MAgĪC: Investigation of Large Language Model Powered Multi-Agent in Cognition, Adaptability, Rationality and Collaboration

Anonymous ACL submission

Abstract

Large Language Models (LLMs) have marked a significant advancement in the field of natural language processing, demonstrating exceptional capabilities in reasoning, tool usage, and memory. As their applications extend into multi-agent environments, a need has arisen for a comprehensive evaluation framework that captures their reasoning, planning, collaboration, and more abilities. This work introduces a novel benchmarking framework specifically tailored to assess LLMs within multi-agent settings, providing quantitative metrics to evaluate their judgment, reasoning, deception, self-awareness, cooperation, coordination, and rationality. We utilize social deduction games, Chameleon and Undercover, alongside game theory scenarios like Cost Sharing, Multi-player Prisoner's Dilemma, and Public Good, to create diverse environments. Our framework is fortified with the probabilistic graphic modeling (PGM) method, enhancing the LLMs' capabilities in navigating complex social and cognitive dimensions. The benchmark evaluates 7 multi-agent systems powered by different LLMs, quantitatively highlighting a significant capability gap over threefold between the strongest, GPT-4, and the weakest, Llama-2-70B. It also confirms that our PGM enhancement boosts the inherent abilities of all selected models by 37% on average. Our codes can be found in the anonymous link. https://anonymous.4open.science/r/magic_anonym-5366

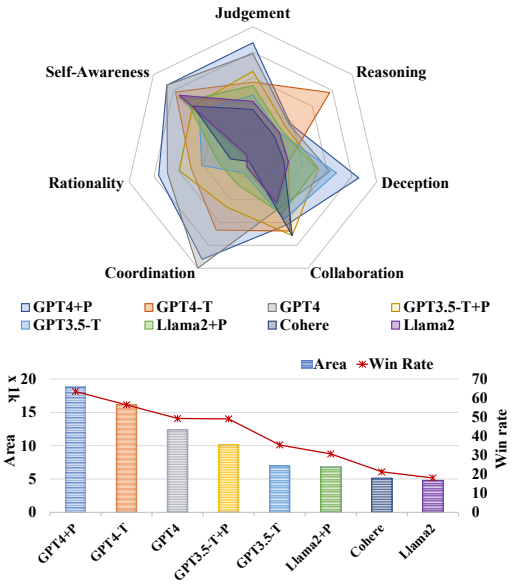

Figure 1: The radar chart depicts LLM's performance on various metrics, with "-T" for "-turbo". The bar chart displays the radar chart's area, with a red line indicating average game-winning rates. A larger area correlates with higher winning rates, validating the effectiveness of the proposed evaluation metrics for assessing LLMs' capabilities. For more information, refer to Sec. 5.

1 Introduction

Large language models (LLMs), particularly ChatGPT and GPT-4 (OpenAI, 2023b), have showcased impressive understanding and generation capabilities. Beyond these fundamental abilities, LLMs also demonstrate promising capabilities in anthropic areas such as reasoning (Wei et al., 2022), planning (Hao et al., 2023), tool usage (Schick et al., 2023), and memorization (Shinn et al., 2023). There is an increasing interest in the investigations of LLMs' behaviors as agents in single or multiple-agent systems with extensive research attention. Noteworthy examples include Generative Agents (Park et al., 2023), Camel (Li et al., 2023a), AutoGPT (Richards, 2023), and Voyager (Wang et al., 2023), all of which are LLM-based agents, have sparked substantial public interest and discourse.

Meanwhile, quantitative assessment of LLMs as agents is crucial for their advancement. Recent benchmarks, such as Liu et al. (2023), evaluate LLM-as-Agent in multi-turn contexts, and a concurrent work Wu et al. (2023) tests them in games requiring reasoning and planning. However, these works focus on understanding and reasoning in specific settings, overlooking capabilities in multi-

agent systems. Other studies, like Agashe et al. (2023) on coordination, Huang et al. (2023) on tool usage, and Fu et al. (2023) on bargaining, just explore specific skills in multi-agent scenarios. Although these studies offer valuable insights into capabilities essential for LLMs navigating complex interactions within multi-agent systems, they are still limited in scope and lack quantitative measurements.

Furthermore, we have observed three characteristics in the interactive multi-agent systems: (1) Agents in these systems often operate within the confines of their local perspectives. However, making wise decisions typically necessitates a good understanding of global information. To overcome this limitation, agents must adeptly discern contexts and reason about the roles or plans of other agents. (2) Contexts are inherently dynamic in multi-agent systems due to the dependent nature of agents' decisions. Success hinges on the ability to swiftly adapt strategies in response to evolving contexts. (3) Collaboration and competition are inevitable when multiple agents try to solve tasks together. The ability to promote cooperation while preserving self-interest is often the ultimate goal of multi-agent systems.

Inspired by the above characteristics, in this work, we first propose a benchmark to comprehensively evaluate the abilities of LLMs as agents within multi-agent systems. We design seven quantitative evaluations from four aspects, cognition, adaptability, rationality, and collaboration. Specifically, (1) **Judgment** and **reasoning** form the core ***cognition*** of agents, crucial for *accurate information estimation* in uncertain scenarios. Judgment evaluates the ratio of the final correct decisions. Reasoning measures the ability to logically analyze other agents' roles and strategy formulation, thus guiding agents to make correct decisions in uncertainty. These combined abilities ensure agents can adeptly handle complex environments, making informed and rational decisions. (2) **Self-awareness** and **deception** are key to *enhanced* ***adaptability*** in agents, vital for multi-agent system. Self-awareness is an assessment of agents' understanding of their capabilities and roles, ensuring the consistency of behaviors towards the target. Deception enables agents to subtly manipulate information in competitive settings, influencing other agents' decisions and gaining advantages in social interactions. (3) ***Rationality*** serves as a metric to gauge the *efficiency* of an agent's behavior.It directs agents towards making decisions with the aim of optimizing their benefits, taking into account the potential actions of other agents rather than resorting to impulsive or uninformed actions. (4) **Cooperation** and **coordination** are two facets of ***collaboration***, essential for effective teamwork in multi-agent systems. Cooperation measures the ability to form successful collaborations, measuring communication and agreeability. Coordination as a finer metric, focuses on aligning and integrating all agents' diverse actions and strategies to facilitate collaboration.

In light of the essential abilities required in multi-agent systems, we further propose a method to enhance LLMs as agents by integrating Bayesian statistical foundations. This novel approach intertwines probabilistic reasoning with LLMs, thereby amplifying their capacity to comprehend intricate scenarios and enabling more informed and strategic decision-making in multi-agent environments. To achieve this, we introduce a Probabilistic Graphical Model (PGM) (Koller and Friedman, 2009) to fortify LLMs' capabilities in global information comprehension. This is achieved by representing complex multi-agent dependencies through multiple random variables arranged in a graphical structure. Conditioned on these well-represented PGMs, agents in multi-agent systems can make more strategic decisions.

In summary, our contributions are as follows:

- We first propose a competition-based benchmark for LLM-powered multi-agent systems by collecting over 100 settings in 5 scenarios and designing 7 metrics to evaluate the true interaction abilities in multi-agent systems.
- We measure 7 LLMs with our benchmark. The results indicate that GPT-4 and GPT-3.5 remain the superior performers, followed by other commercial LLMs - PaLM 2, Claude 2, and Cohere. Llama2 ranks at the lowest.
- We design a PGM-aware agent that integrates LLMs and symbolic reasoning to fortify itself in multi-agent systems. LLMs enhanced with PGM have outperformed their vanilla versions by around 37% over the abilities.

2 Related Work

Emergent Capabilities of LLMs Beyond their core functions, LLMs have shown diverse emergent abilities. Chain of Thought (Wei et al., 2022), Tree

of Thought Yao et al. (2023a), Graph of Thought (Yao et al., 2023b; Besta et al., 2023) and ReAct (Yao et al.) are proposed as effective prompting approaches to enhance the reasoning and planning of LLM. In tool use, "API-bank" (Li et al., 2023b) sets a benchmark for tool-augmented LLMs, while ToolLLM (Qin et al., 2023) offers a framework for this purpose. Furthermore, Reflexion (Shinn et al., 2023) showcases the memory and reflection abilities of LLMs for improved decision-making in subsequent trials. Except for the aforementioned social behavior, Phelps and Russell (2023) investigate the goal-like behavior in economics.

LLMs-Powered Agents Generative Agents (Park et al., 2023) presents a sandbox environment inhabited by 25 AI agents capable of simulating human behavior. These generative agents store extensive records of their experiences, deepening their self-awareness and environmental understanding through reflection, and selectively retrieving information to guide their actions. Auto-GPT (Richards, 2023) showcases the capabilities of the GPT-4 language model in an experimental open-source application. It chains together LLM-generated thoughts to autonomously achieve set goals, demonstrating the model's proficiency in complex tasks. Camel (Li et al., 2023a), explores the "mental" landscape of large language models in AI society, introducing a role-playing agent framework for communication between two AI agents, facilitating nuanced interactions. Voyager (Wang et al., 2023), marks the debut of an LLM-powered lifelong learning agent in Minecraft, capable of continuous exploration, skill acquisition, and novel discovery, all without human intervention.

3 Benchmark

We propose to measure the abilities of various LLMs by putting them into competition in multiple multi-agent scenarios. In this way, we measure the genuine capabilities of LLMs when interacting with multiple agents. To achieve this, we have constructed a comprehensive benchmark that incorporates various competition settings and meticulously designed metrics for each scenario. The detailed constitutes are illustrated in Figure 2.

3.1 Scenarios

As mentioned in Sec. 1, the evaluation of agents in multi-agent systems revolves around crucial attributes such as cognition, adaptability, rationality, and collaboration. To comprehensively assess these capabilities, we present five distinct scenarios. In the game of Chameleon and Undercover, quickly comprehending global information and making corresponding actions are the keys to winning the game. Thus we mainly measure the cognition and adaptability in these two scenarios. Moving to game theory scenarios, which require the agent to make optimal decisions based on the given premise (Myerson, 1991), they are more apt for reflecting rationality and collaboration. As such, we center our evaluation on these latter two attributes in the context of three game theory scenarios. The competition rules are detailed in A.7

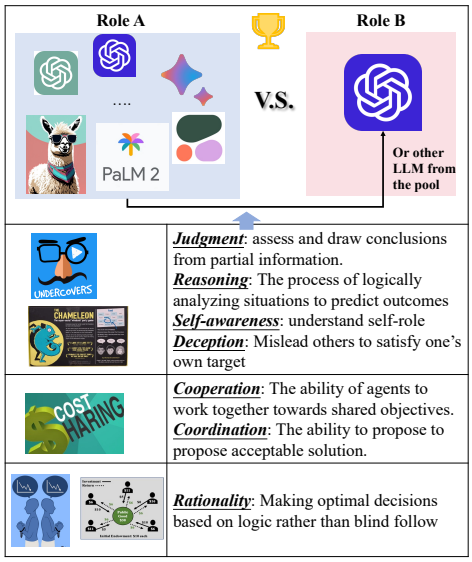

Figure 2: Overview of evaluation setting, scenarios, and proposed metrics.

Chameleon is a social deduction game where players are either a chameleon or a non-chameleon. Players give clues about a secret word; chameleons try to blend in without knowing the word, while non-chameleons aim to expose the chameleon without revealing the word.

Undercover as a similar game, divides players into civilians and undercovers. The word for undercovers is different from civilians, players explore their roles while giving clues, and civilians need to find the undercovers while undercovers should hide themselves.

Cost Sharing involves three parties sharing costs based on their usage of a shared resource. The initial allocation is proposed according to each party's usage, with subsequent negotiations to adjust the distribution to ensure fairness. The game is won by reaching a unanimous agreement on the allocation.

Prisoner's Dilemma extends the classic Prisoner's

Dilemma to a multi-round three-player version. Each participant decides to cooperate or defect in each round with scores determined by collective choices. The game tests players' ability to strategize, foster trust, and navigate group decision-making. The player with the highest total score at the end of the game is declared the winner.

Public Good allows players with fixed initial resources to decide how much to contribute to a common pool at each round. The pool's total is then multiplied, and distributed evenly. The winner is the player possessing the most resources at the end.

3.2 Competition Settings

The core of our benchmark is the *competition settings* and the *evaluation metrics*. We've gathered a collection of competition settings for each scenario. The detailed structures of competition settings and the way of collecting competition settings can be found in A.1. In a competition, the to-be-evaluated Large Language Model (LLM), referred to as the *challenger LLM* in this paper, will be assigned one role, and another LLM will undertake the other role(s). To consistently compare the capabilities of different *challenger LLMs*, we made *challenger LLMs* play with the same LLM, which is GPT-4 as we used in our experiments.

In Chameleon and Undercover, there are clearly two opposite parties, the Chameleon versus Non-Chameleons and the Undercover versus Civilians. Therefore, *challenger LLM* will play each party respectively. For example, the *challenger LLM* plays non-chameleons versus GPT-4 as the chameleon, and the *challenger LLM* plays the chameleon versus GPT-4 as non-chameleons. The win rates of the *challenger LLM* playing different parties will be averaged to indicate its capability.

Cost Sharing has no distinct parties. Therefore, we made the *challenger LLM* as one player to play with other GPT-4-powered players. The final ratio of successful negotiations is defined as the win rate, which measures how much the LLM contributes to the agreement when other players are fixed. Similarly, for public good and prisoners' dilemma, we also made the *challenger LLM* as one of the players and recorded its win rate in these two games. The detailed win rate calculations are presented in A.2.

3.3 Evaluation Metrics

In assessing the seven capabilities within a multi-agent system, the metrics below directly correspond to the core functions necessary for proficient performance in multi-agent environments.

Let $\mathcal{S}$ be the set of roles the challenger LLM will play in all the scenarios. In Chameleon and Undercover, the challenger LLM will play the Chameleon, Non-Chameleons, Undercovers, and Civilians respectively. As for game theory scenarios, the challenger LLM only plays as one of the players. Thus, the length of $\mathcal{S}$ is 7 in our benchmark. For each role, we have defined the criteria for winning and denote the win rate as $w_{s_i}, s_i \in \mathcal{S}$. The detailed definition for winning for each role can be found in A.2.

Win Rate is a straightforward indicator of the success of an LLM in all proposed scenarios.

$$w_r = \frac{1}{|\mathcal{S}|} \sum_{s_i \in \mathcal{S}} w_{s_i} \tag{1}$$

Judgement is the ratio of correct votes, representing the accuracy of an agent's judgment, essential for assessing their ability to distinguish the identities of others based on the partial information provided by other players.

$$S_J = n_{cv}/n_v \tag{2}$$

where n_{cv} and n_v are the number of correct votes and total votes when the challenger LLM are playing civilians and non-chameleons.

Reasoning. The proportion of correct role analyses reflects the agents' ability to logically reason the global status when partially given the information. We let each player analyze other players' roles and think a step further about other players' analysis. We compare these analyses with the gold situation and the subjective analysis of others to decide whether the analysis is right or not. We denote number of these two analysis as n_{gold} and n_{inter}. The number of correct analysis as $n_{\text{c_gold}}$ and $n_{\text{c_inter}}$.

$$S_R = (n_{\text{c_gold}} + n_{\text{c_inter}})/(n_{\text{gold}} + n_{\text{inter}}) \tag{3}$$

Deception evaluates an agent's capability to successfully deceive others by blending in as a chameleon/undercover or causing incorrect code guesses, which is crucial in competitive settings.

$$S_D = n_{\text{wuc}}/n_{\text{uc}} + \lambda(n_{\text{wcg}}/n_{\text{cg}}) \tag{4}$$

where n_{wuc} are the number of wins as the chameleon and undercover, n_{uc} is the total number of competitions as the chameleon and undercover, n_{wcg} is the number of incorrect code guesses, and

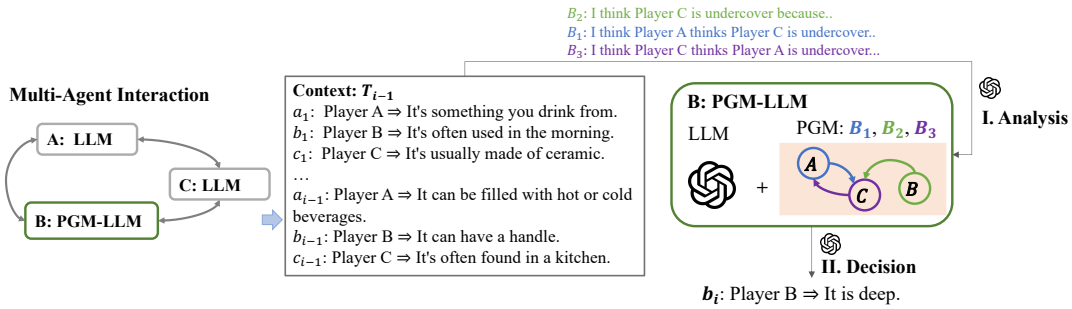

Figure 3: A Decision process of the PGM-aware agent.

n_{cg} is the total number of code guesses. Here we assign $\lambda = 0.25$.

Self-Awareness, correct role identification is critical for agents to function within their capabilities and is indicative of their level of self-awareness.

$$S_{\text{self}} = \mu(n_{\text{crc}}/n_{\text{rc}}) + n_{\text{cru}}/n_{\text{ru}} \tag{5}$$

where n_{crc}, n_{rc} is the number of correct and the total number of role identifications in chameleon, and n_{cru}, n_{ru}. $\mu = 0.6$ is used because it is much easier to identify roles in a chameleon game.

Cooperation. The success in cost-sharing games and prisoner's dilemma quantify how well agents work together, showcasing the collective efficacy of the system.

$$S_{\text{collab}} = n_{\text{wcs}}/n_{\text{cs}} \tag{6}$$

where n_{wcs} and n_{cs} are the number of successful and the total number of cost-sharing games.

Coordination is the ratio of successful collaborations proposed by the challenger LLM.

$$S_{\text{coord}} = n_{\text{pcs}}/n_{\text{wcs}} \tag{7}$$

where n_{pcs} is the number of successful collaborations proposed by the challenger LLM in the cost-sharing games.

Rationality captures the agents' ability to act rationally based according to the rules of the game theory scenarios (Myerson, 1991). Suppose there are $\mathcal{T}_{\text{pd}}$, $\mathcal{T}_{\text{pg}}$ rounds in each competition for Prisoner's Dilemma and Public Good. The Rationality is defined as:

$$S_R = \frac{n_b}{n_{\text{pd}} * \mathcal{T}_{\text{pd}}} + \frac{n_{\text{li}}}{n_{\text{pg}} * \mathcal{T}_{\text{pg}}} \tag{8}$$

where n_b is the round of betray decisions, n_{li} is the round of decisions where the challenger LLM contributes the least in the common pool, n_{pd} and n_{pg} are the number of prisoner's dilemma competitions, and the number of public good competitions, respectively.

4 PGM-Aware Agent

In AI, Bayesian methods embody symbolism, while large language models (LLMs) exemplify connectionism. Despite their individual strengths, effectively combining these approaches remains a challenge. LLMs are proficient in complex language tasks but still struggle with ambiguous relationships and causal reasoning. This shortcoming is especially evident in multi-agent scenarios requiring precise inferential analysis. To address this, we propose integrating Probabilistic Graphical Models (PGMs), classic Bayesian tools adept at depicting dependencies between random variables, to enhance LLMs' analytical and inferential capabilities.

4.1 PGM Structure

We leverage Probabilistic Graphical Models (PGM) to depict intricate dependency relationships among all agents, thereby augmenting the LLMs' comprehension of global information. This heightened understanding subsequently facilitates more informed actions/decisions in the subsequent steps. To ensure wise decision-making for an agent, the PGM should be comprehsive and thorough. For instance, consider the prisoner's dilemma scenario. Before deciding to defect or cooperate, it is crucial to anticipate how others might defect or cooperate and, from others' perspectives, how you will decide. If you anticipate that other players will cooperate and they expect the same from you, but you choose to defect, it can lead to a significant advantage for you. As a result, We design the PGM structure in a two-hop understanding mechanism in which the agent analyze from its own perspective and perspective when it stands in other agents' shoes.

Formally, as shown in Figure 3, suppose there are three players A, B, and C in one game and they've played the game for $i-1$ turns and formed the context $T_{i-1} = \{a_1, b_1, ..., a_{i-1}, b_{i-1}, c_{i-1},\}$.

	Win Rate	Judge.	Reason.	Decept.	Self-aware.	Cooper.	Coord.	Rational.
GPT-4-turbo	**57.2**	81.2	**37.0**	65.0	**55.0**	**66.7**	33.4	**78.1**
GPT-4	53.3	**83.8**	32.3	**75.0**	**55.0**	47.6	**47.6**	69.0
GPT-3.5-turbo	39.3	52.5	24.5	77.5	25.9	57.1	9.50	41.4
Claude 2	34.0	45.0	34.0	25.0	50.0	71.4	23.8	24.3
PaLM 2	33.3	43.8	25.8	32.5	41.1	42.9	14.3	38.1
Cohere	27.3	42.5	27.8	37.5	35.6	71.4	4.80	18.1
Llama-2-70B	26.5	45.0	37.0	40.0	53.2	42.9	4.80	5.20

Table 1: Ability Measurements of LLMs.

Here a_*, b_*, and c_* are the decisions from Players A, B, and C. Player B is a PGM-Aware Agent and it manages three distinct random variables, denoted as B_1, B_2, and B_3. These variables encapsulate Player B's interpretation of global information from the perspectives of the three players. We obtain the estimation for these random variables by prompting LLMs through different prompts, $\mathcal{P}_j^{\text{pgm}}, j \in [1, 2, 3]$:

$$P(B_j) = \text{LLM}(B_j | \mathcal{P}_j^{\text{pgm}}, T_{i-1}) \tag{9}$$

4.2 LLM Decision with PGM

For a basic LLM agent in multi-agent, the inference process is formulated as:

$$P(b_i) = \text{LLM}(b_i | \mathcal{P}, T_{i-1}) \tag{10}$$

where $\mathcal{P}$ is the prompt to let the LLM go to the next step. Our PGM-Aware Agent makes decision conditioned both on the PGM and game contexts, which can be formulated as:

$$P(b_i) = \text{LLM}(b_i | \mathcal{P}^{\text{decision}}, B_1, B_2, B_3, T_{i-1}) \tag{11}$$

where $\mathcal{P}^{\text{decision}}$ is the prompt to guide the LLM to make a decision given both PGM and context in the next step. B_1, B_2, B_3 are the PGM acquired in Equation 10. We have listed the prompts used in basic LLMs and the PGM-Aware Agent in A.7.

5 Experiments

5.1 LLM Leaderboard

We evaluate GPT-3.5-turbo (OpenAI, 2023a), GPT-4 (OpenAI, 2023b), Llama-2-70B (Touvron et al., 2023), PaLM 2 (Anil et al., 2023), Cohere (Cohere, 2023) and Claude 2 (Anthropic, 2023) with our benchmark. In Figure 1, we demonstrate a clear comparison of the capabilities of different LLMs across various metrics. The most prominent performer is the GPT-4-turbo method, showcasing outstanding overall performance with a remarkable win rate of 57.2%. This significantly higher win rate underscores its competitive advantage. Following closely is GPT-4, which achieves a win rate of 53.3%, demonstrating its competitiveness. Furthermore, based on their respective area coverage in the radar chart (Area claculated in the right bar chart), it becomes apparent that GPT-4-turbo outperforms LLaMa-2-70B by more than three times in overall multi-agent capabilities, while GPT-3.5-turbo remains superior to LLaMa-2-70B. We also assess other popular commercial LLMs such as PaLM 2, Claude 2, and Cohere, the experimental results indicate their abilities in multi-agent settings are between GPT-3.5-turbo and Llama-2-70B. Importantly, it can be clearly seen from Figure 1 that the area sizes calculated based on the proposed abilities' values are directly proportional to the winning rates, which verifies the correlation between them and further illustrates our benchmark can effectively assess the LLMs' capabilities.

As demonstrated in Table 1, for a more detailed comparison, we evaluated metrics such as Judgment, Deception, Reasoning, and Self-Awareness within the Chameleon and Undercover scenarios. In these contexts, GPT-4 excelled with impressive scores of 83.8% in Judgment and 75.0% in Deception, solidifying its leadership in these scenarios. Notably, reasoning abilities exhibited the closest performance gap among these models, while deception capabilities showcased significant disparities. Furthermore, when assessing metrics related to collaboration, coordination, and rationality in game theory scenarios like Cost Sharing, Prisoner's Dilemma, and Public Good, GPT-4 and GPT4-turbo continued to shine. It achieved 66.7% in Coordination and the optimal performance of 78.1% in Rationality. In contrast, LLaMa-2-70B, while lagging in overall performance with a win rate of 26.5%, exhibited strengths in specific metrics, such as a relatively high self-awareness score of 53.2%. This score also surpasses GPT-3.5-turbo's 25.9%.

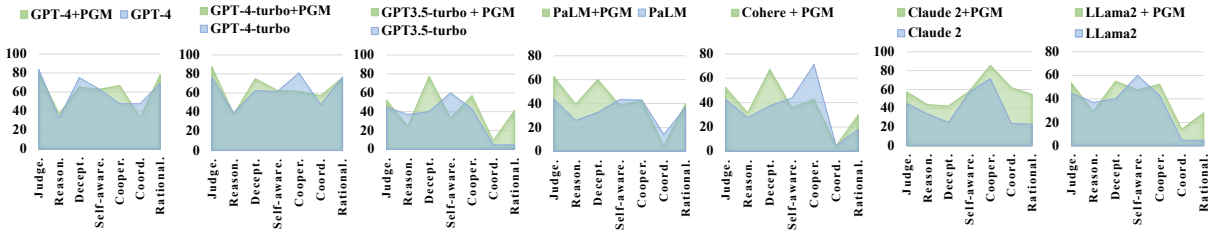

Figure 4: Comparisons on 7 metrics and it's on all metrics

5.2 PGM Enhancement Performance

As shown in Figure 4, the green section highlights the effectiveness of the PGM-aware approach. This enhancement is particularly pronounced in the PaLM, Claude 2, and Llama2 models, as detailed in A.3. Overall, the PGM-aware method has achieved average improvements across all capabilities by a margin of 37%, which is calculated by the difference between the radar areas achieved by vanilla and PGM-aware methods, as illustrated in Figure 1. Accordingly, PGM-aware methods also win more in all the scenarios with an enhancement of 6.57% in win rate. As for each capability, as illustrated in A.3, PGM-aware methods achieve an 8.72% increase in Judgement, confirming the method's ability to strengthen analysis in LLMs. Additionally, Reasoning and Deception abilities have seen increases of approximately 5% and 6%, respectively. Notably, the most significant enhancements are observed in Coordination and Rationality, with improvements of 12.2% and 13%. We also provide some qualitative results to show the influence of PGM in 5.3 and A.4.

5.3 Discussion

The above experimental results prove that the PGM-Aware agent can improve the performance of each metric to varying degrees. The discussion part explores the influence of PGMs from other aspects in each scenario.

RQ1. How PGM of different LLMs help Judgement and Reasoning? In Figure 5, we provide a case of Llama-2-70B, GPT-4, and their PGM-Aware versions playing as non-chameleons versus GPT-4 as the chameleon. For Llama-2-70B and GPT-4, we can find both LLMs failed to win the game because they voted for the wrong chameleon "Player 1". After being equipped with PGM, both models change the game results to Non-chameleons won. If we look into the details of the game process, in the first two clues, the PGMs given by LLama-2-70B all indicate wrong chameleons, as highlighted in red in Figure 5, since "Player 2" is the true chameleon. In the third round, "Player 1" gave another right analysis which successfully changed the PGM to the right indication of the chameleon. However, if we look at the content of the analysis, we find the analysis gave opposite conclusions and explanations. For example, "more suspicious" is "more specific to the secret word". Besides, LLama-2-70B also presents some hallucinations in the game, for example, after Player 1 already gave the clue, the analysis still states "Player 1 hasn't given a clue yet". While GPT-4+PGM's analysis aligns the conclusions and explanations well and has no hallucinations. According to the example, we can find PGM could be helpful for models to make better judgments through clear analysis and PGM is affected by the ability of LLMs. The more powerful the model, the more accurate its judgment and reasoning.

LLM	Cost-Sharing		Prisoner		Public Good	
	WR↑	Cost↓	WR↑	Score↑	WR↑	Payback↑
Llama-2	42.8	37.1	0.0	6.05	0.0	139.1
Llama-2+P	52.4	37.6	38.5	9.86	4.8	109.5
GPT-3.5-T	57.1	37.3	33.3	9.57	9.5	166.2
GPT-3.5-T+P	**71.4**	34.2	52.4	**11.6**	57.1	139.8
GPT-4	47.6	30.5	42.9	9.95	61.9	**175.3**
GPT-4+P	61.9	**30.3**	**76.2**	10.6	**85.7**	144.1

Table 2: Detailed results in game theory scenarios. "Cost", "Score", and "Payback" are the average cost, the final score, and the average payback the challenger LLM got in the Cost sharing, Prisoner's Dilemma and Public Good, respectively.

RQ2. Does Collaboration correlate with Cost in Cost Sharing? As shown in Table 2, we list the win rate(WR) results and several important indicators in each game theory scenario. For cost-sharing, we calculated the average final cost the challenger LLM needs to bear after their negotiations. In the negotiation, this is another target the LLM-powered agent should consider when trying to reach an agreement with other agents. However, these two aspects can contradict each other sometimes. For example, when the player tries to reduce the cost of himself as much as possible, it might be hard for him to achieve agreement with other players. The LLMs need to make a balance

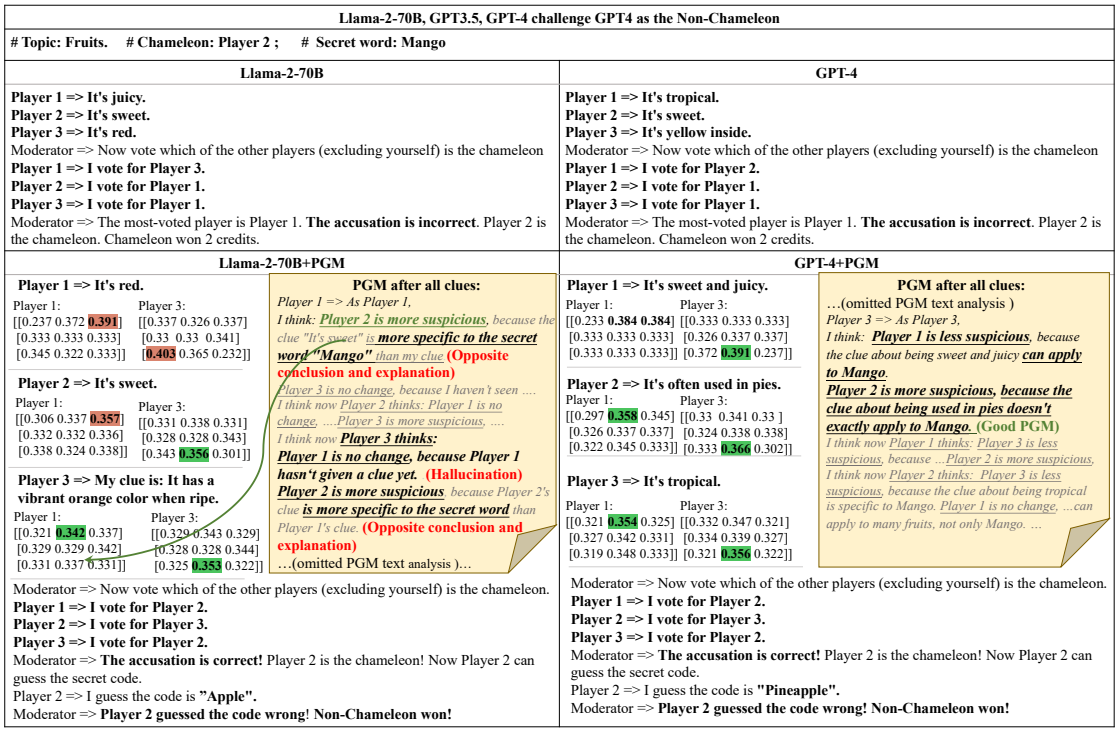

Figure 5: A Chameleon case study on Llama-2-70B, GPT-4 and their PGM-enhanced version (*+PGM).

between these two aspects. According to the results in Table 2, we find that within the models without PGM enhancement, GPT-3.5-turbo won in Win Rate while GPT-4 won in Cost, indicating both models are not well-balanced. If we compare the results with PGM, GPT-4+PGM increases the Win Rate and keeps the cost slightly lower. GPT-3.5-turbo+PGM increases the Win Rate and reduces the cost simultaneously. This proves the effectiveness of PGM enhancement and demonstrates that GPT-3.5-turbo tends to be more collaborative while GPT-4 emphasizes the reduction of cost.

RQ3. Does Rationality correlate with reward? Similar phenomena happen in Prisoner's Dilemma and Public Good as illustrated in Table 2. In these two scenarios, a player is more likely to win when he chooses to betray as a prisoner or chooses to reduce contribution to the common pool in the public good game. The behavior is considered Rational in our metrics. When most of the players are playing rationally, the scores and payback will be much lower, thus approaching the well-known Nash Equilibrium (Kreps, 1989). In the Prisoner's Dilemma, if we compare GPT-3.5+PGM and GPT-4+PGM, GPT-4+PGM won more but got lower scores, showing that GPT-4+PGM made more rational decisions than GPT-3.5-turbo+PGM. In Public Good, we found models with PGM all achieved higher Win Rates but lower payback because they all performed more rationally in this scenario. If we compare the payback within models with or without PGMs, we can observe higher payback for GPT-4 models, which proves that GPT-4 models are more strategic in these games.

6 Conclusion

Our research presents a multi-agent benchmarking framework tailored for evaluating LLMs in multi-agent environments. This framework's incorporation of diverse games and game theory scenarios has enabled a quantitative assessment of LLMs in dimensions of judgment, reasoning, deception, self-awareness, cooperation, coordination, and rationality. The integration of PGM into the current LLMs marks a significant enhancement, enriching the LLMs' capabilities in multi-agent environments. The quantitative analysis of 7 different multi-agent systems powered by various LLMs, including GPT-4-turbo, GPT-4, GPT-3.5-turbo, PaLM 2, Claude 2, Cohere, and Llama2-70B, has revealed their capabilities' disparity. Notably, GPT-4-turbo still emerged as the most capable, outperforming others by a threefold margin. Moreover, our findings confirm that the PGM enhancement amplifies the inherent abilities of these models by 37%. This indicates not only the effectiveness of our benchmarking framework but also the potential of PGM as a tool for advancing LLM capabilities.

Limitation

This paper proposed a benchmark for measuring the ability of LLMs in multi-agent systems. We conclude our limitation as below: Firstly, our investigation of LLMs in multi-agent settings is in its preliminary stage. The scope of game scenarios and topic settings needs to be significantly expanded. Secondly, the PGM-aware method has the potential to enhance LLMs' capabilities in the face of complex multi-agent settings. However, the process of integrating these incremental abilities into LLMs through methods such as fine-tuning requires further exploration.

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A Appendix

A.1 Competition Settings

Setting Definition As shown in Table 3, we present the number of settings, corresponding metrics, and setting samples for each scenario. We build 20 settings for chameleon and undercover, respectively. In each game, Chameleon includes one round of clue giving while undercover contains 2 rounds. For each of the game theory scenarios, we collected 21 settings.

Collection Process In the Chameleon and Undercover scenarios, we've noticed a consistent bias in competition outcomes. Specifically, the Chameleon team has held an advantage in Chameleon, whereas in Undercover, the civilians have tended to win. To rectify this imbalance, we carried out 200 game simulations involving all three players as GPT-4 with randomly chosen topic settings. Through these simulations, we pinpointed 20 topic settings that promote a more equitable win rate between the two roles in both Chameleon and Undercover. In these scenarios, the challenger LLM will play both roles to measure different abilities such as judgment and deception, etc.

For the Cost-Sharing task, we expect all the participating airlines to share a fixed fee, with the specific share of each airline determined by its operational frequencies at the airport. These frequencies encompass various factors such as the number of flights, flight sizes, passenger volumes, and more. To facilitate the task, we asked ChatGPT to create a pool of 20 detailed descriptions of airline operational frequencies. A topic setting with 3 players is then constructed by three airline operational frequency descriptions from the pool, the role, and the position of the test LLM. Since there are 3 positions, we randomly selected 7 groups of airline operational frequency descriptions to form 21 distinct topic settings.

Similarly, for the two-game theory scenarios, we adopt a similar topic construction method as Cost Sharing. In the Prisoner scenario, three players choose to "defect" or "cooperate" for 5 rounds. Each player will get a different score depending on the outcomes of "defect" or "cooperate". The player with the highest cumulative score wins the game. We have devised 7 distinct scoring settings, and the challenger LLM plays the role of each player across these settings, resulting in 21 unique competitions.

In the Public Good game, three players determine the number of points to contribute to a communal pool for 5 rounds. These invested points are multiplied by a specified factor (typically greater than 1), and the resulting sum is equally distributed among all players. Each player's final score comprises their remaining points and the payback from the communal pool. The player achieving the highest score is declared the winner. We establish 7 different multipliers and assign the challenger LLM to play each of the three players in these settings, thus generating an additional 21 competitions.

A.2 Win Rate Definition

In the chameleon, the outcome can be 0: the non-chameleon won, 1: the chameleon won, 2: even voting, and 3: the chameleon guessed right. In these four situations, credits gained by the role chameleon and non-chameleon are $c_{\text{chameleon}} = [0, 1, 2, 1]$ and $c_{\text{non-chameleon}} = [2, 1, 0, 1]$, respectively. Suppose the outcomes of the n competitions are o. The total credits of all the completions are $2n$; the win rate defined in Chameleon is

$$w_{\text{r}} = \frac{\sum_{i \in n} c_{\text{r}}[o_i]}{2n}, \text{r} \in [\text{chameleon}, \text{non-chameleon}]$$

Similarly, in Undercover, the outcome can be 0: undercover won, 1: civilian won, and 2: even voting. The credits for the role undercover and civilians are $c_{\text{undercover}} = [3, 0, 2]$ and $c_{\text{civilian}} = [0, 3, 1]$, respectively.

$$w_{\text{r}} = \frac{\sum_{i \in n} c_{\text{r}}[o_i]}{2n}, \text{r} \in [\text{undercover}, \text{civilian}]$$

The win rate of cost sharing is the success rate of achieving consistency in all competition. In the game theory settings, the win rate is the ratio of the testing player winning the competition.

A.3 PGM Enhancement Performance

We present all the experimental results in Table 4.

A.4 More Case Studies

Deception Another advanced cognitive ability of LLMs extends to their proficiency in strategic deception within a multi-agent framework. In Figure 6, we delve into the dynamics of LLM performance when assuming an undercover role against GPT-4. In this scenario, LLMs are expected to blend in with regular civilians and even give misleading clues to conceal their actual roles. In this example, GPT-3.5-turbo, GPT-3.5-turbo+PGM lost the game, GPT-4 ended with even voting, and GPT-4+PGM won the game. According to their clues, we found models without PGM didn't tend to deceive others, and their clues describe their own words. Within these models, GPT-4 is more cautious when giving clues, while GPT-3.5 often gives very straightforward clues, like "It can be done at a salon or barbershop" and "It can be washed with shampoo" to describe "hair cut".

In contrast, models augmented with PGM adopted a more sophisticated strategy, utilizing PGM analyses to decide their roles, identify potential undercovers, and outline strategic moves for subsequent rounds. This strategic advantage allowed these models to give fake clues to disguise themselves. For instance, as highlighted in Figure 6, "Player 2" portrayed by GPT-3.5-turbo+PGM and GPT-4+PGM introduced deceptive clues, such as "It can be made from human and synthetic hair" and "It can be used to cover baldness," respectively. However, "Player 2" played by GPT-3.5-turbo+PGM, still lost the competition because its first clue looked like copying from "Player 1". "Player 2" played by GPT-4+PGM, won the competition because of its consistently good clues in two rounds. Comparing the results of the three LLMs with PGM, we can conclude that a more capable LLM can benefit more from the PGM structure.

A.5 PGM prompts

We have designed different prompts to test LLMs's ability to make PGM analyses. Two kinds of prompts are used: text-based(**w.PGM**) and direct probability matrix(**w.ProPGM**). In specific, the former lets the LLM analyze global information in text, as shown by the example in Figure 3. The latter requires the LLM to directly give a probabilistic matrix to represent the global information, for example, a matrix [[0.3,0.2.0.5],[0.1,0.4,0.5],[0.3,0.3,0.4]]. The three roles represent B_1,B_2, and B_3 respectively. Each element in a role is the probability of a player being the undercover or the chameleon for example. We compare the vote accuracy and Win rate of these two kinds of prompts in the scenario chameleon, as shown in Figure 7. We found that more capable LLMs, like GPT-4 and GPT-3.5-turbo, both kinds of prompts work well. However, for Llama-2-70B the text PGM analysis performs much better. Therefore, we mainly choose the prompt the LLMs to give text-based PGM analysis to ensure the help of

	Chameleon	Undercover	Cost Sharing	Prisoner's Dilemma	Public Good
Judgement	✓	✓	-	-	-
Reasoning	✓	✓	-	-	-
Deception	✓	✓	-	-	-
Self-Awareness	✓	✓	-	-	-
Collaboration	-	-	✓	✓	✓
Coordination	-	-	✓	-	-
Rationality	-	-	✓	✓	✓
# Rounds	1	2	5	5	5
# Competitions	20	20	21	21	21
Setting sample	`{ "topic": "Fruits", "code": "Grape", "chameleon_name": "Player 3", "first clue": { "Player 1": "It's often used to make wine.", "Player 2": "It's small and round.", "Player 3": "It's red in color." } }`	`{ "undercover_code": "viewer", "non_undercover_code": "listener", "undercover_name": "Player 1" "first clue": { "Player 1": "My clue is: television.", "Player 2": "My clue is: radio.", "Player 3": "My clue is: audience." } }`	`{ "test_player_name": "Player 3", "topic": "Fixed Airport Fee: $1,000,000 Airline Usage Frequency Data: Airline A: Number of Flights: 90/month Number of Passengers: 7,000/month Average Flight Duration: 1.75 hours Flight Size: Primarily small and medium aircraft Airline B: Number of Flights: 50/month Number of Passengers: 7,500/month\Average Flight Duration: 1.5 hours Flight Size: Primarily small aircraft Airline C: Number of Flights: 50/month Number of Passengers: 7,500/month Average Flight Duration: 1.5 hours Flight Size: Primarily small aircraft", "first proposal": { "Player 1": [40,30,30], "Player 2": [50,25,25], "Player 3": [50,25,25] } }`	`{ "test_player_name": "Player 2", "topic_values": { "cooperate": 2, "defect": 1, "one_defect": 4, "two_defect": 2 } }`	`{ "test_player_name": "Player 1", "game_round": 5, "multiplier": 3.5 }`

Table 3: Consolidated Game Settings for Testing Abilities

	Win Rate	Judge.	Reason.	Decept.	Self-aware.	Cooper.	Coord.	Rational.
GPT-4-turbo+PGM	58.3	76.2	39.2	62.5	56.9	**81.0**	47.6	76.7
GPT-4-turbo	57.2	81.2	37.0	65.0	55.0	66.7	33.4	**78.1**
GPT-4+PGM	**63.5**	**87.5**	37.8	**75.0**	**61.3**	61.9	57.1	76.2
GPT-4	58.3	83.8	32.3	**75.0**	55.0	47.6	47.6	69.0
GPT-3.5-turbo+PGM	49.1	65.0	33.5	62.5	36.1	71.4	33.3	59.5
GPT-3.5-turbo	39.3	52.5	24.5	77.5	25.9	57.1	9.50	41.4
Claude 2 + PGM	43.0	57.5	**44.0**	42.5	60.0	85.7	**61.9**	54.8
Claude 2	34.0	45.0	34.0	25.0	50.0	71.4	23.8	24.3
PaLM 2 + PGM	41.4	62.5	39.3	60.0	34.5	42.9	4.80	40.0
PaLM 2	33.3	43.8	25.8	32.5	41.1	42.9	14.3	38.1
Cohere + PGM	35.8	52.5	31.8	67.5	30.4	42.9	4.80	30.0
Cohere	27.3	42.5	27.8	37.5	35.6	71.4	4.80	18.1
Llama-2-70B+PGM	30.8	53.7	29.3	55.0	45.2	52.4	14.3	28.1
Llama-2-70B	26.5	45.0	37.0	40.0	53.2	42.9	4.80	5.20
Average improvement	6.57	8.72	5.21	6.07	0.66	5.46	12.2	13.0

Table 4: Ability Measurement of LLMs.

PGM on all the LLMs.

GPT-3.5-turbo and GPT-4 as Undercover

Moderator => Now the game starts!
Moderator => **[Player 2]Your word is: haircut ;** Moderator => **[Player 1]Your word is: wig**; Moderator => **[Player 3]Your word is: wig**

GPT-3.5-turbo	GPT-4
Player 1 => It's something you wear on your head. **Player 2 => It's something you often get done at a salon or barbershop.** **Player 3 => It can be used to change your appearance.** **Moderator =>** Now we starting another round of giving clue… **Player 1 => It can be of different colors.** **Player 2 => It's something that involves cutting and styling your hair.** **Player 3 => It can be synthetic or natural.** **Player 1 =>** I vote for **Player 2; Player 2 =>** I vote for **Player 3…; Player 3 =>** I vote for **Player 2.** **Moderator =>** The most-voted player is Player 2. **The accusation is correct.** Player 2 is the undercover! **Civilians won!**	**Player 1 => It's something you wear on your head.** **Player 2 => It's something that can change your appearance.** **Player 3 => It's often used in theater productions.** **Moderator =>** Now we starting another round of giving… **Player 1 => It can be made of synthetic materials.** **Player 2 => It's something that can be styled in different ways.** **Player 3 => It's often used by people who have lost their hair.** **Player 1 =>** I vote for **Player 2; Player 2 =>** I vote for **Player 3; Player 3 =>** I vote for **Player 1.** **Moderator => There are even votes.** The accusation does not stand. Player 2 is the undercover.
GPT-3.5-turbo+PGM	**GPT-4+PGM**
Player 1 (clue) => It's something you wear on your head. **Player 2(clue) => It's something you wear on your head, similar to a hat.** **Player 3 (clue) => It can change your appearance** **Moderator =>** Now we starting another round… **Player 1(clue) => It's often used in theater.** **Player 2 (clue) => It can be made from human or synthetic hair.** **Player 3 (clue) => It can come in different colors and styles.** **Player 1 => I vote for Player 2; Player 2 => I vote for Player 3; Player 3 => I vote for Player 2.** **Moderator => The accusation is correct!** Player 2 is the undercover! **Civilians won!** *Player 2 (pgm) => Based on the clues given, Player 1's word could be "wig" and Player 3's word could be "mask" ....As Player 2, your word is "haircut" which does not fit the clues given by Player 1 and Player 3. Therefore, **you can conclude that you are the undercover player in this round**.* *2. I think **Player 1 thinks Player 3** is the undercover because Player 3's ...is more vague and could apply to various items, while Player 1's ...a specific item.* *3. I think **Player 3 thinks Player 1** is the undercover because Player 1's clue ... is more specific and indicates a .. while Player 3's clue ... could apply to multiple items.* *4. ...I will try..clue as "It can be made from human or synthetic hair," which is similar to both a hat and a haircut, but still different enough to not give away my identity ...*	**Player 1 (clue) => It's something you wear on your head.** **Player 2 (clue) => It's something that can change your appearance.** **Player 3 (clue) => It's often used in theatrical performances.** **Moderator =>** Now we starting another round of giving clue… **Player 1 (clue) => It can be made from synthetic materials.** **Player 2 (clue) => It can be used to cover baldness.** **Player 3 (clue) => It comes in different colors and styles.** **Player 1 =>** I vote for **Player 3; Player 2 =>** I vote for **Player 3; Player 3 =>** I vote for **Player 1.** **Moderator =>** The most-voted player is Player 3. **The accusation is incorrect.** Player 2 is the undercover. **Undercover won!** *Player 2 (pgm) => 1. Based on the clues, it seems like Player 1 and Player 3 are describing a wig.... Comparing this to my word, "haircut", it's clear **that I am the undercover** in this game. ...* *2. I think **Player 1 thinks Player 3 is the undercover,** because Player 3's clue about theatrical performances doesn't align as well with the idea of something you wear on your head or something made from synthetic materials.* *3. I think **Player 3 thinks Player 1** is the undercover, because Player 1's clue about synthetic materials doesn't align as well with the idea of something used in theatrical performances.* *4. I think the word of other players may be "wig". I will try to give my next clue as "It can be used to cover baldness", which is similar to them to hide myself.*

Figure 6: A Undercover case study on GPT-3.5-turbo, GPT-4 and their PGM-enhanced version (*+PGM).

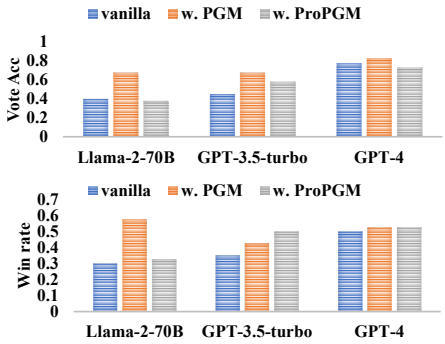

Figure 7: The performance comparison of different prompt designs in Chameleon. "ProPGM" refers to generating a PGM matrix directly with LLM. "PGM" is accumulated PGM extracted from text-based analysis.

A.6 Defect and Investment Tendency

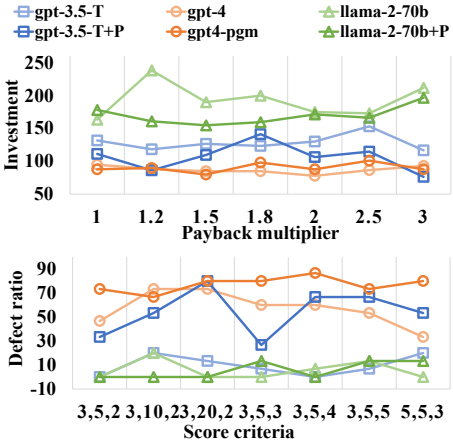

Figure 8: Features in game theory analysis.

LLM awareness of arithmetic. The upper diagram in Figure 8 shows the total investment amount under different payback multipliers. The x-axis is the 7 multipliers used in our topic settings. We told all the participating LLMs in a competition that the total amount of money they can invest is 100. However, except for the model GPT-4, the average amount of total investments of these LLMs almost all exceed 100, which indicates a lack of arithmetic awareness for these LLMs. In contrast, GPT-4 better understands the game rules and masters arithmetic in giving reasonable investment.

LLM behaviors with varying topic settings. To also investigate whether the LLMs will behave differently when given different topic settings. For example, in Public Good, we chose the multipliers [1,1.2,1.5,1.8,2,2.5,3]. We can find in the upper diagram in Figure 8, that even though the multiplier is increasing, all the lines do not show a tendency to increase investment, which proves that all the tested challenger LLMs are not sensitive to the increased possible payback in the communal pool in our current setting. Similar phenomena also happen in Prisoner's Dilemma. As shown by the bottom diagram in Figure 8, the x-axis is the score criteria, which consists of three digits, representing the score a player can get in situations where

the player defects when all the other two defect, the player defects when the other two cooperate, and the player defects when one of the other two defects, respectively.

A.7 Game rules and Prompt Design

We have listed the game rules for our proposed 5 scenarios in Table 5 and 6.

Role	Global Prompt	PGM Prompt	PGM Decision Prompt
Chameleon	You are playing a game of the Chameleon. Here are the game rules: ## Information and roles There are two roles in the game, chameleon and non-chameleon. The topic of the secret word will be first revealed to all the players. Then the secret word will be revealed to non-chameleons. The chameleon does not know the secret word. ## Objectives Your objective in the game depends on your role:- If you are not a chameleon, your goal is to reveal the chameleon without exposing the secret word.- If you are a chameleon, you aim to blend in with other players, avoid being caught, and figure out the secret word. ## Stages There are three stages in the game: 1. The giving clues stage: each player will describe the clues about the secret word. 2. The accusation stage: In this stage, each player will vote for another player who is most likely the chameleon. The chameleon should vote for other players. 3. The guess stage: If the accusation is correct, the chameleon should guess the secret word given the clues revealed by other players.	**Non-Chameleon** {player}, based on all the given clues, evaluate who is more suspicious of being a chameleon from your thoughts and the thoughts of other players you think. If you haven't seen a player's clue, don't think up one for him. You must follow the following format As {player}, I think: {other_player_1} is ..., because ... {other_player_2} is ..., because ... ... As for other players' thoughts: I think now {other_player_1} thinks: {player} is ..., because ... {other_player_2} is ..., because ... ... I think now {other_player_2} thinks: {player} ..., because ... {other_player_1} ..., because ... **Chameleon** {player}, deduce the secret code based only on {other_player_1} and {other_player_2}'s clues and analyze who is more suspicious from the perspective of other players based on all the clues given.	**Non-chameleon:**{player}, you are a non-chameleon, your target is to increase the suspiciousness of the {target_player}. For example, by giving more specific clues than the {target_player}. Remember don't reveal the secret code; **Chameleon:** {player}, you are a chameleon, and your target is to blend in and defraud trust from the {target_player}, try to describe the same word as other players. -If you are the first clue giver, try to be general in case you expose yourself. -If you guessed the secret code, try to give a specific clue about the word. -If other players' clues are general, try to give a clue similar to the {target_player}'s.
Undercover	You are playing a game of the Undercover. Here are the game rules: ## Information and roles There are two roles in the game, undercover and non-undercover. Each Player will get one word but the one undercover gets will be different from the other players'. ## Objectives First, you should guess who is undercover based on everyone's clues. Don't give too specific clues in case expose your own role. After you have guessed your own role, your objective in the game depends on your role:- If you are not undercover, your goal is to reveal the undercover without exposing your own word.- If you are undercover, your aim is to blend in with other players and avoid being caught. ## Stages There are two stages in the game. 1. The giving clues stage: each player will describe the clues in their own words. This stage will continue for several rounds. 2. The accusation stage: In this stage, each player will vote for another player who is most likely undercover. The undercover should vote for other players.	**Is undercover:** You are {player}, your word is {code}, and guess the words other players are describing. Compare your word with other players' clues, if your word is different from what others are describing (even sometimes related), you may be the undercover. Do you think you are the undercover? **Undercover**: "{idx}. You are {player}, your word is {code}, and guess who is more likely to be the undercover from your own perspective. (hint: use the format, I think Player xx is undercover, because...)", "{idx}. You are {player}, your word is {code}. Now try to guess who {other_player} thinks is the undercover. (hint: use the format, I think {other_player} thinks Player xx is undercover, because...)", {idx}. I will try to a more specific clue about my word as... to help identify who is undercover." **Civilian**: {idx}. You are {player}, your word is {code}, and guess who is more likely to be the undercover from your own perspective. (hint: use the format, I think Player xx is undercover, because...), {idx}. You are {player}, your word is {code}, and you think you are undercover. Now try to guess who {other_player} thinks is the undercover. (hint: use the format, I think {other_player} thinks Player xx is undercover, because...), {idx}. I think the word of the other player may be ... I will try to give my next clue as ..., which is similar to them to hide myself.	**Not Sure**: {player}, according to your previous analysis, you are not sure who is undercover, give a not-too-specific clue, and don't reveal the word you have. **Undercover**: {player}, according to your previous analysis, you think you are the undercover, your target is to blend in and defraud trust from the {target_player}. - If you have guessed other players' word, try to give a clue describing others' word; - If you are unsure about others' word, try to give clue similar to the {target_player}'s **Civilian**: {player} according to your previous analysis, you think {target_player} is undercover. Try to increase {target_player}'s suspiciousness. For example, by giving more specific and different clues compared to the clue of {target_player}. You can give a clue based on your above analysis

Table 5: Prompts for Undercover and Chameleon.

Role	Global Prompt	PGM Prompt	PGM Decision Prompt
Airport Fee	You are participating in the 'Airport Fee Allocation' game. You will each represent an airline and split the fixed cost of the airport. ## Information: - The total cost of the airport is fixed, and all airlines must collectively cover this cost. - Airlines have their unique usage frequencies at the airport, determined by factors like flight size and passenger volume. Airlines with higher usage frequencies are responsible for a larger portion of the cost. ## Objectives: - As an airline representative, your goal is to negotiate and agree upon a cost distribution that is both fair and favorable for your airline. ## Rules: The negotiation will continue for {max_turns} rounds. In each round: - Proposal: Each airline proposes a cost distribution. - Vote: Each player must vote for a cost distribution they find acceptable and strive to reach a consensus with other players' votes. - The game ends successfully when all airlines vote for the same proposal. If after {max_turns} rounds of voting no consensus is reached, the game fails. Strive for a successful outcome.	Please try to analyze how will players adjust their proposal from your perspective and other players in the next round. You must follow the template below (make your comments concise): As {player_name}, I think {oth_player1} will... I think {oth_player2} will... As for other players, I think {oth_player1} thinks: {player_name} will... {oth_player2} will... I think {oth_player2} thinks {player_name} will...{oth_player1} will...	According to your own analysis of other players' possible decisions, try to adjust your proposal in the next round so that other players will all agree with you. - If some players stick to their decision and you think it is fair, you can follow their proposals to achieve agreement among you.
Prisoner's Dilemma	You are playing the Iterative Three-Player Prisoners' Dilemma game. Players 1, 2, and 3 are involved in a crime and are interrogated separately. They can either choose to betray the others (Defect, D) or cooperate with them (Cooperate, C). The rewards and penalties are distributed based on the combination of choices made by the three players. The scoring is as follows: - If all three choose to cooperate, they each get {cooperate} points. - If all three choose to defect, they each get {defect} point. - If one chooses to defect and the others choose to cooperate, the betrayer gets {one_defect} points and the collaborators each get 0 points. - If two choose to defect and the other chooses to cooperate, the betrayers each get {two_defect} points, and the collaborator gets 0 points. The game is played for 5 rounds, and the total score is calculated at the end to determine the winner. """	Please try to analyze how other players will choose to defect or corporate in the next round from your own perspective and others' perspectives. You must use the template: In my opinion: As {player_name}, I think {oth_player1} will xx, because... I think {oth_player2} will xx, because... As for other players, I think {oth_player1} thinks: {player_name} will... {oth_player2} will... I think {oth_player2} thinks {player_name} will... {oth_player1} will...	Based on your analysis of others and other's thoughts Remember to be strategic, establishing trust is to defect before others plan to do it to get the highest score. choose "defect" or "cooperate" to achieve the highest points for yourself. You must reply with "defect" or "cooperate"
Public Good	Welcome to the 'Public Good' game. Below are the game's settings and rules: - Each player begins with 100 monetary points. Over {game_round} rounds, you decide how many points to contribute to a communal pool at each round. - After {game_round} rounds of decision-making, the points in the communal pool will be multiplied by a factor of {multiplier} and distributed equally among all players. - A player's final points are the sum of their remaining points and the shared points from the communal pool. The player who gets the highest final points wins the game. - Every player must strategically invest their points to the communal pool to get more shared points and also be the one who invests the least to win the game. - Usually, you can get more payback by investing more when the factor is larger.	Please try to analyze whether other players will "reduce" or "increase" their contributions in the next round from your own perspective and others' perspective. - Remember, the payment in the communal pool is evenly shared by all players at the end of the game, so you need to make sure you invest the least money to get the highest repayment. You must use the template: In my opinion: As {player_name}, I think {oth_player1} will xx, because... I think {oth_player2} will xx, because... As for other players, I think {oth_player1} thinks: {player_name} will... {oth_player2} will... I think {oth_player2} thinks {player_name} will... {oth_player1} will...	Based on your analysis of others and others' thoughts, make the decision about your own contribution to achieving the highest repayment for yourself. Remember - Your total investment should be the least to win the game; - Your target is to get the highest points and also promote the communal repayment to get as many points as possible at the end. You must answer with the template "I contribute xx"

Table 6: Prompts in Cost-Sharing, Prisoner's Dilemma, and Public Good.