for average comparisons per instance. Aug. data represents Augmented data, and Neg. denotes further augmentation with negated relation comparisons.

Data	$s_{tran}(5)$	s_{comm}	# Data	AvgComp/Inst
Raw data	98.4	-	14.8K	6.29
Perturbed data	87.6	-	14.8K	6.29
Aug. data	1	1	30.9K	13.2
Aug. data+Neg.	1	1	61.8K	26.4

Table 3: Instruction-tuning (IT) of Llama-3 Instruct (8B) using different dataset variants (see Section 4.2). Only training on negated relations can enhance negation invariance.

Models	Summarize from Feedback			
	H.	$s_{tran}(5)$	s_{comm}	s_{neg}
Zero-shot inference	64.3	81.1	70.2	63.8
IT Perturbed data	70.3	91.9	88.4	61.0
IT Augmented data	70.1	95.4	91.2	60.8
IT Augmented data + Neg.	69.7	95.2	90.5	87.9

pairwise annotations improves consistency in knowledge-intensive QA tasks. However, real-world preference data are often noisy and self-contradictory (Chowdhery et al., 2022; Ouyang et al., 2022a), especially in more subjective tasks (Bai et al., 2022). The inter-rater agreement rate typically falls between 60% to 80% for preference or evaluation datasets. Prior work (Wang et al., 2024a) has shown that these self-conflicting annotations reduce the efficiency of learning preferences, and we hypothesize that such annotations also contribute to the logical inconsistencies in trained models.

Inspired by the RLHF process, where a reward model is used to establish a complete order of responses, we propose first estimating a coherent ranking from the noisy data. This refined ranking can then be augmented with additional conflict-free pairwise comparisons to align LLMs more effectively. This method is expected to be more efficient than aligning models to noisy and inconsistent annotations directly.

4.1 Estimating Rankings from Noisy Pairwise Annotations Using Win-Loss Rate

Estimating global rankings from noisy pairwise annotations is essentially a rank aggregation problem. We use the win-loss rate method due to its simplicity and two key advantages: **1)** it performs well with partial and sparse comparisons, which is common in real-world preference datasets, and **2)** it remains unaffected by the order in which comparisons are presented. While other rank aggregation methods like ELO or the Bradley-Terry model (Bradley & Terry, 1952) could be explored for improved results, our choice serves as a proof-of-concept rather than an optimization for best performance.

The win-loss rate for each item is calculated as the number of comparisons it wins minus the number of comparisons it loses and then divided by the number of comparisons it participates in. Following that, as shown in Figure 6, we rank the item by the value of its win-loss rate. This way, we can aggregate a full or partial ranking. We then split the ranking into a self-consistent pairwise comparison set. We can further augment the dataset by adding the comparisons using the negated relation.

Table 4: LLMs with better transitivity perform better with PairS. Improved commutativity indicates less need for algorithm calibration. For both PairS methods, we report the average Spearman correlations over 100 runs.

Models	SummEval (Coh)				
	H.	$s_{tran}(5)$	s_{comm}	PairS	PairS calibrated
Mistral-7B	63.6	95.1	59.9	27.7	31.2 +3.5
Phi-3-mini	65.6	92.8	46.9	33.9	38.0 +4.1
GPT-3.5-turbo	66.3	82.5	67.5	33.5	36.3 +2.8
Llama-3-8B	67.8	91.0	76.1	37.7	38.9 +1.2
Phi-3-medium	68.8	96.2	71.0	38.9	41.3 +.2.4

4.2 Experiments

Experimental Setup. We use the Summarize From Feedback dataset (Mostafazadeh et al., 2016b), where human annotators made pairwise comparisons between two summaries based on qualitative factors. More details are available in Appendix §B. Given that the dataset annotations are sparse and relatively clean, we simulate noise by randomly flipping 10% of the training labels. Our data refinement and augmentation techniques are then applied to improve the consistency and quantity of pairwise comparisons, as shown in Table 2.

To assess the effectiveness of our approach, we instruction-tuned three Meta-Llama-3-8B-Instruct models on: **1)** the flipped/perturbed data, **2)** the refined and augmented dataset, and **3)** the 'further augmented' dataset with additional negated relation comparisons. Training parameters are detailed in Appendix § E. We randomly sample 200 instances from the test set and evaluate all models on this subset. We assess the impact of the data augmentation on logical consistency and performance.

Results and Findings. The results shown in Table 3 highlight three key findings: **1)** Zero-shot inference shows considerable logical inconsistency. However, training on perturbed data can already substantially improve both human preference alignment and logical consistency. **2)** Training with the refined and augmented dataset significantly improves transitivity and commutativity, while maintaining strong human alignment. **3)** Only training further on negated relations improves the model performance in negation invariance. However, adding negated relations to the broader dataset may introduce distractions and cause a forgetting effect (Luo et al., 2023), resulting in a slight reduction in performance on other logical properties. Overall, the findings support the effectiveness of our data refinement and augmentation framework in improving the logical consistency of LLMs.

5 Impact of Logical Consistency on Downstream Applications

LLMs are increasingly used as logical operators in high-level algorithms due to their ability to process and understand text semantics. For instance, Qin et al. (2024) use LLMs to enhance document reranking in information retrieval systems. Similarly, Guo et al. (2024) and Yang et al. (2024) utilize LLMs as optimizers in prompt-tuning algorithms, while Liu et al. (2024) and Liusie et al. (2024b) employ LLMs as pairwise comparators to aggregate preference rankings. When LLMs are used as logical operators, maintaining a high level of logical consistency is critical to ensure predictable and efficient decision-making. In this section, we examine how logical consistency influences the performance of LLM-based algorithms in such 'logically grounded' tasks.

5.1 Application Example: Preference Ranking Algorithm

We illustrate the impact of logical consistency through the Pairwise-Preference Search (PairS) algorithm proposed by Liu et al. (2024). PairS is a sorting-based rank aggregation method that uses LLMs as pairwise evaluators (i.e., it is a particular *'LLM-as-a-judge'* algorithm), comparing items based on specific attributes using a merge-sort approach. Its performance is measured by comparing the LLM-generated rankings with human judgments via Spearman's correlation. Sorting algorithms depend heavily on logical properties such as transitivity and commutativity. PairS assumes that LLMs used as evaluators have near-perfect transitivity and commutativity for optimal ranking results.

To evaluate this, we conduct controlled experiments on the coherence aspect of SummEval, using various LLMs with similar accuracy to human judgments. The results are shown in Table 4. As LLMs

often exhibit positional bias, we follow Liu et al. (2024) by reporting both raw and calibrated results. The calibration, performed as described in Zheng et al. (2023), averages the evaluation probabilities across both possible pairwise orders. While this technique increases inference cost, it reduces bias by balancing positional preferences.

Our findings from Table 4 reveal two key insights: **1)** Although Phi-3-mini has slightly lower accuracy with human judgments than GPT-3.5-turbo, its stronger transitivity leads to better ranking performance with or without calibration. **2)** There is a clear correlation between an LLM's commutativity and the performance gains from calibration in the PairS algorithm. LLMs that are more commutative rely less on calibration to achieve optimal performance, and therefore, require less computations.

6 Related Work

Consistency of LLMs has been explored in several contexts before, where previous research has predominantly focused on two domains: *consistency of factual knowledge* and *entailment consistency* across limited statements.

Consistency of Factual Knowledge. Previous work has demonstrated that in knowledge-intensive question-answering tasks, concepts like symmetry and transitivity of knowledge snippets are important (Asai & Hajishirzi, 2020). To this end, a benchmark for studying language model consistency with paraphrased relations (Elazar et al., 2021) has been created. Additionally, different studies have examined whether LLMs have a coherent understanding of knowledge graphs related to real-world entities (Jung et al., 2022; Gu et al., 2023; Kassner et al., 2023; Gu et al., 2023). The 'reverse curse' phenomenon highlights that unlearned factual knowledge cannot be deduced by reversing learned knowledge (Allen-Zhu & Li, 2023; Berglund et al., 2024).

Entailment and Inference Consistency. In Natural Language Inference (NLI), the consistency of transitivity and symmetry in statement pairs has been explored using predefined inference rules (Li et al., 2019; Jang & Lukasiewicz, 2023). Jang et al. (2022) proposed a test set to exam the consistency of LLMs for NLI tasks. Prediction consistency has been improved through the use of adversarial first-order logic examples (Minervini & Riedel, 2018).

Despite the wealth of knowledge regarding the logical consistency of LLMs within specific domains, most studies have concentrated on first-order relations—logical connections or implications directly linking two or three individual statements. Consequently, there is a notable gap in research addressing the consistency and reliability of LLMs when applied to more complex decision-making scenarios or the evaluation of multiple items simultaneously. This limitation suggests a pressing need for further investigation into how LLMs can maintain coherence and consistency in broader, more comprehensive contexts, which is essential for their deployment in practical applications.

7 Conclusion

In this work, we investigated the critical role of logical consistency in enhancing the reliability and trustworthiness of LLMs, especially in their roles as decision-makers and reasoners. We introduced a general framework for quantifying three key properties of logical consistency: transitivity, commutativity, and negation invariance. Through comprehensive evaluation, we demonstrated that these consistency measures serve as strong proxies for assessing the overall reliability of LLMs.

To improve the logical consistency of LLMs, we then proposed a data refinement and augmentation framework, which effectively reduces logical inconsistencies at the source: directly in the data. Our experimental results showed that models trained on this refined and augmented data achieve improved logical consistency without compromising alignment with human preferences. Furthermore, we demonstrated that integrating LLMs with higher logical consistency into a logic-driven algorithm improves both the algorithm's global performance and computational efficiency. This highlights the practical benefits of ensuring logical coherence in LLMs when applied to downstream tasks.

In summary, our work emphasizes the importance of logical consistency as a complementary factor to human alignment in the development of more reliable LLMs. We hope this research will encourage the broader community to recognize the importance of logical consistency as a fundamental aspect of any work aiming towards improved trustworthiness and reliability.

REFERENCES

Marah I Abdin, Sam Ade Jacobs, Ammar Ahmad Awan, et al. Phi-3 technical report: A highly capable language model locally on your phone. *CoRR*, abs/2404.14219, 2024. doi: 10.48550/ARXIV.2404.14219. URL https://doi.org/10.48550/arXiv.2404.14219.

Serge Abiteboul, Richard Hull, and Victor Vianu. *Foundations of databases*, volume 8. Addison-Wesley Reading, 1995.

Zeyuan Allen-Zhu and Yuanzhi Li. Physics of language models: Part 3.2, knowledge manipulation. *CoRR*, abs/2309.14402, 2023. doi: 10.48550/ARXIV.2309.14402. URL https://doi.org/10.48550/arXiv.2309.14402.

Rohan Anil, Sebastian Borgeaud, Yonghui Wu, et al. Gemini: A family of highly capable multimodal models. *CoRR*, abs/2312.11805, 2023a. doi: 10.48550/ARXIV.2312.11805. URL https://doi.org/10.48550/arXiv.2312.11805.

Rohan Anil, Andrew M. Dai, Orhan Firat, et al. Palm 2 technical report. *CoRR*, abs/2305.10403, 2023b. doi: 10.48550/ARXIV.2305.10403. URL https://doi.org/10.48550/arXiv.2305.10403.

Kenneth J Arrow. A difficulty in the concept of social welfare. *Journal of political economy*, 58(4): 328–346, 1950.

Akari Asai and Hannaneh Hajishirzi. Logic-guided data augmentation and regularization for consistent question answering. In Dan Jurafsky, Joyce Chai, Natalie Schluter, and Joel Tetreault (eds.), *Proceedings of the 58th Annual Meeting of the Association for Computational Linguistics*, pp. 5642–5650, Online, July 2020. Association for Computational Linguistics. doi: 10.18653/v1/2020.acl-main.499. URL https://aclanthology.org/2020.acl-main.499.

Yuntao Bai, Andy Jones, Kamal Ndousse, Amanda Askell, Anna Chen, Nova DasSarma, Dawn Drain, Stanislav Fort, Deep Ganguli, Tom Henighan, Nicholas Joseph, Saurav Kadavath, Jackson Kernion, Tom Conerly, Sheer El-Showk, Nelson Elhage, Zac Hatfield-Dodds, Danny Hernandez, Tristan Hume, Scott Johnston, Shauna Kravec, Liane Lovitt, Neel Nanda, Catherine Olsson, Dario Amodei, Tom Brown, Jack Clark, Sam McCandlish, Chris Olah, Ben Mann, and Jared Kaplan. Training a helpful and harmless assistant with reinforcement learning from human feedback, 2022. URL https://arxiv.org/abs/2204.05862.

Lukas Berglund, Meg Tong, Maximilian Kaufmann, Mikita Balesni, Asa Cooper Stickland, Tomasz Korbak, and Owain Evans. The reversal curse: Llms trained on "a is b" fail to learn "b is a". In *The Twelfth International Conference on Learning Representations, ICLR 2024, Vienna, Austria, May 7-11, 2024*. OpenReview.net, 2024. URL https://openreview.net/forum?id=GPKTIktA0k.

Ralph Allan Bradley and Milton E Terry. Rank analysis of incomplete block designs: I. the method of paired comparisons. *Biometrika*, 39(3/4):324–345, 1952.

Tom B. Brown, Benjamin Mann, Nick Ryder, Melanie Subbiah, Jared Kaplan, Prafulla Dhariwal, Arvind Neelakantan, Pranav Shyam, Girish Sastry, Amanda Askell, Sandhini Agarwal, Ariel Herbert-Voss, Gretchen Krueger, Tom Henighan, Rewon Child, Aditya Ramesh, Daniel M. Ziegler, Jeffrey Wu, Clemens Winter, Christopher Hesse, Mark Chen, Eric Sigler, Mateusz Litwin, Scott Gray, Benjamin Chess, Jack Clark, Christopher Berner, Sam McCandlish, Alec Radford, Ilya Sutskever, and Dario Amodei. Language models are few-shot learners. In Hugo Larochelle, Marc'Aurelio Ranzato, Raia Hadsell, Maria-Florina Balcan, and Hsuan-Tien Lin (eds.), *Advances in Neural Information Processing Systems 33: Annual Conference on Neural Information Processing Systems 2020, NeurIPS 2020, December 6-12, 2020, virtual*, 2020. URL https://proceedings.neurips.cc/paper/2020/hash/1457c0d6bfcb4967418bfb8ac142f64a-Abstract.html.

Isabel Cachola, Kyle Lo, Arman Cohan, and Daniel S. Weld. TLDR: extreme summarization of scientific documents. In Trevor Cohn, Yulan He, and Yang Liu (eds.), *Findings of the Association for Computational Linguistics: EMNLP 2020, Online Event, 16-20 November 2020*, volume EMNLP 2020 of *Findings of ACL*, pp. 4766–4777. Association for Computational Linguistics, 2020. doi: 10.18653/V1/2020.FINDINGS-EMNLP.428. URL https://doi.org/10.18653/v1/2020.findings-emnlp.428.

Aakanksha Chowdhery, Sharan Narang, Jacob Devlin, Maarten Bosma, Gaurav Mishra, Adam Roberts, Paul Barham, Hyung Won Chung, Charles Sutton, Sebastian Gehrmann, Parker Schuh, Kensen Shi, Sasha Tsvyashchenko, Joshua Maynez, Abhishek Rao, Parker Barnes, Yi Tay, Noam Shazeer, Vinodkumar Prabhakaran, Emily Reif, Nan Du, Ben Hutchinson, Reiner Pope, James Bradbury, Jacob Austin, Michael Isard, Guy Gur-Ari, Pengcheng Yin, Toju Duke, Anselm Levskaya, Sanjay Ghemawat, Sunipa Dev, Henryk Michalewski, Xavier Garcia, Vedant Misra, Kevin Robinson, Liam Fedus, Denny Zhou, Daphne Ippolito, David Luan, Hyeontaek Lim, Barret Zoph, Alexander Spiridonov, Ryan Sepassi, David Dohan, Shivani Agrawal, Mark Omernick, Andrew M. Dai, Thanumalayan Sankaranarayana Pillai, Marie Pellat, Aitor Lewkowycz, Erica Moreira, Rewon Child, Oleksandr Polozov, Katherine Lee, Zongwei Zhou, Xuezhi Wang, Brennan Saeta, Mark Diaz, Orhan Firat, Michele Catasta, Jason Wei, Kathy Meier-Hellstern, Douglas Eck, Jeff Dean, Slav Petrov, and Noah Fiedel. Palm: Scaling language modeling with pathways, 2022. URL https://arxiv.org/abs/2204.02311.

Paul F. Christiano, Jan Leike, Tom B. Brown, Miljan Martic, Shane Legg, and Dario Amodei. Deep reinforcement learning from human preferences. In Isabelle Guyon, Ulrike von Luxburg, Samy Bengio, Hanna M. Wallach, Rob Fergus, S. V. N. Vishwanathan, and Roman Garnett (eds.), *Advances in Neural Information Processing Systems 30: Annual Conference on Neural Information Processing Systems 2017, December 4-9, 2017, Long Beach, CA, USA*, pp. 4299–4307, 2017. URL https://proceedings.neurips.cc/paper/2017/hash/d5e2c0adad503c91f91df240d0cd4e49-Abstract.html.

Hyung Won Chung, Le Hou, Shayne Longpre, Barret Zoph, Yi Tay, William Fedus, Yunxuan Li, Xuezhi Wang, Mostafa Dehghani, Siddhartha Brahma, et al. Scaling instruction-finetuned language models. *arXiv preprint arXiv:2210.11416*, 2022.

Antonia Creswell, Murray Shanahan, and Irina Higgins. Selection-inference: Exploiting large language models for interpretable logical reasoning. In *The Eleventh International Conference on Learning Representations, ICLR 2023, Kigali, Rwanda, May 1-5, 2023*. OpenReview.net, 2023. URL https://openreview.net/forum?id=3Pf3Wg6o-A4.

Josef Dai, Xuehai Pan, Ruiyang Sun, Jiaming Ji, Xinbo Xu, Mickel Liu, Yizhou Wang, and Yaodong Yang. Safe RLHF: Safe reinforcement learning from human feedback. In *The Twelfth International Conference on Learning Representations*, 2024. URL https://openreview.net/forum?id=TyFrPOKYXw.

DeepSeek-AI. Deepseek-v2: A strong, economical, and efficient mixture-of-experts language model. *CoRR*, abs/2405.04434, 2024. doi: 10.48550/ARXIV.2405.04434. URL https://doi.org/10.48550/arXiv.2405.04434.

Abhimanyu Dubey, Abhinav Jauhri, Abhinav Pandey, et al. The llama 3 herd of models. *CoRR*, abs/2407.21783, 2024. doi: 10.48550/ARXIV.2407.21783. URL https://doi.org/10.48550/arXiv.2407.21783.

Yanai Elazar, Nora Kassner, Shauli Ravfogel, Abhilasha Ravichander, Eduard Hovy, Hinrich Schütze, and Yoav Goldberg. Measuring and improving consistency in pretrained language models. *Transactions of the Association for Computational Linguistics*, 9:1012–1031, 2021. doi: 10.1162/tacl_a_00410. URL https://aclanthology.org/2021.tacl-1.60.

Alexander R. Fabbri, Wojciech Kryściński, Bryan McCann, Caiming Xiong, Richard Socher, and Dragomir Radev. SummEval: Re-evaluating summarization evaluation. *Transactions of the Association for Computational Linguistics*, 9:391–409, 2021. doi: 10.1162/tacl_a_00373. URL https://aclanthology.org/2021.tacl-1.24.

Isabel O. Gallegos, Ryan A. Rossi, Joe Barrow, Md Mehrab Tanjim, Sungchul Kim, Franck Dernoncourt, Tong Yu, Ruiyi Zhang, and Nesreen K. Ahmed. Bias and Fairness in Large Language Models: A Survey. *Computational Linguistics*, 50(3):1097–1179, 09 2024. ISSN 0891-2017. doi: 10.1162/coli_a_00524. URL https://doi.org/10.1162/coli_a_00524.

Yuling Gu, Bhavana Dalvi Mishra, and Peter Clark. Do language models have coherent mental models of everyday things? In Anna Rogers, Jordan L. Boyd-Graber, and Naoaki Okazaki (eds.), *Proceedings of the 61st Annual Meeting of the Association for Computational Linguistics (Volume*

1: Long Papers), ACL 2023, Toronto, Canada, July 9-14, 2023, pp. 1892–1913. Association for Computational Linguistics, 2023. doi: 10.18653/V1/2023.ACL-LONG.106. URL https://doi.org/10.18653/v1/2023.acl-long.106.

Robert M Guion. Validity and reliability. *Handbook of research methods in industrial and organizational psychology*, pp. 57–76, 2004.

Zixian Guo, Ming Liu, Zhilong Ji, Jinfeng Bai, Yiwen Guo, and Wangmeng Zuo. Two optimizers are better than one: Llm catalyst empowers gradient-based optimization for prompt tuning, 2024. URL https://arxiv.org/abs/2405.19732.

Ronan Hamon, Henrik Junklewitz, Ignacio Sanchez, et al. Robustness and explainability of artificial intelligence. *Publications Office of the European Union*, 207:2020, 2020.

Sven Ove Hansson. *The structure of values and norms*. Cambridge university press, 2001.

Jie Huang and Kevin Chen-Chuan Chang. Towards reasoning in large language models: A survey. In Anna Rogers, Jordan L. Boyd-Graber, and Naoaki Okazaki (eds.), *Findings of the Association for Computational Linguistics: ACL 2023, Toronto, Canada, July 9-14, 2023*, pp. 1049–1065. Association for Computational Linguistics, 2023. doi: 10.18653/V1/2023.FINDINGS-ACL.67. URL https://doi.org/10.18653/v1/2023.findings-acl.67.

Dominic Hyde. The sorites paradox. In *Vagueness: A guide*, pp. 1–17. Springer, 2011.

Tomasz Imieliński and Witold Lipski Jr. The relational model of data and cylindric algebras. *Journal of Computer and System Sciences*, 28(1):80–102, 1984.

Myeongjun Jang and Thomas Lukasiewicz. Consistency analysis of chatgpt. In Houda Bouamor, Juan Pino, and Kalika Bali (eds.), *Proceedings of the 2023 Conference on Empirical Methods in Natural Language Processing, EMNLP 2023, Singapore, December 6-10, 2023*, pp. 15970–15985. Association for Computational Linguistics, 2023. doi: 10.18653/V1/2023.EMNLP-MAIN.991. URL https://doi.org/10.18653/v1/2023.emnlp-main.991.

Myeongjun Jang, Deuk Sin Kwon, and Thomas Lukasiewicz. BECEL: Benchmark for consistency evaluation of language models. In Nicoletta Calzolari, Chu-Ren Huang, Hansaem Kim, James Pustejovsky, Leo Wanner, Key-Sun Choi, Pum-Mo Ryu, Hsin-Hsi Chen, Lucia Donatelli, Heng Ji, Sadao Kurohashi, Patrizia Paggio, Nianwen Xue, Seokhwan Kim, Younggyun Hahm, Zhong He, Tony Kyungil Lee, Enrico Santus, Francis Bond, and Seung-Hoon Na (eds.), *Proceedings of the 29th International Conference on Computational Linguistics*, pp. 3680–3696, Gyeongju, Republic of Korea, October 2022. International Committee on Computational Linguistics. URL https://aclanthology.org/2022.coling-1.324.

Albert Q. Jiang, Alexandre Sablayrolles, Arthur Mensch, Chris Bamford, Devendra Singh Chaplot, Diego de Las Casas, Florian Bressand, Gianna Lengyel, Guillaume Lample, Lucile Saulnier, Lélio Renard Lavaud, Marie-Anne Lachaux, Pierre Stock, Teven Le Scao, Thibaut Lavril, Thomas Wang, Timothée Lacroix, and William El Sayed. Mistral 7b. *CoRR*, abs/2310.06825, 2023. doi: 10.48550/ARXIV.2310.06825. URL https://doi.org/10.48550/arXiv.2310.06825.

Jaehun Jung, Lianhui Qin, Sean Welleck, Faeze Brahman, Chandra Bhagavatula, Ronan Le Bras, and Yejin Choi. Maieutic prompting: Logically consistent reasoning with recursive explanations. In Yoav Goldberg, Zornitsa Kozareva, and Yue Zhang (eds.), *Proceedings of the 2022 Conference on Empirical Methods in Natural Language Processing, EMNLP 2022, Abu Dhabi, United Arab Emirates, December 7-11, 2022*, pp. 1266–1279. Association for Computational Linguistics, 2022. doi: 10.18653/V1/2022.EMNLP-MAIN.82. URL https://doi.org/10.18653/v1/2022.emnlp-main.82.

Nora Kassner, Oyvind Tafjord, Ashish Sabharwal, Kyle Richardson, Hinrich Schütze, and Peter Clark. Language models with rationality. In Houda Bouamor, Juan Pino, and Kalika Bali (eds.), *Proceedings of the 2023 Conference on Empirical Methods in Natural Language Processing, EMNLP 2023, Singapore, December 6-10, 2023*, pp. 14190–14201. Association for Computational Linguistics, 2023. doi: 10.18653/V1/2023.EMNLP-MAIN.877. URL https://doi.org/10.18653/v1/2023.emnlp-main.877.

Takeshi Kojima, Shixiang Shane Gu, Machel Reid, Yutaka Matsuo, and Yusuke Iwasawa. Large language models are zero-shot reasoners. In Sanmi Koyejo, S. Mohamed, A. Agarwal, Danielle Belgrave, K. Cho, and A. Oh (eds.), *Advances in Neural Information Processing Systems 35: Annual Conference on Neural Information Processing Systems 2022, NeurIPS 2022, New Orleans, LA, USA, November 28 - December 9, 2022*, 2022. URL http://papers.nips.cc/paper_files/paper/2022/hash/8bb0d291acd4acf06ef112099c16f326-Abstract-Conference.html.

Ashutosh Kumar and Aditya Joshi. Striking a balance: Alleviating inconsistency in pre-trained models for symmetric classification tasks. In Smaranda Muresan, Preslav Nakov, and Aline Villavicencio (eds.), *Findings of the Association for Computational Linguistics: ACL 2022*, pp. 1887–1895, Dublin, Ireland, May 2022. Association for Computational Linguistics. doi: 10.18653/v1/2022.findings-acl.148. URL https://aclanthology.org/2022.findings-acl.148.

Tao Li, Vivek Gupta, Maitrey Mehta, and Vivek Srikumar. A logic-driven framework for consistency of neural models. In Kentaro Inui, Jing Jiang, Vincent Ng, and Xiaojun Wan (eds.), *Proceedings of the 2019 Conference on Empirical Methods in Natural Language Processing and the 9th International Joint Conference on Natural Language Processing, EMNLP-IJCNLP 2019, Hong Kong, China, November 3-7, 2019*, pp. 3922–3933. Association for Computational Linguistics, 2019. doi: 10.18653/V1/D19-1405. URL https://doi.org/10.18653/v1/D19-1405.

Yinhong Liu, Han Zhou, Zhijiang Guo, Ehsan Shareghi, Ivan Vulić, Anna Korhonen, and Nigel Collier. Aligning with human judgement: The role of pairwise preference in large language model evaluators. In *First Conference on Language Modeling*, 2024. URL https://openreview.net/forum?id=9gdZI7c6yr.

Adian Liusie, Yassir Fathullah, and Mark J. F. Gales. Teacher-student training for debiasing: General permutation debiasing for large language models. In Lun-Wei Ku, Andre Martins, and Vivek Srikumar (eds.), *Findings of the Association for Computational Linguistics, ACL 2024, Bangkok, Thailand and virtual meeting, August 11-16, 2024*, pp. 1376–1387. Association for Computational Linguistics, 2024a. doi: 10.18653/V1/2024.FINDINGS-ACL.81. URL https://doi.org/10.18653/v1/2024.findings-acl.81.

Adian Liusie, Potsawee Manakul, and Mark Gales. Llm comparative assessment: Zero-shot nlg evaluation through pairwise comparisons using large language models. In *Proceedings of the 18th Conference of the European Chapter of the Association for Computational Linguistics (Volume 1: Long Papers)*, pp. 139–151, 2024b.

Yun Luo, Zhen Yang, Fandong Meng, Yafu Li, Jie Zhou, and Yue Zhang. An empirical study of catastrophic forgetting in large language models during continual fine-tuning. *arXiv preprint arXiv:2308.08747*, 2023.

Brandon P Miller, Derek D Reed, and Michael Amlung. Reliability and validity of behavioral-economic measures: A review and synthesis of discounting and demand. *Journal of the experimental analysis of behavior*, 120(2):263–280, 2023.

Pasquale Minervini and Sebastian Riedel. Adversarially regularising neural NLI models to integrate logical background knowledge. In Anna Korhonen and Ivan Titov (eds.), *Proceedings of the 22nd Conference on Computational Natural Language Learning, CoNLL 2018, Brussels, Belgium, October 31 - November 1, 2018*, pp. 65–74. Association for Computational Linguistics, 2018. doi: 10.18653/V1/K18-1007. URL https://doi.org/10.18653/v1/k18-1007.

Nasrin Mostafazadeh, Nathanael Chambers, Xiaodong He, Devi Parikh, Dhruv Batra, Lucy Vanderwende, Pushmeet Kohli, and James F. Allen. A corpus and cloze evaluation for deeper understanding of commonsense stories. In Kevin Knight, Ani Nenkova, and Owen Rambow (eds.), *NAACL HLT 2016, The 2016 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, San Diego California, USA, June 12-17, 2016*, pp. 839–849. The Association for Computational Linguistics, 2016a. doi: 10.18653/V1/N16-1098. URL https://doi.org/10.18653/v1/n16-1098.

Nasrin Mostafazadeh, Alyson Grealish, Nathanael Chambers, James Allen, and Lucy Vanderwende. CaTeRS: Causal and temporal relation scheme for semantic annotation of event structures. In Martha Palmer, Ed Hovy, Teruko Mitamura, and Tim O'Gorman (eds.), *Proceedings of the Fourth*

Workshop on Events, pp. 51–61, San Diego, California, June 2016b. Association for Computational Linguistics. doi: 10.18653/v1/W16-1007. URL https://aclanthology.org/W16-1007.

Ramesh Nallapati, Bowen Zhou, Cícero Nogueira dos Santos, Çaglar Gülçehre, and Bing Xiang. Abstractive text summarization using sequence-to-sequence rnns and beyond. In Yoav Goldberg and Stefan Riezler (eds.), *Proceedings of the 20th SIGNLL Conference on Computational Natural Language Learning, CoNLL 2016, Berlin, Germany, August 11-12, 2016*, pp. 280–290. ACL, 2016. doi: 10.18653/V1/K16-1028. URL https://doi.org/10.18653/v1/k16-1028.

Tri Nguyen, Mir Rosenberg, Xia Song, Jianfeng Gao, Saurabh Tiwary, Rangan Majumder, and Li Deng. MS MARCO: A human generated machine reading comprehension dataset. In Tarek Richard Besold, Antoine Bordes, Artur S. d'Avila Garcez, and Greg Wayne (eds.), *Proceedings of the Workshop on Cognitive Computation: Integrating neural and symbolic approaches 2016 co-located with the 30th Annual Conference on Neural Information Processing Systems (NIPS 2016), Barcelona, Spain, December 9, 2016*, volume 1773 of *CEUR Workshop Proceedings*. CEUR-WS.org, 2016. URL https://ceur-ws.org/Vol-1773/CoCoNIPS_2016_paper9.pdf.

OpenAI. GPT-4 technical report. *CoRR*, abs/2303.08774, 2023. doi: 10.48550/ARXIV.2303.08774. URL https://doi.org/10.48550/arXiv.2303.08774.

Long Ouyang, Jeff Wu, Xu Jiang, Diogo Almeida, Carroll L. Wainwright, Pamela Mishkin, Chong Zhang, Sandhini Agarwal, Katarina Slama, Alex Ray, John Schulman, Jacob Hilton, Fraser Kelton, Luke Miller, Maddie Simens, Amanda Askell, Peter Welinder, Paul Christiano, Jan Leike, and Ryan Lowe. Training language models to follow instructions with human feedback, 2022a. URL https://arxiv.org/abs/2203.02155.

Long Ouyang, Jeffrey Wu, Xu Jiang, Diogo Almeida, Carroll L. Wainwright, Pamela Mishkin, Chong Zhang, Sandhini Agarwal, Katarina Slama, Alex Ray, John Schulman, Jacob Hilton, Fraser Kelton, Luke Miller, Maddie Simens, Amanda Askell, Peter Welinder, Paul F. Christiano, Jan Leike, and Ryan Lowe. Training language models to follow instructions with human feedback. In Sanmi Koyejo, S. Mohamed, A. Agarwal, Danielle Belgrave, K. Cho, and A. Oh (eds.), *Advances in Neural Information Processing Systems 35: Annual Conference on Neural Information Processing Systems 2022, NeurIPS 2022, New Orleans, LA, USA, November 28 - December 9, 2022*, 2022b. URL http://papers.nips.cc/paper_files/paper/2022/hash/b1efde53be364a73914f58805a001731-Abstract-Conference.html.

Zhen Qin, Rolf Jagerman, Kai Hui, Honglei Zhuang, Junru Wu, Le Yan, Jiaming Shen, Tianqi Liu, Jialu Liu, Donald Metzler, Xuanhui Wang, and Michael Bendersky. Large language models are effective text rankers with pairwise ranking prompting. In Kevin Duh, Helena Gómez-Adorno, and Steven Bethard (eds.), *Findings of the Association for Computational Linguistics: NAACL 2024, Mexico City, Mexico, June 16-21, 2024*, pp. 1504–1518. Association for Computational Linguistics, 2024. doi: 10.18653/V1/2024.FINDINGS-NAACL.97. URL https://doi.org/10.18653/v1/2024.findings-naacl.97.

Michel Regenwetter, Jason Dana, and Clintin P Davis-Stober. Transitivity of preferences. *Psychological review*, 118(1):42, 2011.

Greg Restall. *An introduction to substructural logics*. Routledge, 2002.

Morgane Rivière, Shreya Pathak, Pier Giuseppe Sessa, et al. Gemma 2: Improving open language models at a practical size. *CoRR*, abs/2408.00118, 2024. doi: 10.48550/ARXIV.2408.00118. URL https://doi.org/10.48550/arXiv.2408.00118.

Frank L Schmidt, Chockaungam Viswesvaran, and Deniz S Ones. Reliability is not validity and validity is not reliability. *Personnel Psychology*, 53(4):901–912, 2000.

Feifan Song, Bowen Yu, Minghao Li, Haiyang Yu, Fei Huang, Yongbin Li, and Houfeng Wang. Preference ranking optimization for human alignment. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 38, pp. 18990–18998, 2024.

Nisan Stiennon, Long Ouyang, Jeffrey Wu, Daniel Ziegler, Ryan Lowe, Chelsea Voss, Alec Radford, Dario Amodei, and Paul F Christiano. Learning to summarize with human feedback. *Advances in Neural Information Processing Systems*, 33:3008–3021, 2020.

Weiwei Sun, Lingyong Yan, Xinyu Ma, Shuaiqiang Wang, Pengjie Ren, Zhumin Chen, Dawei Yin, and Zhaochun Ren. Is chatgpt good at search? investigating large language models as re-ranking agents. In Houda Bouamor, Juan Pino, and Kalika Bali (eds.), *Proceedings of the 2023 Conference on Empirical Methods in Natural Language Processing, EMNLP 2023, Singapore, December 6-10, 2023*, pp. 14918–14937. Association for Computational Linguistics, 2023. doi: 10.18653/V1/2023.EMNLP-MAIN.923. URL https://doi.org/10.18653/v1/2023.emnlp-main.923.

Alfred Tarski. On the calculus of relations. *The journal of symbolic logic*, 6(3):73–89, 1941.

Hugo Touvron, Louis Martin, Kevin Stone, et al. Llama 2: Open foundation and fine-tuned chat models. *CoRR*, abs/2307.09288, 2023. doi: 10.48550/ARXIV.2307.09288. URL https://doi.org/10.48550/arXiv.2307.09288.

Lewis Tunstall, Edward Beeching, Nathan Lambert, Nazneen Rajani, Kashif Rasul, Younes Belkada, Shengyi Huang, Leandro von Werra, Clémentine Fourrier, Nathan Habib, Nathan Sarrazin, Omar Sanseviero, Alexander M. Rush, and Thomas Wolf. Zephyr: Direct distillation of lm alignment, 2023. URL https://arxiv.org/abs/2310.16944.

Amos Tversky. Intransitivity of preferences. *Psychological review*, 76(1):31, 1969.

Binghai Wang, Rui Zheng, Lu Chen, Yan Liu, Shihan Dou, Caishuang Huang, Wei Shen, Senjie Jin, Enyu Zhou, Chenyu Shi, Songyang Gao, Nuo Xu, Yuhao Zhou, Xiaoran Fan, Zhiheng Xi, Jun Zhao, Xiao Wang, Tao Ji, Hang Yan, Lixing Shen, Zhan Chen, Tao Gui, Qi Zhang, Xipeng Qiu, Xuanjing Huang, Zuxuan Wu, and Yu-Gang Jiang. Secrets of rlhf in large language models part ii: Reward modeling, 2024a. URL https://arxiv.org/abs/2401.06080.

Peiyi Wang, Lei Li, Liang Chen, Zefan Cai, Dawei Zhu, Binghuai Lin, Yunbo Cao, Lingpeng Kong, Qi Liu, Tianyu Liu, and Zhifang Sui. Large language models are not fair evaluators. In Lun-Wei Ku, Andre Martins, and Vivek Srikumar (eds.), *Proceedings of the 62nd Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers), ACL 2024, Bangkok, Thailand, August 11-16, 2024*, pp. 9440–9450. Association for Computational Linguistics, 2024b. URL https://aclanthology.org/2024.acl-long.511.

Rui Wang and Ricardo Henao. Unsupervised paraphrasing consistency training for low resource named entity recognition. In Marie-Francine Moens, Xuanjing Huang, Lucia Specia, and Scott Wen-tau Yih (eds.), *Proceedings of the 2021 Conference on Empirical Methods in Natural Language Processing*, pp. 5303–5308, Online and Punta Cana, Dominican Republic, November 2021. Association for Computational Linguistics. doi: 10.18653/v1/2021.emnlp-main.430. URL https://aclanthology.org/2021.emnlp-main.430.

Jason Wei, Maarten Bosma, Vincent Zhao, Kelvin Guu, Adams Wei Yu, Brian Lester, Nan Du, Andrew M. Dai, and Quoc V Le. Finetuned language models are zero-shot learners. In *International Conference on Learning Representations*, 2022a. URL https://openreview.net/forum?id=gEZrGCozdqR.

Jason Wei, Xuezhi Wang, Dale Schuurmans, Maarten Bosma, Brian Ichter, Fei Xia, Ed H. Chi, Quoc V. Le, and Denny Zhou. Chain-of-thought prompting elicits reasoning in large language models. In Sanmi Koyejo, S. Mohamed, A. Agarwal, Danielle Belgrave, K. Cho, and A. Oh (eds.), *Advances in Neural Information Processing Systems 35: Annual Conference on Neural Information Processing Systems 2022, NeurIPS 2022, New Orleans, LA, USA, November 28 - December 9, 2022*, 2022b. URL http://papers.nips.cc/paper_files/paper/2022/hash/9d5609613524ecf4f15af0f7b31abca4-Abstract-Conference.html.

Chengrun Yang, Xuezhi Wang, Yifeng Lu, Hanxiao Liu, Quoc V Le, Denny Zhou, and Xinyun Chen. Large language models as optimizers. In *The Twelfth International Conference on Learning Representations*, 2024. URL https://openreview.net/forum?id=Bb4VGOWELI.

Yue Zhang, Yafu Li, Leyang Cui, Deng Cai, Lemao Liu, Tingchen Fu, Xinting Huang, Enbo Zhao, Yu Zhang, Yulong Chen, Longyue Wang, Anh Tuan Luu, Wei Bi, Freda Shi, and Shuming Shi. Siren's song in the AI ocean: A survey on hallucination in large language models. *CoRR*, abs/2309.01219, 2023. doi: 10.48550/ARXIV.2309.01219. URL https://doi.org/10.48550/arXiv.2309.01219.

Chujie Zheng, Hao Zhou, Fandong Meng, Jie Zhou, and Minlie Huang. Large language models are not robust multiple choice selectors. In *The Twelfth International Conference on Learning Representations*, 2023.

Han Zhou, Xingchen Wan, Yinhong Liu, Nigel Collier, Ivan Vulić, and Anna Korhonen. Fairer preferences elicit improved human-aligned large language model judgments, 2024. URL https://arxiv.org/abs/2406.11370.

A TRANSITIVITY CALCULATION

In this section, we show two algorithms: (i) construct a preference relation graph by LLM's judgements, and (ii) detect cycle in a relation graph.

Algorithm 1 Construct Preference Relation Graph

Objective: Construct a relation graph reflecting the understanding of LLM upon a set of items.
Inputs: $\mathbf{X} = \{x_1, x_2, \ldots, x_N\}$: A set of items; LLM$(\cdot, \cdot|I)$: LLM, for a given instruction prompt I, as a preference operator.
Output: G: A relation graph represented as an adjacency list for each vertex.
Initialize: Initialize the relation graph G such that for each x in X, $G[x] \leftarrow \varnothing$, where $G[x]$ represents the pointed vertexes from vertex x.
for $i \leftarrow 1$ **to** n **do**
  for $j \leftarrow i+1$ **to** n **do**
    $P(x_i \succ x_j) \leftarrow$ LLM$(x_i, x_j|I)$
    if $P(x_i \succ x_j) \geq \frac{1}{2}$ **then**
      $G[x_i] \leftarrow G[x_i] \cup \{x_j\}$
    else
      $G[x_j] \leftarrow G[x_j] \cup \{x_i\}$
    end if
  end for
end for
Return G

Algorithm 2 Check for Cycles in a Directed Graph

Objective: Detect if a directed graph G contains a cycle.
Inputs: G: A directed graph represented as an adjacency list.
Output: True if a cycle exists, False otherwise.
Initialize: $visited \leftarrow \varnothing$, $recStack \leftarrow \varnothing$
for each v in G **do**
  if $v \notin visited$ **and** CycleUtil(v, visited, recStack) **then**
    Return True
  end if
end for
Return False
function CYCLEUTIL(v, visited, recStack)
  $visited \leftarrow visited \cup \{v\}$, $recStack \leftarrow recStack \cup \{v\}$
  for each u in $G[v]$ **do**
    if $u \notin visited$ **and** CycleUtil(u, visited, recStack) **then**
      Return True
    else if $u \in recStack$ **then**
      Return True
    end if
  end for
  $recStack \leftarrow recStack \setminus \{v\}$
  Return False
end function

B DATASET

SummEval (Fabbri et al., 2021) is a summarization meta-evaluation dataset comprising 100 source texts, each paired with 16 summary candidates generated by various language models (LMs). The dataset is annotated based on four criteria: coherence (COH), fluency (FLU), consistency (CON), and relevancy (RE).

NovelEval (Sun et al., 2023) is a document reranking test set consisting of 21 novel questions. This set was constructed by compiling questions and passages from four domains published after the release of GPT-4. For each question, 20 candidate passages were retrieved through Google search, and their relevance was manually labeled.

CaTerS (Mostafazadeh et al., 2016b) is a dataset focused on temporal event ordering. It contains 1,600 annotated sentences derived from 320 five-sentence short stories sampled from the ROCStories corpus (Mostafazadeh et al., 2016a). These stories are organized based on causal and temporal relations. We filtered the dataset to include only instances with seven or more events, resulting in 70 instances.

Summarize from Feedback. (Mostafazadeh et al., 2016b) consists of 64,832 summary comparisons drawn from the TL;DR (Cachola et al., 2020) and CNN/DM datasets (Nallapati et al., 2016). This dataset is divided into two sections: pairwise comparisons and axis annotations. We concentrate on the pairwise comparison section, which is labeled based on the overall quality of two summaries. For training, we utilize the entire training set, while for testing, we randomly sample 200 instances from the validation set.

MS MARCO. (Nguyen et al., 2016) is a widely used large-scale benchmark for training and evaluating retrieval-based question answering systems. Following the methodology of Sun et al. (2023), we leverage the dataset for a document reranking task. Specifically, they randomly sample a subset of queries from the MS MARCO training set, retrieving 10 candidate passages for each query using BM25. Ranking preferences for these passages were distilled using GPT-3.5-turbo. In this work, we randomly sample 1K examples from their query subset and corresponding ranking labels. We then split them into a training set of 800 examples and a test set of 200 examples.

C Large Language Models in Evaluation

Open-Sourced LLMs. The Llama-2-7b-chat-hf and Llama-2-13b-chat-hf models (Touvron et al., 2023) were developed by Meta and are part of the Llama-2 family. Llama-3-8B-Instruct (Dubey et al., 2024) is another model from Meta's Llama series. The Mistral-7B-Instruct-v0.1 (Jiang et al., 2023) model is an instruction-tuned LLM developed by Mistral AI. The zephyr-7b-beta model (Tunstall et al., 2023) is developed by Zephyr AI. The Phi-3-mini-4k-instruct and Phi-3-medium-4k-instruct models (Abdin et al., 2024) are part of the Phi model series developed by Microsoft. The gemma-2-9b-it (Rivière et al., 2024) model is created by the Gemma team.

API-Based LLMs. The GPT-3.5-turbo-0125 model is part of OpenAI's GPT-3.5 series, which is renowned for its strong performance in natural language understanding and generation. Deepseek-chat (DeepSeek-AI, 2024) is another API-based LLM developed by DeepSeek.

D Additional Related Work

Improving Consistency of LLMs. Asai & Hajishirzi (2020) proposed an augmentation method and a training regularization term to improve the logical consistency of language models for QA tasks. Minervini & Riedel (2018) proposed an adversarial training procedure to improve the robustness of NLI models to adversarial examples. Wang & Henao (2021) focus on improving the paraphrasing consistency during training for the task of low-resource Named Entity Recognition (NER).

Kumar & Joshi (2022) performed a symmetric consistency analysis on NLI and semantic textual similarity (STS) tasks in a more conservative manner, arguing that a model should generate not only the same predictions but also the same confidence scores if it is truly input-order invariant. They also observed that pretrained language models violated symmetric consistency and the authors then introduced a consistency regularisation term to compensate for the issue.

Theory of Consistency. The concept of transitivity in preferences has been extensively explored, revealing its significance in both psychological and logical frameworks. Tversky (1969) highlighted various psychological factors that can lead to intransitive preferences, illustrating how human decision-making often deviates from rational models. Building on this, Arrow (1950) introduced Arrow's Impossibility Theorem, which addresses the inherent challenges of constructing social choice functions that maintain transitivity.

Further contributions to the understanding of transitivity include a comprehensive review by Regenwetter et al. (2011), which examines both theoretical and empirical perspectives on the subject. Their work underscores the critical role transitivity plays in preference structures. In a more formal context, Tarski (1941) examined the role of transitivity within relational calculus, an area that is foundational to

many logical systems. This connection demonstrates how transitivity not only influences preferences but also underpins logical reasoning.

Moreover, Hansson (2001) provided insights into the implications of violating transitivity in preferences, showing that such violations can lead to logical inconsistencies within decision theory. This theme is further explored by Hyde (2011), who proved that apparent violations of transitivity can give rise to logical paradoxes. Collectively, these works emphasize the importance of transitivity in ensuring consistency, both in decision-making and logical deduction.

E INSTRUCTION-TUNING: TRAINING DETAILS

The model was fine-tuned using the following hyperparameters. A learning rate of 5×10^{-5} was employed over the course of 2 training epochs. A weight decay of 1×10^{-2} was applied to prevent overfitting. For the LoRA (Low-Rank Adaptation) settings, the rank r was set to 16 and the scaling factor α was set to 64. The batch size during training was 4, with a gradient accumulation step of 2 to effectively handle smaller batches. All training was performed on an A100 machine.

F METRIC COMPUTATION DETAILS AND PROMPT TEMPLATES

Metrics Computation details. For all three datasets, they have the format of each input context is associated with multiple items, and the task is to determine specific relations among them.

For **SummEval**, the input context is a source text, and the task involves determining coherence preferences among multiple summary candidates. In **NovelEval**, the input context is a query, with the task focused on assessing relevance preferences among multiple retrieved documents. For **CaTeRS**, the input context consists of a list of unordered events, with the goal being to infer the temporal or causal order among these events.

To compute the consistency metrics, a full pairwise comparison is conducted between every pair of items, including all permutations of the item set X. Both standard expression ($F(x_i, x_j)$) and its reverse ($\overleftarrow{F}(x_i, x_j)$) are used. This will result two preference matrices as shown in Fig. 3, and can be used to compute the S_{tran}, S_{comm} and S_{neg} following the Equ. 1, Equ. 2 and Equ. 3.

Prompt Templates. we demonstrate the prompt templates we used to perform pairwise comparisons for all three datasets in Fig. 7.

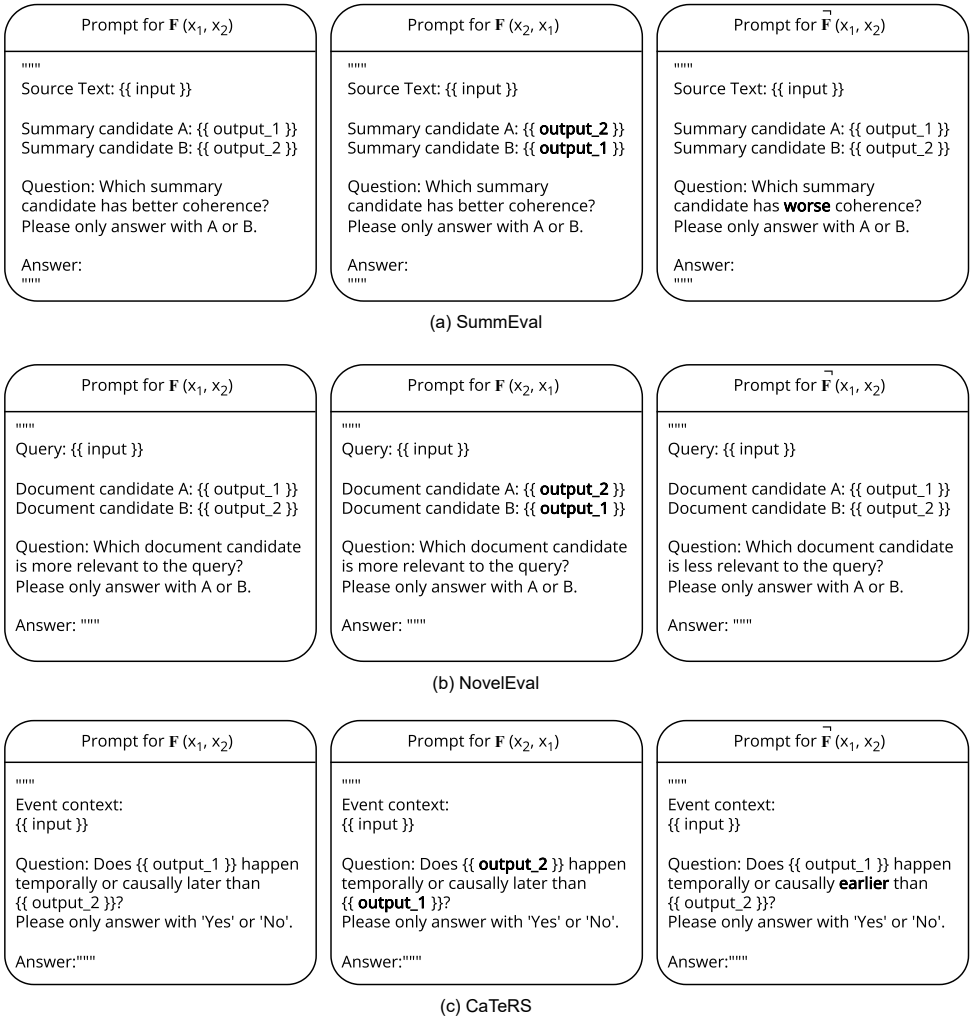

Figure 7: Prompt templates for pairwise comparisons. From left to right: Normal comparison, comparison with reversed item order, and comparison with negated relation.