Distributionally Adaptive Meta Reinforcement Learning

Abstract

Meta-reinforcement learning algorithms provide a data-driven way to acquire poli-
cies that quickly adapt to many tasks with varying rewards or dynamics functions.
However, learned meta-policies are often effective only on the exact task distribu-
tion on which they were trained and struggle in the presence of distribution shift
of test-time rewards or transition dynamics. In this work, we develop a frame-
work for meta-RL algorithms that are able to behave appropriately under test-time
distribution shifts in the space of tasks. Our framework centers on an adaptive
approach to distributional robustness that trains a population of meta-policies to
be robust to varying levels of distribution shift. When evaluated on a potentially
shifted test-time distribution of tasks, this allows us to choose the meta-policy with
the most appropriate level of robustness, and use it to perform fast adaptation. We
formally show how our framework allows for improved regret under distribution
shift, and empirically show its efficacy on simulated robotics problems under a
wide range of distribution shifts.

1 Introduction

The diversity and dynamism of the real world require reinforcement learning (RL) agents that
can quickly adapt and learn new behaviors when placed in novel situations. Meta reinforcement
learning provides a framework for conferring this ability to RL agents, by learning a "meta-policy"
trained to adapt as quickly as possible to tasks from a provided training distribution [39, 10, 33, 48].
Unfortunately, meta-RL agents are prone to overfitting to the distribution of tasks they are trained
on, and have been shown to behave erratically when asked to adapt to tasks beyond the training
distribution [5, 8]. As an example of this negative transfer, consider using meta-learning to teach
a robot to navigate to goals quickly (illustrated in Figure 1). The resulting meta-policy learns to
quickly adapt and walk to any target location specified in the training distribution, but explores poorly
and fails to adapt to any location not in that distribution. Overfitting is particularly problematic
for the meta-learning setting, since the scenarios where we need the ability to learn quickly are
usually exactly those where the agent experiences distribution shift. This type of meta-distribution
shift afflicts a number of real-world problems including autonomous vehicle driving [9], in-hand
manipulation [17, 1], and quadruped locomotion [24, 22, 18], where the test-time task distribution
may not be well represented during training.

In this work, we study meta-RL algorithms that learn meta-policies resilient to task distribution shift
at test time. One approach to enable this resiliency is to leverage the framework of distributional
robustness [37], training meta-policies that prepare for distribution shifts by optimizing the *worst-case*
empirical risk against a set of task distributions which lie within a bounded distance from the original
training task distribution (often referred to as an *uncertainty set*)). This allows meta-policies to
deal with potential test-time task distribution shift, bounding their worst-case test-time regret for

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distributional shifts within the chosen uncertainty set. However, choosing an appropriate uncertainty set can be quite challenging without further information about the test environment, significantly impacting the test-time performance of algorithms under distribution shift. Large uncertainty sets allow resiliency to a wider range of distribution shifts, but the resulting meta-policy adapts very slowly at test time; smaller uncertainty sets enable faster test-time adaptation, but leave the meta-policy brittle to task distribution shifts. Can we get the best of both worlds?

Our key insight is that we can prepare for a variety of potential test-time distribution shifts by constructing and training against different uncertainty sets at training time. By preparing for adaptation against each of these uncertainty sets, an agent is able to adapt to a variety of potential test-time distribution shifts by adaptively choosing the most appropriate level of distributional robustness for the test distribution at hand. We introduce a conceptual framework called distributionally adaptive meta reinforcement learning, formalizing this idea. At train time, the agent learns robust meta-policies with widening uncertainty sets, preemptively accounting for different levels of test-time distribution shift that may be encountered. At test time, the agent infers the level of distribution shift it is faced with, and then uses the corresponding meta-policy to adapt to the new task (Figure 2). In doing so, the agent can adaptively choose the best level of robustness for the test-time task distribution, preserving the fast adaptation benefits of

Figure 1: **Failure of Typical Meta-RL.** On meta-training tasks, π_{meta} explores effectively and quickly learns the optimal behavior (top row). When test tasks come from a slightly larger task distribution, exploration fails catastrophically, resulting in poor adaptation behavior (bottom row).

meta RL, while also ensuring good asymptotic performance under distribution shift. We instantiate a practical algorithm in this framework (DiAMetR), using learned generative models to imagine new task distributions close to the provided training tasks that can be used to train robust meta-policies.

The contribution of this paper is to propose a framework for making meta-reinforcement learning resilient to a variety of task distribution shifts, and DiAMetR, a practical algorithm instantiating the framework. DiAMetR trains a population of meta-policies to be robust to different degrees of distribution shifts and then adaptively chooses a meta-policy to deploy based on the inferred test-time distribution shift. Our experiments verify the utility of adaptive distributional robustness under test-time task distribution shift in a number of simulated robotics domains.

2   Related Work

Meta-reinforcement learning algorithms aim to leverage a distribution of training tasks to "learn a reinforcement learning algorithm", that is able to learn as quickly on new tasks drawn from the same distribution. A variety of algorithms have been proposed for meta-RL, including memory-based [7, 25], gradient-based [10, 35, 13] and latent-variable based [33, 48, 47, 11] schemes. These algorithms show the ability to generalize to new tasks drawn from the same distribution, and have been applied to problems ranging from robotics [27, 47, 18] to computer science education [43]. This line of work has been extended to operate in scenarios without requiring any pre-specified task distribution [12, 16], in offline settings [6, 28, 26] or in hard (meta-)exploration settings [49, 46], making them more broadly applicable to a wider class of problems. However, most meta-RL algorithms assume source and target tasks are drawn from the same distribution, an assumption rarely met in practice. Our work shows how the machinery of meta-RL can be made compatible with distribution shift at test time, using ideas from distributional robustness. Some recent work shows that model based meta-reinforcement learning can be made to be robust to a particular level distribution shift [23, 20] by learning a shared dynamics model against adversarially chosen task distributions. We show that we can build model-free meta-reinforcement learning algorithms, which are not just robust to a particular level of distribution shift, but can adapt to various levels of shift.

Distributional robustness methods have been studied in the context of building supervised learning systems that are robust to the test distribution being different than the training one. The key idea is to train a model to not just minimize empirical risk, but instead learn a model that has the lowest worst-case empirical risk among an "uncertainty-set" of distributions that are boundedly close to the empirical training distribution [37, 21, 2, 15]. If the uncertainty set and optimization are chosen carefully, these methods have been shown to obtain models that are robust to small amounts

of distribution shift at test time [37, 21, 2, 15], finding applications in problems like federated learning [15] and image classification [21]. This has been extended to the min-max robustness setting for specific algorithms like model-agnostic meta-learning [3], but are critically dependent on correct specification of the appropriate uncertainty set and applicable primarily in supervised learning settings. Alternatively, several RL techniques aim to directly tackle the robustness problem, aiming to learn policies robust to adversarial perturbations [41, 45, 32, 31]. [44] conditions the policy on uncertainty sets to make it robust to different perturbation sets. While these methods are able to learn conservative, robust policies, they are unable to adapt to new tasks as DiAMetR does in the meta-reinforcement learning setting. In our work, rather than choosing a single uncertainty set, we learn many meta-policies for widening uncertainty sets, thereby accounting for different levels of test-time distribution shift.

3 Preliminaries

Meta-Reinforcement Learning aims to learn a fast reinforcement learning algorithm or a "meta-policy" that can quickly maximize performance on tasks $\mathcal{T}$ from some distribution $p(\mathcal{T})$. Formally, each task $\mathcal{T}$ is a Markov decision process (MDP) $\mathcal{M} = (\mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma, \mu_0)$; the goal is to exploit regularities in the structure of rewards and environment dynamics across tasks in $p(\mathcal{T})$ to acquire effective exploration and adaptation mechanisms that enable learning on new tasks much faster than learning the task naively from scratch. A meta-policy (or fast learning algorithm) π_{meta} maps a history of environment experience $h \in (\mathcal{S} \times \mathcal{A} \times \mathcal{R})^*$ in a new task to an action a, and is trained to acquire optimal behaviors on tasks from $p(\mathcal{T})$ within k episodes:

$$\min_{\pi_{\text{meta}}} \mathbb{E}_{\mathcal{T} \sim p(\mathcal{T})} \left[\text{Regret}(\pi_{\text{meta}}, \mathcal{T}) \right],$$

$$\text{Regret}(\pi_{\text{meta}}, \mathcal{T}) = J(\pi_{\mathcal{T}}^*) - \mathbb{E}_{a_t^{(i)} \sim \pi_{\text{meta}}(\cdot | h_t^{(i)}), \mathcal{T}} \left[\frac{1}{k} \sum_{i=1}^{k} \sum_{t=1}^{T} r_t^{(i)} \right], \quad J(\pi_{\mathcal{T}}^*) = \max_{\pi} \mathbb{E}_{\pi, \mathcal{T}} [\sum_t r_t]$$

$$\text{where } h_t^{(i)} = (s_{1:t}^{(i)}, r_{1:t}^{(i)}, a_{1:t-1}^{(i)}) \cup (s_{1:T}^{(j)}, r_{1:T}^{(j)}, a_{1:T}^{(j)})_{j=1}^{i-1}. \tag{1}$$

Intuitively, the meta-policy has two components: an exploration mechanism that ensures that appropriate reward signal is found for all tasks in the training distribution, and an adaptation mechanism that uses the collected exploratory data to generate optimal actions for the current task. In practice, the meta-policy may be represented explicitly as an exploration policy conjoined with a policy update[10, 33], or implicitly as a black-box RNN [7, 48]. We use the terminology "meta-policies" interchangeably with that of "fast-adaptation" algorithms, since our practical implementation builds on [30] (which represents the adaptation mechanism using a black-box RNN). Our work focuses on the setting where there is potential drift between $p_{\text{train}}(\mathcal{T})$, the task distribution we have access to during training, and $p_{\text{test}}(\mathcal{T})$, the task distribution of interest during evaluation.

Distributional robustness [37] learns models that do not minimize empirical risk against the training distribution, but instead prepare for distribution shift by optimizing the *worst-case* empirical risk against a set of data distributions close to the training distribution (called an *uncertainty set*):

$$\min_{\theta} \max_{\phi} \mathbb{E}_{x \sim q_\phi(x)} [l(x; \theta)] \quad \text{s.t.} \quad D(p_{\text{train}}(x) || q_\phi(x)) \leq \epsilon \tag{2}$$

This optimization finds the model parameters θ that minimizes worst case risk l over distributions $q_\phi(x)$ in an ϵ-ball (measured by an f-divergence) from the training distribution $p_{\text{train}}(x)$.

4 Distributionally Adaptive Meta-Reinforcement Learning

In this section, we develop a framework for learning meta-policies, that given access to a training distribution of tasks $p_{\text{train}}(\mathcal{T})$, is still able to adapt to tasks from a test-time distribution $p_{\text{test}}(\mathcal{T})$ that is similar but not identical to the training distribution. We introduce a framework for distributionally adaptive meta-RL below and instantiate it as a practical method in Section 5.

4.1 Known Level of Test-Time Distribution Shift

We begin by studying a simplified problem where we can exactly quantify the degree to which the test distribution deviates from the training distribution. Suppose we know that p_{test} satisfies

Figure 2: During meta-train, DiAMetR learns a meta-policy $\pi_{\text{meta}}^{\epsilon_1}$ and task distribution model $\mathcal{T}_\omega(s, a, z)$ on train task distribution. Then, it uses the task distribution model to imagine different shifted test task distributions on which it learns different meta-policies $\{\pi_{\text{meta}}^{\epsilon_i}\}_{i=2}^{M}$, each corresponding to a different level of robustness. During meta-test, it chooses an appropriate meta-policy based on inferred test task distribution shift with Thompson's sampling and then quickly adapts the selected meta-policy to individual tasks.

$D(p_{\text{test}}(\mathcal{T})\|p_{\text{train}}(\mathcal{T})) < \epsilon$ for some $\epsilon > 0$, where $D(\cdot\|\cdot)$ is a probability divergence on the set of task distributions (e.g. an f-divergence [34] or a Wasserstein distance [40]). A natural learning objective to learn a meta-policy under this assumption is to minimize the *worst-case* test-time regret across any test task distribution $q(\mathcal{T})$ that is within some ϵ divergence of the train distribution:

$$\min_{\pi_{\text{meta}}} \mathcal{R}(\pi_{\text{meta}}, p_{\text{train}}(\mathcal{T}), \epsilon),$$
$$\mathcal{R}(\pi_{\text{meta}}, p_{\text{train}}(\mathcal{T}), \epsilon) = \max_{q(\mathcal{T})} \mathbb{E}_{\mathcal{T} \sim q(\mathcal{T})}\left[\text{Regret}(\pi_{\text{meta}}, \mathcal{T})\right] \quad \text{s.t. } D(p_{\text{train}}(\mathcal{T})\|q(\mathcal{T})) \leq \epsilon \quad (3)$$

Solving this optimization problem results in a meta-policy that has been trained to adapt to tasks from a *wider* task distribution than the original training distribution. It is worthwhile distinguishing this robust meta-objective, which incentivizes a *robust adaptation mechanism* to a wider set of tasks, from robust objectives in standard RL, which produce base policies robust to a wider set of dynamics conditions. The objective in Eq 3 incentivizes an agent to explore and adapt more broadly, not act more conservatively as standard robust RL methods [32] would encourage. Naturally, the quality of the robust meta-policy depends on the size of the uncertainty set. If ϵ is large, or the geometry of the divergence poorly reflect natural task variations, then the robust policy will have to adapt to an overly large set of tasks, potentially degrading the speed of adaptation.

4.2 Handling Arbitrary Levels of Distribution Shift

In practice, it is not known how the test distribution p_{test} deviates from the training distribution, and consequently it is challenging to determine what ϵ to use in the meta-robustness objective. We propose to overcome this via an adaptive strategy: to train meta-policies for *varying* degrees of distribution shift, and at test-time, inferring which distribution shift is most appropriate through experience.

We train a population of meta-policies $\{\pi_{\text{meta}}^{(i)}\}_{i=1}^{M}$, each solving the distributionally robust meta-RL objective (eq 3) for a different level of robustness ϵ_i:

$$\left\{\pi_{\text{meta}}^{\epsilon_i} := \arg\min_{\pi_{\text{meta}}} \mathcal{R}(\pi_{\text{meta}}, p_{\text{train}}(\mathcal{T}), \epsilon_i)\right\}_{i=1}^{M} \quad \text{where } \epsilon_M > \epsilon_{M-1} > \ldots > \epsilon_1 = 0 \quad (4)$$

In choosing a spectrum of ϵ_i, we learn a set of meta-policies that have been trained on increasingly large set of tasks: at one end ($i = 1$), the meta-policy is trained only on the original training distribution, and at the other ($i = M$), the meta-policy trained to adapt to any possible task within the parametric family of tasks. These policies span a tradeoff between being robust to a wider set of task distributions with larger ϵ (allowing for larger distribution shifts), and being able to adapt quickly to any given task with smaller ϵ (allowing for better per-task regret minimization).

With a set of meta-policies in hand, we must now decide how to leverage test-time experience to discover the right one to use for the actual test distribution p_{test}. We recognize that the problem of policy selection can be treated as a stochastic multi-armed bandit problem (precise formulation in Appendix C), where pulling arm i corresponds to running the meta-policy $\pi_{\text{meta}}^{\epsilon_i}$ for an entire meta-episode (k task episodes). If a zero-regret bandit algorithm (eg: Thompson's sampling [42]) is used , then after a certain number of test-time meta episodes, we can guarantee that the meta-policy selection mechanism will converge to the meta-policy that best balances the tradeoff between adapting quickly while still being able to adapt to all the tasks from $p_{\text{test}}(\mathcal{T})$.

To summarize our framework for distributionally adaptive meta-RL, we train a population of meta-policies at varying levels of robustness on a distributionally robust objective that forces the learned adaptation mechanism to also be robust to tasks not in the training task distribution. At test-time, we use a bandit algorithm to select the meta-policy whose adaptation mechanism has the best tradeoff between robustness and speed of adaptation specifically on the test task distribution. Combining distributional robustness with test-time adaptation allows the adaptation mechanism to work even if distribution shift is present, while obviating the decreased performance that usually accompanies overly conservative, distributionally robust solutions.

4.3   Analysis

To provide some intuition on the properties of this algorithm, we formally analyze adaptive distributional robustness in a simplified meta RL problem involving tasks $\mathcal{T}_g$ corresponding to reaching some unknown goal g in a deterministic MDP $\mathcal{M}$, exactly at the final timestep of an episode. We assume that all goals are reachable, and use the family of meta-policies that use a stochastic exploratory policy π until the goal is discovered and return to the discovered goal in all future episodes. The performance of a meta-policy on a task $\mathcal{T}_g$ under this model can be expressed in terms of the state distribution of the exploratory policy: $\text{Regret}(\pi_{\text{meta}}, \mathcal{T}_g) = \frac{1}{d_\pi^T(g)}$. This particular framework has been studied in [12, 19], and is a simple, interpretable framework for analysis.

We seek to understand performance under distribution shift when the original training task distribution is relatively concentrated on a subset of possible tasks. We choose the training distribution $p_{\text{train}}(\mathcal{T}_g) = (1 - \beta)\text{Uniform}(\mathcal{S}_0) + \beta\text{Uniform}(\mathcal{S} \backslash \mathcal{S}_0)$, so that p_{train} is concentrated on tasks involving a subset of the state space $\mathcal{S}_0 \subset \mathcal{S}$, with β a parameter dictating the level of concentration, and consider test distributions that perturb under the TV metric. Our main result compares the performance of a meta-policy trained to an ϵ_2-level of robustness when the true test distribution deviates by ϵ_1.

Proposition 4.1. *Let $\overline{\epsilon_i} = \min\{\epsilon_i + \beta, 1 - \frac{|\mathcal{S}_0|}{|\mathcal{S}|}\}$. There exists $q(\mathcal{T})$ satisfying $D_{TV}(p_{train}, q) \leq \epsilon_1$ where an ϵ_2-robust meta policy incurs excess regret over the optimal ϵ_1-robust meta-policy:*

$$\mathbb{E}_{q(\mathcal{T})}[\text{Regret}(\pi_{meta}^{\epsilon_2}, \mathcal{T}) - \text{Regret}(\pi_{meta}^{\epsilon_1}, \mathcal{T})] \geq \left(c(\epsilon_1, \epsilon_2) + \frac{1}{c(\epsilon_1, \epsilon_2)} - 2\right)\sqrt{\overline{\epsilon_1}(1 - \overline{\epsilon_1})|\mathcal{S}_0|(|\mathcal{S}| - \mathcal{S}_0|)}$$

(5)

The scale of regret depends on $c(\epsilon_1, \epsilon_2) = \sqrt{\frac{\overline{\epsilon_2}^{-1} - 1}{\overline{\epsilon_1}^{-1} - 1}}$, a measure of the mismatch between ϵ_1 and ϵ_2.

We first compare robust and non-robust solutions by analyzing the bound when $\epsilon_2 = 0$. In the regime of $\beta \ll 1$, excess regret scales as $\mathcal{O}(\epsilon_1\sqrt{\frac{1}{\beta}})$, meaning that the robust solution is most necessary when the training distribution is highly concentrated in a subset of the task space. At one extreme, if the training distribution contains no examples of tasks outside $\mathcal{S}_0$ ($\beta = 0$), the non-robust solution incurs *infinite excess regret*; at the other extreme, if the training distribution is uniform on the set of all possible tasks ($\beta = 1 - \frac{|\mathcal{S}_0|}{|\mathcal{S}|}$), *robustness provides no benefit*.

We next quantify the effect of mis-specifying the level of robustness in the meta-robustness objective, and what benefits *adaptive* distributional robustness can confer. For small β and fixed ϵ_1, the excess regret of an ϵ_2-robust policy scales as $\mathcal{O}(\sqrt{\max\{\frac{\epsilon_2}{\epsilon_1}, \frac{\epsilon_1}{\epsilon_2}\}})$, meaning that excess regret gets incurred if the meta-policy is trained either to be too robust ($\epsilon_2 \gg \epsilon_1$) or not robust enough $\epsilon_1 \gg \epsilon_2$. Compared to a fixed robustness level, our strategy of training meta-policies for a sequence of robustness levels $\{\epsilon_i\}_{i=1}^M$ ensures that this misspecification constant is at most the relative spacing between robustness levels: $\max_i \frac{\epsilon_i}{\epsilon_{i-1}}$. This enables the distributionally adaptive approach to *control* the amount of excess regret by making the sequence more fine-grained, while a fixed choice of robustness incurs larger regret (as we verify empirically in our experiments as well).

5   DiAMetR: A Practical Algorithm for Meta-Distribution Shift

In order to instantiate our distributionally adaptive framework into a practical algorithm, we must address how task distributions should be parameterized and optimized over . We must also address

Algorithm 1 DiAMetR: Meta-training phase

1: Given: $p_{\text{train}}(\mathcal{T})$, Return: $\{\pi_{\text{meta},\theta}^{\epsilon_i}\}_{i=1}^{M}$
2: $\pi_{\text{meta},\theta}^{\epsilon_1}, \mathcal{D}_{\text{Replay-Buffer}} \leftarrow$ Solve equation 1 with off-policy RL[2]
3: Reward distribution r_ω / Dynamic distribution p_ω, prior $p_{\text{train}}(z) \leftarrow$ Solve eq 6 using $\mathcal{D}_{\text{Replay-Buffer}}$
4: **for** ϵ in $\{\epsilon_2, \ldots, \epsilon_M\}$ **do**
5:     Initialize q_ϕ, $\pi_{\text{meta},\theta}^{\epsilon}$ and $\lambda \geq 0$.
6:     **for** iteration $n = 1, 2, \ldots$ **do**
7:         **Meta-policy:** Update $\pi_{\text{meta},\theta}^{\epsilon}$ using off-policy RL[2] [30]

$$\theta := \theta + \alpha \nabla_\theta \mathbb{E}_{\mathcal{T} \sim p_{\text{train}}(\mathcal{T})}[\mathbb{E}_{\pi_{\text{meta},\theta}^{\epsilon}, \mathcal{T}}[\frac{n w_{\mathcal{T}}}{k \sum_{i=1}^{n_{\text{tr}}} w_{\mathcal{T}_i}} \sum_{i=1}^{k} \sum_{t=1}^{T} r_t^{(i)}]]$$

$$\theta := \theta + \alpha \nabla_\theta \mathbb{E}_{z \sim q_\phi(z)}[\mathbb{E}_{\pi_{\text{meta},\theta}^{\epsilon}, \mathcal{P}}[\frac{1}{k} \sum_{i=1}^{k} \sum_{t=1}^{T} r_\omega(s_t^{(i)}, a_t^{(i)}, z)]]$$

$$\theta := \theta + \alpha \nabla_\theta \mathbb{E}_{z \sim q_\phi(z)}[\mathbb{E}_{\pi_{\text{meta},\theta}^{\epsilon}, p_\omega(\cdot, \cdot, z)}[\frac{1}{k} \sum_{i=1}^{k} \sum_{t=1}^{T} r_t^{(i)}]]$$

8:         **Adversarial task distribution:** Update q_ϕ using Reinforce [38]

$$\phi := \phi - \alpha \nabla_\phi (\mathbb{E}_{\mathcal{T} \sim p_{\text{train}}(\mathcal{T})}[\mathbb{E}_{\pi_{\text{meta},\theta}^{\epsilon}, \mathcal{T}}[\frac{n w_{\mathcal{T}}}{k \sum_{i=1}^{n_{\text{tr}}} w_{\mathcal{T}_i}} \sum_{i=1}^{k} \sum_{t=1}^{T} r_t^{(i)}]] + \lambda D_{\text{KL}}(p_{\text{train}}(\mathcal{T}) \| q_\phi(\mathcal{T}))$$

$$\phi := \phi - \alpha \nabla_\phi (\mathbb{E}_{z \sim q_\phi(z)}[\mathbb{E}_{\pi_{\text{meta},\theta}^{\epsilon}, \mathcal{P}}[\frac{1}{k} \sum_{i=1}^{k} \sum_{t=1}^{T} r_\omega(s_t^{(i)}, a_t^{(i)}, z)]] + \lambda D_{\text{KL}}(p_{\text{train}}(z) \| q_\phi(z))$$

$$\phi := \phi - \alpha \nabla_\phi (\mathbb{E}_{z \sim q_\phi(z)}[\mathbb{E}_{\pi_{\text{meta},\theta}^{\epsilon}, p_\omega(\cdot, \cdot, z)}[\frac{1}{k} \sum_{i=1}^{k} \sum_{t=1}^{T} r_t^{(i)}]] + \lambda D_{\text{KL}}(p_{\text{train}}(z) \| q_\phi(z))$$

9:         **Lagrange constraint multiplier:** Update λ to enforce $D_{\text{KL}}(p_{\text{train}} \| q_\phi) < \epsilon$,

$$\lambda :=_{\lambda \geq 0} \lambda + \alpha(D_{\text{KL}}(p_{\text{train}}(\mathcal{T}) \| q_\phi(\mathcal{T})) - \epsilon)$$
$$\lambda :=_{\lambda \geq 0} \lambda + \alpha(D_{\text{KL}}(p_{\text{train}}(z) \| q_\phi(z)) - \epsilon) \quad \lambda :=_{\lambda \geq 0} \lambda + \alpha(D_{\text{KL}}(p_{\text{train}}(z) \| q_\phi(z)) - \epsilon)$$

10:     **end for**
11: **end for**

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how the robust meta-RL problem can be solved with stochastic gradient methods. We first introduce the individual components of task parameterization and robust optimization, and describe the overall algorithm in Algorithm 1 and 2.

5.1 Parameterizing Task Distributions

Handling in-support distribution shifts: For handling in-support task distribution shifts, we propose to represent new task distributions as re-weighted training task distribution $q(\mathcal{T}) \propto w_{\mathcal{T}} p_{\text{train}}(\mathcal{T})$ where $w_{\mathcal{T}} > 0$ is a parameter. Since we have a finite set of training tasks, say $\{\mathcal{T}_i\}_{i=1}^{n_{\text{tr}}}$, new task distributions become $q(\mathcal{T}_i) = \frac{w_{\mathcal{T}_i}}{\sum_{i=1}^{n_{\text{tr}}} w_{\mathcal{T}_i}}$. With a slight abuse of notation, we can write empirical training task distribution as $p_{\text{train}}(\mathcal{T}_i) = \frac{1}{n_{\text{tr}}}$. We can use KL divergence to measure the divergence between training task distribution and test task distribution $D(p_{\text{train}}(\mathcal{T}) \| q_\phi(\mathcal{T})) = \sum_{i=1}^{n_{\text{tr}}} \frac{1}{n_{\text{tr}}} \log \frac{\sum_{i=1}^{n_{\text{tr}}} w_{\mathcal{T}_i}}{n_{\text{tr}} w_{\mathcal{T}_i}}$. We collectively represent the parameters $\phi = \{w_{\mathcal{T}_i}\}_{i=1}^{n_{\text{tr}}}$.

Handling out-of-support distribution shift: For handling out-of-support task distribution shifts, we propose to learn a probabilistic model of the training task distribution, and use the learned latent representation as a space on which to parameterize uncertainty sets over new task distributions. Specifically, we jointly train a task encoder $q_\psi(z|h)$ that encodes an environment history into the latent space, and a decoder $\mathcal{T}_\omega(s, a, z)$ mapping a latent vector z to a property of the task using a dataset of trajectories collected from the training tasks. To describe the exact form of $\mathcal{T}_\omega$, we consider how tasks can differ and list two scenarios: (1) Tasks differ in reward functions: $\mathcal{T}_\omega$ takes form of reward functions $r_\omega(s, a, z)$ that maps a latent vector z to a reward function and (2) Tasks differ in dynamics: $\mathcal{T}_\omega$ takes form of dynamics $p_\omega(s, a, z)$ that maps a latent vector z to a dynamics function.

(a) Ant navigation (b) Wind navigation (c) Fetch reach (d) Block push

Figure 3: The agent needs to either navigate in absence of winds, navigate in presence of winds, move its gripper or push the block to an unobserved target location, indicated by green sphere, by exploring its environment and experiencing reward. While tasks vary in reward functions for `Ant navigation`, `Fetch reach` and `Block push`, they vary in dynamics function for `Wind navigation`.

This generative model can be trained as a standard latent variable model by maximizing a standard evidence lower bound (ELBO), trading off reward prediction and matching a prior $p_{\text{train}}(z)$ (chosen to be the unit gaussian).

$$\min_{\omega,\psi} \mathbb{E}_{h \sim \mathcal{D}} \left[\mathbb{E}_{z \sim q_\psi(z|h)} \left[\sum_{t=1}^{T} l(\mathcal{T}_\omega(s_t, a_t, z), h, t) \right] + D_{\text{KL}}(q_\psi(z|h) \| \mathcal{N}(0, I)) \right]$$

$$l(\mathcal{T}_\omega(s_t, a_t, z), h, t) = (r_\omega(s_t, a_t, z) - r_t)^2 \quad \text{when rewards differ}$$

$$l(\mathcal{T}_\omega(s_t, a_t, z), h, t) = \|p_\omega(s_t, a_t, z) - s_{t+1}\|^2 \quad \text{when dynamics differ} \tag{6}$$

Having learned a latent space, we can parameterize new task distributions $q(\mathcal{T})$ as distributions $q_\phi(z)$ (the original training distribution corresponds to $p_{\text{train}}(z) = \mathcal{N}(0, I)$, and measure the divergence between task distributions as well using the KL divergence in this latent space $D(p_{\text{train}}(z) \| q_\phi(z))$.

5.2 Training and test-time selection of meta-policies

Learning Robust Meta-Policies: Given this task parameterization, the next question becomes how to actually perform the robust optimization laid out in Eq:3. The distributional meta-robustness objective can be modelled as an adversarial game between a meta-policy $\pi_{\text{meta}}^\epsilon$ and a task proposal distribution $q(\mathcal{T})$. As described above, this task proposal distribution is parameterized as a distribution over latent space $q_\phi(z)$, while $\pi_{\text{meta}}^\epsilon$ is parameterized a typical recurrent neural network policy as in [30]. We parameterize $\{\pi_{\text{meta}}^{\epsilon_i}\}_{i=1}^{M}$ as a discrete set of meta-policies, with one for each chosen value of ϵ.

This leads to a simple alternating optimization scheme (see Algorithm 1), where the meta-policy is trained using a standard meta-RL algorithm (we use off-policy RL[2] [30] as a base learner), and the task proposal distribution with an constrained optimization method (we use dual gradient descent [29]). Each iteration n, three updates are performed: 1) the meta-policy π_{meta} updated to improve performance on the current task distribution, 2) the task distribution $q(z)$ updated to increase weight on tasks where the current meta-policy adapts poorly and decreases weight on tasks that the current meta-policy can learn, while staying close to the original training distribution, and 3) a penalty coefficient λ is updated to ensure that $q(z)$ satsifies the divergence constraint.

Test-time meta-policy selection: Since test-time meta-policy selection can be framed as a multi-armed bandit problem, we use a generic Thompson's sampling [42] algorithm (see Algorithm 2). Each meta-episode n, we sample a meta-policy $\pi_{\text{meta}}^\epsilon$ with probability proportional to its estimated average episodic reward, run the sampled meta-policy $\pi_{\text{meta}}^\epsilon$ for an meta-episode (k environment episodes) and then update the estimate of the average episodic reward. Since Thompson's sampling is a zero-regret bandit algorithm, it will converge to the meta-policy that achieves the highest average episodic reward and lowest regret on the test task distribution.

Algorithm 2 DiAMetR: Meta-test phase

1: Given: $p_{\text{test}}(\mathcal{T})$, $\Pi = \{\pi_{\text{meta},\theta}^{\epsilon_i}\}_{i=1}^{M}$
2: Initialize `TS = Thompson-Sampler()`
3: **for** meta-episode $n = 1, 2, \ldots$ **do**
4: Choose meta-policy $i = $ `TS.sample()`
5: Run $\pi_{\text{meta},\theta}^{\epsilon_i}$ for meta-episode
6: `TS.update(`
    `arm=i,`
    `reward=meta-episode return)`
7: **end for**

---

6 Experimental Evaluation

We aim to comprehensively evaluate DiAMetR and answer the following questions: **(1)** Do meta-policies learned via DiAMetR allow for quick adaptation under different distribution shifts in the

271 test-time task distribution? **(2)** Does learning for multiple levels of robustness actually help the
272 algorithm adapt more effectively than a particular chosen uncertainty level? **(3)** Does proposing
273 uncertainty sets via generative modeling provide useful distributions of tasks for robustness?

274 **Setup.** We use DiAMetR on four continuous control environments: `Ant navigation` (controlling a
275 four-legged robotic quadruped), `Wind navigation` [6] (controlling a linear system robot in presence
276 of wind), `Fetch reach` and `Block push` [13] (Figures 3a to 3d) (see Appendix D for details about
277 reward function and dynamics). We design various meta RL tasks from these environments. Each
278 meta RL task has a train task distribution $\mathcal{T}_i \sim p_{\text{train}}(\mathcal{T})$ such that each task $\mathcal{T}_i$ either parameterizes
279 a reward function $r_i(s, a) \coloneqq r(s, a, \mathcal{T}_i)$ or a dynamics function $p_i(s, a) \coloneqq p(s, a, \mathcal{T}_i)$. $\mathcal{T}_i$ itself
280 remains unobserved, the agent simply has access to reward values and executing actions in the
281 environment. The learned meta-policies are evaluated on different distributionally shifted test task
282 distributions $\{p_{\text{test}}^i(\mathcal{T})\}_{i=1}^K$ which are either in-support or out-of-support of training task distribution.
283 In all meta RL tasks, the train and test task distribution is determined by the distribution of an
284 underlying task parameter (i.e. wind velocity $w_\mathcal{T}$ for `Wind navigation` and target location $s_\mathcal{T}$ for
285 other environments), which either determines the reward function or the dynamics function. While
286 tasks vary in reward functions for `Ant navigation`, `Fetch reach` and `Block push`, they vary in
287 dynamics function for `Wind navigation` (exact task distributions in Table 2). We use 4 random
288 seeds for all our experiments and include the standard error bars in our plots.

6.1 Adaptation to Varying Levels of Distribution Shift

290 During meta test, given a test task distribution $p_{\text{test}}(\mathcal{T})$, DiAMetR uses Thompson sampling to select
291 the appropriate meta-policy $\pi_{\text{meta},\theta}^\epsilon$ within $N = 250$ meta episodes. $\pi_{\text{meta},\theta}^\epsilon$ can then solve any new
292 task $\mathcal{T} \sim p_{\text{test}}(\mathcal{T})$ within 1 meta episode (k environment episode). To test DiAMetR's ability to adapt
293 to varying levels of distribution shift, we evaluate it on different test task distributions, as detailed in
294 Table 2. We compare DiAMetR with meta RL algorithms such as (off-policy) RL^2 [30], VariBAD [48]
295 and HyperX [49]. Since DiAMetR uses 250 meta-episodes to adaptively choose a meta-policy during
296 test time, we finetune RL^2, VariBAD and HyperX with 250 meta-episodes of test task distribution
297 to make the comparisons fair (see Appendix H for the finetuning curves). Figure 4 and Figure 6
298 show that DiAMetR outperforms RL^2, VariBAD and HyperX on *out-of-support* and *in-support*
299 shifted test task distributions respectively (see Appendix F for extensive evaluations on *in-support*
300 distribution shifts). Furthermore, the performance gap between DiAMetR and other baselines increase
301 as distribution shift between test task distribution and train task distribution increases. Naturally,
302 the performance of DiAMetR also deteriorates as the distribution shift is increased, but as shown in
303 Fig 4 and Fig 6, it does so much more slowly than other algorithms. We also evaluate DiAMetR on
304 train task distribution to see if it incurs any performance loss. Figure 4 shows that DiAMetR either
305 matches or outperforms RL^2, VariBAD, and HyperX on the train task distribution. We refer readers
306 to Appendix I for ablation studies and further experimental evaluations.

Figure 4: We evaluate DiAMetR and meta RL algorithms (RL^2, VariBAD and HyperX) on training task
distribution and different test task distributions. DiAMetR outperforms RL^2, VariBAD and HyperX on train
distributions and different test distributions. The first point p_1 on the horizontal axis indicates the task parameter
(Δ) distribution $\mathcal{U}(0, p_1)$ and the subsequent points p_i indicate task parameter (Δ) distribution $\mathcal{U}(p_{i-1}, p_i)$.
While task parameter for `wind navigation` is wind velocity $w_\mathcal{T}$, it is target location $s_\mathcal{T}$ for other environments.
Table 2 details the task distributions used in this evaluation.

6.2 Analysis of Tasks Proposed by Latent Conditional Uncertainty Sets

308 We visualize the imagined test reward distribution for various distribution shifts. Specifically, we
309 create a heatmap of imagined test reward functions. Figure 5 visualizes the imagined test reward
310 distribution in `Ant-navigation` environment in increasing order of distribution shifts with respect

Figure 5: Imagined test reward distributions in `Ant-navigation` environment in increasing order of distribution shifts. Train reward distribution is uniform within the red circle.

Figure 6: We evaluate DiAMetR on different *in-support* shifted test task distributions. DiAMetR either matches or outperforms RL^2, VariBAD and HyperX on these test task distributions. Furthermore, adaptively choosing an uncertainty set for DiAMetR policy (Adapt) during test time allows it to better adapt to test time distribution shift than choosing an uncertainty set beforehand (Mid). Choosing a large uncertainty set for DiAMetR policy (Conservative) leads to a conservative behavior and hurts its performance when test time distribution shift is low.

to train reward distribution (with distribution shift parameter ϵ increasing from left to right). The train distribution of rewards has uniformly distributed target locations within the red circle. As clearly seen in Figure 5, as we increase the distribution shifts, the learned reward distribution model imagines more target locations outside the red circle.

6.3 Analysis of Importance of Multiple Uncertainty Sets

DiAMetR meta-learns a family of adaptation policies, each conditioned on different uncertainty set. As discussed in section 4, selecting a policy conditioned on a large uncertainty set would lead to overly conservative behavior. Furthermore, selecting a policy conditioned on a small uncertainty set would result in failure if the test time distribution shift is high. Therefore, we need to adaptively select an uncertainty set during test time. To validate this phenomenon empirically, we performed an ablation study in Figure 6 and Figure 7. As clearly visible, adaptively choosing an uncertainty set during test time allows for better test time distribution adaptation when compared to selecting an uncertainty set beforehand or selecting a large uncertainty set. These results suggest that a combination of training robust meta-learners and constructing various uncertainty sets allows for effective test-time adaptation under distribution shift. DiAMetR is able to avoid both overly conservative behavior and under-exploration at test-time.

Figure 7: Adaptively choosing an uncertainty set for DiAMetR policy (Adapt) during test time allows it to better adapt to test time distribution shift than choosing an uncertainty set beforehand (Mid). Choosing a large uncertainty set for DiAMetR policy (Conservative) leads to a conservative behavior and hurts its performance when test time distribution shift is low.

7 Discussion

In this work, we discussed the challenge of distribution shift in meta-reinforcement learning and showed how we can build meta-reinforcement learning algorithms that are robust to varying levels of distribution shift. We show how we can build distributionally "adaptive" reinforcement learning algorithms that can adapt to varying levels of distribution shift, retaining a tradeoff between fast learning and maintaining asymptotic performance. We then show we can instantiate this algorithm practically by parameterizing uncertainty sets with a learned generative model. We empirically showed that this allows for learning meta-learners robust to changes in task distribution.

There are several avenues for future work we are keen on exploring, for instance extending adaptive distributional robustness to more complex meta RL tasks, including those with differing transition dynamics. Another interesting direction would be to develop a more formal theory providing adaptive robustness guarantees in meta-RL problems under these inherent distribution shifts.

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