EXPLORING THE LIMITS OF DIFFERENTIALLY PRIVATE DEEP LEARNING WITH GROUP-WISE CLIPPING

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Paper under double-blind review

ABSTRACT

Differentially private deep learning has recently witnessed advances in computational efficiency and privacy-utility trade-off. We explore whether further improvements along the two axes are possible and provide affirmative answers leveraging two instantiations of *group-wise clipping*. To reduce the compute time overhead of private learning, we show that *per-layer clipping*, where the gradient of each neural network layer is clipped separately, allows clipping to be performed in conjunction with backpropagation in differentially private optimization. This results in private learning that is as memory-efficient and almost as fast per training update as non-private learning for many workflows of interest. While per-layer clipping with constant thresholds tends to underperform standard flat clipping, per-layer clipping with adaptive thresholds matches or outperforms flat clipping under given training epoch constraints, hence attaining similar or better task performance within less wall time. To explore the limits of scaling (pretrained) models in differentially private deep learning, we privately fine-tune the 175 billion-parameter GPT-3. We bypass scaling challenges associated with clipping gradients that are distributed across multiple devices with *per-device clipping* that clips the gradient of each model piece separately on its host device. Privately fine-tuning GPT-3 with per-device clipping achieves a task performance at $\epsilon = 1$ better than what is attainable by non-privately fine-tuning the largest GPT-2 on a summarization task.

1 INTRODUCTION

Recent works on deep learning with differential privacy (DP) have substantially improved the computational efficiency (Subramani et al., 2021; Anil et al., 2021) and privacy-utility trade-off (Li et al., 2022a; Yu et al., 2022; De et al., 2022; Mehta et al., 2022), resulting in cost-effective private learning workflows with favourable utility under common levels of privacy guarantee. Common to most of these works is the use of differentially private stochastic gradient descent (DP-SGD) which clips per-example gradients (herein referred to as *flat clipping*) and noises their average before performing the parameter update based on a minibatch (Song et al., 2013; Bassily et al., 2014; Abadi et al., 2016). We explore whether further improvements in computational efficiency and privacy-utility trade-off are possible and provide affirmative answers for both directions, leveraging two instantiations of *group-wise clipping* for DP-SGD.

DP-SGD is known to be computationally costly due to clipping per-example gradients. Instantiating per-example gradients and (potentially) normalizing them can incur both high memory and time costs in standard machine learning frameworks (Paszke et al., 2019; Frostig et al., 2018), and thus private machine learning with DP-SGD is reportedly much more memory demanding and/or slower than its non-private counterpart (Carlini et al., 2019; Hoory et al., 2021). Recent works have considerably improved the memory and time efficiency of DP-SGD with better software primitives (Subramani et al., 2021) and algorithms (Yousefpour et al., 2021; Lee & Kifer, 2021; Li et al., 2022b; Bu et al., 2022). Nevertheless, private learning still shows non-trivial increases in either memory usage or compute time when compared to non-private learning head-to-head. For instance, better software primitives do not eliminate the inherent increase in memory spending (Subramani et al., 2021), and improved algorithms only remove this memory overhead at the cost of extra runtime (Li et al., 2022b). The first research question we study is therefore

Can private learning be as memory and time efficient (per-epoch) as non-private learning?

We answer the above question affirmatively by giving an efficient implementation of *per-layer clipping* which had been studied in past works but not from a computational efficiency perspective (McMahan et al., 2018b; Dupuy et al., 2022). Clipping per-example gradients of separate neural networks layers (e.g., linear, convolution) separately allows clipping to be performed in conjunction with backpropagation. This results in private learning that is as memory-efficient and almost as time-efficient per training update as non-private learning for many small to moderate scale workflows of practical interest. While per-layer clipping with static clipping thresholds chosen by hand tends to underperform flat clipping, we show that per-layer clipping with adaptively estimated thresholds matches or outperforms flat clipping under given training epoch constraints, hence attaining similar or better task performances with less wall time.

DP-SGD is known to (possibly) incur substantial performance losses compared to non-private learning. To improve the privacy-utility trade-off, several past works have leveraged large-scale publicly pretrained models (Yu et al., 2022; Li et al., 2022b; De et al., 2022; Mehta et al., 2022). These works observe that the privacy-utility trade-off improves with the use of larger (and thus better) pretrained models.[1] We extend this research and study a second research question

Can the privacy-utility trade-off be further improved with even better / larger pretrained models?

To study this, we scale DP fine-tuning to work with one of the largest and most performant pretrained language models to date—the 175 billion-parameter GPT-3. Weights of this model cannot be hosted in the memory of a single accelerator (e.g., GPU) and must be distributed across multiple devices. This presents challenges for flat clipping which calls for communicating per-example gradient norms across devices. To bypass these challenges, we turn to *per-device clipping*, where each device is prescribed a clipping threshold for clipping per-example gradients of the hosted model piece. Per-device clipping incurs no additional communication cost and allowed us to obtain with GPT-3 a private fine-tuning performance at $\epsilon = 1$ that is better than what is attainable by non-privately fine-tuning the largest GPT-2 on a challenging summarization task.

Our contributions are summarized below.

(1) We show per-layer clipping enables clipping to be done in conjunction with backpropagation in DP optimization and results in private learning that is as memory-efficient and almost as fast per training update as non-private learning for many small to moderate scale workflows of interest.

(2) We show adaptive per-layer clipping matches or outperforms flat clipping under fixed training epoch constraints, and thus attains similar or better task performances with less wall time.

(3) We bypass scaling challenges associated with communicating per-example gradient norms with per-device clipping, with which we scale DP fine-tuning to work with the 175 billion-parameter GPT-3 and obtain improved task performance for a challenging summarization task at $\epsilon = 1$.

2 PRELIMINARIES

This section aims to cover background on gradient clipping in DP optimization and explain why alternate group-wise clipping strategies can be attractive from a computational efficiency standpoint.

Our work trains machine learning models (more precisely deep neural networks) with optimizers that guarantee DP. For completeness, we recap the definition of approximate DP / (ϵ, δ)-DP below.

Definition 2.1 ((ϵ, δ)-DP)**.** A randomized algorithm $\mathcal{M} : \mathcal{X} \to \mathcal{Y}$ is (ϵ, δ)-DP if for all neighboring datasets $\mathbb{D}, \mathbb{D}' \in \mathcal{X}$ and all $Y \subset \mathcal{Y}$, it holds that $\mathbb{P}\left(\mathcal{M}(\mathbb{D}) \in Y\right) \leq \exp(\epsilon) \cdot \mathbb{P}\left(\mathcal{M}(\mathbb{D}') \in Y\right) + \delta$.

In our case, $\mathcal{M}$ is an optimization algorithm which outputs the learned model parameters $\boldsymbol{\theta}$. Widely used DP optimizers (e.g., DP-SGD) usually introduce two additional steps before each parameter update to privatize gradients: (1) clip per-example gradients of a minibatch by their Euclidean norms according to some threshold C; (2) add Gaussian noise to the sum of clipped gradients. In practice, the clipping threshold C can be either a tunable hyperparameter or set to some (privatized) statistic

[1] While size does not equate quality, there is strong correlation between the two under currently popular pretraining techniques (Liu et al., 2019; Brown et al., 2020). We're optimistic that future smaller models pretrained with improved techniques can be as performant as current large models (Hoffmann et al., 2022).

estimated from data. The standard deviation of the Gaussian noise is determined by the clipping threshold C and the noise multiplier σ, the latter of which is set by a privacy accounting procedure given target privacy parameters (ϵ, δ), the number of iterations T, and the subsampling rate ρ (Abadi et al., 2016; Mironov, 2017; Dong et al., 2021; Gopi et al., 2021). We now present background on different per-example gradient clipping strategies in DP optimization.

Flat Clipping. This is the clipping scheme used in the original DP-SGD algorithm (Abadi et al., 2016). Here, the gradient of example s_i's loss $\ell(\boldsymbol{\theta}, s_i)$ with respect to model parameters $\boldsymbol{g}^{(i)} := \partial\ell(\boldsymbol{\theta}, s_i)/\partial\boldsymbol{\theta}$ is normalized if its magnitude exceeds the threshold C. Thus, the actual contribution (up to scaling) of the ith instance to the noisy gradient is $\widetilde{\boldsymbol{g}}^{(i)} := \boldsymbol{g}^{(i)} \cdot \min\{1, C/\|\boldsymbol{g}^{(i)}\|\}$. Flat clipping cannot be performed until the gradient norms $\{\|\boldsymbol{g}^{(i)}\|\}_i$ are computed. Since the latter quantities are only known after backpropagation completes, flat clipping necessitates a second-round of computation after backpropagation to conditionally rescale the gradients. This is a source of overhead and presents complications when model weights don't fit on a single device.

Group-Wise Clipping. This scheme partitions the set of parameters $\boldsymbol{\theta} \in \mathbb{R}^d$ into K disjoint groups $\{\boldsymbol{\theta}_k\}_{k=1}^K$ with $\boldsymbol{\theta}_k \in \mathbb{R}^{d_k}$ for $k \in [K]$. For each group k, the scheme prescribes a clipping threshold C_k. Denote example s_i's gradient for the kth group by $\boldsymbol{g}_k^{(i)} := \partial\ell(\boldsymbol{\theta}, s_i)/\partial\boldsymbol{\theta}_k$. Under group-wise clipping, the kth clipped gradient for s_i is $\tilde{\boldsymbol{g}}_k^{(i)} := \boldsymbol{g}_k^{(i)} \cdot \min\{1, C_k/\|\boldsymbol{g}_k^{(i)}\|\}$. Next, we present two instantiations of group-wise clipping that are computationally advantageous in different settings.

3 Efficient Private Learning with Adaptive Per-layer Clipping

The first instantiation of group-wise clipping we study is per-layer clipping which clips gradients of separate neural network layers separately. This scheme had been presented in past works (McMahan et al., 2018a;b; Dupuy et al., 2022), but neither its computational properties nor its performance implications had been carefully studied. We show that with a proper implementation, per-layer clipping can be as memory-efficient and almost as time-efficient as non-private training. Additionally, we confirm that per-layer clipping with hand-set thresholds underperforms flat clipping and demonstrate that adaptively setting these thresholds eliminates potential performance losses.

3.1 Per-layer Clipping DP-SGD Can Be Almost as Efficient as Non-private SGD

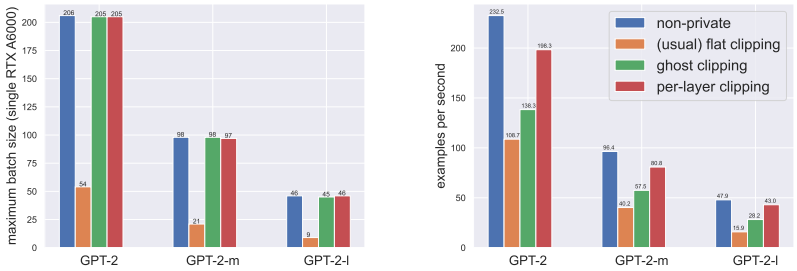
Per-layer clipping groups together parameters of a neural network layer (e.g., linear, convolution) and prescribes each of the K layers of the network a clipping threshold C_k to clip the gradient of that layer. This directly implies that gradient clipping for any layer can be performed as soon as backpropagation reaches that layer (to construct per-example gradients or norms) when parameter sharing is absent, and is unlike flat clipping which cannot be performed until backpropagation completes entirely.[2]

Our efficient implementation of per-layer clipping clips layer-wise gradients as soon as the gradient with respect to outputs of that layer are returned from backpropagation. The operations of clipping and summing per-example gradients can be fused once input activations, output gradients, and per-example gradient norms are known. In addition, per-example gradient norms can be cheaply computed without materializing actual per-example gradients in memory (Li et al., 2022b, Section 4). This implementation results in private training that is as memory-efficient as non-private training since per-example gradients are not instantiated, and almost as time-efficient per update since the extra computation involving gradient norm and gradient scaling are typically cheap. Figure 1 shows that carefully implemented per-layer clipping matches the memory profile and almost matches the training throughput of non-private learning for an autoregressive fine-tuning task with GPT-2 (we followed the same experimental protocol as that of Section 4 in (Li et al., 2022b) for a fair comparison).

3.2 Fixed Per-layer Clipping May Hurt the Utility

Despite the computational advantages, per-layer clipping with fixed clipping thresholds set by hand (which we refer to as *fixed per-layer clipping*) reportedly underperforms flat clipping (McMahan

[2]Note the DP learning library Opacus (Yousefpour et al., 2021) has a per-layer clipping optimizer which supports clipping each layer with a separate threshold. But this implementation conducts per-layer clipping after backpropagation completes and instantiates all per-example gradients before clipping, which is inefficient.



(a) Memory (b) Throughput

Figure 1: Private learning with per-layer clipping can be almost as efficient as non-private learning (the throughput gap is less than 15% in this case). Here, "(usual) flat clipping" refers to the implementation which first creates and stores in memory all per-example gradients (e.g., adopted in Opacus (Yousefpour et al., 2021)). Ghost clipping (Li et al., 2022b) is based on flat clipping but avoids materializing per-example gradients by performing an additional backward pass each time.

et al., 2018b). To further verify this and remove confounding effects of potentially suboptimal hyperparameters, we compare fixed per-layer clipping against (fixed) flat clipping on two tasks: 1) training wide ResNet (WRN16-4) (Zagoruyko & Komodakis, 2016) from scratch to classify CIFAR-10 images, and 2) fine-tuning the pretrained RoBERTa-base for classifying sentiment on SST-2. We carefully tuned the clipping thresholds and learning rate for both clipping methods; see Appendix A for details. Tables 1a and 1b confirm that per-layer clipping with hand-set fixed thresholds underperforms flat clipping.

Table 1: Fixed per-layer clipping underperforms (fixed) flat clipping.

(a) CIFAR-10 validation accuracy over 3 seeds.

Model	Method	$\epsilon = 3$	$\epsilon = 8$
WRN16-4	Fixed per-layer	60.6	67.8
	Fixed flat	63.1	73.9

(b) SST-2 validation accuracy over 3 seeds.

Model	Method	$\epsilon = 3$	$\epsilon = 8$
RoBERTa-base	Fixed per-layer	89.4	89.7
	Fixed flat	91.0	91.7

To understand why fixed per-layer clipping gives worse performance, we plot the per-layer gradient norms of randomly sampled CIFAR-10 examples for privately training WRN16-4 in Figure 2 (see Appendix B for the setup). We observe that the general magnitudes of per-layer gradient norms change dramatically across training. Early on, gradient norms are generally uniformly low across all layers. As training proceeds, gradient norms for layers close to the input gradually become high. We present additional evidence for this phenomenon with language model fine-tuning in Appendix B.

These observations suggest that clipping with fixed layer-wise thresholds likely removes the structural relation between gradients of different layers. This incurs an extra source of bias in addition to the usual bias of flat clipping that alters the relation of gradients across samples, and makes balancing clipping bias and privacy noise throughout training even more challenging. This motivates us to set the thresholds based on some adaptively estimated statistic of layer-wise gradients.

3.3 Adaptive Per-layer Clipping Can Be as Effective as Flat Clipping

To overcome the above performance issues of fixed per-layer clipping, we consider per-layer clipping with adaptive clipping thresholds that we herein refer to as *adaptive per-layer clipping*. Our hope is that the adaptive thresholds can track gradient norm shift, capture gradient structure, and consequently mitigate the structural bias caused by clipping gradients of separate layers separately. One candidate statistic for setting the adaptive threshold is some quantile of gradient norms. Notably, Andrew et al. (2019) provided an effective way of estimating quantiles privately for flat clipping via online convex optimization. We adapt their algorithm to the per-layer setup, and let each layer maintain an online estimate of a target gradient norm quantile.

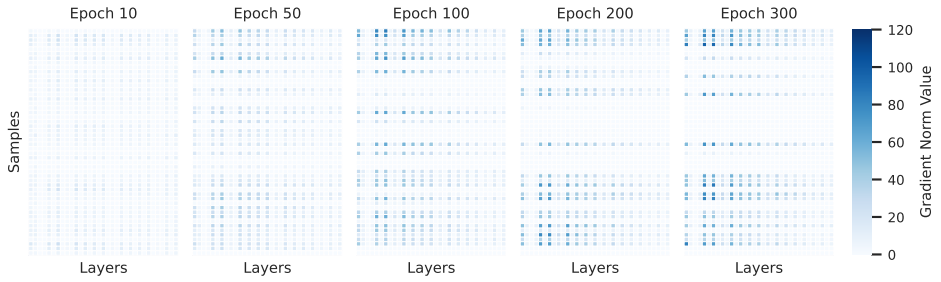


Figure 2: The distribution of per-layer gradient norms shifts substantially across training. Each column represents one layer, and each row represents one example. Layers of the neural network are placed from input (left) to output (right). Darker colors indicate higher values of gradient norms.

Two questions naturally arise with this formulation: 1) How should the per-layer gradient norm quantiles be privately estimated, and 2) how should the noise levels for different layers be decided? We separately address the two questions below. Algorithm 1 delineates the overall procedure in pseudocode, where per-layer gradient clipping in conjunction with backpropagation occurs on lines 7-12, adaptive private quantile estimation occurs on lines 15-18, and noise allocation occurs on line 13. The pseudocode is based on DP-SGD, but the core ideas naturally apply to private versions of other first-order optimizers (e.g., DP-Adam).

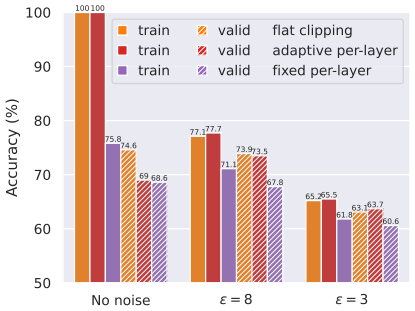


Figure 3: Adaptive per-layer clipping eliminates performance losses experienced by fixed per-layer clipping.

Estimating Quantiles Privately. We allocate some privacy budget (in practice r =1% to 10% of total budget) to estimate a target quantile of each layer's gradient norms. Clipping thresholds $C_1, ..., C_K$ are then set to these estimated quantiles. We record the number of gradients clipped before each parameter update and adjust the clipping threshold based on whether too many or too few are clipped. The central quantity which needs to be privatized is then the fraction of clipped gradients. We introduce the additional noise multiplier σ_b to privatize this fraction statistic (used in Gaussian mechanism). The new noise multiplier σ_{new} (based on $1 - r$ fraction of total budget) for noising parameter updates is computed with the following proposition, whose proof we defer to Appendix D.

Proposition 3.1. *Let σ be the original noise multiplier for noising parameter updates to achieve a certain level of differential privacy (without private quantile estimation) and σ_b be chosen for noising quantile estimates (release of the latter consumes r fraction of the privacy budget). Then the new noise multiplier σ_{new} for noising parameter updates (consuming $1 - r$ fraction of the budget) is*

$$\sigma_{new} = (\sigma^{-2} - K/(2\sigma_b)^2)^{-1/2}. \tag{3.1}$$

Remark 3.1. The private quantile estimation for K groups costs a fraction $r = K\sigma^2/(4\sigma_b^2)$ of privacy budget (in terms of Rényi differential privacy (Mironov, 2017)).

Allocating Noise. The original Gaussian mechanism adds isotropic noise to statistics before their release (Dwork et al., 2014) which results in different coordinates experiencing the same amount of noise. Yet, simply scaling different components with public quantities (before adding noise) allows different components to experience different levels of noise. As an example, let $\gamma_1, \cdots, \gamma_K$ be coefficients for scaling, and recall that $\tilde{\boldsymbol{g}}_k$ is the sum of clipped gradients for layer / group k. Then, applying the Gaussian mechanism to the scaled $\hat{\boldsymbol{g}} := (\hat{\boldsymbol{g}}_1, ..., \hat{\boldsymbol{g}}_K)$, where $\hat{\boldsymbol{g}}_k := \tilde{\boldsymbol{g}}_k/\gamma_k$, and rescaling back the privatized quantities afterwards ends up adding noise to $\tilde{\boldsymbol{g}}_k$ that has standard deviation proportional to γ_k.

Among the possible ways of choosing $\{\gamma_1, ..., \gamma_K\}$, we outline two simple approaches that we found to be effective for different reasons in our empirical studies. We use the global strategy in all but experiments with GPT-3. Appendix E includes empirical studies of alternate strategies.

- *Global strategy*: $\gamma_k = 1$ for $k \in [K]$. This strategy adds the same amount of noise to every component. The total noise has squared ℓ_2 norm $V_G \propto (\sum_k C_k^2) \cdot (\sum_k d_k)$.
- *Equal budget strategy*: $\gamma_k = C_k$ for $k \in [K]$. Each group has the same amount of privacy budget. The total noise has squared ℓ_2 norm $V_E \propto K \sum_{k=1}^{K} d_k C_k^2$.

Algorithm 1 DP-SGD with adaptive per-layer clipping

INPUT: Private dataset $\mathbb{D} = \{s_i\}_{i=1}^N$; initial iterate $\boldsymbol{\theta}_0$; number of iterations T; learning rate η_t; learning rate for quantile estimation η; privacy parameters ϵ, δ; per-layer parameters $\{\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_K\}$; initial clipping thresholds $\{C_1, \ldots, C_K\}$; weighting factors $\{\gamma_1, \ldots, \gamma_K\}$; target quantile q; sampling rate $\rho = B/N$.
$\sigma \leftarrow$ PrivacyAccountant$(\epsilon, \delta, \rho, T)$
Choose σ_b as the noise multiplier for private quantile estimation
Compute the new noise multiplier σ_{new} for gradient privatization with equation 3.1
for $t = 0$ to $T - 1$ **do**
  Sample a minibatch $\mathcal{S}_t$ with sampling rate ρ and perform the forward pass
  for $k = K$ to 1 **do**
    Compute per-sample gradient norms $\{\|\boldsymbol{g}_k^{(i)}\|\}_{i \in \mathcal{S}_t}$ given activations and output gradients
    Compute $\tilde{\boldsymbol{g}}_k \leftarrow \sum_{i \in \mathcal{S}_t} \tilde{\boldsymbol{g}}_k^{(i)} = \sum_{i \in \mathcal{S}_t} \boldsymbol{g}_k^{(i)} \cdot \min\{1, C_k / \|\boldsymbol{g}_k^{(i)}\|\}$ with fused operation
    Record $\bar{b}_k \leftarrow \sum_{i \in \mathcal{S}_t} \mathbb{1}[\|\boldsymbol{g}_k^{(i)}\| \leq C_k]$ for quantile estimation
    Perform usual backpropagation to obtain input gradients if $k > 1$
  end for
  Draw $\boldsymbol{z} \leftarrow (\boldsymbol{z}_1, ..., \boldsymbol{z}_K)$, where $\boldsymbol{z}_k \sim \mathcal{N}\left(0, \sigma_{\text{new}}^2 S^2 \gamma_k^2 \boldsymbol{I}_{d_k}\right)$ and $S = (\sum_{k=1}^{K} C_k^2 / \gamma_k^2)^{1/2}$
  $\boldsymbol{\theta}_{t+1} \leftarrow \boldsymbol{\theta}_t - \eta_t \left(\tilde{\boldsymbol{g}} + \boldsymbol{z}\right) / B$.
  for $k = 1$ to K **do**
    Draw $z_k \sim \mathcal{N}(0, \sigma_b^2)$ and set $\tilde{b}_k \leftarrow (\bar{b}_k + z_k)/B$
    $C_k \leftarrow C_k \cdot \exp(-\eta(\tilde{b}_k - q))$
  end for
end for
return θ_T (or some average of all iterates)

With the tools of quantile estimation and noise allocation, we show that adaptive per-layer clipping matches the performance of flat clipping. Here, we compare adaptive per-layer clipping against fixed per-layer clipping and flat clipping with and without noise for training WRN16-4 on CIFAR-10 (setup details in Appendix A). Figure 3 shows that the performance of adaptive per-layer clipping roughly matches that of flat clipping, while fixed adaptive clipping suffers large performance drops. Section 5 includes additional results to validate this point.

4 Efficient Private Pipeline Parallelism with Per-device Clipping

Past works have shown that DP fine-tuning yields improved privacy-utility trade-offs with the use of larger / better pretrained models. We study whether this trend continues to hold as one leverages pretrained models even larger than those previously presented by scaling DP training to work with one of the largest pretrained language models to date—the 175 billion-parameter GPT-3. The sheer size of this model presents challenges in computational efficiency, since model weights cannot be fit on a single device (e.g., GPU) and existing approaches for distributing computation across devices doesn't tend to play well with flat per-example gradient clipping.

We base our distributed DP training strategy off the popular *pipeline parallelism* used in non-private training of models that don't fit on single devices (Huang et al., 2019; Rasley et al., 2020).[3] We summarize the idea of pipeline parallelism here and defer to the cited works for the specifics. Pipeline parallelism first partitions the model into chunks of consecutive layers / blocks and distributes each onto a single accelerator. Forward computation with a microbatch (created through splitting a minibatch) then chains together local computations with each model piece (hosted on each accelerator)

[3] Alternate parallelization schemes can be more flat clipping friendly (e.g., FSDP (Zhao et al., 2022)), but current open source implementations of these schemes are generally not light-weight fine-tuning friendly.

by communicating activations across accelerators. Backward computation (backpropagation) roughly reverses the above process, but on each accelerator, intermediate forward activations of the model piece are recomputed to reduce peak memory (Huang et al., 2019, Section 2.3). Most importantly, pipeline parallelism simultaneously performs computation with different microbatches on different accelerators to reduce the overall idle time. Devices synchronize after all microbatches finish their forward and backward computation and before the optimizer invokes the parameter update.

Flat clipping necessitates computing per-example gradient norms. This calls for the communication of per-example norms of local gradients on each device and leads to an inherent source of overhead. We outline two viable approaches for accomplishing this communication, both of which unfortunately would lead to non-trivial slowdowns as well as complications in implementation. The first approach synchronizes all devices after the full backward pass finishes for each microbatch (within a minibatch) so that each device will retain the (same) gradient norms necessary for clipping. This approach incurs as many extra synchronization steps as the number of microbatches per processed minibatch and substantially reduces training efficiency when the number of microbatches is large. To reduce the frequency of synchronization, a second approach instead asks devices to retain their local gradient norms and only synchronizes after the last microbatch has been processed. This approach, however, requires either per-example gradients be offloaded to CPU (due to limited accelerator memory) and transported back for clipping after synchronization, or per-example gradients be rematerialized through another round of backpropagation.

As the first attempt at experimenting with DP fine-tuning on huge models, we instead turn to an alternative *per-device clipping* scheme, where each device is prescribed a clipping threshold for clipping per-example gradients of the hosted model piece. Leveraging the equal budget strategy, the noise level added to gradients on each device is agnostic of the clipping thresholds of other devices (thus, no extra communication incurred). We present the full pseudocode of the algorithm in Appendix C. Notably, per-device clipping with DP LoRA fine-tuning allowed us to obtain improved results for a challenging summarization task (see Section 5.3).

5 Experiments

We have verified that per-layer clipping enjoys efficiency advantage over flat clipping. We now demonstrate that the utility of adaptive per-layer clipping is not compromised. Our experiments cover training wide ResNets from scratch, fine-tuning RoBERTa models on GLUE tasks, and fine-tuning GPT-2 and GPT-3 models for table-to-text generation and summarization tasks. For private quantile estimation, we use the geometric update rule by Andrew et al. (2019) and set $\eta = 0.3$ for all experiments. Reported numbers of this work are averaged over three seeds unless otherwise stated.

5.1 Privately Learn WideResNets for CIFAR-10 Classification

We train a wide ResNet (WRN16-4, 2.8M trainable parameters) (Zagoruyko & Komodakis, 2016) from scratch for CIFAR-10 classification with differential privacy. We follow the implementation setting by (De et al., 2022), e.g., batch normalization are replaced with group normalization and weight standardization is applied for convolutional layers, except that we do not use augmentation multiplicity for simplicity. We set privacy parameter $\delta = 10^{-5}$ and choose ϵ from $\{1, 3, 5, 8\}$, which are typical privacy parameters used in previous works.

We compare the performance of adaptive per-layer clipping with that of flat clipping. For both algorithms, we use hyperparameters suggested by De et al. (2022) and tune learning rates. We use a fraction $r = 0.01$ of privacy budget for quantile estimation and choose the target quantile q from $\{0.5, 0.6, 0.7\}$. For both algorithms we train for 300 epochs. We summarize the details in Appendix A.1. From Table 2, we can see that adaptive per-layer clipping achieves training and validation accuracy on par with flat clipping for multiple choices of ϵ.

5.2 Privately Fine-tune RoBERTa on GLUE Tasks

We fine-tune the RoBERTa-base (125M) and RoBERTa-large (355M) (Liu et al., 2019) on SST-2, QNLI, QQP, and MNLI from the GLUE benchmark (Wang et al., 2018) with differential privacy. We let $\delta = 1/n^{1.1}$ and $\epsilon \in \{3, 8\}$, where n is the size of training set. We tune the learning rate, batch

Table 2: Adaptive per-layer achieves accuracy (in %) on par with flat clipping for CIFAR-10 task.

Method	$\epsilon = 1$		$\epsilon = 3$		$\epsilon = 5$		$\epsilon = 8$	
	Train	Valid	Train	Valid	Train	Valid	Train	Valid
Flat clipping (De et al., 2022)	44.9	44.7	65.2	63.1	72.3	70.1	77.1	73.9
Adaptive per-layer	49.1	48.6	65.5	63.7	72.3	69.5	77.7	73.5

size, and target quantile on SST-2's training set and transfer the best hyerparameters to other datasets. The number of epochs is set to be 20. We use a fraction $r = 0.1$ of privacy budget for private quantile estimation and choose the target quantile from $\{0.5, 0.75, 0.85\}$. See Appendix A.2 for details.

From Table 3, we can see that adaptive per-layer clipping achieves better accuracy than baselines for majority of the settings. We also report results of using different epochs ($\{10, 20, 30\}$) on SST-2 in Table 4, which suggests that per-layer clipping is on par with flat clipping for all training epochs and justifies that the adaptive per-layer clipping indeed uses less wall time compared with flat clipping.

Table 3: Adaptive per-layer clipping matches the accuracy (in %) of flat clipping for GLUE tasks.[4]

Model	Method	$\epsilon = 3$				$\epsilon = 8$			
		MNLI-(m/mm)	QQP	QNLI	SST-2	MNLI-(m/mm)	QQP	QNLI	SST-2
RoBERTa-base	Yu et al. (2021b)	-	-	-	-	80.1	85.5	87.2	91.6
	Li et al. (2022b)	82.47/82.10	85.41	84.62	86.12	83.30/83.13	86.15	84.81	85.89
	Yu et al. (2022)	-	-	-	-	83.5	85.7	87.3	92.2
	Adaptive per-layer	82.83/83.27	85.67	86.13	92.03	83.70/83.97	86.23	87.13	92.40
RoBERTa-large	Yu et al. (2021b)	-	-	-	-	86.1	86.7	90.0	93.0
	Li et al. (2022b)	85.53/85.81	86.65	88.94	90.71	86.28/86.54	87.49	89.42	90.94
	Yu et al. (2022)	-	-	-	-	87.8	87.4	90.8	95.3
	Adaptive per-layer	87.10/87.20	86.80	89.80	93.87	87.67/87.57	87.20	90.77	94.03

Table 4: Adaptive per-layer clipping matches or outperforms flat clipping with regards to SST-2 dev set accuracy (in %) under fixed fine-tuning epoch (E) constraints.

Model	Method	$\epsilon = 3$			$\epsilon = 8$		
		$E = 10$	$E = 20$	$E = 30$	$E = 10$	$E = 20$	$E = 30$
RoBERTa-base	Flat clipping	90.53	90.76	91.27	91.00	91.30	91.90
	Adaptive per-layer	91.30	91.57	92.10	91.50	91.96	92.60
RoBERTa-large	Flat clipping	93.00	93.50	93.90	93.50	93.87	94.00
	Adaptive per-layer	92.80	93.63	93.67	93.43	94.00	94.03

5.3 Privately Fine-tune on Langauge Generation Tasks

Table-To-Text Generation. We compare adaptive per-layer clipping against flat clipping for full fine-tuning with GPT-2 on the E2E (Novikova et al., 2017) and DART (Nan et al., 2020) table-to-text generation tasks. Since Li et al. (2022b) performed extensive tuning on these tasks for flat clipping, we recall their results here. For runs with adaptive per-layer clipping, we reused hyperparameter values tuned for SST-2, but re-tuned the target quantile with the E2E validation set and constrained the batch size and number of training epochs to be the same as that for flat clipping; see Appendix A.3 for details. Table 5 confirms that DP learning with adaptive per-layer clipping performs comparably to flat clipping under a given epoch constraint.

[4]Results in Yu et al. (2021b; 2022) on MNLI are the average of the matched and mismatched accuracy.

Table 5: Adaptive per-layer clipping matches the performance of flat clipping for full fine-tuning under the same number of training epochs for common privacy levels. Results based on fine-tuning GPT-2 on E2E and DART. Numbers in the column "flat" are reported by Li et al. (2022b).

Metric	DP Guarantee	E2E		DART	
		Adaptive per-layer	Flat	Adaptive per-layer	Flat
BLEU	$\epsilon = 3$	61.101	61.519	31.851	31.025
	$\epsilon = 8$	63.416	63.189	34.166	35.057
	non-private	-	69.463	-	42.783
ROUGE-L	$\epsilon = 3$	65.120	65.670	51.519	52.063
	$\epsilon = 8$	66.689	66.429	52.877	54.576
	non-private	-	71.359	-	56.717

Dialog Summarization. We use the SAMSum dialog summarization task as a testbed for studying model scaling (Gliwa et al., 2019).[5] This task is more challenging than previously tested ones since its training set is small (less than 15k examples) and inputs are long. We fine-tune both GPT-2-xl and the (original) 175 billion-parameter GPT-3 with LoRA (Hu et al., 2021) with and without DP, and compare them against in-context learning with GPT-3 (Brown et al., 2020). Table 6 shows that GPT-3 fine-tuned at $\epsilon = 1$ outperforms non-privately fine-tuned GPT-2-xl and in-context learning with 4 demonstrations (the maximum that can be fitted within the context window of 2048 tokens). See Appendix C for more details.

Table 6: Fine-tuning GPT-3 with DP LoRA achieves improved privacy-utility trade-off. Metrics are ROUGE scores.

Model+Method+DP guarantee	R-1	R-2	R-L
Flat clipping			
GPT-2-xl LoRA ($\epsilon = 0.25$)	8.0	2.7	6.8
GPT-2-xl LoRA ($\epsilon = 1$)	30.0	11.0	25.3
GPT-2-xl LoRA ($\epsilon = 4$)	35.4	14.3	29.9
GPT-2-xl LoRA (non-private)	46.2	23.7	39.4
Per-device clipping			
GPT-3 LoRA ($\epsilon = 0.25$)	42.0	20.4	35.7
GPT-3 LoRA ($\epsilon = 1$)	48.0	25.4	41.3
GPT-3 LoRA ($\epsilon = 4$)	48.5	26.4	42.0
GPT-3 LoRA (non-private)	53.8	29.8	45.9
GPT-3 0-shot	27.4	9.0	20.9
GPT-3 4-shot	42.1	19.6	34.2

6 Related Work

Training large deep learning models with DP has gained momentum in the recent years. For instance, Anil et al. (2021) privately pretrained BERT models, and Kurakin et al. (2022) privately trained deep ResNets on ImageNet. Recent works have also investigated private fine-tuning (Kerrigan et al., 2020; Tian et al., 2021; Senge et al., 2021; Hoory et al., 2021; Basu et al., 2021; Yu et al., 2021b) and observed that one can achieve favourable privacy-utility trade-offs with large pretrained models for image classification (Luo et al., 2021; Tramèr & Boneh, 2021; Golatkar et al., 2022; De et al., 2022; Mehta et al., 2022) and tasks in NLP (Yu et al., 2022; Li et al., 2022b;a). Group-wise clipping schemes considered in our work improve the efficiency of DP-SGD and further this line of research by making scaling private learning easier.

Several works considered adjusting the clipping threshold of DP-SGD adaptively during training (Andrew et al., 2019; Pichapati et al., 2019; Asi et al., 2021). The most related to us is that by Andrew et al. (2019), where authors set the threshold for flat clipping as some privately estimated quantile of gradient norms. They showed that doing so eased hyperparameter tuning without affecting the final model performance. Different from these works, ours considers per-layer clipping, where adapting the clipping threshold plays a more crucial role for obtaining good utility. More discussion on related work is in Appendix G.

7 Conclusion and Limitation

We showed that group-wise clipping schemes are effective tools to improve the efficiency of DP-SGD for small to moderate scale workflows, and to avoid communication overheads in private distributed pipeline parallel training of models that do not fit on single accelerators. We showed that adaptive clipping algorithms can mitigate known utility losses associated with using fixed and hand-tuned thresholds. Designing group-wise clipping algorithms that can pareto-dominate flat clipping in terms of privacy vs utility (or show such schemes are unlikely to exist) is an interesting future direction. One limitation of group-wise clipping algorithms arise from the potential increase in the number of hyperparameters. This could lead to a need of additional tuning when optimal hyperparameters differ across tasks and domains. Another limitation of per-group clipping is that it gives limited efficiency improvements in non-distributed settings when light-weight fine-tuning methods are used.

[5] We believe the chance of contamination occurring for this task to be small. See Appendix C for discussions.

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A EXPERIMENT DETAILS

A.1 SETUP FOR CIFAR-10 CLASSIFICATION WITH WRN16-4 MODEL

In the paper, we have used CIFAR-10 classification task with wide residual network (WRN16-4) to demonstrate some points. To increase the reproducibility, we describe the detailed setting of these experiments.

We modify the standard WRN16-4 (Zagoruyko & Komodakis, 2016) following the suggestions of De et al. (2022), i.e., replacing batch normalization with group normalization and using weight standardization for convoluntional weights, except that we do not use augmentation multiplicity for simplicity. Specifically, for flat clipping, De et al. (2022) uses a handed tuned clipping threshold $C = 1$ and learning rate $\text{lr} = 4$. Due to the fact that the learning rate and clipping thresholds jointly affect the performance in a complex way, we set the fixed per-layer clipping thresholds and the adaptive per-layer clipping thresholds so that they both have equivalent global threshold $C = 1$. For fixed per-layer clipping, each layer clipping threshold is $C/\sqrt{K}$ where K is the number of layers. For adaptive clipping thresholds $C_1, ..., C_K$, we rescale them $\tilde{C}_k := C \cdot C_k/\sqrt{\sum_k C_k^2}$ for all $k \in [K]$.

To make the comparison more fair, we carefully tune the hyper-parameters of fixed per-layer clipping in Table 1, Table 11 and Figure 3. We find that small fixed thresholds can improve the performance of fixed per-layer clipping. We try different C's from $\{1.0, 0.5, 0.1, 0.05\}$ while making $C \cdot \text{lr}$ constant (which is critical for SGD), and set clipping threshold $C_k = C/\sqrt{K}$ for each layer. Finally, for fixed per-layer clipping, We choose the best hyper-parameter combinations ($C = 0.05, \text{lr} = 40$) and ($C = 0.1, \text{lr} = 20$) for $\epsilon = \{3, 8\}$, respectively.

For both fixed and adaptive per-layer clippings, we use the global strategy for noise allocation, i.e., $\gamma_k = 1$ for all $k \in [K]$. Moreover, we use the same optimizer, weight decay, momentum, learning rate schedule, batch size and max epochs as flat clipping, as shown in Table 7. We tune the learning rate from two choices $\{2, 4\}$ for all three algorithms. For adaptive per-layer clipping, we use a fraction $r = 0.01$ of privacy budget to estimate quantiles and quantile learning rate $\eta = 0.3$. We tune the target quantile from three choices $\{0.5, 0.6, 0.7\}$. We will evaluate the hyperparameter sensitivity in ablation study (Section F). Hyperparameters are tuned by training from scratch on training set and evaluating on test set. We use the best hyperparameter combinations for different ϵ respectively and report the test set accuracy of the last epoch in Table 2.

Table 7: Hyper-parameters of flat clipping and per-layer clipping for WRN16-4 on CIFAR-10.

	Adaptive per-layer	Fixed per-layer	Flat clipping
Optimizer		SGD	
Weight Decay		0	
Momentum		0	
Learning Rate Schedule		Constant	
Batch Size		4096	
Max Epochs		300	
Learning Rate	$\{2, 4\}$	equivalent $\{2, 4\}$	$\{2, 4\}$
Allocation Method	Global	Global	-
Private Quantile Relative Budget r	1%	-	-
Quantile Learning Rate η	0.3	-	-
Target Quantile q	$\{0.5, 0.6, 0.7\}$	-	-

A.2 SETUP FOR GLUE TASKS WITH ROBERTA MODELS

To evaluate the performance of adaptive per-layer clipping, we conduct well-controlled experiments on the GLUE tasks with RoBERTa-base and RoBERTa-large models.

The optimizer setup and dropout are same for adaptive per-layer clipping, fixed per-layer clipping, flat clipping and adaptive flat clipping, as shown in Table 8.

For per-layer clipping, we use global strategy for noise allocation, i.e., $\gamma_k = 1$ for all $k \in [K]$.

We tune other hyperparameters: peak learning rate, batch size, initial clipping thresholds for fixed per-layer clipping, target quantile q for adaptive per-layer clipping, as shown in the bottom half of Table 8.

To tune hyperparameters fairly, we split the training set of SST-2 dataset into two parts: a new training set containing 80% of original training set and a new dev set containing 20% of original training set. We select the best hyperparameters by watching the best performance on the new dev set, average over 3 different seeds. Table 8 shows the best hyperparameter combination we eventually use for adaptive and fixed per-layer, with pretrained RoBERTa models.

In our experiments, we just adopted private quantile relative budget $r = 10\%$ and fixed this hyper-parameter. As Figure 6 suggests, using a smaller relative budget ratio like $r = 1\%$ or $r = 5\%$ may produce slightly better results.

We transfer these hyperparameters to the remaining tasks. Specifically, we follow Li et al. (2022b) and keep the sampling rate the same across different datasets.

For these GLUE tasks, we find that more epochs will consistently improve the performance for both flat and per-layer clipping. However, to balance the performance and training time, we fix the max epochs to be 20 in all setups. We report the best accuracy on the validation set for each task and setup. The comparison results are presented in Table 3.

As the performance always gets improved if you train longer, we also compare the performance of flat clipping and adaptive per-layer clipping given various max epochs, in Table 4. Note that we only train self-attention layers and classification head of RoBERTa models for both methods in (and only in) this part to reduce the computational costs of fine-tuning with flat clipping.

Table 8: Hyper-parameters of per-layer clipping for RoBERTa on SST-2 dataset, where the **text in bold** denotes the hyper-parameters we eventually use.

Model	RoBERTa-base		RoBERTa-large	
Method	Adaptive	Fixed	Adaptive	Fixed
Optimizer	Adam			
Adam (β_1, β_2)	(0.9, 0.98)			
Adam ϵ	10^{-6}			
Weight Decay	0			
Warm-up Ratio	0.06			
Learning Rate Schedule	Linear Decay			
Dropout	0.1			
Attention Dropout	0.1			
Max Epochs	20			
Peak Learning Rate	$\{1, \mathbf{2}, 4\} \times 10^{-4}$	$\{\mathbf{1}, 2, 4\} \times 10^{-4}$	$\{1, \mathbf{2}, 4\} \times 10^{-4}$	$\{1, \mathbf{2}, 4\} \times 10^{-4}$
Batch Size	$\{1, \mathbf{2}, 4\} \times 2^9$	$\{\mathbf{1}, 2, 4\} \times 2^9$	$\{\mathbf{1}, 2, 4\} \times 2^9$	$\{\mathbf{1}, 2, 4\} \times 2^9$
Allocation Method	Global			
Init Threshold	1.0	{**0.1**, 0.5, 1.0}	1.0	{0.1, 0.5, **1.0**}
Private Quantile Relative Budget r	10%	-	10%	-
Quantile Learning Rate η	0.3	-	0.3	-
Target Quantile q	{0.5, 0.75, **0.85**}	-	{0.5, 0.75, **0.85**}	-

A.3 Setup for Language Generation Tasks with GPT-2

We reused most of the hyperparameters specific to adaptive quantile estimation based on tuning results on SST-2. We retuned the target quantile parameter as we observed that optimal values of this parameter tend to be different for different tasks. To ensure a fair comparison against full fine-tuning, we constrain the runs with adaptive per-layer clipping to have the same batch size and training epochs as in (Li et al., 2022b). We adopted the default values set by the Hugging Face `transformers` library for Adam's β_1, β_2, and ϵ. Table 9 contains the full set of hyperparameters.

B Additional Experiments on Gradient Norm Shift

In this section, we illustrate the distribution of gradient norms shift in both CIFAR-10 training and SST-2 fine-tuning. To visualize the gradient norms, we first randomly select some samples from the

Table 9: Hyperparameters for full fine-tuning GPT-2 with adaptive per-layer clipping. Numbers in bold are best performing hyperparameters used for reporting final results.

Model	GPT-2	
Dataset	E2E	DART
Optimizer	Adam	
Adam (β_1, β_2)	(0.9, 0.999)	
Adam (ϵ)	1e-8	
Weight Decay	0	
Learning Rate Schedule	Linear Decay	
Batch Size	1000	1500
Max Epochs	10	15
Learning Rate	2×10^{-3}	
Allocation Method	Global	
Init Threshold	0.01	
Private Quantile Relative Budget r	1%	
Quantile Learning Rate η	0.3	
Target Quantile q	$\{\mathbf{0.3}, 0.5, 0.7, 0.9\}$	reuse best of left

training set, and take the checkpoints at different epochs of a privately trained model with adaptive per-layer clipping and the privacy parameter $\epsilon = 8$. For each sample, we compute the gradient norm of each layer. Specifically, for CIFAR-10, we ramdonly select 32 samples and place layers of WRN16-4 from input (left) to output (right) in Figure 2.

For SST-2, we randomly select 4,096 samples and some layers in the RoBERTa-base model, and plot the histogram of the per-sample per-layer gradient norms in Figure 4 across the first few epochs. It is worth noting that we found that for fine-tuning SST-2 with RoBERTa-base, it is true for many layers that the 85% clipping threshold (see red dashed line in Figure 4) is just the point can split samples into a group with small gradient norms and a group with large ones.

Both of Figure 2 and Figure 4 demonstrate that the distribution of gradient norms is complex and may related to many factors: (1) **Iterations & Samples**: gradient norms are small and spread out across layers in the early epochs, and as the training process goes on, per-sample gradients become divided, the large becomes larger and the small becomes smaller; (2) **Layers**: gradient norms of layers close to the input are larger than those of layers close to output, it is more prominent in the later stages of training, but it's aligned well across samples.

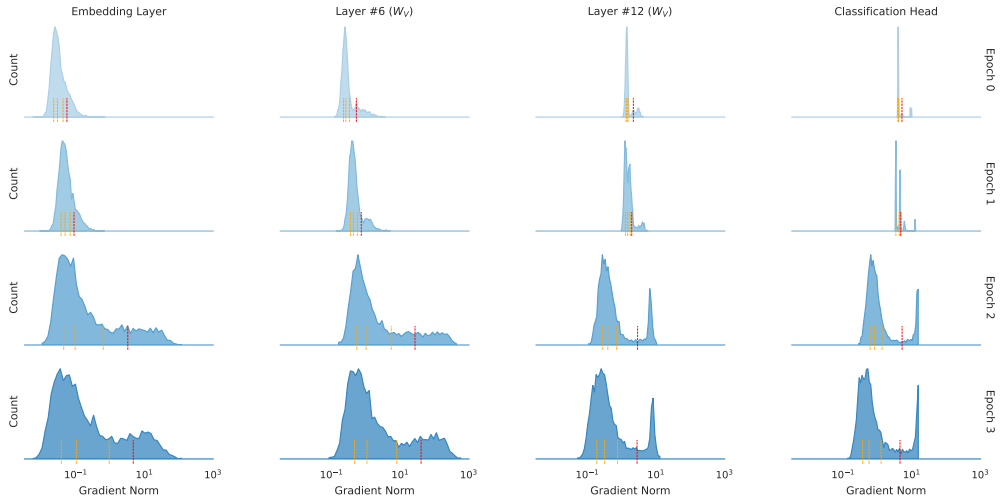


Figure 4: The distribution of gradient norms of 4,096 randomly selected samples shifts across the fine-tuning procedure for the setup of SST-2 with RoBERTa-base. We show the gradient statistics of (1) the embedding layer, (2) the value matrix W_V of the 6th and 12th self-attention module, (3) the final classification layer. Quantiles $\{25\%, 50\%$ (median), $75\%, 85\%\}$ of the gradient norms are marked with dashed lines, from left to right.

C MORE ON PER-DEVICE CLIPPING AND EXPERIMENTS WITH GPT-3

C.1 ADDITIONAL COMMENTS ON PER-DEVICE CLIPPING

Algorithm 2 details the private pipeline parallel training procedure with per-device clipping covered in the main text. The algorithm adopts the equal budget noise allocation strategy to avoid incurring extra communication across devices. For simplicity, the pseudocode covers a single update and omits any subprocedures for adapting clipping thresholds. Extending this pseudocode to adaptive threshold clipping based on quantile estimation is straightforward. Lastly, the pseudocode assumes that the model has already been partitioned into chunks of consecutive layers, where each chunk $\boldsymbol{\theta}_k$ is hosted on device k.

Algorithm 2 Single Update of Private Pipeline Parallel Training With Per-Device Clipping

INPUT: Minibatch $\mathcal{S}$; iterate $\boldsymbol{\theta}$; per-device parameters $\{\boldsymbol{\theta}_1, \cdots, \boldsymbol{\theta}_K\}$ hosted on K devices; clipping thresholds $\{C_1, \ldots, C_K\}$; noise multiplier σ; learning rate η; number of microbatches per minibatch J
Partition minibatch $\mathcal{S}$ into J microbatches $\{\mathcal{S}_1, \cdots, \mathcal{S}_J\}$
Create an empty execution schedule $\mathcal{C}$
for $j = 1$ to J **do**
  Stage microbatch $\mathcal{S}_j$'s `LocalForward` and `LocalBackward` calls in the schedule $\mathcal{C}$, ensuring the stages are executed sequentially for this microbatch
end for
Organize the schedule $\mathcal{C}$ based on pipeline parallel rules, allowing different devices to process different microbatches simultaneously
Execute the schedule $\mathcal{C}$
Synchronize all devices
for $k = 1$ to K **do**
  $\boldsymbol{\theta}'_k \leftarrow \boldsymbol{\theta}_k - \eta u_k$.
end for
return $\boldsymbol{\theta}' = (\boldsymbol{\theta}'_1, \cdots, \boldsymbol{\theta}'_K)$

Algorithm 3 `LocalForward`

INPUT: Device id k; microbatch index j
Wait for activations $a_{k-1}^{(j)}$ from device $k-1$ if $k > 1$; otherwise transfer microbatch $\mathcal{S}_j$ onto device k
Perform forward pass with $a_{k-1}^{(j)}$ and model piece $\boldsymbol{\theta}_i$ (stored on device k) to obtain outputs $a_k^{(j)}$
Store a copy of $a_k^{(j)}$ on CPU if not already saved
Communicate activations $a_k^{(j)}$ to device $k+1$ if $k < K$

Algorithm 4 `LocalBackward`

INPUT: Device id k; microbatch index j
Transfer activations $a_{k-1}^{(j)}$ from CPU to device k
Wait for output gradients $o_k^{(j)}$ if device $k < K$
Rematerialize activations by performing extra forward pass
Clip and sum per-example gradients $\{\tilde{\boldsymbol{g}}_k^{(i)}\}_i$ of local model piece $\boldsymbol{\theta}_k$ by back-propagating based on $o_k^{(j)}$ or the loss values with threshold C_k; add this to a local accumulator u_k (stored on device k)
If $j = 1$, add noise to accumulator u_k to guarantee DP
Compute gradients with respect to input $o_{k-1}^{(j)}$ and communicate this to device $k-1$ if $k > 1$.

C.2 FINE-TUNING GPT-3 ON SAMSUM

Note the term "GPT-3" in the literature is used in multiple occasions and can refer to multiple models. Our experiments are based on fine-tuning or prompting the original GPT-3 model (Brown et al.,

2020) and not the more recent variants which had been fine-tuned or adapted in some way (e.g., instruct-GPT-3 (Ouyang et al., 2022) labeled with prefix `instruct-` in OpenAI API).

Larger models are known to have better fine-tuned performance when inputs and outputs are formatted as instructions and responses (Wei et al., 2021; Sanh et al., 2021). We observed similar results when fine-tuning with differential privacy, and thus augmented the training and test sets by prepending the inputs with the instruction "Summarize the following dialogue" and the outputs with the delimiter "TL;DR". To ensure a fair comparison, we used this instruction-augmented dataset for all experiments.

For decoding from models, we used beam search with a beam size of 4 for both GPT-3 and GPT-2-xl (including in-context learning experiments).

To ensure we account for the variability in performance with different prompts for in-context learning, we sampled 3 sets of prompts for the 4-shot learning experiments and reported the average metric over runs.

Without access to the original pretraining corpus, we cannot completely rule out the possibility of data contamination, which refers to the unfortunate outcome that parts of the fine-tuning or evaluation data occur in the pretraining corpus. Nevertheless, we believe the chances of this happening are small due to two reasons. First, zero-shot prompting GPT-3 with both low temperature sampling and beam search based on instruction-augmented inputs tended to result in completions which either repeated or extended the instruction or the dialog (e.g., "the following is a dialog between..."), or attempted to continue the dialog but digressed. In the limited number of examples we inspected, we were unable to find an instance where the output looked similar to a high-quality summary. Second, we looked up the initial time when the SAMSum paper was released to arXiv (late Nov. 2019). Given that the GPT-3 model we based our experiments off were pretrained with shards of Common Crawl uploaded (possibly) at the end of 2019 (Brown et al., 2020), we performed simple searches of the SAMSum paper with their url index in the Dec. 2019 crawl archive of Common Crawl and were not able to find the link of the paper. Notably the SAMSum dataset was crafted by linguists and highly curated (as opposed to collected based on web data).

For fine-tuning GPT-3 with DP LoRA on SAMSum, we reused hyperparameters adopted by Hu et al. (2021), but re-tuned the learning rate based on preliminary runs for another dataset. We set all per-device clipping threshold to be 1e-5 and adopted the equal budget noise allocation strategy for simplicity. We fine-tuned for 5 epochs in all runs (both GPT-3 and GPT-2-xl; both private and non-private).

For the DP LoRA fine-tuning runs, we used a machine with 16 V100 GPUs each with 32 gigabytes of VRAM. This enabled LoRA fine-tuning with a rank of 32 with a microbatch size of 1 under pipeline parallelism. Fine-tuning with DP LoRA for 5 epochs on SAMSum's training set took 15 hours, and decoding with test inputs using beam search further took another 22 hours.

D PROOFS

We present the proof for Proposition 3.1. For easy reference, we restate the proposition here.

Proposition 3.1. *Let σ be the original noise multiplier for noising parameter updates to achieve a certain level of differential privacy (without private quantile estimation) and σ_b be chosen for noising quantile estimates (release of the latter consumes r fraction of the privacy budget). Then the new noise multiplier σ_{new} for noising parameter updates (consuming $1-r$ fraction of the budget) is*

$$\sigma_{new} = (\sigma^{-2} - K/(2\sigma_b)^2)^{-1/2}. \tag{3.1}$$

Proof. The proof is based on direct calculation, a simple version of which is given in Andrew et al. (2019). First, we note that the clip counts $b_k^{(i)}$ is either 0 or 1 (see line 10 in Algorithm 1). One can make it to be symmetric by using $b_k^{(i)} - \frac{1}{2}$, whose sensitivity is $\frac{1}{2}$. Suppose the gradient has sensitivity S.

For the Gaussian mechanism, to keep the privacy budget constant we have

$$S^2/(S\sigma)^2 = S^2/(S\sigma_{\text{new}})^2 + K \cdot \frac{(1/2)^2}{\sigma_b^2}.$$

Simplifying the above expression, we get σ_{new}.

We can further compute the fraction r of budget that is used to privately estimate quantiles by $r = K \cdot \frac{(1/2)^2}{\sigma_b^2}/(1/\sigma^2)$. We can also derive the value of σ_b given r from the above formula. □

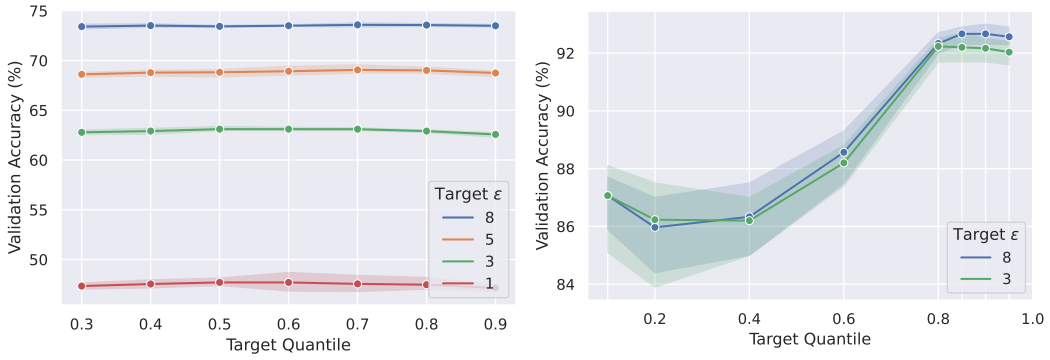
E NOISE ALLOCATION COMPARISON

We compare the noise allocation strategies empirically. Apart from the *global strategy* where $\gamma_k = 1$ for all $k \in [K]$ and the *equal budget strategy* where $\gamma_k = C_k$ for all $k \in [K]$ that are discussed in Section 3.3, we also consider another *weighted strategy*: $\gamma_k = C_k/\sqrt{d_k}$ for $k \in [K]$. In this case, the number of parameter plays a role so that each coordinate would roughly have the same signal to noise ratio and the total noise has squared ℓ_2 norm $V_E \propto (\sum_{k=1}^K d_k) \cdot (\sum_{k=1}^K C_k^2)$.

We fine-tune RoBERTa-base models on the SST-2 sentence classification task. The hyper-parameters are searched for each strategy separately where the ranges follow Appendix A. Results are presented in Table 10. We can see that three strategies achieves comparable performance and the global strategy is slightly better. Therefore, we use global strategy for all the experiment except for GPT-3 where the equal budget strategy is used to eliminate the concern of communication across devices.

Table 10: Accuracy (in %) on SST-2 dev set with different noise allocation strategies.

Model	Strategy	$\epsilon = 3$		$\epsilon = 8$	
		Training	Validation	Training	Validation
RoBERTa-base	Global	**89.2**	**92.0**	**89.7**	**92.3**
	Equal Budget	88.3	91.4	89.0	91.7
	Weighted (equal SNR)	89.0	91.7	89.6	91.9



(a) CIFAR-10

(b) SST-2

Figure 5: Validation accuracy (in %) with different target quantiles.

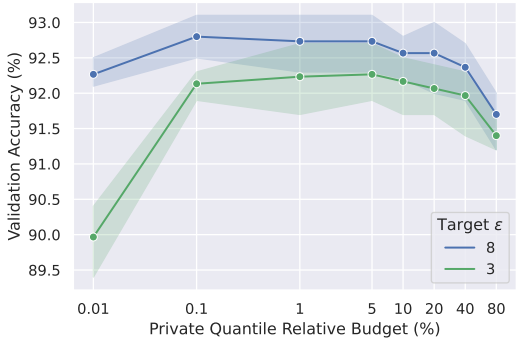


Figure 6: Validation accuracy (in %) on SST-2 of using different budgets for quantile estimation.

F Ablation Studies

Here we conduct ablation studies to see 1) the influence of using different quantiles to perform clipping; 2) the influence of varying the privacy budget for quantile estimation; 3) whether adaptive flat clipping significantly better than fixed flat clipping.

Clipping with Different Target Quantiles. We use different target quantiles for clipping on both WRN16-4 and RoBERTa-base. We choose the quantile for CIFAR-10 from $\{0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$ and that for SST-2 from $\{0.05, 0.2, 0.4, 0.6, 0.85, 0.9, 0.95\}$. Other hyperparameters are the same as those in Section 5.1 and 5.2. We plot the results in Figure 5. On CIFAR-10, the accuracy is robust to the choice of target quantile on all values considered. On SST-2, all quantiles around 0.9 give good performance. This suggests that setting the target quantile according to the model accuracy is a good default choice for the classification tasks. For generation task, we tune the target quantile as a hyper-parameter in general.

Different Budgets for Quantile Estimation. We show the influence of using different privacy budgets to estimate the target quantile. We fine-tune RoBERTa-base models on SST-2. The fraction of privacy budget for quantile estimation r is from $\{0.01\%, 0.1\%, 1\%, 5\%, 10\%, 20\%, 40\%, 80\%\}$. We plots the results in Figure 6. The performance is good for a wide range of r. When $\epsilon = 8$, using r as small as 0.01% still gives good accuracy. This further confirms the finding in Andrew et al. (2019) that quantiles can be estimated quite accurately with small privacy budget. Therefore we only need to split negligible budget for the private quantile estimation without affecting much the noises added to the model updates.

Adaptive Per-layer Clipping vs Adaptive Flat Clipping. We have verified that adaptive per-layer clipping can match the performance of well-tuned flat clipping in Section 5. To really justify value

of adaptive per-layer clipping, we need to demonstrate that adaptive flat clipping does not achieve significantly better performance than fixed flat clipping. We run experiments on the CIFAR-10 task with WRN16-4 and the SST-2 task with RoBERTa-base model. Their results are presented in Table 11a and Table 11b. We can see that adaptivity also helps flat clipping but the improvement is not statistically significant. The performance of adaptive per-layer clipping is on par with that of adaptive flat clipping as well.

Table 11: Adaptivity helps flat clipping but not as much as for per-layer clipping. Averaged accuracy and standard deviation are given by 3 independent runs.

(a) CIFAR-10

Method	$\epsilon = 3$	$\epsilon = 8$
Flat clipping		
fixed	$63.1_{(0.22)}$	$73.9_{(0.87)}$
adaptive	-	-
	+0	+0
Per-layer clipping		
fixed	$60.6_{(0.79)}$	$67.8_{(1.20)}$
adaptive	$63.7_{(0.34)}$	$73.5_{(0.87)}$
	+3.1	+5.7

(b) SST-2

Method	$\epsilon = 3$	$\epsilon = 8$
Flat clipping		
fixed	$91.0_{(0.51)}$	$91.7_{(0.76)}$
adaptive	$92.2_{(0.70)}$	$92.6_{(0.46)}$
	+1.2	+0.9
Per-layer clipping		
fixed	$89.4_{(1.04)}$	$89.7_{(0.70)}$
adaptive	$92.0_{(0.23)}$	$92.4_{(0.44)}$
	+2.6	+2.7

G Additional Related Work

Faster DP-SGD. Improving the efficiency of DP-SGD is an active research area. One line of works improve the implementation without changing the algorithm such as using better parallelism and compile-time optimization (Subramani et al., 2021; Anil et al., 2021). Subramani et al. (2021) show that the running time of carefully implemented DP-SGD is comparable to non-private SGD for small-size models. However, the high memory cost of storing per-example gradients still limits the throughput of DP-SGD when the model size is large (Li et al., 2022b). Another line of works avoids instantiating per-example gradients by running backpropagation twice (Goodfellow, 2015; Lee & Kifer, 2021; Bu et al., 2021; Li et al., 2022b; Bu et al., 2022). The high-level idea is to compute or estimate per-example gradient norms in the first backpropagation and reweight loss functions before the second backpropagation. Although these works achieve memory efficiency, they add computational overhead because of the additional backpropagation.

DP-SGD with Per-layer Clipping. Per-layer clipping (or more generally group-wise clipping) has been studied in Abadi et al. (2016); McMahan et al. (2018a;b); Dupuy et al. (2022). However, the advantage of per-layer clipping has not yet been fully understood because of two reasons. Firstly, previous work does not focus on computational efficiency, leaving the empirical advantage of group-wise clipping unexplored. Secondly, these works simply adopt fixed thresholds for per-layer clipping and hence generally observe performance drops compared to flat clipping. In this work, we use adaptive thresholds to improve the privacy-utility tradeoff of per-layer clipping. Moreover, we give an efficient implementation of per-layer clipping to demonstrate its superior empirical advantage.

Privacy Attacks Against Deep Models. Deep models may unintentionally leak sensitivie information about their training data (Shokri et al., 2017; Hitaj et al., 2017; Zhu et al., 2019; Song et al., 2019; Carlini et al., 2020; Choquette-Choo et al., 2021; Carlini et al., 2022; Balle et al., 2022). For instance, Carlini et al. (2020) show that GPT-2 outputs its training data when short prefixs are provided. Training deep models with differential privacy has become a popular choice to prevent data leakage (Abadi et al., 2016; Papernot et al., 2017; McMahan et al., 2018b; Zhu et al., 2020). In addition to theoretical guarantee, differentially private models are also very robust to empirical privacy attacks (Bernau et al., 2019; Carlini et al., 2019; Yu et al., 2021b).

Adapting to the Geometry of Gradients in DP-SGD. Using adaptive clipping thresholds in DP-SGD fits more broadly into a line of work that adapts the geometry of gradients to clipping and noising. The gradients of machine learning models usually have much smaller intrinsic dimensions

than the model sizes. This property has been used to prove better theoretical bounds for DP-SGD or improve its empirical performance (Kairouz et al., 2020; Song et al., 2020; Zhou et al., 2021; Yu et al., 2021a; Li et al., 2022a; Ma et al., 2022).