INFLUENCE ESTIMATION FOR GENERATIVE ADVERSARIAL NETWORKS

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ABSTRACT

Identifying harmful instances, whose absence in a training dataset improves model performance, is important for building better machine learning models. Although previous studies have succeeded in estimating harmful instances under supervised settings, they cannot be trivially extended to generative adversarial networks (GANs). This is because previous approaches require that (1) the absence of a training instance directly affects the loss value and that (2) the change in the loss directly measures the harmfulness of the instance for the performance of a model. In GAN training, however, neither of the requirements is satisfied. This is because, (1) the generator's loss is not directly affected by the training instances as they are not part of the generator's training steps, and (2) the values of GAN's losses normally do not capture the generative performance of a model. To this end, (1) we propose an influence estimation method that uses the Jacobian of the gradient of the generator's loss with respect to the discriminator's parameters (and vice versa) to trace how the absence of an instance in the discriminator's training affects the generator's parameters, and (2) we propose a novel evaluation scheme, in which we assess harmfulness of each training instance on the basis of how GAN evaluation metric (e.g., inception score) is expect to change due to the removal of the instance. We experimentally verified that our influence estimation method correctly inferred the changes in GAN evaluation metrics. Further, we demonstrated that the removal of the identified harmful instances effectively improved the model's generative performance with respect to various GAN evaluation metrics.

1 INTRODUCTION

Generative adversarial networks (GANs) proposed by Goodfellow et al. (2014) are a powerful subclass of generative model, which is successfully applied to a number of image generation tasks (Antoniou et al., 2017; Ledig et al., 2017; Wu et al., 2016). The expansion of the applications of GANs makes improvements in the generative performance of models increasingly crucial.

An effective approach for improving machine learning models is to identify training instances that harm the model performance. Traditionally, statisticians manually screen a dataset for harmful instances, which misguide a model into producing biased predictions. Recent *influence estimation* methods (Khanna et al., 2019; Hara et al., 2019) automated the screening of datasets for deep learning settings, in which the sizes of both datasets and data dimensions are too large for users to manually determine the harmful instances. Influence estimation measures the effect of removing an individual training instance on a model's prediction without the computationally prohibitive cost of model retraining. The recent studies identified harmful instances by estimating how the loss value changes if each training instance is removed from the dataset.

Although previous studies have succeeded in identifying the harmful instances in supervised settings, the extension of their approaches to GAN is non-trivial. Previous approaches require that (1) the existence or absence of a training instance directly affects a loss value, and that (2) the decrease in the loss value represents the harmfulness of the removed training instance. In GAN training, however, neither of the requirements is satisfied. (1) As training instances are only fed into the discriminator, they only indirectly affect the generator's loss, and (2) the changes in the losses of GAN do not necessarily capture how the removed instances harm the generative performance. This

is because the ability of the loss to evaluate the generator is highly dependent on the performance of the discriminator.

To this end, (1) we propose an influence estimation method that uses the Jacobian of the gradient of the discriminator's loss with respect to the generator's parameters (and vice versa), which traces how the absence of an instance in the discriminator's training affects the generator's parameters. In addition, (2) we propose a novel evaluation scheme to judge if an instance is harmful or not on the basis of *influence on GAN evaluation metric*, that is, how a GAN evaluation metric (e.g., inception score (Salimans et al., 2016)) changes if a given training instance is removed from the dataset. We identify harmful instances by estimating the influence on GAN evaluation metric by leveraging our influence estimation method.

We verified that the proposed influence estimation method correctly estimated the influence on GAN evaluation metrics across different settings of the dataset, model architecture, and GAN evaluation metrics. We also demonstrated that removing harmful instances, which were identified by the proposed method, effectively improved various GAN evaluation metrics.

Our contributions are summarized as follows:

- We propose an influence estimation method that uses the Jacobian of the gradient of the discriminator's loss with respect to the generator's parameters (and vice versa), which traces how the absence of an instance in the discriminator's training affects the generator's parameters.
- We propose a novel evaluation scheme to judge if an instance is harmful or not on the basis of influence on GAN evaluation metrics rather than that on the loss value, and to leverage the proposed influence estimation method to identify harmful instances.
- We experimentally verified that our influence estimation method correctly inferred the influence on GAN evaluation metrics. Further, we demonstrated that the removal of the harmful instances suggested by the proposed method effectively improved the generative performance with respect to various GAN evaluation metrics.

2 PRELIMINARIES

Notation For column vectors $\boldsymbol{a}, \boldsymbol{b} \in \mathbb{R}^p$, we denote the inner product by $\langle \boldsymbol{a}, \boldsymbol{b} \rangle = \sum_{i=1}^{p} a_i b_i$. For a function $f(\boldsymbol{a})$, we denote its gradient with respect to $\boldsymbol{a}$ by $\nabla_{\boldsymbol{a}} f(\boldsymbol{a})$. We denote the identity matrix of size p by $\boldsymbol{I}_p$, the zero vector of length p by $\mathbf{0}_p$, and the ones vector of length p by $\mathbf{1}_p$.

Generative Adversarial Networks (GAN) For simplicity, we consider an unconditional GAN that consists of the generator $G : \mathbb{R}^{d_{\boldsymbol{z}}} \to \mathbb{R}^{d_{\boldsymbol{x}}}$ and the discriminator $D : \mathbb{R}^{d_{\boldsymbol{x}}} \to \mathbb{R}$, where $d_{\boldsymbol{z}}$ and $d_{\boldsymbol{x}}$ are the number of dimensions of latent variable $\boldsymbol{z} \sim p(\boldsymbol{z})$ and data point $\boldsymbol{x} \sim p(\boldsymbol{x})$, respectively. The parameters of generator $\boldsymbol{\theta}_G \in \mathbb{R}^{d_G}$ and discriminator $\boldsymbol{\theta}_D \in \mathbb{R}^{d_D}$ are learned though the adversarial training; G tries to sample realistic data while D tries to identify whether the data is real or generated.

Formulation of GAN Objectives For the generality, we adopt the formulation of Gidel et al. (2018) in which G and D try to minimize $\mathscr{L}_G$ and $\mathscr{L}_D$, respectively, to obtain the following *Nash equilibrium* $(\boldsymbol{\theta}_G^*, \boldsymbol{\theta}_D^*)$:

$$\boldsymbol{\theta}_G^* \in \arg\min_{\boldsymbol{\theta}_G} \mathscr{L}_G\left(\boldsymbol{\theta}_G, \boldsymbol{\theta}_D^*\right) \text{ and } \boldsymbol{\theta}_D^* \in \arg\max_{\boldsymbol{\theta}_D} \mathscr{L}_D\left(\boldsymbol{\theta}_G^*, \boldsymbol{\theta}_D\right). \tag{1}$$

For the latter part of this paper, we use a coupled parameter vector $\boldsymbol{\theta} := (\boldsymbol{\theta}_G, \boldsymbol{\theta}_D)^\top \in \mathbb{R}^{d_{\boldsymbol{\theta}} = d_G + d_D}$ when we refer to the whole parameters of GAN.

In this paper, we assume that $\mathscr{L}_G$ and $\mathscr{L}_D$ have the following forms[1]:

$$\mathscr{L}_G\left(\boldsymbol{\theta}\right) := \mathbb{E}_{\boldsymbol{z}\sim p(\boldsymbol{z})}\left[f_G\left(\boldsymbol{z};\boldsymbol{\theta}\right)\right], \quad \mathscr{L}_D\left(\boldsymbol{\theta}\right) := \mathbb{E}_{\boldsymbol{z}\sim p(\boldsymbol{z})}\left[f_D^{[\boldsymbol{z}]}\left(\boldsymbol{z};\boldsymbol{\theta}\right)\right] + \mathbb{E}_{\boldsymbol{x}\sim p(\boldsymbol{x})}\left[f_D^{[\boldsymbol{x}]}\left(\boldsymbol{x};\boldsymbol{\theta}\right)\right]. \tag{2}$$

We can recover the original minimax objective by taking $f_G\left(\boldsymbol{z};\boldsymbol{\theta}\right) = \log\left(1 - D_{\boldsymbol{\theta}_D}\left(G_{\boldsymbol{\theta}_G}\left(\boldsymbol{z}\right)\right)\right)$, $f_D^{[\boldsymbol{z}]} = -f_G$, and $f_D^{[\boldsymbol{x}]}\left(\boldsymbol{x};\boldsymbol{\theta}\right) = -\log D_{\boldsymbol{\theta}_D}\left(\boldsymbol{x}\right)$.

[1]This covers the common settings of GAN objectives: the non-zero-sum game proposed by Goodfellow et al. (2014), Wasserstein distance (Arjovsky et al., 2017), and the least squares loss (Mao et al., 2017).

Adversarial SGD (ASGD) To make our derivation easier to understand, we newly formulate the parameter update of a GAN trained by stochastic gradient descent, which we call adversarial SGD (*ASGD*). For simplicity, this paper considers simultaneous training, in which the generator and the discriminator are simultaneously updated in one step. We denote the dataset by $\mathcal{D}_{\boldsymbol{x}} := \{\boldsymbol{x}_n \sim p(\boldsymbol{x})\}_{n=1}^{N}$, which consists of N data points. Let $\mathcal{S}_t \subset \{1, \dots, N\}$ be a set of sample indices at the t-th step. We assume that the mini-batch of the t-th step consists of instances $\{\boldsymbol{x}_i\}_{i \in \mathcal{S}_t}$ and a set of latent variables $\mathcal{Z}_t = \{\boldsymbol{z}_l^{[t]} \sim p(\boldsymbol{z})\}_{l=1}^{|\mathcal{S}_t|}$, which are sampled independently at each step t. We denote the mean of $\mathscr{L}_G$ and $\mathscr{L}_D$ across the mini-batch by $\overline{\mathscr{L}}_G(\mathcal{Z}; \boldsymbol{\theta}) := \frac{1}{|\mathcal{Z}|} \sum_{\boldsymbol{z} \in \mathcal{Z}} f_G(\boldsymbol{z}; \boldsymbol{\theta})$ and $\overline{\mathscr{L}}_D(\mathcal{S}, \mathcal{Z}; \boldsymbol{\theta}) := \frac{1}{|\mathcal{Z}|} \left(\sum_{\boldsymbol{z} \in \mathcal{Z}} f_D^{[\boldsymbol{z}]}(\boldsymbol{z}; \boldsymbol{\theta}) + \sum_{i \in \mathcal{S}} f_D^{[\boldsymbol{x}]}(\boldsymbol{x}_i; \boldsymbol{\theta}) \right)$, respectively. The t-th step of ASGD updates the coupled parameters by $\boldsymbol{\theta}^{[t+1]} = \boldsymbol{\theta}^{[t]} - \boldsymbol{B}_t g\left(\mathcal{S}_t, \mathcal{Z}_t; \boldsymbol{\theta}^{[t]}\right)$, where

$$\boldsymbol{B}_t := \begin{pmatrix} \eta_G^{[t]} \boldsymbol{I}_{d_G} & \boldsymbol{O} \\ \boldsymbol{O} & \eta_D^{[t]} \boldsymbol{I}_{d_D} \end{pmatrix} \in \mathbb{R}^{d_{\boldsymbol{\theta}} \times d_{\boldsymbol{\theta}}}, \quad g(\mathcal{S}, \mathcal{Z}; \boldsymbol{\theta}) := \begin{pmatrix} \nabla_{\boldsymbol{\theta}_G} \overline{\mathscr{L}}_G(\mathcal{Z}; \boldsymbol{\theta}) \\ \nabla_{\boldsymbol{\theta}_D} \overline{\mathscr{L}}_D(\mathcal{S}, \mathcal{Z}; \boldsymbol{\theta}) \end{pmatrix} \in \mathbb{R}^{d_{\boldsymbol{\theta}}}. \tag{3}$$

$\eta_G^{[t]} \in \mathbb{R}^+$ and $\eta_D^{[t]} \in \mathbb{R}^+$ are the learning rates of the t-th step for $\boldsymbol{\theta}_G$ and $\boldsymbol{\theta}_D$, respectively.

3 Proposed Method

This section explains the two main contributions of our paper: the influence estimation method for GANs that predicts how the removal of a training instance changes the output of the generator and the discriminator (Section 3.1), and two important parts of our instance evaluation scheme, that are, the definition of influence on GAN evaluation metric and its estimation algorithm (Section 3.2).

3.1 Influence Estimation for GAN

We refer to influence estimation as the estimation of changes in a model's output under a training instance's absence. As the model's output changes through the changes in the model's parameters, we start with the definition of ASGD-Influence, which represents the changes in parameters, and then formulate its estimator.

ASGD-Influence ASGD-Influence is defined on the basis of the following *counterfactual ASGD*. Let $\theta_{-j}^{[t]}$ denote the parameters at t-th step trained without using j-th training instance. Counterfactual ASGD starts optimization from $\boldsymbol{\theta}_{-j}^{[1]} = \boldsymbol{\theta}^{[1]}$ and updates the parameters of the t-th step by $\boldsymbol{\theta}_{-j}^{[t+1]} = \boldsymbol{\theta}_{-j}^{[t]} - \boldsymbol{B}_t g\left(\mathcal{S}_t \setminus \{j\}, \mathcal{Z}_t; \boldsymbol{\theta}_{-j}^{[t]}\right)$. We define ASGD-Influence $\Delta\boldsymbol{\theta}_{-j}$ as the parameter difference between counterfactual ASGD and ASGD at the final step $t = T$, namely $\Delta\boldsymbol{\theta}_{-j} := \boldsymbol{\theta}_{-j}^{[T]} - \boldsymbol{\theta}^{[T]}$.

Estimator of ASGD-Influence Our estimator uses approximation of mean of the gradient. Let $\left(\nabla_{\boldsymbol{\theta}_G} \overline{\mathscr{L}}_G(\mathcal{Z}; \boldsymbol{\theta}), \nabla_{\boldsymbol{\theta}_D} \overline{\mathscr{L}}_D(\mathcal{S}, \mathcal{Z}; \boldsymbol{\theta})\right)^\top$ be the joint gradient vector of the mini-batch. We introduce the Jacobian of the joint gradient vector of the t-th mini-batch with respect to $\boldsymbol{\theta}$:

$$\boldsymbol{J}_t := \begin{pmatrix} \boldsymbol{J}_{GG}^{[t]} & \boldsymbol{J}_{GD}^{[t]} \\ \boldsymbol{J}_{DG}^{[t]} & \boldsymbol{J}_{DD}^{[t]} \end{pmatrix} = \begin{pmatrix} \nabla_{\boldsymbol{\theta}_G}^2 \overline{\mathscr{L}}_G\left(\mathcal{Z}_t; \boldsymbol{\theta}^{[t]}\right) & \nabla_{\boldsymbol{\theta}_D} \nabla_{\boldsymbol{\theta}_G} \overline{\mathscr{L}}_G\left(\mathcal{Z}_t; \boldsymbol{\theta}^{[t]}\right) \\ \nabla_{\boldsymbol{\theta}_G} \nabla_{\boldsymbol{\theta}_D} \overline{\mathscr{L}}_D\left(\mathcal{S}_t, \mathcal{Z}_t; \boldsymbol{\theta}^{[t]}\right) & \nabla_{\boldsymbol{\theta}_D}^2 \overline{\mathscr{L}}_D\left(\mathcal{S}_t, \mathcal{Z}_t; \boldsymbol{\theta}^{[t]}\right) \end{pmatrix}. \tag{4}$$

When we assume both $\mathscr{L}_G(\boldsymbol{\theta})$ and $\mathscr{L}_G(\boldsymbol{\theta})$ are second-order differentiable with respect to $\boldsymbol{\theta}$, the first-order Taylor approximation gives $g\left(\mathcal{S}_t, \mathcal{Z}_t; \boldsymbol{\theta}_{-j}^{[t]}\right) - g\left(\mathcal{S}_t, \mathcal{Z}_t; \boldsymbol{\theta}^{[t]}\right) \approx \boldsymbol{J}_t \left(\boldsymbol{\theta}_{-j}^{[t]} - \boldsymbol{\theta}^{[t]}\right)$. With this approximation, we have

$$\begin{aligned} \boldsymbol{\theta}_{-j}^{[t+1]} - \boldsymbol{\theta}^{[t+1]} &= \left(\boldsymbol{\theta}_{-j}^{[t]} - \boldsymbol{\theta}^{[t]}\right) - \boldsymbol{B}_t \left(g\left(\mathcal{S}_t, \mathcal{Z}_t; \boldsymbol{\theta}_{-j}^{[t]}\right) - g\left(\mathcal{S}_t, \mathcal{Z}_t; \boldsymbol{\theta}^{[t]}\right)\right) \\ &\approx \left(\boldsymbol{I}_{d_{\boldsymbol{\theta}}} - \boldsymbol{B}_t \boldsymbol{J}_t\right) \left(\boldsymbol{\theta}_{-j}^{[t]} - \boldsymbol{\theta}^{[t]}\right) \quad, \forall j \notin \mathcal{S}_t. \end{aligned} \tag{5}$$

For simplicity, we first focus on 1-epoch ASGD in which each instance appears only once. Let $\pi(j)$ be the step where the j-th instance is used. Considering the absence of

$\nabla_{\boldsymbol{\theta}_D} f_D^{[\boldsymbol{x}]}(\boldsymbol{x}_j;\boldsymbol{\theta}^{[\pi(j)]})$ in the $\pi(j)$-th step of counterfactual ASGD, we have $\boldsymbol{\theta}_{-j}^{[\pi(j)+1]} - \boldsymbol{\theta}^{[\pi(j)+1]} = \frac{\eta_D^{[\pi(j)]}}{|\mathcal{S}_{\pi(j)}|}\left(\mathbf{0}_{d_G},\ \nabla_{\boldsymbol{\theta}_D} f_D^{[\boldsymbol{x}]}(\boldsymbol{x}_j;\boldsymbol{\theta}^{[\pi(j)]})\right)^\top$. By denoting $\boldsymbol{Z}_t := \boldsymbol{I}_{d_{\boldsymbol{\theta}}} - \boldsymbol{B}_t\boldsymbol{J}_t$ and recursively applying the approximation (5), we obtain

$$\Delta\boldsymbol{\theta}_{-j} \approx \frac{\eta_D^{[\pi(j)]}}{|\mathcal{S}_{\pi(j)}|}\boldsymbol{Z}_{T-1}\boldsymbol{Z}_{T-2}\cdots\boldsymbol{Z}_{\pi(j)+1}\begin{pmatrix}\mathbf{0}_{d_G}\\ \nabla_{\boldsymbol{\theta}_D} f_D^{[\boldsymbol{x}]}\left(\boldsymbol{x}_j;\boldsymbol{\theta}^{[\pi(j)]}\right)\end{pmatrix}. \tag{6}$$

For the practical situation of K-epoch ASGD, in which the j-th instance is sampled K times at $t = \pi_1(j), \ldots, \pi_K(j)$, the estimator of the ASGD-Influence is given by

$$\Delta\hat{\boldsymbol{\theta}}_{-j} := \sum_{k=1}^{K}\left(\prod_{s=1}^{T-\pi_k(j)-1}\boldsymbol{Z}_{T-s}\right)\frac{\eta_D^{[\pi_k(j)]}}{|\mathcal{S}_{\pi_k(j)}|}\begin{pmatrix}\mathbf{0}_{d_G}\\ \nabla_{\boldsymbol{\theta}_D} f_D^{[\boldsymbol{x}]}\left(\boldsymbol{x}_j;\boldsymbol{\theta}^{[\pi_k(j)]}\right)\end{pmatrix}. \tag{7}$$

Linear Influence To estimate the influence on outputs, we introduce *linear influence* $L_{-j}^{[T]}(\boldsymbol{u}) := \langle \boldsymbol{u}, \Delta\boldsymbol{\theta}_{-j}\rangle$ of a given query vector $\boldsymbol{u} \in \mathbb{R}^{d_{\boldsymbol{\theta}}}$. If we take $\boldsymbol{u} = \nabla_{\boldsymbol{\theta}} f_G\left(\boldsymbol{z};\boldsymbol{\theta}^{[T]}\right)$, the linear influence approximates the influence on the generator's loss $L_{-j}^{[T]}(\boldsymbol{u}) \approx f_G\left(\boldsymbol{z};\boldsymbol{\theta}_{-j}^{[T]}\right) - f_G\left(\boldsymbol{z};\boldsymbol{\theta}^{[T]}\right)$.

Let $\left(\boldsymbol{u}_G^{[t]\top} \in \mathbb{R}^{d_G}, \boldsymbol{u}_D^{[t]\top} \in \mathbb{R}^{d_D}\right) := \boldsymbol{u}^\top\boldsymbol{Z}_{T-1}\boldsymbol{Z}_{T-2}\cdots\boldsymbol{Z}_{t+1}$. The linear influence of the j-th instance is approximated by the proposed estimator:

$$L_{-j}^{[T]}(\boldsymbol{u}) \approx \left\langle \boldsymbol{u}, \Delta\hat{\boldsymbol{\theta}}_{-j}\right\rangle = \sum_{k=1}^{K}\frac{\eta_D^{[\pi_k(j)]}}{|\mathcal{S}_{\pi_k(j)}|}\left\langle \boldsymbol{u}_D^{[\pi_k(j)]}, \nabla_{\boldsymbol{\theta}_D} f_D^{[\boldsymbol{x}]}\left(\boldsymbol{x}_j;\boldsymbol{\theta}^{[\pi_k(j)]}\right)\right\rangle. \tag{8}$$

The estimation algorithm consists of two phases; *training phase* performs K-epoch ASGD by storing information $\mathcal{A}^{[t]} \leftarrow (\mathcal{S}_t, \eta_G^{[t]}, \eta_D^{[t]}, \boldsymbol{\theta}^{[t]}, \mathcal{Z}_t)$ and *inference phase* calculates (8) using $\mathcal{A}^{[1]}, \ldots, \mathcal{A}^{[T-1]}$. See Appendix A for the detailed algorithm.

3.2 Influence on GAN Evaluation Metric

This section explains our proposal of a new evaluation approach for data screening for GANs. Firstly we propose to evaluate harmfulness of an instance on the basis of influence on GAN evaluation metrics. Secondly we propose to leverage the influence-estimation algorithm explained in Section 3.1 to identify harmful instances with respect to the GAN evaluation metrics.

Influence on GAN Evaluation Metric Let $V(\mathcal{D})$ be a GAN evaluation metric that maps a set of data points $\mathcal{D} := \{\tilde{\boldsymbol{x}}_m \in \mathbb{R}^{d_{\boldsymbol{x}}}\}_{m=1}^M$ into a scalar value that gives the performance measure of G. Let generated dataset $\mathcal{D}_G(\mathcal{Z};\boldsymbol{\theta}_G) := \{G(\boldsymbol{z};\boldsymbol{\theta}_G) |\ \boldsymbol{z} \in \mathcal{Z}\}$. Using a set of latent variables $\mathcal{Z}' := \{\boldsymbol{z}'_n \sim p(\boldsymbol{z})\}_{n=1}^{N'}$ that is sampled independently from the training, we define the influence on GAN evaluation metric by

$$\Delta V_{-j}^{[T]} := V\left(\mathcal{D}_G\left(\mathcal{Z}';\boldsymbol{\theta}_{G,-j}^{[T]}\right)\right) - V\left(\mathcal{D}_G\left(\mathcal{Z}';\boldsymbol{\theta}_G^{[T]}\right)\right), \tag{9}$$

where $\boldsymbol{\theta}_{G,-j}^{[T]}$ and $\boldsymbol{\theta}_G^{[T]}$ are the generator parameters of counterfactual ASGD and the ASGD of the T-th step, respectively.

Estimation Algorithm In order to build the estimation algorithm of the influence on GAN evaluation metric, we focus on an important property of some common evaluation metrics for which the gradient with respect to the element of their input $\nabla_{\tilde{\boldsymbol{x}}_m} V(\mathcal{D})$ is computable. For example, Monte Carlo estimation of inception score has a form of $\exp(\frac{1}{|\mathcal{D}|}\sum_{\tilde{\boldsymbol{x}}_m\in\mathcal{D}} \mathbb{KL}(p_c(y|\tilde{\boldsymbol{x}}_m)\|p_c(y))$ where p_c is a distribution of class label y drawn by a pretrained classifier. When the classifier is trained using back-propagation, $\nabla_{\tilde{\boldsymbol{x}}_m} V(\mathcal{D})$ is computable.

Here, we assume $V(\mathcal{D})$ is first-order differentiable with respect to $\tilde{\boldsymbol{x}}_m$. From the chain rule, we have a gradient of the GAN evaluation metrics with respect to $\boldsymbol{\theta}$:

$$\nabla_{\boldsymbol{\theta}} V(\mathcal{D}_G(\mathcal{Z}';\boldsymbol{\theta}_G^{[T]})) = \begin{pmatrix}\sum_{n=1}^{N'}\nabla_{\boldsymbol{\theta}_G}\nabla_{\tilde{\boldsymbol{x}}_n} V\left(\mathcal{D}_G\left(\mathcal{Z}';\boldsymbol{\theta}_G^{[T]}\right)\right)\\ \mathbf{0}_{d_D}\end{pmatrix}. \tag{10}$$

Our estimation algorithm performs the inference phase of linear influence taking $\boldsymbol{u} = \nabla_{\boldsymbol{\theta}} V(\mathcal{D}_G(\mathcal{Z}'; \boldsymbol{\theta}_G^{[T]}))$ in order to obtain the approximation $L_{-j}^{[T]}(\nabla_{\boldsymbol{\theta}} V(\mathcal{D}_G(\mathcal{Z}'; \boldsymbol{\theta}_G^{[T]}))) \approx \Delta V_{-j}^{[T]}$.

4 Related Studies

SGD-Influence Hara et al. (2019) proposed a novel definition of the influence called *SGD-Influence* and its estimator, which greatly inspired us to propose the influence estimation method for GANs. Suppose a machine learning model with parameters $\boldsymbol{\phi} \in \mathbb{R}^{d_\phi}$ is trained to minimize the mean of the loss $\frac{1}{N}\sum_{n=1}^{N} \mathscr{L}(\chi_n; \boldsymbol{\phi})$ across the training instances $\chi_1, \ldots, \chi_N$. Let the mean of the loss of the mini-batch $\overline{\mathscr{L}}(\mathcal{S}; \boldsymbol{\phi}) := \frac{1}{|\mathcal{S}|}\sum_{i \in \mathcal{S}} \mathscr{L}(\chi_i; \boldsymbol{\phi})$. They introduced two SGD steps with learning rate $\eta_t \in \mathbb{R}^+$: SGD given by $\boldsymbol{\phi}^{[t+1]} = \boldsymbol{\phi}^{[t]} - \eta_t \nabla_{\boldsymbol{\phi}} \overline{\mathscr{L}}\left(\mathcal{S}_t; \boldsymbol{\phi}^{[t]}\right)$, and counterfactual SGD given by $\boldsymbol{\phi}_{-j}^{[t+1]} = \boldsymbol{\phi}_{-j}^{[t]} - \eta_t \nabla_{\boldsymbol{\phi}} \overline{\mathscr{L}}\left(\mathcal{S}_t \setminus \{j\}; \boldsymbol{\phi}_{-j}^{[t]}\right)$. Their estimator of SGD-Influence $\boldsymbol{\phi}_{-j}^{[T]} - \boldsymbol{\phi}^{[T]}$ is based on the following approximation:

$$\boldsymbol{\phi}_{-j}^{[t+1]} - \boldsymbol{\phi}^{[t+1]} \approx \left(\boldsymbol{I}_{d_\phi} - \eta_t \nabla_{\boldsymbol{\phi}}^2 \overline{\mathscr{L}}\left(\mathcal{S}_t; \boldsymbol{\phi}^{[t]}\right)\right)\left(\boldsymbol{\phi}_{-j}^{[t]} - \boldsymbol{\phi}^{[t]}\right) \quad , \forall j \notin \mathcal{S}_t. \tag{11}$$

Hara et al. (2019) also identified harmful instances for classification based on linear influence of the cross-entropy loss estimated using a validation dataset. Removing the estimated harmful instances with their approach demonstrated improvements in the classification accuracy.

Our approach differs from Hara et al. (2019)'s work in two ways. Firstly, our approach uses the Jacobian of the joint gradient vector $\boldsymbol{J}_t$ instead of the Hessian of the mean loss $\nabla_{\boldsymbol{\phi}}^2 \overline{\mathscr{L}}\left(\mathcal{S}_t; \boldsymbol{\phi}^{[t]}\right)$. As long as $\mathscr{L}_G \neq \mathscr{L}_D$, $\boldsymbol{J}_t$ is asymmetric and inherently different from the Hessian. Moreover, a source of the asymmetry $\boldsymbol{J}_{GD}^{[t]}$ plays an important role in transferring the effect of removal of a training instance from the discriminator to the generator. Let $\boldsymbol{\theta}_{G,-j}^{[t]} - \boldsymbol{\theta}_G^{[t]} \in \mathbb{R}^{d_G}$ and $\boldsymbol{\theta}_{D,-j}^{[t]} - \boldsymbol{\theta}_D^{[t]} \in \mathbb{R}^{d_D}$ be ASGD-Influence on $\boldsymbol{\theta}_G$ and $\boldsymbol{\theta}_D$ of the t-th step, respectively. The upper blocks of (5) can be rewritten as

$$\boldsymbol{\theta}_{G,-j}^{[t+1]} - \boldsymbol{\theta}_G^{[t+1]} \approx \left(\boldsymbol{I}_{d_D} - \eta_G^{[t]} \boldsymbol{J}_{GG}^{[t]}\right)\left(\boldsymbol{\theta}_{G,-j}^{[t]} - \boldsymbol{\theta}_G^{[t]}\right) + \eta_G^{[t]} \boldsymbol{J}_{GD}^{[t]}\left(\boldsymbol{\theta}_{D,-j}^{[t]} - \boldsymbol{\theta}_D^{[t]}\right). \tag{12}$$

Note that $\boldsymbol{J}_{GD}^{[t]}$ transfers the t-th step of ASGD-Influence on $\boldsymbol{\theta}_D$ to the next step of ASGD-Influence on $\boldsymbol{\theta}_G$. The Hessian of Hara et al. (2019), which uses a single combination of the parameters and the loss function, cannot handle this transfer between the two models. Secondly, we use the influence on GAN evaluation metrics for identifying harmful instances rather than that on the loss value. This alleviates the problem of the GAN's loss not representing the generative performance.

Influence Function Koh & Liang (2017) proposed influence estimation method that incorporated the idea of influence function (Cook & Weisberg, 1980) in robust statistics. They showed that influences on parameters and predictions can be estimated with the influence function assuming the satisfaction of the optimality condition and strong convexity of the loss function. They also identified harmful instances on the basis of the influence on the loss value, assuming consistency of the loss value with the task performance.

Our influence estimation method is designed to eliminate these assumptions because normally GAN training does not satisfy the assumptions regarding the optimality condition, the convexity in the loss function, and the consistency of the loss value with the performance.

5 Experiments

We evaluated the effectiveness of the proposed method in two aspects: the accuracy of influence estimation on GAN evaluation metrics (Section 5.1), and the improvement in generative performance by removing estimated harmful instances (Section 5.2)

GAN Evaluation Metrics In both experiments, we used three GAN evaluation metrics: average log-likelihood (ALL), inception score (IS), and Fréchet inception distance (FID) (Heusel et al., 2017). ALL is the de-facto standard for evaluating generative models (Tolstikhin et al., 2017). Let

$\mathcal{D}'_{\boldsymbol{x}} := \{\boldsymbol{x}'_n \sim p(\boldsymbol{x})\}_{n=1}^{N'}$, which is sampled separately from the training dataset $\mathcal{D}_{\boldsymbol{x}}$. ALL measures the likelihood of the true data under the distribution that is estimated from generated data using kernel density estimation. We calculated ALL of $\mathcal{D}'_{\boldsymbol{x}}$ under the distribution estimated from generated dataset $\mathcal{D}_G(\mathcal{Z}'; \boldsymbol{\theta}_G^{[T]})$. Recall $\mathcal{Z}'$ is the set of latent variables sampled independently from the training (Section 3.2). FID measures Fréchet distance between two sets of feature vectors of real images $\mathcal{D}'_{\boldsymbol{x}}$ and those of generated images $\mathcal{D}_G(\mathcal{Z}'; \boldsymbol{\theta}_G^{[T]})$. The feature vectors are calculated on the basis of a pre-trained classifier. Larger values of ALL and IS and a smaller value of FID indicate the better generative performance. See Appendix C.1 for the detailed setting of each GAN evaluation metric.

5.1 Experiment 1: Estimation Accuracy

We ran the influence estimation method on GANs to estimate influence on various GAN evaluation metrics, and then compared the estimated influence with true influence. The detailed setup can be found in Appendix C.2.

Setup ALL is known to be effective for low-dimensional data distributions (Borji, 2018) and both FID and IS are effective for image distributions. We thus prepared two different setups: fully-connected GAN (FCGAN) trained with 2D multivariate normal distribution (2D-Normal) for ALL, and DCGAN (Radford et al., 2015) trained with MNIST (LeCun et al., 1998) for IS and FID. IS and FID require classifiers to obtain class label distribution and feature vectors, respectively. We thus trained CNN classifier of MNIST[2] using $\mathcal{D}'_{\boldsymbol{x}}$. We set $N = 10\text{k}$ and $N' = |\mathcal{D}'_{\boldsymbol{x}}| = |\mathcal{Z}'| = 10\text{k}$.

The experiment was conducted as follows. Firstly, we ran the K-epoch of the training phase of linear influence with the training dataset $\mathcal{D}_{\boldsymbol{x}}$. We determined $K = 50$ since we observed the convergence of GAN evaluation metrics at $K = 50$. For IS and FID, we trained the classifier using $\mathcal{D}'_{\boldsymbol{x}}$ and corresponding labels. We then randomly selected 200 target instances from $\mathcal{D}_{\boldsymbol{x}}$. We obtained estimated influence on GAN evaluation metrics of each target instance by performing the inference phase of linear influence with $\boldsymbol{u} = \nabla_{\boldsymbol{\theta}} V(\mathcal{D}_G(\mathcal{Z}'; \boldsymbol{\theta}_G^{[T]}))$. The true influence of each target instance was computed by running the counterfactual ASGD.

We used the same evaluation measures as the previous work (Hara et al., 2019): Kendall's Tau and the Jaccard index. Kendall's Tau measures the ordinal correlation between the estimated and true influence on GAN evaluation metrics. It has a value of 1 when the orders of the two sets of values are identical. For the Jaccard index, we selected 10 instances with the largest positive and largest negative influence values to construct a set of 20 critical instances. The Jaccard index is equal to 1 when a set of estimated critical instances is identical to that of true critical instances.

To investigate the relationship between a number of tracing back steps and the estimation accuracy, we also evaluated the influence on GAN evaluation metrics of k-epoch ASGD. In k-epoch training, both inference phase of linear influence and the counterfactual ASGD traced back only $k \leq K$ epochs from the latest epoch K. We varied $k = 1, 5, 10, 20, 50$ and ran the experiment ten times for each k by changing the random seeds of the experiments.

Results and Discussion Figure 1 shows the average Kendal's Tau and the Jaccard index of the repeated experiments. Hereinafter, we use $p < .05$ to judge the statistical significance of the results. For all k, Kendall's Tau and the Jaccard index of estimated influence on ALL were statistically significantly better than the result in which the order of estimated influence values were random (random case). Even in the more difficult setups of IS and FID, which handled the high-dimensional dataset and complex architecture, the results were statistically significantly better than that of the random case except for Jaccard index of IS with $k = 50$. We also observed the estimation accuracy dropped as k increased. This reflects the nature of our estimator that recursively performs linear approximation as many times as the number of steps. We thus conclude that when the required number of tracing back steps is small enough, our influence estimation method is effective and the estimated influence on GAN evaluation metrics are useful for identifying harmful instances.

[2] Although the original IS and FID use Inception Net (Szegedy et al., 2016) trained with ImageNet, we instead adopted a domain-specific classifier as encouraged by several studies (Zhou et al., 2018; Liu et al., 2018) to alleviate the domain mismatch with ImageNet.

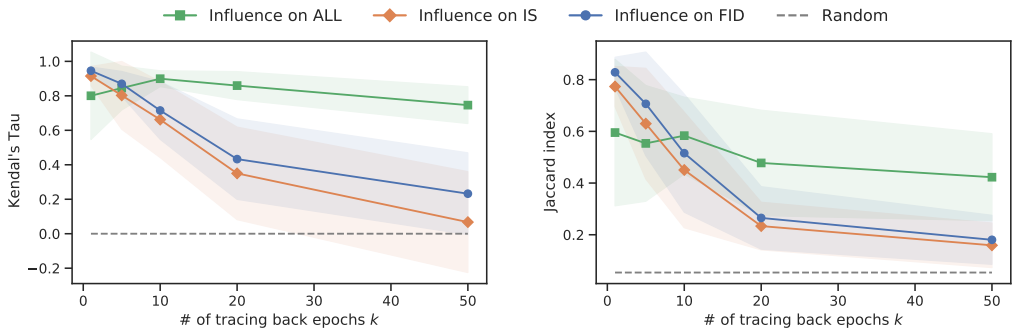

Figure 1: Average Kendall's Tau (±std) (left) and the Jaccard index (±std) (right) calculated from true and estimated influence on ALL, IS, and FID.

5.2 Experiment 2: Data Cleansing

We investigated if removing identified harmful instances actually improved the generative performance to evaluate the effectiveness of our proposed method for *data cleansing*. We define data cleansing as an attempt to improve GAN evaluation metrics by removing a set of training instances. See appendix C.3 for the detailed settings.

Setup We studied the data cleansing for the two setups explained in the previous section: 2D-Normal with FCGAN and MNIST with DCGAN. We mostly followed the settings of Section 5.1 but set training dataset size $N = 50\text{k}$ for both setups.

We identified harmful instances in 2D-Normal training dataset using estimated influence on ALL, and those in MNIST using estimated influence on IS and FID. We considered an training instance was harmful when it had negative (positive) influence on FID (ALL or IS).

For both setups, we also selected instances using baseline approaches: anomaly detection method, influence on the discriminator loss, and random values. For anomaly detection, we adopted isolation forest (Liu et al., 2008). Isolation forest fitted the model using the data points of $\mathcal{D}_{\boldsymbol{x}}$ for 2D-Normal and feature vectors of the classifier of $\mathcal{D}_{\boldsymbol{x}}$ for MNIST. We adopted the selection based on the influence on the discriminator loss to verify our assumption that the influence on the loss does not represent the harmfulness of the instances. Influence on the discriminator loss was calculated on the expected loss of $\mathscr{L}_D(\boldsymbol{\theta})$ with $\mathcal{D}_G(\mathcal{Z}'; \boldsymbol{\theta}_G^{[T]})$ and $\mathcal{D}'_{\boldsymbol{x}}$. We considered instances with negative influence were harmful.

We conducted the experiments as follows. After the training phase of K epoch, we determined $n_h < N$ harmful instances with the proposed approach and baselines. Then, we ran counterfactual ASGD with the determined harmful instances excluded. For the reliable estimation accuracy of influence and reasonable costs of the computation and storage, the inference phase traced back only 1-epoch from the last epoch, and counterfactual ASGD only re-ran the latest epoch.

We refer to the generator of the final model as the cleansed generator and denote its parameters by $\boldsymbol{\theta}_G^\star$. We evaluated the cleansed generator with test GAN evaluation metrics $V(\mathcal{D}_G(\mathcal{Z}_{test}); \boldsymbol{\theta}_G^\star))$, in which a set of test latent variables $\mathcal{Z}_{test}$ was obtained by sampling N_{test} times from $p(\boldsymbol{z})$ independently from $\mathcal{Z}'$ and $\mathcal{Z}_1, \ldots, \mathcal{Z}_T$. ALL and FID used N_{test} data points newly sampled from 2D-Normal and the original test dataset of MNIST, respectively. We set $N_{test} = 10\text{k}$. We tested with various n_h. We ran the experiment 15 times for each n_h with different random seeds.

Results and Discussion Figure 2 shows the average test GAN evaluation metrics of the repeated experiments for each selection approach. For the data cleansing on 2D-Normal, the proposed approach with influence on ALL showed statistically significant improvement from the original model and it outperformed the baselines (Figure 2a). For the MNIST setup, our approach with influence on FID and IS statistically significantly improved FID (Figure 2c) and IS (Figure 2b), respectively. They also outperformed the baselines. In addition, the results indicate that data cleansing based on the influence on a specific GAN evaluation metric is also effective for another metric that is not used

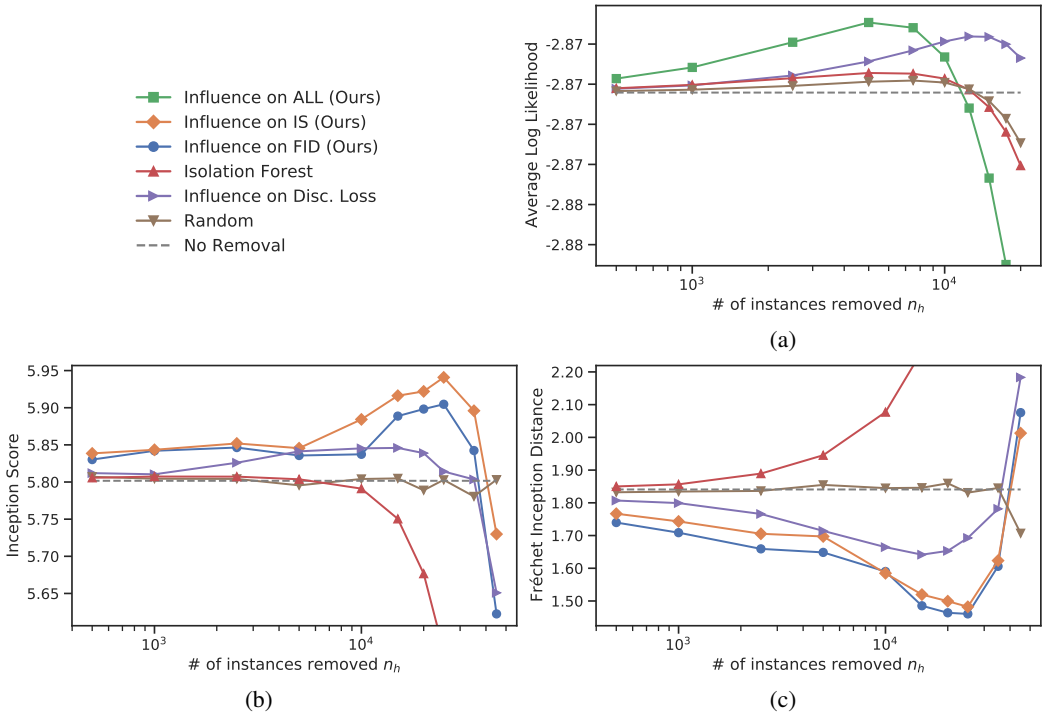

Figure 2: Average test ALL (a), IS (b), and FID (c) after the data cleansing. Larger values in (a) and (b), a smaller value in (c) indicate the better generative performance. Error bars and plots of too large or small values are omitted for better visibility. See Appendix C.3 for full results.

for the selection; removing harmful instances based on the influence on FID (IS) statistically significantly improved IS (FID). However, we make no claim that the proposed method can improve all the other evaluation metrics, such as Kullback-Leibler divergence. This is because all the current GAN evaluation metrics have their own weaknesses (e.g., IS fails to detect whether a model is trapped into one bad mode (Zhou et al., 2018)), and the proposed method based on those GAN evaluation metrics cannot inherently avoid their weaknesses. These improvements thus can be observed only in a subclass of GAN evaluation metrics. Further evaluation of data cleansing with our method should incorporate the future improvements of the GAN evaluation metrics.

While the improvements were smaller than the proposed approach, we also observed that data cleansing based on the influence on the discriminator loss improved all the GAN evaluation metrics. This counter-intuitive result indicates that the discriminator loss weakly measures the performance of the generator that is trained along with the discriminator.

6 Conclusion

We proposed an influence estimation method for GAN that uses the Jacobian of the gradient of the discriminator's loss with respect to the generator's parameters (and vice versa), which traces how the absence of an instance in the discriminator's training affects the generator's parameters. We also proposed a novel evaluation scheme to judge if an instance is harmful or not on the basis of the influence on GAN evaluation metrics rather than that on the loss value, and to leverage the proposed influence estimation method to identify harmful instances. We experimentally verified that estimated and true influence on GAN evaluation metrics had statistically significant correlation. We also demonstrated removing identified harmful instances effectively improved the generative performance with respect to various GAN evaluation metrics.

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Algorithm 1 Training Phase

Initialize the parameter $\boldsymbol{\theta}^{[1]}$
Initialize the sequence as null: $\mathcal{A} \leftarrow \emptyset$
for $t = 1, 2, \ldots, T-1$ **do**
  // sample latent variables
  $\mathcal{Z}_t = \{\boldsymbol{z}_l^{[t]} \sim p(\boldsymbol{z})\}_{l=1}^{|\mathcal{S}_t|}$
  // store information
  $\mathcal{A}^{[t]} \leftarrow \left(\mathcal{S}_t, \eta_G^{[t]}, \eta_D^{[t]}, \boldsymbol{\theta}^{[t]}, \mathcal{Z}_t\right)$
  // update parameters
  $\boldsymbol{\theta}^{[t+1]} = \boldsymbol{\theta}^{[t]} - \boldsymbol{B}_t g\left(\mathcal{S}_t, \mathcal{Z}_t; \boldsymbol{\theta}^{[t]}\right)$
end for

Algorithm 2 Inference Phase

Require: $\boldsymbol{u} = \left(\boldsymbol{u}_G \in \mathbb{R}^{d_G},\ \boldsymbol{u}_D \in \mathbb{R}^{d_D}\right)^\top$
  Initialize the influence: $L_{-j}^{[T]}(\boldsymbol{u}) \leftarrow 0$
  for $t = T-1, T-2, \ldots, 1$ **do**
    // load information
    $\left(\mathcal{S}_t, \eta_G^{[t]}, \eta_D^{[t]}, \boldsymbol{\theta}^{[t]}, \mathcal{Z}_t\right) \leftarrow \mathcal{A}^{[t]}$
    // update the linear influence of jth instance
    if $j \in \mathcal{S}_t$ **then**
      $L_{-j}^{[T]}(\boldsymbol{u}) \mathrel{+}= \frac{\eta_D^{[t]}}{|\mathcal{S}_t|} \left\langle \boldsymbol{u}_D, \nabla_{\boldsymbol{\theta}_D} f_D^{[\boldsymbol{x}]}(\boldsymbol{x}_j; \boldsymbol{\theta}^{[t]}) \right\rangle$
    end if
    // update $\boldsymbol{u}$
    $\boldsymbol{u} \mathrel{-}= \boldsymbol{u}^\top \boldsymbol{B}_t \boldsymbol{J}_t$
  end for

A Algorithm for Linear Influence

The proposed estimation algorithm of linear influence, which is explained in Section 3.1, is divided into the training phase (Algorithm 1) and inference phase (Algorithm 2).

The training phase executes ASGD training while storing the mini-batch indices $\mathcal{S}_t$, the learning rate $\eta_G^{[t]}$, $\eta_D^{[t]}$, the parameters $\boldsymbol{\theta}^{[t]}$ and the sampled latent variable $\mathcal{Z}_t$ into the information $\mathcal{A}^{[t]}$ for each step.

In the inference phase, $L_{-j}^{[T]}(\boldsymbol{u})$ is estimated by the recursive calculation. First, we set $L_{-j}^{[T]}(\boldsymbol{u})$ to 0 and set the query vector $\boldsymbol{u}$. The information $\mathcal{A}^{[t]}$, which is obtained in the training phase, is read in the order of $t = T-1, T-2, \ldots, 1$. When $j \in \mathcal{S}_t$, $L_{-j}^{[T]}(\boldsymbol{u})$ is updated using (8). Let $\boldsymbol{u}_t = \left(\boldsymbol{u}_G^{[t]}, \boldsymbol{u}_D^{[t]}\right)^\top$. Each step updates $\boldsymbol{u}$ based on $\boldsymbol{u}_{t+1} = \boldsymbol{u}_t^\top \left(\boldsymbol{I}_{d_{\boldsymbol{\theta}}} - \boldsymbol{B}_t \boldsymbol{J}_t\right)$. A naive calculation of $\boldsymbol{u}_t^\top \boldsymbol{J}_t$ requires $O\left(d_{\boldsymbol{\theta}}^2\right)$ memory to store the matrix $\boldsymbol{J}_t$, which can be prohibitive for very large models. We can avoid this difficulty by directly computing $\boldsymbol{u}_t^\top \boldsymbol{J}_t$ without the explicit computation of $\boldsymbol{J}_t$. Because $\boldsymbol{u}_t^\top \boldsymbol{J}_t = \nabla_{\boldsymbol{\theta}} \left\langle \boldsymbol{u}_t, (\nabla_{\boldsymbol{\theta}_G} \overline{\mathscr{L}}_G, \nabla_{\boldsymbol{\theta}_D} \overline{\mathscr{L}}_D)^\top \right\rangle$, we need only to compute the derivative of the inner product of $\boldsymbol{u}_t$ and the joint gradient vector.

B Other Related Works

Anomaly Detection A typical approach for identifying harmful instances is outlier detection. Outlier detection is used to remove abnormal instances from the training set before training the model to ensure that the model is not affected by the abnormal instances. For tabular data, there are several popular methods, such as One-class support vector machine (Schölkopf et al., 2001), local outlier factor (Breunig et al., 2000), and isolation forest (Liu et al., 2008). Although these methods can find abnormal instances, they are not necessarily harmful for the resulting models, as we showed in the experiment.

Training GAN from Noisy Images One typical type of data that harm generative performance is noisy images. AmbientGAN (Bora et al., 2018) and noise-robust GAN (Kaneko & Harada, 2020) are learning algorithms that make it possible to train a clean image generator from noisy images. The difference between these studies and ours is that these studies assume that the noise (e.g., Gaussian noise on pixels) given independently from the data distribution of the clean images is the only problem. However, some instances can affect the performance even if the instances are drawn only from the data distribution, which is the case robust statistics (Huber, 2004) typically focuses on. Our experiment 5.2 indicates that the model performance depends not only on noisy images but also on a non-negligible number of harmful instances in the original dataset.

Table 1: Model architecture of CNN classifier of MNIST in Section 5.1 and 5.2.

Stage	Operation	Stride	Filter Shape	Bias	Norm.	Activation	Output
0	Input	-	-	-	-	-	[28, 28, 1]
1	Conv2D	1	[5, 5]	✓	-	Sigmoid	[25, 25, 8]
2	Conv2D	1	[5, 5]	✓	-	Sigmoid	[12, 12, 8]
3	MaxPooling	2	[2, 2]	-	-	Sigmoid	[392]
4	Linear	1	-	✓	-	Sigmoid	[128]
5	Linear	1	-	✓	-	Sigmoid	[10]

Table 2: Hyper parameters in Section 5.1.

	K	$\eta_G^{[t]}$	$\eta_D^{[t]}$	N	N'	$\mathcal{S}_t$	γ	h_G	h_D
2D-Normal	50	10^{-3}	10^{-3}	10k	10k	100	10^{-3}	32	64
MNIST	50	10^{-3}	10^{-3}	10k	10k	100	10^{-3}	8	8

C Detailed Experimental Settings and results

C.1 GAN evaluation metrics

We adopted Gaussian kernel with the band-width 1 for kernel density estimation used in ALL. The architecture of CNN classifier of MNIST used for IS and FID can be found in Table 1. We selected the output of the 4-th layer for the feature vectors for FID.

C.2 Experiment 1: estimation accuracy

Setup In the experiment of Section 5.1, we adopted the hyper parameters shown in Table 2. We trained fullly-connected GAN (FCGAN) for 2D multivariate normal distribution, in which the both G and D has 1 hidden layer of h_G and h_D units, respectively (Table 3). 2D-Normal is given by $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, in which the mean vector $\boldsymbol{\mu} = \mathbf{1}_2$ and the covariance matrix $\boldsymbol{\Sigma} = ((1, 0.8), (0.8, 1))^\top$. DCGAN consists of transposed convolution (or deconvolution) layers and convolution layers (Table 4). The channels of the both layers in G and D were determined by h_G and h_D, respectively. We used Layer Normalization (Ba et al., 2016) for the layers shown in Table 4 for the stability of the training. We also introduced the L2-norm regularization with the rate $\gamma \in \mathbb{R}^+$ for all the kernels of both FCGAN and DCGAN. We used the non-zero-sum game objective of the original paper (Goodfellow et al., 2014) in which G tries to minimize $-D_{\boldsymbol{\theta}_D}\left(G_{\boldsymbol{\theta}_G}\left(\boldsymbol{z}\right)\right)$ for both models.

C.3 Experiment 2: data cleansing

Setup We adopted the same architecture as the Section 5.1 (Table 3) for FCGAN and slightly larger architecture (Table 4) in which h_G and h_D are larger (Table 5) for DCGAN. Other hyper parameters followed Table 5.

Results Table 6 shows the improvements of ALL from the original model in the 2D-Normal case. Table 7 and 8 show the improvements of IS and FID from the original model in the MNIST case, respectively. In Table 6, 7 and 8, we highlighted the improvements that were statistically significant with the significant level 0.05.

Table 3: Model Architecture of FCGAN in Section 5.1 and 5.2.

Net.	Stage	Operation	Bias	Activation	Output
-	0	Input	-	-	[10]
G	1	Linear	✓	ReLU	$[h_G]$
G	2	Linear	✓	Tanh	[2]
D	3	Linear	✓	ReLU	$[h_D]$
D	4	Linear	✓	Sigmoid	[1]

Table 4: Model Architecture of DCGAN in Section 5.1 and 5.2.

Net.	Stage	Operation	Stride	Filter Shape	Bias	Norm.	Activation	Output
-	0	Input	-	-	-	-	-	[32]
G	1	Deconv2D	1	[2, 2]	✓	✓	Sigmoid	[2, 2, h_G]
G	2	Deconv2D	1	[3, 3]	✓	✓	Sigmoid	[4, 4, h_G]
G	3	Deconv2D	2	[3, 3]	✓	✓	Sigmoid	[9, 9, h_G]
G	4	Deconv2D	1	[2, 2]	✓	✓	Sigmoid	[10, 10, h_G]
G	5	Deconv2D	1	[3, 3]	✓	✓	Sigmoid	[12, 12, h_G]
G	6	Deconv2D	2	[3, 3]	✓	✓	Sigmoid	[25, 25, h_G]
G	7	Deconv2D	1	[4, 4]	✓	✓	Sigmoid	[28, 28, h_G]
G	8	Conv2D	1	[1, 1]	✓	-	Tanh	[28, 28, 1]
D	9	Conv2D	1	[4, 4]	✓	✓	Sigmoid	[25, 25, h_D]
D	10	Conv2D	2	[3, 3]	✓	✓	Sigmoid	[12, 12, h_D]
D	11	Conv2D	1	[3, 3]	✓	✓	Sigmoid	[10, 10, h_D]
D	12	Conv2D	1	[2, 2]	✓	✓	Sigmoid	[9, 9, h_D]
D	13	Conv2D	2	[3, 3]	✓	✓	Sigmoid	[4, 4, h_D]
D	14	Conv2D	1	[3, 3]	✓	✓	Sigmoid	[2, 2, h_D]
D	15	Conv2D	1	[2, 2]	✓	✓	Sigmoid	[1, 1, h_D]
D	16	Linear	-	-	✓	-	Sigmoid	[1]

Table 5: Hyper parameters in Section 5.2.

	K	$\eta_G^{[t]}$	$\eta_D^{[t]}$	N	N'	$\mathcal{S}_t$	γ	h_G	h_D
2D-Normal	70	10^{-3}	10^{-3}	10k	10k	100	10^{-3}	32	64
MNIST	20	10^{-3}	10^{-3}	10k	10k	100	10^{-3}	32	32

Table 6: Improvements of test average log-likelihood [10^{-3}] ($\pm$std) after the data cleansing (2D-Normal). The metric value is highlighted when the improvement is statistically significant with the significant level 0.05

	n_h									
	0.5k	1.0k	2.5k	5.0k	7.5k	10.0k	12.5k	15.0k	17.5k	20.0k
Influence on ALL	**+0.09 (0.06)**	**+0.16 (0.12)**	**+0.31 (0.27)**	**+0.44 (0.50)**	+0.40 (0.73)	+0.22 (0.99)	-0.10 (1.28)	-0.53 (1.60)	-1.07 (1.95)	-1.67 (2.33)
Influence on Disc. loss	**+0.02 (0.03)**	**+0.04 (0.05)**	**+0.11 (0.10)**	**+0.19 (0.19)**	**+0.26 (0.28)**	**+0.32 (0.39)**	**+0.35 (0.51)**	+0.35 (0.64)	+0.30 (0.79)	+0.22 (0.95)
Isolation Forest	+0.03 (0.05)	+0.05 (0.11)	+0.09 (0.27)	+0.12 (0.54)	+0.12 (0.79)	+0.09 (1.05)	+0.02 (1.31)	-0.09 (1.58)	-0.25 (1.86)	-0.46 (2.16)
Random	+0.01 (0.04)	+0.02 (0.08)	+0.04 (0.19)	+0.07 (0.39)	+0.08 (0.61)	+0.06 (0.83)	+0.02 (1.07)	-0.05 (1.34)	-0.16 (1.61)	-0.31 (1.91)

Table 7: Improvements of test inception score ($\pm$std) after the data cleansing (MNIST). The metric value is highlighted when the improvement is statistically significant with the significant level 0.05

	n_h									
	0.5k	1.0k	2.5k	5.0k	10.0k	15.0k	20.0k	25.0k	35.0k	45.0k
Influence on FID	+0.03 (0.07)	+0.04 (0.09)	+0.04 (0.17)	+0.03 (0.25)	+0.04 (0.24)	**+0.09 (0.13)**	**+0.10 (0.12)**	**+0.10 (0.13)**	+0.04 (0.17)	-0.18 (0.28)
Influence on IS	**+0.04 (0.05)**	+0.04 (0.08)	+0.05 (0.14)	+0.04 (0.23)	+0.08 (0.15)	**+0.11 (0.13)**	**+0.12 (0.14)**	**+0.14 (0.14)**	+0.09 (0.25)	-0.07 (0.24)
Influence on Disc. Loss	+0.01 (0.03)	+0.01 (0.05)	**+0.02 (0.03)**	**+0.04 (0.04)**	**+0.04 (0.05)**	**+0.04 (0.06)**	**+0.04 (0.06)**	+0.01 (0.06)	+0.00 (0.07)	-0.15 (0.11)
Isolation Forest	+0.00 (0.02)	+0.01 (0.02)	+0.01 (0.04)	+0.00 (0.05)	-0.01 (0.06)	-0.05 (0.08)	-0.13 (0.13)	-0.23 (0.18)	-0.67 (0.33)	-1.70 (0.75)
Random	+0.01 (0.02)	+0.00 (0.01)	+0.00 (0.02)	-0.01 (0.04)	+0.00 (0.04)	+0.00 (0.05)	-0.01 (0.09)	+0.00 (0.06)	-0.02 (0.07)	+0.00 (0.10)

Table 8: Improvements of test FID (±std) after the data cleansing (MNIST). The metric value is highlighted when the improvement is statistically significant with the significant level 0.05

	n_h									
	0.5k	1.0k	2.5k	5.0k	10.0k	15.0k	20.0k	25.0k	35.0k	45.0k
Influence on FID	**-0.10 (0.13)**	**-0.13 (0.18)**	**-0.18 (0.28)**	-0.19 (0.46)	-0.25 (0.45)	**-0.36 (0.35)**	**-0.38 (0.36)**	**-0.38 (0.37)**	-0.23 (0.46)	+0.23 (0.60)
Influence on IS	**-0.07 (0.10)**	**-0.10 (0.14)**	**-0.14 (0.22)**	-0.14 (0.37)	**-0.26 (0.28)**	**-0.32 (0.29)**	**-0.34 (0.30)**	**-0.36 (0.30)**	-0.22 (0.45)	+0.17 (0.49)
Influence on Disc. Loss	-0.03 (0.06)	-0.04 (0.08)	**-0.07 (0.07)**	**-0.13 (0.10)**	**-0.18 (0.12)**	**-0.20 (0.13)**	**-0.19 (0.14)**	**-0.15 (0.14)**	-0.06 (0.12)	+0.34 (0.19)
Isolation Forest	+0.01 (0.03)	+0.02 (0.03)	+0.05 (0.06)	+0.10 (0.08)	+0.24 (0.15)	+0.42 (0.22)	+0.73 (0.37)	+1.09 (0.54)	+2.56 (0.85)	+6.99 (3.57)
Random	-0.01 (0.04)	-0.01 (0.02)	-0.00 (0.04)	+0.01 (0.06)	+0.00 (0.07)	+0.01 (0.08)	+0.02 (0.16)	-0.01 (0.09)	+0.00 (0.13)	**-0.13 (0.18)**