Weighted sampling based reward distribution preserving coreset compression for experience replay buffer in lifelong deep reinforcement learning for medical imaging

Author name(s) withheld EMAIL(S) WITHHELD
Address withheld

Editors: Under Review for MIDL 2023

Abstract

Deep reinforcement learning with lifelong learning is a challenging area of active research. One method to achieve lifelong learning is to introduce an Experience Replay Buffer (ERB) in the reinforcement learning framework. ERBs encapsulate knowledge from previous rounds and can be used in future training. However, the ERB can vary in size, resulting in a potentially large buffer. To that end, we propose a coreset based ERB compression method to reduce the size of the ERBs while preserving previous knowledge obtained.

We test the coreset based ERB lifelong deep reinforcement learning on the segmentation of the brain tumor (BRATS) dataset for the task of ventricle localization. The conventional and coreset lifelong learning models (at compression ratios of 10x, 20x, 30x, and 40x) were iteratively trained on a sequence of 10 different imaging environments. The coreset lifelong learning models with compression ratios of 10x, 20x, 30x, and 40x localized the ventricle with a mean pixel distance of 12.93, 13.46, 17.75, and 18.55, respectively compared to the baseline performance of 10.87 for the conventional lifelong learning model. Our results demonstrate that the proposed coreset based ERB compression method can achieve up to 10x compression without a significant drop in performance.

Keywords: Deep reinforcement learning, lifelong learning, continual learning, medical imaging, coresets, clustering

1. Introduction

The field of radiology is moving towards implementation of artificial intelligence (AI) methods for radiologists to use in advanced reading rooms. Deep reinforcement learning (DRL) is an evolving area of research within the field of AI that deals with the development of AI systems that can learn from experience (mimicking the human learning process). DRL has produced excellent results across diverse domains (Mnih et al., 2013; Li et al., 2016; Silver et al., 2017; Sallab et al., 2017). The property of learning from experience makes DRL algorithms ideal for deployment into radiological decision support systems, where the DRL models can learn how to map the intra- and inter-structural relationships within different radiological images. The use of DRL in radiological applications is an emerging area of active research with new techniques being developed for anatomical landmark localization, image segmentation, registration, treatment planning, and assessment (Ghesu et al., 2017; Tseng et al., 2017; Maicas et al., 2017; Ma et al., 2017; Alansary et al., 2018; Ali et al., 2018; Alansary et al., 2019; Vlontzos et al., 2019; Allioui et al., 2022; Zhang et al., 2021; Joseph Nathaniel Stember, 2022).

© 2023 Anonymous.

In medical imaging applications, the same task might be present within different radiological imaging environments. For example, brain imaging will involve different imaging orientations (axial, sagittal, or coronal), different imaging sequences (ie, MRI: T1, T2, FLAIR, PWI), and modalities (PET, CT) or different pathologies (low-grade or high-grade gliomas), potentially resulting in large environments, depending on the application. Furthermore, the imaging environments for medical imaging tasks are constantly evolving, i.e., newer environments might be present at future time points due to change in image acquisition parameters or the introduction of newer imaging sequences. As a result, a model trained in an old environment may fail when evaluated in a new unseen environment. The model could then be fine-tuned using different learning methods to work in the new environment, However, these methods could potentially result in catastrophic forgetting, where the model is no longer capable of operating in the original environment. Such an example is shown in Figure 1.

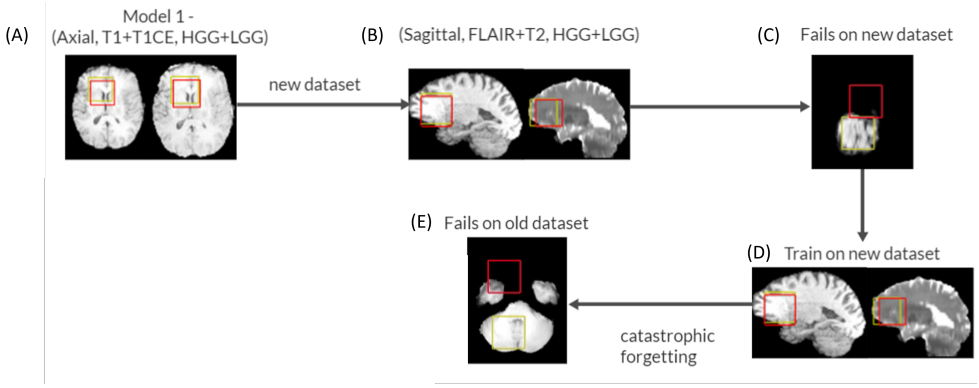

Figure 1: Illustration of potential catastrophic forgetting in dynamically evolving medical imaging environments. (A) Baseline deep reinforcement learning model trained for ventricle localization on an environment consisting of T1-weighted pre- and post-contrast enhanced images in the axial orientation. (B) The trained model encounters a new environment where, the new dataset consists of both T2-weighted and FLAIR MRI in the sagittal orientation. (C) The baseline model fails in the new environment due lack of training with similar data during training. (D) The baseline model is fine-tuned on the new dataset. (E) The fine-tuning results in catastrophic forgetting so that the fine-tuned model fails in the original environment.

The challenges of catastrophic forgetting could be addressed using lifelong deep reinforcement learning. Elastic weight consolidation (EWC) (Kirkpatrick et al., 2017) and selective experience replay (Rolnick et al., 2019) are two commonly used techniques for integrating lifelong learning with reinforcement learning. Elastic weight consolidation aims to preserve the network parameters learned in the previous tasks while learning a new task. On the other hand, selective experience replay aims to recount selected experiences from previous tasks to avoid catastrophic forgetting. Furthermore, selective experience replay based techniques are model agnostic and allow experiences to be shared across different

models. However, storing experiences from all previous tasks make lifelong learning using selective experience replay computationally very expensive and impractical as the number of tasks increase. To that end, we develop a reward distribution-preserving coreset compression technique based on weighted sampling to compress experience replay buffers stored for lifelong learning without sacrificing the performance, as shown in Figure 2. We evaluated the proposed coreset based ERB compression technique for the task of ventricle localization in brain MRI across a sequence twenty-four different imaging environments consisting of a combination of different MRI sequences, diagnostic pathologies, and imaging orientations.

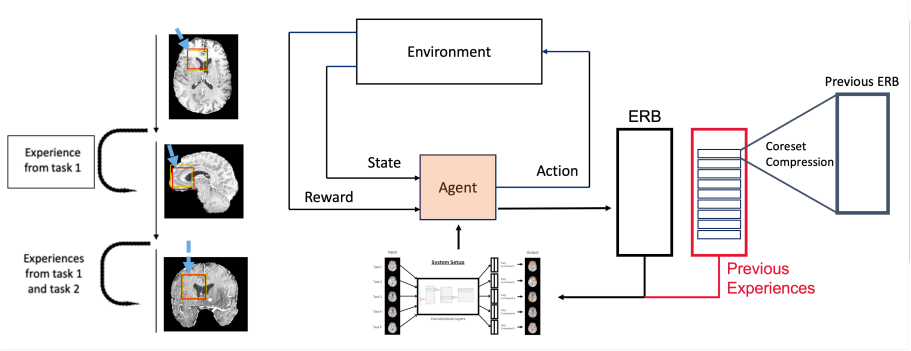

Figure 2: A schematic of the coreset-compressed lifelong deep reinforcement learning setup for training deep reinforcement learning models. ERB=Experience Replay Buffer

2. Related Work

The majority of the previous work on experience replay sampling is built on top of the work by Schaul et. al. (Schaul et al., 2015), in which the authors proposed a method to use temporal difference (TD) error to prioritize the more valuable experiences. Further work explored the idea of prioritized experience replay and developed new methods that improved the results (Pan et al., 2022; Ramicic et al., 2022; Hessel et al., 2018). However, they all need an extra calculation for absolute TD error, which is constantly updated during the network's training session, since the TD error requires the Q function, and the Q function updates after every training iteration.

Our proposed algorithm does not require additional information from the training session when the experience replay was produced, nor does it require the model parameters from the training session. It only requires the experience replay buffer for compression. This is an asynchronous process and can be used to learn from reinforcement learning models that just use an experience replay buffer, but do not have any additional implementation.

3. Methods

3.1. Lifelong deep reinforcement learning

We implemented a deep learning framework based on the deep Q-network (DQN) algorithm, as illustrated in Figure 2. The 3D DQN model implemented in this work was adapted from

(Mnih et al., 2013; Alansary et al., 2018; Vlontzos et al., 2019; Parekh et al., 2020). The environment was represented by the 3D imaging volume and the agent, by a 3D bounding box with six possible actions, $a \in \{$x++, x--, y++, y--, z++, z--$\}$. The state is defined by the current location (or a sequence of locations), where each location is represented by a 3D bounding box. The reward is defined by the difference in the distance between the agent's location and the target landmark before and after the agent's action. The state-action-reward-resulting state $[s, a, r, s']$ tuples resulting from the DRL agent's interaction with the environment over many episodes were used to populate the experience replay buffer (ERB).

To perform lifelong learning, we implemented a selective experience replay buffer to collect a trajectory of experience samples across the model's training history. The model attempts to learn a generalized representation of its current and previous tasks by sampling a batch of experience from both its current task's experience replay buffer (ERB) as well as from its history of previous tasks' experience replays during training.

3.2. Weighted sampling-based reward distribution preserving coreset ERB compression

We proposed and introduce a new method to compress the ERB based on coresets for lower storage cost, less communication cost, and preservation of information: given an ERB of size N and compression rate of R, we use k-means++ clustering to partition the points in the ERB to N/R clusters based on their rewards. We use k-means++ clustering because we want to manually pick the number of clusters to be N/R as it reflects the compression ratio. Moreover, since the reward distribution for ERBs is known, does not have outliers, and is only 1-dimensional, k-means++ clustering can guarantee convergence on a good label assignment very fast and accurately. We pick the closest point to the center of these clusters to be in the compressed ERB. Additionally, we give these points in the compressed ERB a weight parameter that is equal to the number of points in the clusters they are in. When the compressed ERB is received by an agent, the agent will unpack it by repeating the points in the compressed ERB multiple times according to their weight parameter. For example, if a point $[s, a, r, s']$ has a weight of 2 in the compressed ERB, then it will be repeated twice in the unpacked ERB. This method guarantees an approximate reward distribution similar to the original reward distribution, as illustrated in Figure 3.

4. Experiment and Data

4.1. Clinical Data

We used the brain tumor segmentation (BRATS) dataset for evaluating the coreset compression method (Menze et al., 2014). The dataset consisted of 285 patients with T1-weighted pre- and post-contrast enhanced, T2-weighted, and Fluid Attenuated Inversion Recovery (FLAIR) sequences. All the images were acquired in the axial orientation. In this work, we selected a random subset of 100 patients from the BRATS dataset for the development, training, and evaluation of different DRL models. This subset consisted of 60 patients with high-grade glioma (HGG) and 40 patients with low-grade glioma (LGG). Of the 100 patients, 80 were used for training and 20 were used for evaluation. The training dataset consisted of 48 patients with HGG and 32 patients with LGG tumors. The test dataset

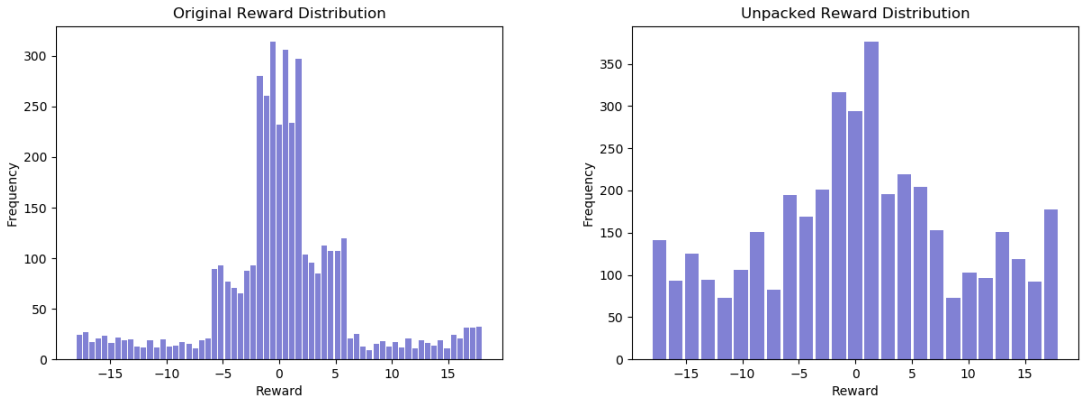

Figure 3: Reward distribution of original ERB (left) vs. compressed and unpacked ERB with a compression ratio of 10x (right)

consisted of 12 patients with HGG and 8 patients with LGG tumors. We synthetically resliced the axial images into coronal and sagittal views and reconstructed each dataset in all three imaging orientations, resulting in a total of twenty-four imaging environments ($2\text{pathologies} \times 4\text{sequences} \times 3\text{orientations} = 24$). We used the top left ventricle as the task for this experiment, resulting in a total of 24 task-environment pairs, as shown in Figure 4.

4.2. Training Protocol

We trained 5 lifelong deep reinforcement learning models (no-compression, 10x compression, 20x compression, 30x compression, and 40x compression) to test the performance of our coreset compression algorithm. The models are trained for the localization of the top left ventricle in a randomly sampled 10 task-environment pairs out of the 24 task-environment pairs. The models were trained for four epochs with a batch size of 48. The agent's state was represented as a bounding box of size 45x45x11 with a frame history length of four. The models were iteratively trained for the localization of the top left ventricle in one imaging environment at a time, resulting in 10 rounds of training. The no-compression model used complete ERBs generated by previous training rounds for the next training round. The 10x, 20x, 30x, and 40x compression models used 10x, 20x, 30x, and 40x compressed ERBs, respectively, and unpacked based on weights in the ERBs from all previous rounds for the next training round.

4.3. Performance Evaluation

The performance metric was set as the terminal Euclidean distance between the agent's prediction and the target landmark. We performed Wilkoxon ranksum tests to compare the performance of the no-compression model with the compression models at different compression rates. The p-value for statistical significance was set to $p \leq 0.05$.

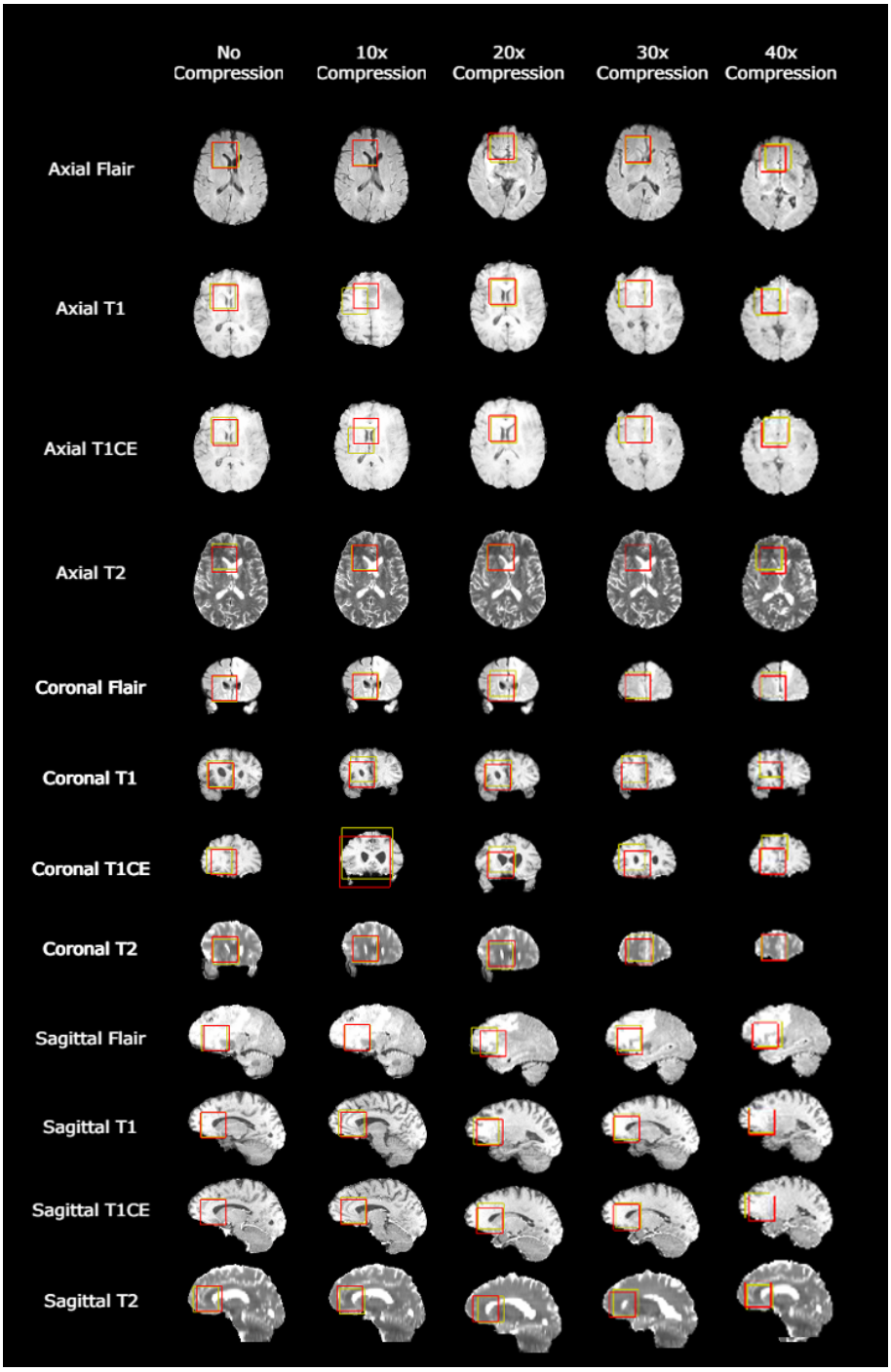

Figure 4: Illustration of various task-environment pairs in our dataset and the performance of no compression and the proposed coreset-compression method at different compression rates. The red and yellow bounding boxes represent the ground truth and the agent's prediction, respectively.

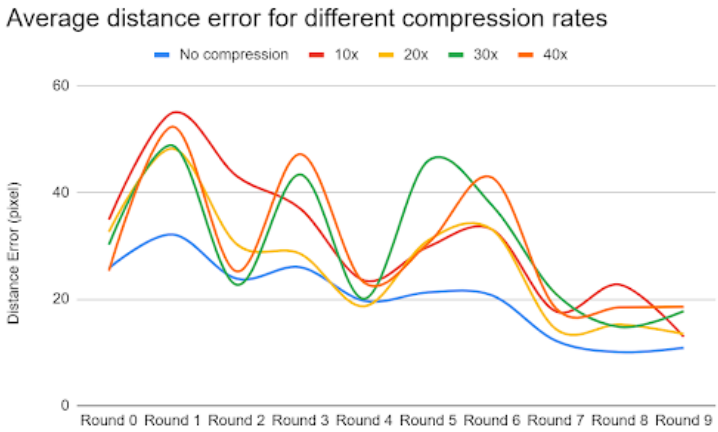

Figure 5: Illustration of the average Euclidean distance error on testing images (24 task-environment pairs) after every training round.

5. Results

We sequentially trained each selective experience replay based lifelong reinforcement model (no compression, 10x compression, 20x compression, 30x compression, and 40x compression) on 10 distinct task-environment pairs, one pair each round. After every round of training, the models were evaluated across all 24 task-environment pairs in the test set. Figure 4 illustrates the performance of all five models across a subset of twelve task-environment pairs after 10 rounds of training. As shown in Figure 4, both the compressed and uncompressed models demonstrated excellent continual learning performance across all task-environment pairs. The average Euclidean distance errors for no compression, 10x compression, 20x compression, 30x compression, and 40x compression were 10.87, 12.93 ($p = 0.16$), 13.46 ($p = 0.0003$), 17.75 ($p \leq 0.0001$), and 18.55 ($p \leq 0.0001$) respectively. There was no significant difference in the performance between no compression and 10x compression, with the performance slightly degrading with higher compression ratios. Figure 5 compares the performance at different compression levels across different rounds. The performance after 10 rounds of training across different compression levels has been illustrated as a box plot in Figure 6. The original sizes of the ERBs tested are 90MB, 10x compression results in 9MB, 20x compression results in 4.5MB, 30x compression results in 3MB, and 40x compression results in 2MB.

6. Conclusion

In conclusion, we propose a coreset-based ERB compression technique for increased computational efficiency and scalability of selective experience replay based deep lifelong reinforcement learning with excellent performance. In particular, our weighted sampling-based reward distribution-preserving coreset ERB compression showed excellent performance for shrinking the size of saved experiences from previous tasks for lifelong deep reinforcement

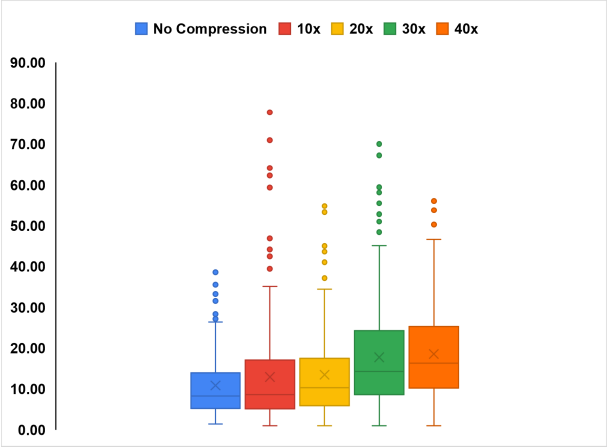

Figure 6: Illustration of the Euclidean distance error on testing images (24 task-environment pairs) after ten rounds of training.

learning in medical imaging. Our results demonstrated that experience replay buffers can be compressed up to 10x without any significant drop in performance.

Experience replay has been one of the major methods for lifelong reinforcement learning because it is model agnostic and because different experiences can be shared between models for collaborative lifelong learning. However, the computational complexity of storing multiple experiences from previous tasks makes experience replay less attractive as the number of tasks increases. Many techniques have been developed in the literature for sampling experience replay (Schaul et al., 2015; Pan et al., 2022; Ramicic et al., 2022; Hessel et al., 2018). However, they required additional calculations that constantly update during the network's training and require the Q function. In contrast, our proposed coreset-based compression techniques asynchronously compress the ERB, without the need for extra information from the training session where the ERB is produced or the model parameters that came from the training session. This provides more flexibility to the training and is less computationally intensive compared to other methods.

There are certain limitations to our study. This work demonstrates our technique in a single-agent deep reinforcement learning model and in only one application. In the future, we plan to evaluate the coreset-based ERB compression technique in a multi-agent setup where the size of the ERB linearly increases with the number of agents in the model across a diverse set of applications.

Acknowledgments

Acknowledgments withheld.

References

Amir Alansary, Loic Le Folgoc, Ghislain Vaillant, Ozan Oktay, Yuanwei Li, Wenjia Bai, Jonathan Passerat-Palmbach, Ricardo Guerrero, Konstantinos Kamnitsas, Benjamin

Hou, et al. Automatic view planning with multi-scale deep reinforcement learning agents. In *International Conference on Medical Image Computing and Computer-Assisted Intervention*, pages 277–285. Springer, 2018.

Amir Alansary, Ozan Oktay, Yuanwei Li, Loic Le Folgoc, Benjamin Hou, Ghislain Vaillant, Konstantinos Kamnitsas, Athanasios Vlontzos, Ben Glocker, Bernhard Kainz, and Daniel Rueckert. Evaluating Reinforcement Learning Agents for Anatomical Landmark Detection. *Medical Image Analysis*, 2019.

Issa Ali, Gregory R Hart, Gowthaman Gunabushanam, Ying Liang, Wazir Muhammad, Bradley Nartowt, Michael Kane, Xiaomei Ma, and Jun Deng. Lung nodule detection via deep reinforcement learning. *Frontiers in oncology*, 8:108, 2018.

Hanane Allioui, Mazin Abed Mohammed, Narjes Benameur, Belal Al-Khateeb, Karrar Hameed Abdulkareem, Begonya Garcia-Zapirain, Robertas Damaševi˘cius, and Rytis Maskeliunas. A multi-agent deep reinforcement learning approach for enhancement of covid-19 ct image segmentation. *Journal of Personalized Medicine*, 12:1–23, 2022.

Olivier Bachem, Mario Lucic, and Andreas Krause. Practical coreset constructions for machine learning, 2017.

Dan Feldman. Core-sets: An updated survey. *WIREs Data Mining Knowl. Discov.*, 10(1), 2020.

Dan Feldman, Melanie Schmidt, and Christian Sohler. Turning big data into tiny data: Constant-size coresets for k-means, pca, and projective clustering. *SIAM J. Comput.*, 49 (3):601–657, 2020.

Florin-Cristian Ghesu, Bogdan Georgescu, Yefeng Zheng, Sasa Grbic, Andreas Maier, Joachim Hornegger, and Dorin Comaniciu. Multi-scale deep reinforcement learning for real-time 3d-landmark detection in ct scans. *IEEE transactions on pattern analysis and machine intelligence*, 41(1):176–189, 2017.

Matteo Hessel, Joseph Modayil, Hado van Hasselt, Tom Schaul, Georg Ostrovski, Will Dabney, Dan Horgan, Bilal Piot, Mohammad Azar, and David Silver. Rainbow: Combining improvements in deep reinforcement learning. In *Proceedings of the Thirty-Second AAAI Conference on Artificial Intelligence and Thirtieth Innovative Applications of Artificial Intelligence Conference and Eighth AAAI Symposium on Educational Advances in Artificial Intelligence*, AAAI'18/IAAI'18/EAAI'18. AAAI Press, 2018. ISBN 978-1-57735-800-8.

Hrithwik Shalu Joseph Nathaniel Stember. Deep reinforcement learning with automated label extraction from clinical reports accurately classifies 3d mri brain volumes. *Journal of Digital Imaging*, pages 1–11, 2022.

James Kirkpatrick, Razvan Pascanu, Neil Rabinowitz, Joel Veness, Guillaume Desjardins, Andrei A Rusu, Kieran Milan, John Quan, Tiago Ramalho, Agnieszka Grabska-Barwinska, et al. Overcoming catastrophic forgetting in neural networks. *Proceedings of the national academy of sciences*, 114(13):3521–3526, 2017.

Jiwei Li, Will Monroe, Alan Ritter, Michel Galley, Jianfeng Gao, and Dan Jurafsky. Deep reinforcement learning for dialogue generation. *arXiv preprint arXiv:1606.01541*, 2016.

Kai Ma, Jiangping Wang, Vivek Singh, Birgi Tamersoy, Yao-Jen Chang, Andreas Wimmer, and Terrence Chen. Multimodal image registration with deep context reinforcement learning. In *International Conference on Medical Image Computing and Computer-Assisted Intervention*, pages 240–248. Springer, 2017.

Gabriel Maicas, Gustavo Carneiro, Andrew P Bradley, Jacinto C Nascimento, and Ian Reid. Deep reinforcement learning for active breast lesion detection from dce-mri. In *International conference on medical image computing and computer-assisted intervention*, pages 665–673. Springer, 2017.

Bjoern H Menze, Andras Jakab, Stefan Bauer, Jayashree Kalpathy-Cramer, Keyvan Farahani, Justin Kirby, Yuliya Burren, Nicole Porz, Johannes Slotboom, Roland Wiest, et al. The multimodal brain tumor image segmentation benchmark (brats). *IEEE transactions on medical imaging*, 34(10):1993–2024, 2014.

Volodymyr Mnih, Koray Kavukcuoglu, David Silver, Alex Graves, Ioannis Antonoglou, Daan Wierstra, and Martin Riedmiller. Playing atari with deep reinforcement learning. *arXiv preprint arXiv:1312.5602*, 2013.

Yangchen Pan, Jincheng Mei, Amir massoud Farahmand, Martha White, Hengshuai Yao, Mohsen Rohani, and Jun Luo. Understanding and mitigating the limitations of prioritized experience replay. In *The 38th Conference on Uncertainty in Artificial Intelligence*, 2022. URL https://openreview.net/forum?id=HBlNGvIicg9.

Vishwa S Parekh, Vladimir Braverman, Michael A Jacobs, et al. Multitask radiological modality invariant landmark localization using deep reinforcement learning. In *Medical Imaging with Deep Learning*, pages 588–600. PMLR, 2020.

Mirza Ramicic, Vaclav Smidl, and Andrea Bonarini. Informed sampling of prioritized experience replay. In *2022 IEEE International Conference on Development and Learning (ICDL)*, pages 215–222, 2022. doi: 10.1109/ICDL53763.2022.9962235.

David Rolnick, Arun Ahuja, Jonathan Schwarz, Timothy Lillicrap, and Gregory Wayne. Experience replay for continual learning. *Advances in Neural Information Processing Systems*, 32, 2019.

Ahmad EL Sallab, Mohammed Abdou, Etienne Perot, and Senthil Yogamani. Deep reinforcement learning framework for autonomous driving. *Electronic Imaging*, 2017(19): 70–76, 2017.

Tom Schaul, John Quan, Ioannis Antonoglou, and David Silver. Prioritized experience replay. *CoRR*, abs/1511.05952, 2015.

David Silver, Julian Schrittwieser, Karen Simonyan, Ioannis Antonoglou, Aja Huang, Arthur Guez, Thomas Hubert, Lucas Baker, Matthew Lai, Adrian Bolton, et al. Mastering the game of go without human knowledge. *Nature*, 550(7676):354–359, 2017.

Huan-Hsin Tseng, Yi Luo, Sunan Cui, Jen-Tzung Chien, Randall K Ten Haken, and Issam El Naqa. Deep reinforcement learning for automated radiation adaptation in lung cancer. *Medical physics*, 44(12):6690–6705, 2017.

Athanasios Vlontzos, Amir Alansary, Konstantinos Kamnitsas, Daniel Rueckert, and Bernhard Kainz. Multiple landmark detection using multi-agent reinforcement learning. In *International Conference on Medical Image Computing and Computer-Assisted Intervention*, pages 262–270. Springer, 2019.

Quan Zhang, Qian Du, and Guohua Liu. A whole-process interpretable and multi-modal deep reinforcement learning for diagnosis and analysis of alzheimer's disease. *Journal of Neural Engineering*, 18:1–19, 2021.

Appendix A. Background on Coresets

Coresets are an important concept in data science because they present a technique for representing datasets with a large number of observations as weighted datasets with a smaller number of observations. Coresets are frequently used as a pre-processing technique for dimensionality reduction to significantly improve the runtime or memory cost of downstreams tasks, such as clustering, regression, or low-rank approximation. See the surveys (Feldman, 2020; Bachem et al., 2017) for a more comprehensive introduction and discussion on coresets, their applications, and the state-of-the-art techniques.

More formally, we are given a dataset $X = x_1, \ldots, x_n$ such that $x_i \in \mathbb{R}^d$ for all $i \in [n]$. In other words, the dataset X consists of n observations, each with d features. The goal is to minimize the function $f(X, q, u)$ across all possible values q in some query space Q. Here we use u to denote the function that assigns unit weight to all of the n points in X. For example, if the task was to do k-means clustering, then Q would be the set consisting of all sets of k points from $\mathbb{R}^d$, i.e., the set of all possible sets of centers for the clustering problem.

A coreset construction produces a weighted subset Y consisting of m observations from the original dataset X. Let w be the function that assigns weights to points in Y. Given an approximation parameter $\epsilon > 0$, the coreset with multiplicative error $(1 + \epsilon)$ satisfies

$$(1 - \epsilon) f(Y, q, w) \leq f(X, q, u) \leq (1 + \epsilon) f(Y, q, w),$$

for all $q \in Q$. Similarly, a coreset with additive error $E > 0$ satisfies

$$|f(Y, q, w) - f(X, q, u)| \leq E.$$

Since the coreset is defined with respect to f, then it naturally follows that there may be drastically different coreset constructions depending on the task at hand, i.e., the function f to be minimized.

The definition can then be naturally generalized to the setting where X is a set of weighted points. Formally, we have the following:

Definition 1 (Coreset) *Given a set X with weight function u and an accuracy parameter $\epsilon > 0$, we say a set Y with weight function w is an $(1+\epsilon)$-multiplicative coreset for a function f, if for all queries q in a query space Q, we have*

$$(1-\epsilon)f(Y,q,w) \leq f(X,q,u) \leq (1+\epsilon)f(Y,q,w).$$

A standard approach in coreset constructions is to first assign a *sensitivity* $s(x_i)$ to each point x_i with $i \in [n]$. The sensitivity is intuitively a value that quantifies the "importance" of the point x_i. The coreset construction then samples a fixed number m of points from X, with probabilities proportional to their sensitivity. That is, for each of the m sampled points p, we have that $\Pr[p = x_i] \propto s_i$. In fact, the following statement shows this procedure can generate a coreset even if we only have approximations to the sensitivities of each point for the task of k-means clustering:

Theorem 2 (Theorem 35 in (Feldman et al., 2020)) *Let $C > 1$ be a universal constant and for each $i \in [n]$, let $q(x_i)$ be a C-approximation to the sensitivity $s(x_i)$ for any point x_i. Let $T = \sum_{i=1}^{n} q(x_i)$. Then sensitivity sampling $m = O\left(\frac{Tk}{\epsilon^2}\log^2 k\right)$ points with replacement, i.e., choosing each of the m points to be x_i with probability proportional to $q(x_i)$ and then rescaling by the sampling probability, outputs a $(1+\epsilon)$-coreset for k-means clustering with probability $\frac{2}{3}$.*

However, it turns out that even approximating the sensitivities of each point may be time-consuming. Thus for real-world datasets, it is often more practical to follow procedures that capture the main intuition behind sensitivity sampling without actually performing sensitivity sampling.

Uniform sampling. One reason to assign different sensitivities to each observation is the following. Suppose there exist $n-1$ similar observations and a single drastically different observation. It may be possible the drastically different observation is an outlier that should be ignored, but it may also be possible that the different observation is a crucial but rare counterexample that behaves differently from the rest of the population, in which case the counterexample needs to be sampled into the coreset in order to maintain representation. Thus in this case, the sensitivity of the different observation should be much larger than the sensitivities of the remaining points.

However, if the dataset is roughly uniform, then no particular example stands out, and so intuitively, the sensitivities of the points are also roughly uniform. In this case, the intuition behind sensitivity sampling is also achieved through uniform sampling, i.e., selecting a number of observations uniformly at random.

Group sampling. A frequent phenomenon is that the observations in the dataset can be clustered or partitioned into a number of groups, so that the behavior of the observations is similar within a group but drastically different across groups. In this case, the sensitivities within each group are similar, but the sensitivities across different groups can also vary greatly. Hence, sensitivity sampling would sample roughly an equal number of observations from each group.

If a natural partition of the observations into the groups is known a priori, then sensitivity sampling can be reasonably simulated by first partitioning the observations into

these groups. We can then sample a fixed number of points from each group, uniformly at random. Specifically, if the coreset construction for sensitivity sampling intended to store m points and k groups were formed from the natural partition of the observations, then we can sample $\frac{m}{k}$ points from each of the k groups. That is, for each of the k groups, $\frac{m}{k}$ points are selected uniformly at random from this group and then weighted proportional to the number of points in the group.