When to intervene: learning optimal intervention policies for critical events

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Abstract

Providing timely interventions before the onset of a critical event, such as a system failure is an important problem in many industrial settings. Before the onset of the critical event, usually, systems exhibit behavioral changes which often manifest as stochastic co-variate observations which may be leveraged to trigger intervention. In this paper, for the first time, we formulate the problem of finding an optimally timed intervention (OTI) policy as minimizing the expected residual time to event, subject to a constraint on the probability of missing the event. Existing machine learning approaches to intervention on critical events focus on predicting event occurrence within a pre-defined window (a classification problem) or predicting time-to-event (a regression problem). Interventions are then triggered by setting model thresholds. These are heuristic-driven, lacking guarantees regarding optimality. To model the evolution of system behavior, we introduce the concept of a survival process. We show that the OTI problem is equivalent to an optimal stopping problem on the associated survival process. This key link has not been explored in literature. Under Markovian assumptions on the survival process, we show that an OTI policy at any time is completely determined by the conditional survival function at that time. We proceed to analytically characterize the optimal stopping policy, enabling practical survival-model-based critical event intervention algorithms. Further, we show that our framework includes, as a special case, the important class of neural survival processes generated by recurrent neural networks(RNNs). To model neural survival processes we propose a dynamic deep recurrent survival analysis (DDRSA) architecture, introducing an RNN encoder into the static DRSA setting. Finally, we demonstrate RNN-based OTI policies with practical experiments and show that they outperform popular intervention methods.

1 Introduction

We consider a problem that involves the monitoring of observations from a discrete-time stochastic process with the goal of intervening to prevent the occurrence of a critical event that will happen in the future. A critical event is missed (loosely termed a false negative) when intervention is not triggered before the critical event occurs. The general objective is the design of algorithms that can be used to trigger a timely intervention (the objective may be to maximize useful life, minimize overall domain specific costs e.t.c.), subject to false negative constraints.

This problem has significant practical significance in a number of fields. Critical events could be the failure of a machine or key component, customer churn, loan default, or even death. Intervening on such events ahead of time, in an optimal manner is of great significance to businesses and society, due to the potential of mitigating the impact of the occurrences of these events.

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1.1 Interventions on critical events

The monitored stochastic process is represented by a discrete, temporal sequence of random variables, where $X_j \in \mathbb{R}^d$ represents an observation at time-step j. Let $\mathcal{X}_j = \{X_s : 1 \leq s \leq j\}$ represent the observation history until time step j. The critical event occurs at a random time-step L in the series. The sequence of observations terminates after the occurrence of the critical event. An intervention at time step j is considered early if the intervention occurs at a time-step $j < L$. An intervention is considered late (the critical event is effectively missed) if no intervention occurs at any time-step $j < L$.

We consider a data-driven sequential decision rules (intervention policies) that are defined as follows:

Definition 1.1 (Intervention policies)**.** An intervention policy is a sequence of functions $\boldsymbol{\varphi} = (\varphi_1, \varphi_2, \cdots \varphi_j, \cdots)$, where $\varphi_j : \mathcal{X}_j \mapsto \{0, 1\}$ maps the history of observations at a time step j to a decision of whether to intervene ($\varphi_j(\mathcal{X}_j) = 1$) or not ($\varphi_j(\mathcal{X}_j) = 0$).

The intervention policies we consider in this paper are terminal, i.e: once a decision to intervene is triggered, the policy stops. In effect, this means that intervention triggers after the first trigger are ignored for a particular realization of the process[1].

Definition 1.2 (policy stopping time)**.** Intervention on a monitored process happens at a random time step $\tau = \tau(\mathcal{X}_j) := \min\{j \geq 1 : \varphi_j(\mathcal{X}_j) = 1\}$. We call this time the stopping[2] time of the policy.

Note that the stopping time random variable may be understood as a general mapping of observation history to an intervention time, while an intervention policy maps observation history to a decision to intervene. The two concepts are equivalent. Indeed, we have $\varphi_j(\mathcal{X}_j) = \mathbb{1}(\tau(\mathcal{X}_j) = j)$.

1.2 Optimally timed intervention (OTI) on critical events

We seek to find intervention policy (or equivalently, the stopping time) that minimizes expected time-to-event subject to the constraint that the probability of missing the critical event is at most $\alpha \in (0, 1]$.

We now formally state the optimally timed intervention (OTI) problem for the first time as:

$$\tau^* = \arg \inf_{\tau \in \mathcal{M}_\alpha} \mathbb{E}[(L - \tau)^+] \text{ where: } \mathcal{M}_\alpha = \{1 \leq \tau < L_{max} : \mathbb{P}(L \leq \tau) \leq \alpha\} \tag{1}$$

Optimal interventions are efficient in the sense that they optimize the tradeoff between expected time-to-event and the probability of missing the event (the miss-rate). For a given expected time-to-event, the miss-rate is minimized and for a given miss-rate the expected time-to-event is minimized.

Note that there exists a multiplier C_α, such that the OTI problem may be expressed in the following equivalent, unconstrained form:

$$\tau^* = \arg \inf_{\tau} \mathbb{E}[(L - \tau)^+] + C_\alpha \mathbb{P}(L \leq \tau) \text{ for some } C_\alpha > 0 \qquad \text{(OTI problem)}$$

C_α can be directly be interpreted as the risk (cost) of missing the critical event. Changing C_α allows one to vary the risk tradeoff between the miss-rate and expected time-to-event.

1.3 Related work

Window based critical event prediction: Intervention is a binary decision. At each time-step a decision must be made to intervene or not. This is often modeled directly as a binary classification problem where we look for event occurrence within a specified future window. In window based intervention (WBI), time-steps where an event is present within a lookahead period are labeled as positive class examples and the rest are labeled as negative examples. A classifier may then be learnt to predict the probability of the positive class. An optimal threshold on this probability is then tuned

[1] we may easily generalize this by considering a hold time before the policy is re-activated. The sequences of observations starting at policy reactivations are simply treated as independent realizations of the process in this scenario

[2] a stopping time in probability theory is random a variable τ such that $\mathbb{1}(\tau = n)$ is a function of $\mathcal{X}_n$

over a validation set that measures costs of triggered interventions directly to choose an optimal trigger threshold. WBI policies are standard practice in predictive maintenance[31, 1, 20, 1, 36, 38, 12].

Time-to-Event prediction (Survival analysis): Classical survival analysis deals with predicting the time to an event (ex: death, machine failure) given a vector of observations (co-variates) obtained from a subject in the presence of incomplete (censored) observations [21]. The goal is to estimate functions such as the hazard rate (rate of death as a function of time) or survival function (probability of survival past a given time) and understand the impact of co-variates on these functions. Classical model-based approaches for modeling co-variate impact on survival such as the Cox proportional hazards model [9] and the accelerated failure time model [39] have been extended in recent work using machine learning approaches [40, 16, 4, 28, 18]. In some cases, the impact of a co-variate on the survival function may vary over time (for instance, the impact of smoking may increase with age), resulting in approaches that extend survival analysis to model the time-varying effects of covariates [33, 2]. These have deep learning counterparts [29, 11] that use recurrent neural networks to model the survival function. Note that even in this case co-variates for a subject themselves are static and doe not evolve over time. It is the impact of the co-variates that is time-varying. Recent work in survival analysis considers longitudinal data which are sequential observations taken over time for each subject. The regression approaches in this case are based on the concept that degradation or deterioration of a subject's health follows a stochastic process and failure occurs when the process first reaches a failure state or threshold (a first hitting time). This requires underlying parametric stochastic process models for the joint evolution of the time varying observations and the time to event [13, 19]. More recent work has applied modern deep learning approaches to this problem [14, 17, 15]. Corresponding work also exists in the area of machine failure prediction [24]. The relevant papers to our work are the recurrent neural network approaches [17, 24] that operate on longitudinal data to predict the probability distribution of failure time given previous history of co-variates for any subject, at each observation time. While these methods are mostly complementary to the primary goal of this paper where we focus on obtaining optimal intervention policies, we do propose improvements in to these methods in Section 4.1 to overcome the distributional assumptions in [24] and difficulty with long sequences [17]. In practice, intervention policies based on survival analysis are typically just thresholds on the predicted time-to-event.

Optimal stopping problems: Optimal stopping problems [37, 8, 3] are a subclass of stochastic control problems with only two possible actions:stop and continue. The goal is to stop taking observations when the current reward (or cost) exceeds (or falls below) expected future reward (or cost). Optimal stopping techniques have been applied to several problems such as sequential hypothesis testing [37, 10, 6], early change point detection [27, 35, 34, 30] and early classification [23]. Optimal stopping policies for machine maintenance have been studied in reliability theory [5, 22], where the goal is to find the best stopping time to replace/maintain a machine based on condition. However, the approaches in such cases are limited to simple probabilistic models of machine degradation with a single or few co-variates. General model-free reinforcement learning approaches to the optimal stopping problem are generally feasible in situations where we have a lot of data or in a model-based setting where the underlying randomness can efficiently be simulated and the model can be used to draw Monte-Carlo samples [7]. This is not an option in most critical event prediction settings when the critical event data is often scarce and no simple model exists for the evolution of the co-variates.

1.4 Outline of contributions

The OTI problem is hard since system evolution towards a critical event is a non-stationary in general with typically scarce occurences of critical events. Current modeling approaches are challenged either because they make significant overly simplifying assumptions on the underlying process or because scarce events precludes powerful model-free approaches.

To our knowledge, we are the first to frame the OTI problem and link it to both survival analysis and optimal stopping theory. This allows, for the first time, the ability to model complex non-stationary system evolution towards scarce critical events with the ability to intervene optimally.

- A process under observation may exhibit degradation/disorder as it evolves towards a critical event. We capture this feature by introducing the concept of a survival (or hazard rate) process, which is a stochastic process modeling the evolution of conditional survival functions as new observations are taken. A key contribution of our work is to formalize the

OTI problem in equation (OTI problem) and show that the OTI problem is equivalent to an optimal stopping problem on the underlying survival (or hazard rate) process (Proposition 2.1). This allows us to tap into optimal stopping theory [37, 8, 3] to unlock the structure of OTI policies and also leverage computational methods of estimating/learning survival/hazard rate functions. These are key to our development of a general machine learning approach to the OTI problem.

- Under Markovian assumptions on the survival process, we show that the optimal intervention policy decision rule at each time step can be analytically determined by the conditional survival (or equivalently, hazard rate) function at that time step (Theorem 3.1). Further we show that the optimal policy is non-stationary in general and reduces to a variable threshold on the expected time-to-event(Corollary 3.1.2), showing that static threshold policies are sub-optimal in general.
- We show that our theory applies to an important class of survival and hazard rate processes generated by recurrent neural networks (Section 4). We term these processes Neural survival and hazard rate processes respectively. This allows us to apply survival analysis methods to learn survival processes and analytically compute optimal intervention policies for critical events. We adapt the state of the art deep dynamic survival analysis (DRSA) work [29] to our setting. Unlike DRSA, our dynamic DRSA (DDRSA) setting requires modeling dynamic survival and hazard rate functions. We do this by adapting the DRSA architecture to our dynamic covariate evolution setting using an encoder-decoder seq2seq architecture.

2 OTI as an optimal stopping problem on a survival or hazard rate process

In this section we establish a link between the OTI problem, survival analysis and optimal stopping theory. We require a few definitions.

2.1 Preliminaries

We model the degradation of the monitored stochastic process towards the critical event by the corresponding evolution of underlying survival functions or hazard rate functions conditioned on observation history. To this end, we will use the following definitions of a conditional *hazard rate function* $h_j(\cdot)$, and *survival function* $S_j(\cdot)$:

$$h_j(k) := \mathbb{P}(L = j + k | \mathcal{X}_j, L \geq j + k, L \geq j) \qquad S_j(k) := \mathbb{P}(L > j + k | \mathcal{X}_j, L > j) \tag{2}$$

Note that we have omitted the implied conditioning on the the hazard and survival functions, so that $h_j(k|\mathcal{X}_j)$ is written as $h_j(k)$ for convenience. Also, we have $h_j(-1) := 0$ and $S_j(-1) := 1$ reflecting the fact that the critical event has not occured until time step j. We now define the concept of survival and hazard rate processes:

Definition 2.1 (Survival Process)**.** Consider the random vector constructed from the survival function $S_j(\cdot)$ given by $\mathbf{S}_j = [S_j(0), S_j(1) \cdots S_j(L_{max} - 1)]^T$. We call the sequence of tuples $\{(\mathcal{X}_j, \mathbf{S}_j)\}$ a survival process.

Definition 2.2 (Hazard Rate Process)**.** Consider the random vector constructed from the conditional hazard rate function $h_j(\cdot)$ given by $\mathbf{H}_j = [h_j(0), h_j(1) \cdots h_j(L_{max} - 1)]^T$. We call the sequence of tuples $\{(\mathcal{X}_j, \mathbf{H}_j)\}$ a hazard rate process.

Thus, survival and hazard rate processes model the relation between the evolution of co-variates and onset of the critical event.

2.2 Optimal intervention on a survival or hazard rate process

Proposition 2.1 (Optimal intervention on a survival or hazard rate process)**.** Given a survival process $\{(\mathcal{X}_j, \mathbf{S}_j)\}$ or a hazard rate process $\{(\mathcal{X}_j, \mathbf{H}_j)\}$, the intervention risk at a stopping time τ is given equivalently by:

$$R^S_\tau(\mathbf{S}_\tau) = \sum_{k=0}^{L_{max}} S_\tau(k) + C_\alpha[1 - S_\tau(0)] = R^H_\tau(\mathbf{H}_\tau) = \sum_{k=0}^{L_{max}} \prod_{m=0}^{k} (1 - h_\tau(k)) + C_\alpha h_\tau(0) \tag{3}$$

In this setting, the optimal critical event intervention problem of equation (OTI problem) reduces to the following optimal stopping problems[3] on the corresponding survival or hazard rate processes:

$$\tau^* = \arg \inf_{1 \leq \tau < L_{max}} \mathbb{E}\left[R_\tau^S(\mathbf{S}_\tau)\right] = \arg \inf_{1 \leq \tau < L_{max}} \mathbb{E}\left[R_\tau^H(\mathbf{H}_\tau)\right] \tag{4}$$

The proof is established in Appendix A. The first term in these risks may be viewed as the cost of intervention when the critical event does not occur at time τ, while the second term is the cost of intervention when the critical event does occur at time τ.

3 Analytic solution to the OTI problem for Markov survival processes

In order to solve the OTI optimal stopping problem we make the assumption that the survival and hazard rate processes we deal with satisfy the Markov property:

Definition 3.1 (Markov survival or hazard rate process)**.** Consider the survival process $\{(\mathcal{X}_j, \mathbf{S}_j)\}$ or the hazard rate process $\{(\mathcal{X}_j, \mathbf{H}_j)\}$. These are said to be Markov survival and Markov hazard rate processes respectively, if we have:

$$\mathbb{P}(X_{j+1}|\mathcal{X}_j) = \mathbb{P}(X_{j+1}|X_j) \tag{5}$$

Further, the process is termed *homogeneous* if $\mathbb{P}(X_{j+1}|X_j)$ does not depend on j.

With this assumption in place we can state the central result of this paper:

Theorem 3.1 (OTI policy for a Markov hazard rate process)**.** *Given a Markov hazard rate process $\{(\mathcal{X}_j, \mathbf{H}_j)\}$ and associated derived risk process risk process $\{R_j^H(\mathbf{H}_j)\}$, the optimal intervention policy that solves the problem posed in Proposition 2.1 is a decision rule that calls for intervention at stopping time τ^* such that:*

$$\tau^* = \min\{1 \leq j \leq L_{max} : R_j^H(\mathbf{H}_j) \leq V_j(\mathbf{H}_j)\} \tag{6}$$

for a sequence of value functions $V_j(\mathbf{H}_j)$ given by:

$$V_j(\mathbf{H}_j) = \min_{l \geq 0} \left\{ \sum_{k=0}^{L_{max}-l-1} \prod_{m=0}^{k} (1 - h_j(l+k)) + \sum_{m=0}^{l} \left[\prod_{k=0}^{m-1} (1 - h_j(k)) h_j(m) \right] C_\alpha \right\} \tag{7}$$

Further if the Markov hazard rate process is homogeneous the optimal policy is stationary, with value function $V(\mathbf{H}_j)$ that does not depend on j.

The proof of this theorem is given in Appendix A. The following corollary establishing a similar result for Markov survival processes follows:

Corollary 3.1.1 (OTI policy for a Markov survival process)**.** *Given a Markov survival process $\{(\mathcal{X}_j, \mathbf{S}_j)\}$ and associated derived risk process risk process $\{R_j^S(\mathbf{S}_j)\}$, the optimal intervention policy that solves the problem posed in Proposition 2.1 is a decision rule that calls for intervention at stopping time τ^* such that:*

$$\tau^* = \min\{1 \leq j \leq L_{max} : R_j^S(\mathbf{S}_j) \leq V_j(\mathbf{S}_j)\} \tag{8}$$

for a sequence of value functions $V_j(\mathbf{S}_j)$ given by:

$$V_j(\mathbf{S}_j) = \min_{l \geq 0} \left\{ \sum_{k=0}^{L_{max}-l-1} S_j(l+k) + \sum_{k=0}^{l} \left[S_j(k-1) - S_j(k)\right] S_j(k-1) C_\alpha \right\} \tag{9}$$

Further if the Markov survival process is homogeneous the optimal policy is stationary, with value function $V(\mathbf{S}_j)$ that does not depend on j.

This theorem tells us that in the case of a Markov survival or hazard rate processes, the optimal policy may be constructed explicitly with knowledge of only the conditional survival or hazard rate functions at that time. The proof of the above theorem leverages fundamental results in optimal stopping theory [8, 3] and is given in Appendix A. The *value* function $V_j(\cdot)$ gives the minimum expected cost of

[3] Problems of the form $\inf_\tau \mathbb{E}\left[R_\tau(Y_\tau)\right]$ where $R(\cdot)$ is a cost/risk function, and $\{Y_j\}$ is a stochastic process

continuing to take observations, having obtained a conditional survival ($\mathbf{S}_j$) or hazard rate function ($\mathbf{H}_j$) at time j. It is clear that the the optimal policy is non-stationary in general. Also note that putting $l = 0$ in the value function recovers the instantaneous risk at time step j as the first term over which the minimum is computed in the value function. The value function gives the best stopping risk over all stopping decisions in the present and future. The following Corollary allows a further simplification of the optimal policy:

Corollary 3.1.2 (OTI policy as a time varying threshold on time-to-event)**.** *Let T_j be the expected time to event at time j. The OTI policy of Theorem 3.1 may be simplified to the following form:*

$$\tau^* = \min\{1 \leq j \leq L_{max} : T_j \leq V_j^{'}(\mathbf{H}_j)\} \tag{10}$$

where

$$V_j^{'}(\mathbf{H}_j) = \min_{l \geq 0} \left\{ \sum_{k=0}^{L_{max}-l-1} \prod_{m=0}^{k} (1 - h_j(l+k)) + \sum_{m=1}^{l} \left[\prod_{k=0}^{m-1} (1 - h_j(k)) h_j(m) \right] C_\alpha \right\} \tag{11}$$

with T_j given by:

$$T_j = \sum_{k=0}^{L_{max}} \prod_{m=0}^{k} (1 - h_j(k)) = \sum_{k=0}^{L_{max}} S_j(k) \tag{12}$$

The proof is given in Appendix A. This Corollary clearly shows that static threshold TTE policies are sub-optimal in general. The value functions in these results contain two terms. The first term, which is the risk of stopping early before the critical event which intuitively gets smaller as j progresses toward the critical event. The second term is the risk of missing the critical event that depends on the cost of missing the event. It is high where the onset of the critical event is expected.

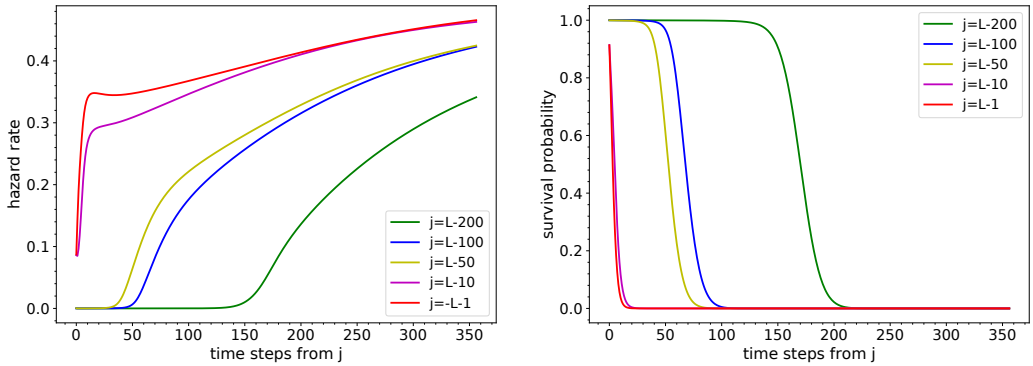
4 Neural survival processes

The previous theory applied to Markov survival processes. We show that this is not a significant restriction. In fact, the class of survival and hazard rate processes that can be modeled by recurrent neural networks (RNNs) also satisfy our Markovian assumptions. To see this, note that RNNs may be viewed as a feature transformation, mapping $\mathcal{X}_j$ to features $\mathcal{Z}_j = \{Z_s : 1 \leq s \leq j\}$ where Z_j is the hidden state vector of the RNN at time step j. If we output survival or hazard rate vectors as functions of the hidden state we generate *neural* survival and hazard rate process given by $\{(\mathcal{Z}_j, \mathbf{S}_j)\}$ or $\{(\mathcal{Z}_j, \mathbf{H}_j)\}$ respectively. Due to the recurrent dynamics of the RNN, the final state encode all history, therefore $\mathbb{P}(Z_{j+1}|\mathcal{Z}_j) = \mathbb{P}(Z_{j+1}|Z_j)$ and the processes are Markovian and stationary (since the processes depend entirely on the hidden state evolution and not time-step). However, RNN models can be made non-stationary by encoding time into the inputs.

4.1 Dynamic deep recurrent survival analysis (DDRSA)

Since the OTI theory developed in the previous section directly applies to neural survival processes, we may fit a neural survival process model to data and then apply the optimal OTI policy to intervene. In the static co-variate setting, Ren et. al. [29] developed a deep recurrent neural architecture for survival analysis. In this work the authors use an RNN to map a static feature vector X_0 to a conditional hazard rate vector $\mathbf{H}_0$) by using an RNN that generates a sequence of hidden states $\{Z_{0k}\}$. The kth hidden state vector Z_{0k} is mapped through a dense layer D with a sigmoid activation to produce conditional hazard rate $h_0(i) = \sigma(D(Z_{0k}))$. The model is fit using a novel sequential loss, producing state of the art survival analysis results without requiring assumptions on distribution of time-to-event probabilities [29].

Unlike DRSA, our dynamic DRSA (DDRSA) setting requires modeling dynamic survival and hazard rate functions. We do this by adapting the DRSA architecture to our dynamic covariate evolution setting. While there are other attempts at modeling the dynamic evolution of survival functions [17, 24] they either make distributional assumptions [24] or have difficulty with long sequences due to large softmax layer [17]. Our adaptation of DRSA uses an encoder RNN that maps co-variate history $\mathcal{X}_j$ to an single encoder hidden state vector Q_j. This vector is then passed into a decoder which has a standard DRSA architecture, producing decoder hidden states Z_{jk} and corresponding conditional hazard rates $h_j(k) = \sigma(D(Z_{jk}))$. The model is then learnt end-to-end using the DRSA loss. DDRSA can be seen as a generative model for a Markov hazard rate process. We use the DDRSA architecture in all our experiments.



(a) Progression of survival functions

(b) Progression of conditional hazard rates

Figure 1: Example of typical evolution of estimated survival functions and conditional hazard rates modeled by DDRSA with progression in time step j towards the critical event.

5 Experiments

The goal of this section is twofold:

1. To complement the theory developed in the earlier sections with examples to validate and build intuition into survival and hazard rate processes how the optimal policy works to produce interventions. (Section 5.1)
2. To demonstrate that the theory does indeed translate into an effective practical methodology embodied by the DDRSA-RNN that can model survival processes and generate interventions using the OTI policy. We call this combined approach the OTI-DDRSA-RNN. We show that OTI-DDRSA-RNN outperforms static threshold policies that are either window based (WBI-RNN) or time-to-event based (TTE-RNN)(Section 5.2).

Experiments are performed on two real-world datasets to validate our goals:

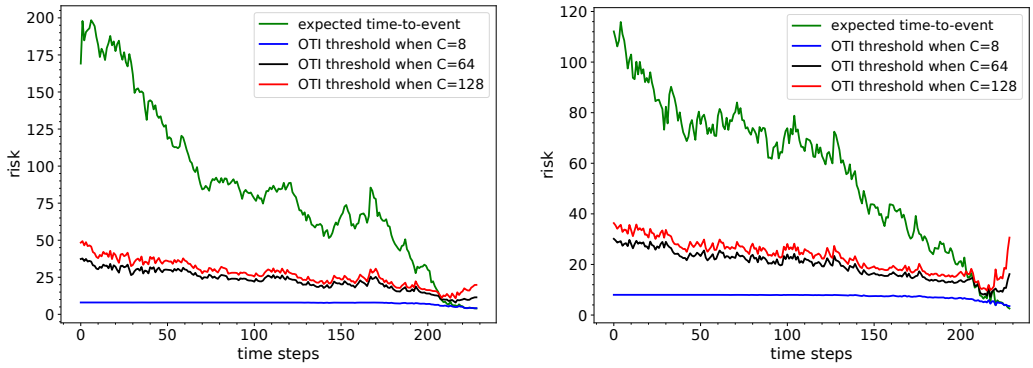
Turbofan Engine Failure Data [26, 32]: This dataset from NASA provides train and test data to predict Turbofan engine failures. The training set has simulated multivariate time-series observations from 21 sensors on a fleet of 218 engines. Each engine operates normally at the start and develops a fault at some point resulting in eventual failure. The test set consists of 218 partial engine time-series with remaining useful life (in engine cycles) indicated in each case. We could not use the test data as provided since it is missing part of every time series close to failure. We standardized the sensor observations to prepare them for input into our models.

Azure Predictive Maintenance Guide Data[25]: This is a dataset from a guide provided by Microsoft as a case study in failure prediction. The data comes from multiple sources including time-series of voltage, rotation, pressure and vibration measurements collected from 100 machines in real time averaged over every hour, error logs, machine information (type, age etc.) and failure records indicating four different component failures. We adopt the same pre-processing, feature generation as given in the guide. Also, we downsample the data to take one sample every 15 hours.

For both datasets, we train on 70% of randomly selected co-variate time-series sequences and hold out 30% of the sequences for testing. From the training set a further 30% of the sequences are set aside as validation data to tune model parameters and policy thresholds. This process is repeated to produce 5 random splits. Model selection details and parameter settings are discussed in Appendix B.

5.1 Discussion of OTI interventions on real data

We fit the DDRSA-RNN on the covariate time-series leading upto failure and predict on a single test set sequence. This produces, at each time step j of the sequence, an estimated conditional hazard rate vector $\mathbf{H}_j$ (representing the conditional future hazard rates), which gives us a probabilistic estimate of the onset of the critical event at each time step, taking into account co-variate history upto that



(a) OTI policy on sequence A

(b) OTI policy on sequence B

Figure 2: The OTI policy in action on two sequences from the NASA Turbofan dataset. The OTI policy calls for intervention when the estimated time-to-event falls at or below a dynamicaly estimated policy threshold. As the cost C of missing the critical event increases, interventions are triggered earlier. The policy thresholds naturally adapt to the behavior of an individual sequence.

time step. The conditional hazard rates may then be converted into a corresponding survival function $\mathbf{S}_j$ at each time step. Figure 1 shows the evolution of estimated conditional hazard rate functions and survival functions. We see that the onset of the critical event is clearly reflected in the progression of the estimated vectors $\mathbf{H}_j$ and $\mathbf{S}_j$.

Next, we demonstrate the working of the OTI-policy, which was shown in Section 3 to be an adaptive threshold on the estimated time to event. Figure 2 plots the estimated time-to-event (estimated from $\mathbf{H}_j$ or $\mathbf{S}_j$ on two sequences as we progress toward the critical event. We also plot the adaptive threshold given by the OTI policy, calculated using Corrolary 3.1.2. An intervention is triggered when our time-to-event estimate falls on or below the adaptive threshold.

Consider what happens as the cost of missing the critical event is varied. First, note that since time-to-event is independent of C, only the threshold changes. On both sequences we see that increasing the cost of missing the critical event increases the threshold causing earlier intervention as we would expect. Also, note how the threshold adapts to the sequence. For example at the end of sequence B the threshold (which signifies continuation risk) rises rapidly while this feature is not observed on sequence A which is subject to different co-variate observations.

5.2 Performance Evaluation

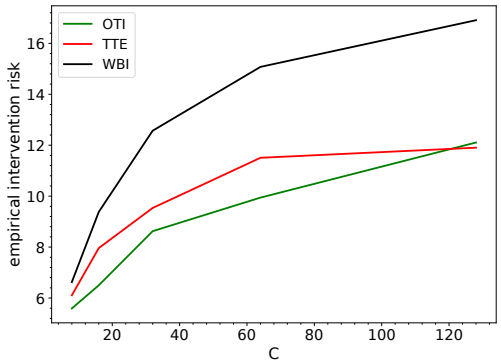
In this section we will evaluate the performance of the proposed OTI-DDRSA-RNN vs. two static threshold policy baselines: A window based intervention RNN (WBI-RNN) and a time-to-event based intervention RNN (TTE-RNN). All RNNs share the same encoder architecture which is an LSTM with 128 step lookback and hidden state dimension of 16 units.

WBI-RNN: The WBI-RNN uses a dense layer with a sigmoid activation to convert co-variate history encoded in the encoder final hidden-state into a probability that the critical event appears within a window of size W. A threshold is used on this probability to trigger interventions once the probability is greater than the threshold. The window size and threshold are selected by cross-validation (see Apendix B).

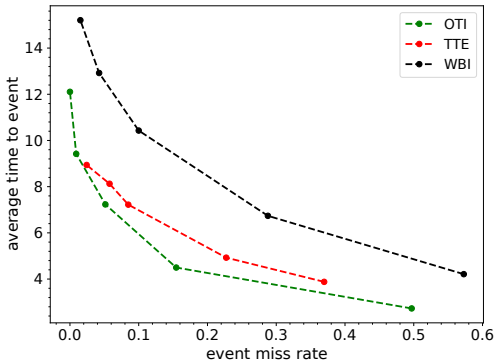
TTE-RNN: The TTE-RNN uses the same DDRSA-RNN architecture, sharing both encoder and decoder. However, interventions are trigerred with a static threshold that is set via cross-validation similar to the WBI-RNN.

All methods are evaluated using an empirical intervention policy risk (of policy φ) defined over a range of costs:

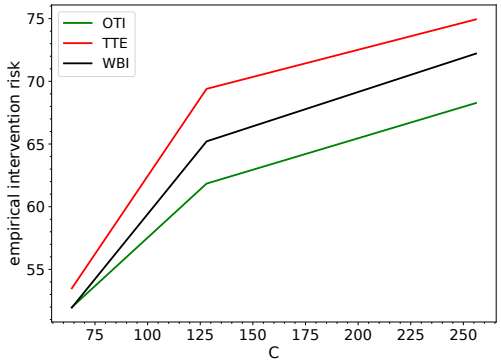
$$R_{emp}(\varphi) = \frac{1}{N}\sum_{i=1}^{N}(L_i - \tau_i)\mathbb{1}(L_i \geq \tau_i) + C_\alpha \mathbb{1}(L_i \leq \tau_i) \tag{13}$$



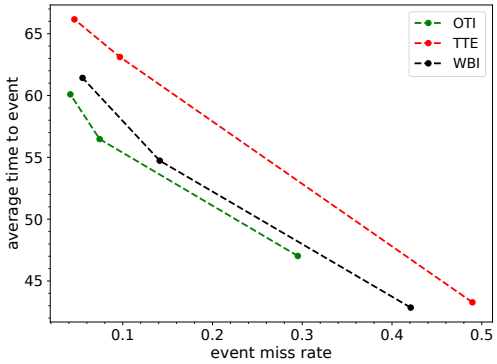
(a) NASA turbofan data: policy risk



(b) NASA turbofan data: tradeoff plot



(c) Azure PM guide data: policy risk



(d) Azure PM guide data: tradeoff plot

Figure 3: Performance comparison of OTI-DDRSA-RNN, WBI-RNN and TTE-RNN policies on the NASA Turbofan (top row) and Azure PM (bottom row) datasets. Average empirical intervention risk is plotted vs. cost C of missing the critical event (left column). The corresponding tradeoff between the probability of missing an event and the expected time-to-event (right column).

where $(L = L_i, \tau = \tau_i)$ are realized outcomes for the ith series.

All methods were trained on training splits and tuned on validation splits. Policy thresholds for the WBI-RNN and TTE-RNN that were determined on the different validation splits are averaged to get a final threshold setting. Allmodels are then re-trained on the entire training data (including validation splits) and tested on the test splits. Model parameter initialization seeds were also randomized between splits. Results reported are averaged over the 5 test splits.

Figure 3 summarizes the evaluation results. We see that the OTI-DDRSA-RNN outperforms the static threshold policies on both datasets having lower empirical intervention risk over a range of costs (Figure 3(a) and Figure 3(c)) reflected in a better tradeoff between time-to-event and critical event miss-rate. Regarding the tradeoff plots, a lower frontier is preferred, since it imples a lower time-to-event for a given miss-rate and a lower miss-rate for a given time-to-event. On the NASA dataset the TTE-RNN outperformed WBI-RNN while on the Azure PM data the WBI policy performed better.

Limitations Since both TTE and OTI policies are model based, the quality of the policies depend on the accuracy of the underlying survival analysis. A key challenge in this domain is the scarcity of publicly available datasets.

Supplemental Material The Appendix included with this submission as supplemental material includes complete proofs of all theorems, and further details on the experiments including model selection.

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   (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? **[TODO]**YES
   (b) Did you describe the limitations of your work? **[TODO]**YES
   (c) Did you discuss any potential negative societal impacts of your work? **[TODO]**No obvious negative societal impact
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