VARIANCE-BASED GRADIENT COMPRESSION FOR EFFICIENT DISTRIBUTED DEEP LEARNING

ABSTRACT

Due to the substantial computational cost, training state-of-the-art deep neural networks for large-scale datasets often requires distributed training using multiple computation workers. However, by nature, workers need to frequently communicate gradients, causing severe bottlenecks, especially on lower bandwidth connections. A few methods have been proposed to compress gradient for efficient communication, but they either suffer a low compression ratio or significantly harm the resulting model accuracy, particularly when applied to convolutional neural networks. To address these issues, we propose a method to reduce the communication overhead of distributed deep learning. Our key observation is that gradient updates can be delayed until an unambiguous (high amplitude, low variance) gradient has been calculated. We also present an efficient algorithm to compute the variance and prove that it can be obtained with negligible additional cost. We experimentally show that our method can achieve very high compression ratio while maintaining the result model accuracy. We also analyze the efficiency using computation and communication cost models and provide the evidence that this method enables distributed deep learning for many scenarios with commodity environments.

1 INTRODUCTION

Deep neural networks are attracting attention because of their outstanding prediction power in many application fields such as image recognition, natural language processing, and speech recognition. In addition, software frameworks are publicly available, making it easier to apply deep learning. However, their crucial drawback is the substantial computational cost on training. For example, it takes over a week to train ResNet-50 on the ImageNet dataset. Such long training time limits the number of trials possible when creating models.

Therefore, we must conduct distributed training using multiple computation workers (e.g., multiple GPUs in different nodes). However, by nature, workers need to frequently communicate gradients, which yields a severe bottleneck for scalability, especially when using lower bandwidth connections. For example, when using 1000BASE-T Ethernet, communication takes at least ten times longer than forward and backward computation for ResNet-50, making multiple nodes impractical. High performance interconnects such as InfiniBand and Omni-Path are an order of magnitude more expensive than commodity interconnects, which limits research and development of deep learning using large-scale datasets to a small number of researchers.

Although several methods have been proposed to compress gradient for efficient communication, they either suffer a low compression ratio or significantly harm the resulting model accuracy, particularly when applied to convolutional neural networks. There are mainly two lines of research: quantization and sparsification. Quantization-based methods include *1-bit SGD* (Seide et al., 2014) and *TernGrad* (Wen et al., 2017). Though they achieve small loss of accuracy by using at least one bit for each parameter, the compression ratio is limited. Sparsification-based methods include Strom (2015) and *QSGD* (Alistarh et al., 2017). While they can achieve high compression ratio, as we will see in our experiments, they harm the resulting model accuracy or suffer a low compression ratio, particularly when applied to convolutional neural networks.

To address these issues, we propose a new gradient compression algorithm to reduce the communication overhead of distributed deep learning. The proposed method belongs to the sparsification

approaches. Our key observation is that the variance of the gradient for each parameter point over iterations is a useful signal for compression. As almost all previous approaches of both sparsification and quantization only look at the magnitude of gradient, we believe that we are opening a new door for this field. In addition, we also show that our method can be combined with previous compression methods to further boost performance. We also present an efficient algorithm to compute the variance and prove that it can be obtained with negligible additional cost.

We experimentally demonstrate that our method can achieve a high compression ratio while maintaining result model accuracy. We also analyze the efficiency using computation and communication cost models and provide evidence that our method enables distributed deep learning for many scenarios with commodity environments.

Organization. The remainder of this paper is organized as follows: Section 2 provides the definitions and notations used in this paper. Section 3 reviews related work in this field. Section 4 presents the proposed method. Section 5 analyzes performance. Section 6 shows our experimental results, and we conclude in Section 7.

2 PRELIMINARIES

In this section, we describe an overview of distributed deep learning and parameter updates with compressed gradients.

2.1 CHALLENGES IN DATA PARALLEL STOCHASTIC GRADIENT DESCENT

In data parallel distributed Stochastic Gradient Descent (SGD), all workers have identical copies of the same model and calculate gradients using different subsets of training data. Gradients are shared across all workers, and each worker updates its local model using the shared gradients.

There are two well-known approaches to communication of gradients: synchronous and asynchronous. Even though our method can be applied to both of them, we focus on the synchronous approach in this paper. Each worker computes gradients and shares them with other workers using a synchronized group communication routine in every single training iteration, typically using a communication routine known as allreduce.

The challenge is that the communication is possibly a severe bottleneck in a training process. Gradients typically consist of tens of millions of floating point values so the total size of exchanged data can be large. For example, the model size of ResNet-50 (He et al. (2016)) is over 110 MB, and the size of gradients becomes large accordingly. Thus, the communication time has a significant effect on the total training time in environments equipped with a commodity interconnect hardware, such as 1Gb Ethernet. Also, in the synchronous approach, workers have to wait for completion of communication and their computing resources including GPUs are idle, which is a significant performance loss.

2.2 PROBLEM FORMULATION

In basic procedures of SGD, model parameters are updated as

$$x_{t+1} = x_t - \gamma \nabla f_t(x_t),$$

where x_t and $\nabla f_t(x_t)$ are model parameters and calculated gradients in time step t, respectively. f_t is a loss function and it differs between samples used in a mini-batch. γ is a step size.

To reduce an amount of data to be exchanged over a network, either quantization or sparsification or both are used as explained in Sec. 3.

3 RELATED WORK

There are two main approaches to gradient compression: quantization-based approaches and sparsification-based approaches. Quantization-based approaches reduce communication cost by expressing each gradient with fewer bits. If a baseline uses 32-bit floating points in communication,

then it can reduce the amount of communication by up to 32 times. Seide et al. (2014) showed that neural networks can be trained using only one sign bit per parameter. There are two key techniques in their algorithm. First, different values are used for the absolute value the encoded gradients are decoded for each column of weight matrix. Second, quantization errors are added to the gradients calculated in the next step. Its effectiveness has been experimentally verified through speech models. Wen et al. (2017) proposed TernGrad to encode gradients with 2 bits per parameter. The algorithm is characterized by its theoretically-guaranteed convergence and reported that it can successfully train GoogLeNet (Szegedy et al., 2015) on ImageNet with an average loss of accuracy of less than 2%.

As a second approach, sparsification-based approaches reduce communication cost by sending only a small fraction of gradients. Even though they require sending not only the values of gradients but also parameters' indexes, their strong sparsification reduces transmission requirements significantly. Strom (2015) proposed sending only gradients whose absolute values are greater than a user-defined threshold. The algorithm sends only sign bits and encoded indexes of parameters. Gradients are decoded up to the threshold and quantization errors are added to the gradients calculated in the next step as 1-bit stochastic gradients. Its effectiveness has also been experimentally verified on speech applications. Dryden et al. (2016) extended Strom's method. They proposed to use an adaptive threshold instead of using a user-defined threshold. They also introduced repeated sparsification of gradients in order to combine the algorithm with an efficient communication algorithm. Alistarh et al. (2017) proposed QSGD. QSGD stochastically rounds gradients to linearly quantized values, which are calculated in the algorithm. Their work enjoys strong theoretical properties in convex optimizations. Furthermore, they can control the trade-off between accuracy and compression. On the other hand, Strom's method does not work with small or large thresholds.

4 PROPOSED METHODS

In this section, we describe the proposed method. An efficient implementation and combination with other compression methods are also explained.

Our work belongs to the sparsification-based approaches. Previous works on this direction have focused on gradients with small magnitudes, and they rounded them to zero to sparsify. Our work diverges at this point. We propose using approximated variances of gradients instead of magnitudes. In our method, ambiguous gradients are not transmitted until clarified by additional data. Our method significantly reduces communication while maintaining accuracy. This method enables shifting the balance between accuracy and compression, as necessary. Furthermore, our work can be combined with sparsity-promoting quantization like QSGD. We also show that our work can be combined with Strom's method.

4.1 KEY CONCEPTS

The key idea of our method is delaying sending ambiguously estimated gradients. We consider a gradient to be ambiguous when its amplitude is small compared to its variance over the data points. We extend the standard updating method to the following:

$$x_{t+1} - \gamma r_{t+1} = x_t - \gamma(\nabla f_t(x_t) + r_t).$$

This extension follows Seide et al. (2014) and Strom (2015).

In previous works, the approximation errors are accumulated in r_t and used in future updates. In each step, parameters are updated only with approximated gradients represented by less number of bits. In our work, we interpret r_t as delayed update, not approximation errors.

We send the gradient of i-th parameter only when it satisfies the following criterion

$$\frac{\alpha'}{|B|} V_B[\nabla_i f_z(x)] < (\nabla_i f_B(x))^2, \tag{1}$$

where $|B|$ is a size of the mini-batch and α' is a hyper parameter representing required estimation accuracy. z is a each sample and B is a mini-batch, respectively and f_z and f_B are corresponding loss functions. $V_B[\nabla_i f_z(x)]$ is the sample variance of i-th parameter's gradients over a mini-batch B.

If we do not send some gradients, we add them to the next batch and recalculate (1) with increased batch size. For example, if we postpone sending a gradient 9 times consequently, the criterion (1) is calculated as if 10 times larger batch than usual mini-batch in the next step. Note that even though the criterion (1) is calculated as if we used a larger batch, what is used for an update is not the mean of the mini-batches across steps but the sum of them.

Following lemma supports our formulation.

Lemma 4.1. *(De et al., 2017) A sufficient condition that* $-\nabla f_B(x)$ *is a descent direction is*

$$\|\nabla f_B(x) - \nabla f(x)\|_2^2 < \|\nabla f_B(x)\|_2^2. \tag{2}$$

By the weak law of large numbers, the lhs can be estimated as follows.

$$E[\|\nabla f_B(x) - \nabla f(x)\|_2^2] \sim \frac{1}{|B|} V_B[\nabla f_z(x)]$$

Thus our formulation with $\alpha' > 1$ corresponds to an estimation of the sufficient condition (2) that a gradient decreases the loss function. Gradients become more likely to be sent as sample size increases. However, if once gradients are estimated with too high variances, it takes too long time for the gradient to be sent. Thus, we decay variance at every step. Details are described in subsection 4.4.

4.2 QUANTIZATION AND PARAMETER ENCODING

After deciding which gradients to send, each worker sends pairs of a gradient value and its parameter index as Strom (2015) and Alistarh et al. (2017). We further quantize each gradient value to 4-bit so that we can represent each pair in 32-bit as Strom (2015). The 4-bit consists of one sign bit and three exponent bits.

Our quantization for exponent bits is as follows. For a weight matrix W (or a weight tensor in CNN), let M be the maximum absolute value of gradients in it. First, for each gradient g_i in the weight matrix, if $|g_i|$ is larger than $2^{\lfloor \log_2 M_i \rfloor}$ truncate it to $2^{\lfloor \log_2 M_i \rfloor}$, otherwise, round to the closer value of $2^{\lfloor \log_2 |g_i| \rfloor}$ or $2^{\lceil \log_2 |g_i| \rceil}$. Let g_i' be the preprocessed gradient. Next, calculate a integer $d := \lfloor \log_2 M \rfloor - \log_2 g_i'$. If $d > 7$ then we do not send the value, otherwise, encode the integer from 0 to 7 using 3 bits. $\lfloor \log_2 M \rfloor$ is also sent for every weight matrix. An efficient implementation is presented in subsection 4.4.

Because our method is orthogonal to the quantization shown above, we can reduce communication cost further using sparsity promoting quantization methods such as QSGD. However, we used the quantization to show that enough level of sparsity is gained solely by our quantization method because it dumps only a small fraction of gradients as a byproduct of quantization. We show how to combine our method with a method in Strom (2015) later in this paper because the way of the combination is less obvious. We use a naive encoding for parameter indexes because the rest 28-bits are enough. We can further reduce the number of bits by compressing parameter indexes (Strom, 2015; Alistarh et al., 2017).

4.3 COMMUNICATION BETWEEN WORKERS

In distributed deep learning, the most important operation is to take the global mean of the gradients calculated in each worker. The operation is referred to as "allreduce." It consists of three steps: (1) collects all local arrays in each worker, (2) reduce them using a given arithmetic operator, which is summation in this case, and (3) broadcast the result back to all workers so that all workers obtain the identical copies of the array.

Conventional data parallel deep learning applications can enjoy the benefit of highly optimized allreduce implementations thanks to the fact that only the sum of the values has to be kept during the communication. However, after applying the proposed method to the local gradients, they are converted to a sparse data structure so the allreduce operation can no longer apply.

Dryden et al. (2016) and Aji & Heafield (2017) proposed sparsifying gradients multiple times to utilize a kind of allreduce for sparsification-based compressions. However, the accuracy is possibly

degraded when the gradients are highly compressed through repetitive compressions. Instead, we adopt allgatherv for communication, where each worker just sends the calculated gradients to other workers. We avoid to encode and decode gradients multiple times by allgatherv. In allgatherv communication cost hardly increase from a kind of allreduce because index overlap, which is needed for summation, rarely occur if the compression ratio is sufficiently higher than the number of workers. Some optimization methods, such as ADAM (Ba & Kingma, 2015). require parameter updates and postprocessing.They are done locally after the communication.

4.4 Efficient implementation

We first describe the efficient computation of the criterion (1) and second how to quantize gradients without additional floating points operations. We can efficiently compare squared mean and variance in the criterion (1) by just comparing squared mean of gradients and sum of squared gradients. That is,

$$\left(\sum_{z \in B} \frac{1}{|B|} \nabla_i f_z(x)\right)^2 > \alpha \sum_{z \in B} \left(\frac{1}{|B|} \nabla_i f_z(x)\right)^2$$

achieves our goal. Thus, we have to maintain only the sum of gradients and sum of squared gradients. Details are described in Appendix A. Decay of variance, described in subsection 4.1, is accomplished by multiplying hyperparameter $\zeta(< 1)$ to the sum of squared gradients at every step. Fig. 1 shows the final algorithm.

For our quantization, we can make use of the representation of floating points and we can achieve above quantization with only binary operations and integer arithmetics. We can calculate $2^{\lfloor \log_2 x \rfloor}$ by just masking mantissa to 0. We can also round values as described in subsection 4.2 by adding one to the most sinificant bit of mantissa as if x is an unsigned integer and then masking mantissa to 0.

4.5 Hybrid algorithm

We describe the way to combine our method with Strom's method. It becomes problematic how to modify variance when only parts of gradients are exchanged. We solve the issue simply by modifying a^2 to $(a - b)^2$. Let S be the value sent in a step (i.e. threshold, -threshold or 0). We correct squared gradients $\sum(\nabla_i f)^2$ to $\sum(\nabla_i f)^2 - 2S\sum(\nabla_i f) + S^2$. Fig. 2 shows the algorithm. We show the effectiveness of this combined algorithm by experiments in Sec. 6. Combinations with other works like QSGD and TernGrad are rather straightforward and we do not explore further in this paper.

5 Performance analysis

We estimate speedup of each iteration by gradient compression with a performance model of communication and computation. The total speed up of the whole training process is just the summation of each iteration because we adopt a synchronized approach.

In the communication part, the pairs of the quantized values and the parameter indexes in each node are broadcast to all nodes. The i-th node's input data size $n_i(i = 1, ..., p)$ may be different among nodes, where p denotes the number of nodes. An MPI function called allgatherv realizes such an operation. Because recent successful image recognition neural network models like VGG (Simonyan & Zisserman, 2015) or ResNet (He et al., 2016) have a large number of parameters, the latency term in communication cost can be ignored even if we achieve very high compression ratio such as $c > 1,000$. In such cases, a type of collective communication algorithms called the ring algorithm is efficient (Thakur et al., 2005). Its bandwidth term is relatively small instead its latency term is proportional to p. Although the naive ring allgatherv algorithm costs unacceptable $O(\max_i n_i \cdot p)$ time, Träff et al. (2008) proposed a method to mitigate it by dividing large input data, which is called pipelined ring algorithm. For example, an allgatherv implementation in MVAPICH adopts pipelined ring algorithm for large input data.

The calculation of variance of gradients dominates in the additional cost for the computation part of the proposed method. The leading term of the number of multiply-add operations in it is $2N|B|$,

Algorithm 1: Basic

hyperparam: B = batch size, ζ, α
foreach *parameters*:
    $r_i = 0$;
    $v_i = 0$;
while *not converged*:
    CalcGrad();
    foreach *parameters*:
        r_i += $\sum \frac{\nabla_i f_z}{B}$;
        v_i += $\sum(\frac{\nabla_i f_z}{B})^2$;
        if $r_i^2 > \alpha v_i$:
            Encode(residual);
            $r_i = 0$;
            $v_i = 0$;
        else:
            v_i *= ζ;
    CommunicateAndUpdate();

Figure 1: Basic algorithm of our variance-based compression. ζ and α are hyperparameters. Recommended value for ζ is 0.999. α controls compression and accuracy. $\nabla_i f_z$ denotes gradients of parameter i for each sample z in mini-batch. CalcGrad() is backward and forward computaion. Encode() includes quantization and encoding of indexes. CommunicateAndUpdate() requires sharing gradients and decode, then update parameters.

Algorithm 2: Hybrid

hyperparam: B = batch size, ζ, α, τ
foreach *parameters*:
    $r_i = 0$;
    $v_i = 0$;
while *not converged*:
    CalcGrad();
    foreach *parameters*:
        r_i += $\sum \frac{\nabla_i f_z}{B}$;
        v_i += $\sum(\frac{\nabla_i f_z}{B})^2$;
        if $|r_i| > \tau$ *and* $r_i^2 > \alpha v_i$:
            Encode(Sign(r_i));
            r_i −= Sign(r_i) τ;
            $v_i =$
                $\max(v_i - 2|r_i|\tau + \tau^2, 0)$;
        v_i *= ζ;
    CommunicateAndUpdate();

Figure 2: Hybrid algorithm of our variance-based compression and Strom's method. τ is a user-defined threshold required in Strom's method. Other parameters and notations are the same with Fig. 1.

where N and $|B|$ are the number of parameters and the local batch size, respectively. Other terms such as determination of sent indexes and application of decay are at most $O(N)$. Therefore hereafter we ignore the additional cost for the computation part and concentrate to the communication part.

We discuss the relationship between the compression ratio and the speedup of the communication part. As stated above, we ignore the latency term. The baseline is ring allreduce for uncompressed gradients. Its elapsed time is $T_r = 2(p-1)Ns\beta/p$, where s and β are the bit size of each parameter and the transfer time per bit, respectively. On the other hand, elapsed time of pipelined ring allgatherv is $T_v = \sum \lceil ((n_i/m) - 1)m\beta \rceil$, where m is the block size of pipelining. Defining c as the averaged compression ratio including change of the number of bits per parameter, T_v is evaluated as $T_v \leq (\sum n_i + (p-1)m)\beta = (Nsp/c + (p-1)m)\beta$. If we set m small enough, relative speedup is $T_r/T_v \geq 2(p-1)c/p^2$. Therefore we expect linear speedup in $c > p/2$ range.

6 EXPERIMENTS AND RESULTS

In this section, we experimentally evaluate the proposed method. Specifically, we demonstrate that our method can significantly reduce the communication cost while maintaining test accuracy. We also show that it can reduce communication cost further when combined with other sparsification methods, and even improves test accuracy in some settings.

We used CIFAR-10 (Krizhevsky, 2009) and ImageNet (Russakovsky et al., 2015), the two most popular benchmark datasets of image classification. We fixed the hyperparameter ζ in Fig. 1 and Fig. 2 to 0.999 in all experiments. We evaluated gradient compression algorithms from the following two viewpoints: accuracy and compression ratio. The accuracy is defined as the test accuracy at the last epoch, and the compression ratio is defined as the number of the total parameters of networks

Table 1: Training of a VGG-like network on CIFAR-10. τ denotes the threshold in Strom's method. α is the hyperparameter of our method described in the criterion (3). The number of bits of QSGD refers the number of bits to express gradients except for the sign bits. For each configuration, the median of the accuracy from five independent runs is reported. The compression column lists the compression ratio defined at the beginning of Sec. 6.

Method	Adam		Momentum SGD	
	Accuracy	Compression	Accuracy	Compression
no compression	88.1	1	91.7	1
Strom, $\tau = 0.001$	62.8	88.5	84.8	6.53
Strom, $\tau = 0.01$	85.0	230	10.6	991
Strom, $\tau = 0.1$	88.0	6,943	71.6	8,485
our method, $\alpha = 1$	88.9	78.1	89.3	30.8
our method, $\alpha = 1.5$	88.6	224	88.5	87.2
our method, $\alpha = 2.0$	88.5	433	88.7	170
hybrid, $\tau = 0.01, \alpha = 2$	85.0	1,942	87.6	984
hybrid, $\tau = 0.1, \alpha = 2$	88.2	12,822	87.1	12,397
QSGD, 2bit	85.3	36.0	88.3	40.3
QSGD, 3bit	85.5	48.2	89.2	40.1
QSGD, 4bit	87.0	32.7	91.2	27.5

divided by the average number of parameters sent. We do not consider the size of other unessential information required for the communication because they are negligible. In addition, we do not care the number of bits to express each gradient because we pack both a gradient and a parameter index into a 32 bit word as Strom (2015) in all experiments. Please note that, as all methods use allgatherv for communication, communication cost increases in proportion to the number of workers. Thus, high compression ratio is required to achieve sufficient speed up when using tens or hundreds of workers.

6.1 CIFAR-10

For experiments on CIFAR-10, we used a convolutional neural network similar to VGG (Simonyan & Zisserman, 2015). The details of the network architecture are described in Appendix B. We trained the network for 300 epochs with weight decay of 0.0005. A total number of workers was 8 and batch size was 64 for each worker. We applied no data augmentation to training images and just center-cropped both training and test images into 32x32. We used two different optimization methods: Adam (Ba & Kingma, 2015) and momentum SGD (Sutskever et al., 2013). For Adam, we used Adam's default parameter described in Ba & Kingma (2015). For momentum SGD, we set the initial learning rate to 0.4 and halved it at every 25 epochs. For each configuration, we report the median of the accuracy from five independent runs. Compression ratios are calculated based on the execution that achieved the reported accuracy.

Table 1 summarizes the results. Our method successfully trained the network with slight accuracy gain for the Adam setting and 2 to 3 % of accuracy degradation for the Momentum SGD setting. Compression ratios were also sufficiently high, and our method reduced communication cost beyond quantization-based approaches described in section 3. Even though QSGD with more than 4 bits achieved higher accuracy than our method in the Momentum SGD setting, our method achieved better accuracy in the Adam setting. We emphasize that, in both settings, our method reduced the communication cost more aggressively. Furthermore, we can combine QSGD with our method in a straightforward manner. On the other hand, Strom's method caused significant accuracy degradation. Counter-intuitively, the hybrid algorithm improved its accuracy, in addition to the further reduction of communication.

Our hypothesis for this phenomena is as follows. During the training with Momentum SGD, learning rate changed at every 25 epochs. Thus, if the rise of the learning curve is too slow, it is difficult to

Table 2: Training ResNet50 on ImageNet. τ denotes a threshold in Strom's method. α is the hyperparameter of our method described in the criterion (3). Accuracy is the test accuracy at the last epoch. Compression refers compression ratio defined in the beginning of Sec. 6.

Method	Accuracy	Compression
no compression	76.0	1
Strom, $\tau = 0.001$	75.2	2.14
Strom, $\tau = 0.01$	75.5	35.2
Strom, $\tau = 0.1$	75.5	2,002
our method, $\alpha = 1$	74.8	54.2
our method, $\alpha = 1.5$	75.1	163
our method, $\alpha = 2.0$	75.4	313
hybrid, $\tau = 0.01, \alpha = 2$	75.0	471
hybrid, $\tau = 0.1, \alpha = 2$	75.1	4,345

catch up later. We suppose that Strom's method is not good at beginning of training, which leads to the reported accuracy degradation. Now, we explain why the hybrid algorithm did not cause the problem. In Strom's algorithm, when a large positive gradient appears, it has no choice but send positive values for consequent steps even if calculated gradients in following mini-batches have negative values. On the other hand, in the hybrid algorithm, if following gradients have the different sign with a residual, the residual is not likely to be sent. We assume that this effect helped the training procedure and lead to better accuracy. We also would like to mention the difficulty of hyperparameter tuning in Strom's method. As Table 1 shows, using lower threshold does not necessarily always lead to higher accuracy. This is because the hyperparameter controls both its sparsification and quantization. Thus, users do not have idea whether we should use larger or smaller value as a threshold to maintain accuracy.

6.2 IMAGENET

As larger scale experiments, we trained ResNet-50 (He et al., 2016) on ImageNet. We followed training procedure of Goyal et al. (2017) including optimizer, hyperparameters and data augmentation. We used batch size 32 for each worker and used 16 workers.

Table 2 summarizes the results. Our method and the hybrid algorithm both successfully finished training with only slight accuracy degradation and significant reduction of communication cost. Note that training configuration is highly optimized to training without any compression. For reference, the original paper of ResNet-50 reports its accuracy as 75.3% (He et al., 2016). Wen et al. (2017) reports that it caused up to 2% accuracy degradation in training with GoogLeNet (Szegedy et al., 2015) on ImageNet and our method causes no more degradation compared to quantization-based approaches. The hybrid algorithm is especially attractive as it achieved far better compression compared to Strom's method with only slight accuracy degradation. This indicates that the property of the proposed method is orthogonal to sparsification-based methods, and thus their combination performs very well.

7 CONCLUSION

We proposed a novel method for gradient compression. Our method can reduce communication cost significantly with no or only slight accuracy degradation. Contributions of our work can be summarized in the following three points. First, we proposed a novel measurement of ambiguity (high variance, low amplitude) to determine when a gradient update is required. Second, we showed the application of this measurement as a threshold for updates significantly reduces update requirements, while providing comparable accuracy. Third, we demonstrated this method can be combined with other efficient gradient compression approaches to further reduce communication cost.

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A   A SIMPLIFIED VIEW OF THE VARIANCE-BASED CRITERION

We derive that the criterion (3) corresponds to the criterion (1).

$$\left(\sum_{z \in B} \frac{1}{|B|} \nabla_i f_z(x)\right)^2 > \alpha \sum_{z \in B} \left(\frac{1}{|B|} \nabla_i f_z(x)\right)^2$$

$$\Longleftrightarrow \quad \frac{|B| - \alpha}{|B|} \left(\sum_{z \in B} \frac{1}{|B|} \nabla_i f_z(x)\right)^2 > \alpha \frac{1}{|B|} \left(\frac{1}{|B|} \sum_{z \in B} (\nabla_i f_z(x)) - \sum_{z \in B} \left(\frac{1}{|B|} \nabla_i f_z(x)\right)^2\right)$$

$$\Longleftrightarrow \quad \frac{|B| - \alpha}{|B|} \left(\sum_{z \in B} \frac{1}{|B|} \nabla_i f_z(x)\right)^2 > \alpha \frac{1}{|B|^2} \sum_{z \in B} \left(\nabla_i f_z(x) - \frac{1}{|B|} \sum_{z \in B} (\nabla_i f_z(x))\right)^2$$

$$\Longleftrightarrow \quad \frac{|B| - \alpha}{|B|} \left(\sum_{z \in B} \frac{1}{|B|} \nabla_i f_z(x)\right)^2 > \alpha \frac{|B| - 1}{|B|^2} \frac{1}{|B| - 1} \sum_{z \in B} \left(\nabla_i f_z(x) - \frac{1}{|B|} \sum_{z \in B} (\nabla_i f_z(x))\right)^2$$

$$\Longleftrightarrow \quad \frac{|B| - \alpha}{|B|} \left(\sum_{z \in B} \frac{1}{|B|} \nabla_i f_z(x)\right)^2 > \alpha \frac{|B| - 1}{|B|^2} V_B[\nabla_i f_z(x)]$$

$$\Longleftrightarrow \quad \nabla_i f_B(x)^2 > \alpha \frac{|B| - 1}{|B| - \alpha} \frac{1}{|B|} V_B[\nabla_i f_z(x)]$$

$V_B[\nabla_i f_z(x)]/|B|$ is an estimated variance of the means of gradients in mini-batches with size $|B|$. For $\alpha = 1$, the above criterion reduces to

$$\nabla f_B(x)^2 > \frac{1}{|B|} V_B[\nabla f_z(x)],$$

which is an estimated version of the sufficient condition (2). The term $(|B| - 1)/(|B| - \alpha)$ is approximately 1 in most settings.

B   ARCHITECTURE OF VGG-LIKE NETWORK USED ON CIFAR-10

Table 3 shows the network architecture used for experiments on CIFAR-10. All convolutional layers are followed by batch normalization and ReLU activation. The code is available in examples of Chainer (Tokui et al., 2015).

Table 3: The network architecture for the CIFAR-10 classification task

input (3x32x32)
conv3-64
dropout(0.3)
conv3-64
maxpool
conv3-128
dropout(0.4)
conv3-128
maxpool
conv3-256
dropout(0.4)
conv3-256
dropout(0.4)
conv3-256
maxpool
conv3-512
dropout(0.4)
conv3-512
dropout(0.4)
conv3-512
maxpool
conv3-512
dropout(0.4)
conv3-512
dropout(0.4)
conv3-512
maxpool
dropout(0.5)
fully connected 512
bn
relu
dropout(0.5)
fully connected 10
softmax