Generalization in Cooperative Multi-Agent Systems

Anonymous authors
Paper under double-blind review

Abstract

Collective intelligence is a fundamental trait shared by many species that has allowed them to thrive in diverse environmental conditions. From simple organisations in an ant colony to complex systems in human groups, collective intelligence is vital for solving many survival tasks. Such natural systems are flexible to changes in their structure: they generalize well when the abilities or number of agents change, which we call *Combinatorial Generalization* (CG). CG is a highly desirable trait for autonomous systems as it can increase their utility and deployability across a wide range of applications. While recent works addressing specific aspects of CG have shown impressive results on complex domains, they provide no performance guarantees when generalizing to novel situations. In this work, we shed light on the theoretical underpinnings of CG for cooperative multi-agent systems (MAS). Specifically, we study generalization bounds under a linear dependence of the underlying dynamics on the agent capabilities, which can be seen as a generalization of Successor Features to MAS. We then extend the results first for Lipschitz and then arbitrary dependence of rewards on team capabilities. Finally, empirical analysis on various domains using the framework of multi-agent reinforcement learning highlights important desiderata for multi-agent algorithms towards ensuring CG.

1 Introduction

Imagine attending a football summer camp. The coach decides to split the participating players into random teams for practice. While each player has different capabilities (e.g., defending, dribbling, speed, and pace), they quickly adapt to the other players in the team to facilitate the common objective of outscoring their opponents. Furthermore, they smoothly adjust to unexpected events such as a player getting hurt and retiring with substitution, which forces them to change their behaviours and adjust their roles. Similarly, they rapidly adjust to changes in team size (as a result of a player being sent off or new players joining the team).

Such adaptations are typically possible for two reasons. First, the players understand each others' *capabilities*, including how a change in capabilities affects the underlying environment and chances of success. Second, players have coordination protocols for adapting to the changes, both explicitly (e.g., communicating the game plan) or implicitly (inferring capabilities from observations, e.g., passing the ball to a player going in for an attack). This phenomenon, which we call *Combinatorial Generalization* (CG), is not specific to football or humans, and organisms in general manifest abilities to adapt in almost every situation requiring team efforts (Crozier et al., 2010; Nouyan et al., 2009; Anderson & McMillan, 2003).

In order to capture specific aspects of CG, recent methods in multi-agent reinforcement learning (MARL) utilize advances in deep learning architectures, such as graph neural networks Ryu et al. (2020) and attention mechanisms Iqbal et al. (2021), as well as extensively tuned training regimes, such as a mixture of human and generated data, self-play, and population-based training (Vinyals et al., 2019; OpenAI et al., 2019). While these methods show impressive empirical performance on complex domains, they provide little insight into aspects of when and how much generalization to expect. These are crucial for deploying agents in the real world due to practical considerations like tolerance and minimum expected performance in unseen settings. Additionally, the problem of sample-efficient generalization, already hard for single-agent RL (Mahajan & Tulabandhula, 2017; Du et al., 2020; Ghosh et al., 2021; Malik et al., 2021), is particularly challenging in the multi-agent case. Specifically, even when the underlying task remains the same, agents in MARL typically need to be trained from scratch for different team compositions. Moreover, across similar tasks with similar team compositions, there is a lack of modularity for sharing knowledge to enable quick learning (Wang et al., 2020).

Thus, we posit that a theoretical understanding of generalization in multi-agent systems (MAS) can help address both of the above-mentioned issues: it can provide important performance guarantees for practical deployment and can additionally inform better algorithm design to ensure sample efficiency.

We first highlight the key properties that make CG particularly difficult for MAS:

- **P1:** The capabilities of agents can come from infinite sets, e.g., maximum permissible torque for an agent joint which can take values in a continuous set.
- **P2:** Combinatorial blow-up in the number of possible teams (w.r.t. agent capabilities) given a team size.
- **P3:** The capabilities need to be grounded w.r.t. the dynamics of the environment, ie. the agent needs to infer how the capability affects the long term utility in terms of joint rewards and transitions. This becomes increasingly hard with team size (similar to credit assignment).
- **P4:** Team sizes can vary across different tasks.
- **P5:** Agents need to infer the capabilities of teammates in settings where it is hidden, in a potentially non-stationary environment.

P2-**P4** particularly distinguish CG from single-agent generalization, highlighting its combinatorial nature. Furthermore, **P5** requires agents to adapt to changing teammate policies, making the problem harder.

In this work, we analyse multi-agent generalization by modelling the dependence of underlying environment rewards and transitions on agent capabilities. We first look at generalization bounds for the case when the environment dynamics are linear with respect to the agent capabilities. We elucidate how this generalizes the successor feature (SF) framework Barreto et al. (2016) to the multi-agent case. We provide theoretical bounds for generalization between team compositions, transfer of optimal policy from one team to another and changes to optimal values arising from agent addition and elimination under this framework. Next, we bound the performance gap as a result of an error in estimating the agent capabilities, which covers scenarios such as lossy or inaccurate communication. Furthermore, we provide bounds for optimal value deviation when the dynamics themselves are approximately linear. Finally, we elucidate how the bounds can be extended to Lipschitz rewards (Appendix A.6) and then extend this framework to study arbitrary dependence of rewards on capabilities to shed light on when generalization can be difficult (Appendix A.7). Our results apply to various training and deployment settings in MAS and are agnostic to the type of algorithm used (MARL or other forms of policy search methods). Finally, we empirically analyse popular methods in MARL on tasks of varying difficulty in terms of generalization and discuss important desiderata to be met for better generalization.

2 Background and Formulation

Multi-Agent Reinforcement Learning

We model the cooperative multi-agent task as a decentralized partially observable MDP (Dec-POMDP) Oliehoek & Amato (2016). A Dec-POMDP is formally defined as a tuple $G = \langle S, U, P, R, Z, O, n, \rho, \gamma \rangle$. S is the state space of the environment, ρ is the initial state distribution. At each time step t, every agent $i \in \mathcal{A} \equiv \{1, ..., n\}$ chooses an action $u^i \in U$ which forms the joint action $\mathbf{u} \in \mathbf{U} \equiv U^n$. $P(s'|s, \mathbf{u}) : S \times \mathbf{U} \times S \to [0, 1]$ is the state transition function. $R(s) : S \to [0, 1]$ is the reward function shared by all agents and $\gamma \in [0, 1)$ is the discount factor. A Dec-POMDP is *partially observable* (Kaelbling et al., 1998): each agent i does not have access to the full state and instead samples observations $z \in Z$ according to observation distribution $O(s, i) : S \times \mathcal{A} \to \mathcal{P}(Z)$. Without loss of generality (WLOG), we assume the state is a represented as a k-dimensional feature vector $S \subset [0, 1]^k$ and similarly observations $Z \subset [0, 1]^l$. When the observation function O is identity, the problem becomes a multi-agent MDP (MMDP). Similarly, when the observations are invertible for each agent, so that the observation space is partitioned w.r.t. S, i.e., $\forall i \in \mathcal{A}, \forall s_1, s_2 \in S, \forall z_i \in Z, P(z_i|s_1) > 0 \wedge s_1 \neq s_2 \implies P(z_i|s_2) = 0$, we classify the problem as a multi-agent richly observed MDP (M-ROMDP) Mahajan et al. (2021). The action-observation history for an agent i is $\tau^i \in T \equiv (Z \times U)^*$. We use u^{-i} to denote the action of all the agents other than i and similarly for the policies π^{-i}. The value of a policy is defined as $V^\pi = \mathbb{E}_{\pi,\rho}\left[\sum_{t=0}^{\infty} \gamma^t R_{\mathcal{T}}(s_t)\right]$. Similarly, the

joint action-value function given a policy π is defined as: $Q^\pi(s_t, \mathbf{u}_t) = \mathbb{E}_\pi \left[\sum_{k=0}^{\infty} \gamma^k R(s_{t+k}) | s_t, \mathbf{u}_t\right]$. The goal is to find the optimal policy π^* corresponding to the optimal value function V^*.

MARL with Agent Capabilities

We now extend the MARL problem setting for generalisation where agents can have different capabilities. To this end, we assume that each agent in the task can be characterised by a d-dimensional *capability vector* $c \in \mathcal{C}$, which governs its contribution to rewards and transition dynamics (and thus its policy/behaviour denoted as $\pi^i(\ .\ ; c)$). Without loss of generality, we assume $\mathcal{C} \subseteq \Delta_{d-1}$ (the $d-1$ dimensional simplex). Intuitively, an agent's capability reflects the abilities of an agent along various properties that may be important for solving the collective task (e.g., an agent's speed, health recovery, and accuracy). We next assume an unknown probability distribution $\mathcal{M} : \mathcal{C}^n \to \mathbb{R}^+$ with support $Sup(\mathcal{M})$ over a subset of the joint capability space $\mathcal{C}^n$. Any $\mathcal{T}$ sampled from $\mathcal{M}$ can be seen as a tuple of capability vectors $\mathcal{T} = (c_i)_{i=1}^n$, one for each agent in the team. We augment the Dec-POMDP with $\mathcal{T}$: $G = \langle S, U, P_\mathcal{T}, R_\mathcal{T}, Z, O, n, \rho, \gamma, \mathcal{T}\rangle$ and call it a *variation* for the MARL setting [1]. Thus $\mathcal{T}$ defines the rewards and transition dynamics of the underlying MMDP (ie. $R_\mathcal{T}(s) = \langle f(\mathcal{T}) \cdot s\rangle$ where $\langle\cdot\rangle$ is the dot product[2] and $f : \mathcal{C}^n \to \mathbb{R}^k$ and similarly for transitions). Our goal is then to find algorithms, which when trained on a small number of *variations* sampled from $\mathcal{M} : \{\mathcal{T}^j\}_{j=1}^M$, generalise well to unseen team variations in $\mathcal{M}$. i.e., we want to maximise the expected value over the team variation distribution,

$$\max_\pi \ \mathbb{E}_{\mathcal{T}\sim\mathcal{M}} \left[\mathbb{E}_{\pi(\cdot;\mathcal{T}), P_\mathcal{T}, \rho} \left[\sum_{t=0}^{\infty} \gamma^t R_\mathcal{T}(s_t)\right]\right], \tag{1}$$

where $\pi = \{\pi^i\}_{i=1}^n$ is a group of n agents. The challenge here arises because of two main factors. First, the agents do not have any prior knowledge about what these capability vectors mean, and are thus required to learn their semantics (also called grounding). Second, in the setting where the agents cannot observe the capability vectors (including possibly their own), they have to infer and learn protocols for sharing them with each other in order to generalize in a zero-shot setting.

Successor Features

Thus successor features (SF) framework Dayan (1993); Barreto et al. (2016; 2018; 2020) assumes that the rewards in an MDP can be decomposed as $r(s) = \boldsymbol{\phi}(s)^\top \mathbf{w}$, where $\boldsymbol{\phi}(s) \in \mathbb{R}^d$ are features of s and $\mathbf{w} \in \mathbb{R}^d$ are weights[3]. When no assumption is made about $\boldsymbol{\phi}(s)$, any reward function can be recovered using this representation. The value function then follows

$$\begin{aligned} V^\pi(s) &= \mathrm{E}^\pi \left[r_{t+1} + \gamma r_{t+2} + ... \,|\, S_t = s\right] \\ &= \mathrm{E}^\pi \left[\boldsymbol{\phi}_{t+1}^\top \mathbf{w} + \gamma \boldsymbol{\phi}_{t+2}^\top \mathbf{w} + ... \,|\, S_t = s\right] \\ &= \boldsymbol{\psi}^\pi(s)^\top \mathbf{w}. \end{aligned}$$

Here $\boldsymbol{\psi}^\pi(s)$ is called the *successor feature* of s under policy π. The ith component of SF $\boldsymbol{\psi}^\pi(s)$ provides the expected discounted sum of ϕ_i when following policy π from s.

3 Analysis

As mentioned before, we are interested in understanding how the long term joint-utility of a cooperative group changes with changes happening in the group. Our analysis here focuses on the generalisation properties w.r.t. $\mathcal{M}$. We focus on the case of MMDPs for ease of exposition, but similar results for the more general cases can be obtained by suitable assumptions for identifiability of the state (e.g., M-ROMDP in Mahajan et al. (2021)). Our results are applicable irrespective of whether agents can observe the capabilities. They are also agnostic to the training and deployment regimes (e.g., centralized or decentralized) and the algorithm being used to find the policy. **All the proofs can be found in Appendix A.** For the analysis we assume

[1] Agent capabilities can also be interpreted as the contexts, see Hallak et al. (2015)
[2] Note that this is still the most general form as states can be encoded as one-hot vectors, see Barreto et al. (2016).
[3] Similar formulations hold WLOG for $\boldsymbol{\phi}$(s,a),$\boldsymbol{\phi}$(s,a,s')

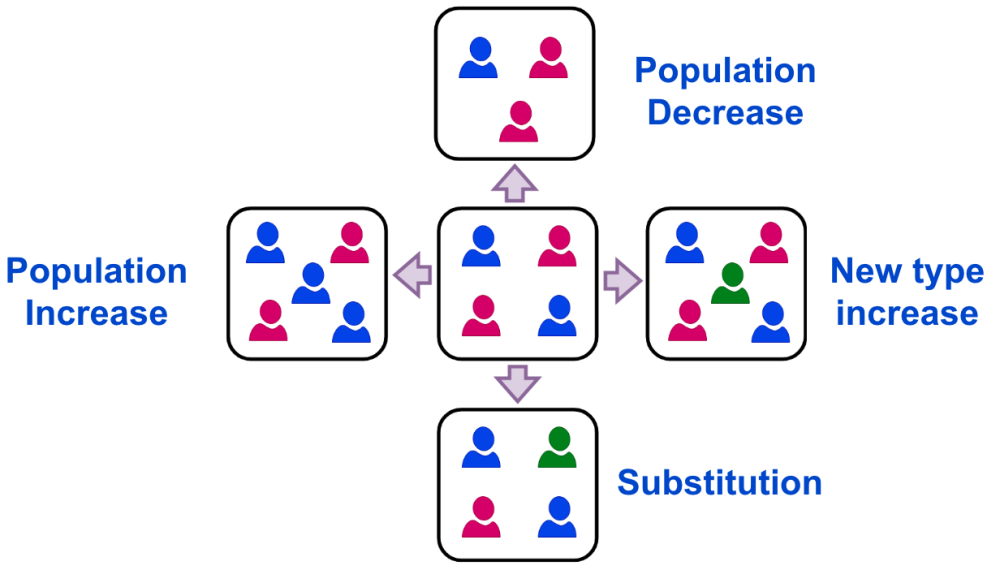


Figure 1: Combinatorial Generalization in MAS, various settings.

that the rewards and transitions depend linearly on the agents capabilities c_i :

$$R_{\mathcal{T}}(s) = \sum_{i=1}^{n} a_i \langle c_i \cdot W_R s \rangle \tag{2}$$

$$P_{\mathcal{T}}(s'|s, \mathbf{u}) = \sum_{i=1}^{n} a_i \langle c_i \cdot W_P(s', s, \mathbf{u}) \rangle \tag{3}$$

where $W_R \in \mathbb{R}^{dk}$ is the reward kernel of the MMDP and defines the dependence of the rewards on each capability component. Similarly in Eq. (3), $W_P : S \times \mathbf{U} \times S \times \{1..d\} \to [0, 1]$ defines the transition kernel of the MMDP so that $P_j(\cdot|s, \mathbf{u}) \triangleq W_P(s, \mathbf{u}, j) \in \Delta_{|S|-1}, j \in \{1..d\}$ give the next state distribution as directed by the jth component of the capability and agent i's propensity (unweighted) to make the state transition to s' is given by $\left\langle c_i \cdot \left[P_1(s'|s, \mathbf{u}) \dots P_d(s'|s, \mathbf{u})\right]\right\rangle = \langle c_i \cdot W_P(s', s, \mathbf{u})\rangle$. Finally $(a_i)_{i=1}^n \in \Delta_{n-1}$ are the *influence weights* of agents which quantify the influence of agent i in determining the rewards and transitions. Under the linear setting, given a policy π and capabilities $\mathcal{T}$ we have that value function satisfies $V_{\mathcal{T}}^{\pi} = \sum_{i=1}^{n} a_i \langle c_i \cdot W_R \mu_{\mathcal{T}}^{\pi} \rangle$ where $\mu_{\mathcal{T}}^{\pi} = E_{\rho, P_{\mathcal{T}}, \pi}[\gamma^t s_t]$ are the expected discounted state features and similarly for a given state s, $V_{\mathcal{T}}^{\pi}(s) = \sum_{i=1}^{n} a_i \langle c_i^T W_R \cdot \mu_{\mathcal{T}}^{\pi}(s) \rangle$ where $\mu_{\mathcal{T}}^{\pi}(s) = E_{P_{\mathcal{T}}, \pi}[\gamma^t s_t | s_0 = s]$. The linear formulation for dynamics generalizes the successor feature Barreto et al. (2016) formulation to the MAS setting, this can be seen by noting that when the dependence of transition dynamics on capabilities is dropped (Eq. (3)) and only single agent is considered (by considering a one-hot a), we get the successor feature formulation with capability of the nonzero a_i interpreted as the task weight in (Barreto et al., 2016)(see Section 2). We now present the first result concerning the difference between the optimal values of two different team compositions:

Theorem 1 (Generalisation between team compositions)**.** *Let team compositions $\mathcal{T}^x, \mathcal{T}^y \in \mathcal{C}^n$ with influence weights $a^x, a^y \in \Delta_{n-1}$, $s_{max} = \max_s ||W_R s||_1$, $V_{mid} = \frac{1}{2} \max_s V_{\mathcal{T}^y}^*(s)$, Then*[4]*:*

$$|V_{\mathcal{T}^x}^* - V_{\mathcal{T}^y}^*| \leq \frac{s_{max} + \gamma d V_{mid}}{\gamma(1-\gamma)} \Psi, \text{ where}$$

$$\Psi = \left[|\sum_i a_i^x (\mathcal{T}_i^x - \mathcal{T}_i^y)|_\infty + |\sum_i (a_i^x - a_i^y) \mathcal{T}_i^y|_\infty \right] \tag{4}$$

[4]for $\gamma \in (0, \frac{\sqrt{5}-1}{2})$ we can replace $\frac{1}{\gamma(1-\gamma)}$ by $\frac{1+\gamma}{1-\gamma}$

Theorem 1 gives an interesting decomposition of an upper bound to the difference of the optimal values between the two team compositions. The first terms in the square brackets on the RHS denote contributions arising purely from substituting the old capacities with the new one. The second term denotes the contribution arising from a change in how much influence the agents have over the dynamics of the MMDP.

Corollary 1.1 (Change in optimal value as a result of agent substitution)**.** *Let $\mathcal{T} \in \mathcal{C}^n$ be a team composition with influence weights $a \in \Delta_{n-1}$. If agent i is substituted with i' keeping a_i unchanged such that $|\mathcal{T}_{i'} - \mathcal{T}_i|_\infty \leq \epsilon_C$ then the new team ($\mathcal{T}'$) optimal value follows:*

$$|V^*_{\mathcal{T}'} - V^*_{\mathcal{T}}| \leq \frac{(s_{max} + \gamma d V_{mid}) a_i \epsilon_C}{\gamma(1-\gamma)}$$

We define an important policy concept which captures the absolute optimality for an oracle with access to the capabilities. For the ease of exposition we consider fixed influence weights a and define a metric on the joint capability space as $d_a(\mathcal{T}^x, \mathcal{T}^y) = |\sum_i a_i(\mathcal{T}^x_i - \mathcal{T}^y_i)|_\infty$. We similarly generalize this metric to distances between sets by taking the infimum of the distances between pairs of points in the cross product $d_a(\mathcal{M}_x, \mathcal{M}_y) \triangleq \inf_{\mathcal{T}^x \in \mathcal{M}_x, \mathcal{T}^y \in \mathcal{M}_y} d_a(\mathcal{T}^x, \mathcal{T}^y)$.

Definition 1 (Absolute Oracle)**.** *Let $\pi^*_{\mathcal{M}}$ be the oracle policy which optimizes Eq.* (1) *ie. $\pi^*_{\mathcal{M}}$ is the multiplexer policy which given a team composition $\mathcal{T}$ behaves identically to the optimal policy for $\mathcal{T}^j$ where $\mathcal{T}^j \in \arg\min_{\mathcal{T}^l \in Sup(\mathcal{M})} d_a(\mathcal{T}^l, \mathcal{T})$.*

We now answer the question of what happens when agents are trained on specific capabilities but the learnt policy is used on potentially unseen capabilities (this could occur, e.g., due to changes in hardware components).

Theorem 2 (Transfer of optimal policy)**.** *Let $\mathcal{T}^x, \mathcal{T}^y \in \mathcal{C}^n$, $a^x, a^y \in \Delta_{n-1}$, $s_{max} = \max_s ||W_R s||_1$, $V_{mid} = \frac{1}{2}\max_s V^*_{\mathcal{T}^y}(s)$. Let π^*_y be the optimal policy for the team composed of agents with capabilities $\mathcal{T}^y$ and influence weights a^y. Then:*

$$V^*_{\mathcal{T}^x} - V^{\pi^*_y}_{\mathcal{T}^x} \leq 2\frac{s_{max} + \gamma d V_{mid}}{\gamma(1-\gamma)}\Psi,$$

where Ψ is defined as in Eq. (4)*.*

Corollary 2.1 (Out of distribution performance)**.** *Let $\mathcal{T} \notin Sup(\mathcal{M})$ be an out of distribution task, we then have that the performance of the absolute oracle policy on $\mathcal{T}$ satisfies:*

$$V^*_{\mathcal{T}} - V^{\pi^*_{\mathcal{M}}}_{\mathcal{T}} \leq 2\frac{s_{max} + \gamma d V_{mid}}{\gamma(1-\gamma)} d_a(\mathcal{T}, Sup(\mathcal{M})),$$

We now address the scenarios when the team population changes.

Theorem 3 (Population decrease bound)**.** *For the team composition $\mathcal{T} \in \mathcal{C}^n$ with influence weights $a \in \Delta_{n-1}$. If agent n is eliminated followed by a renormalization of influence weights, we have that for the remaining team ($\mathcal{T}^- \triangleq (\mathcal{T})_{i=1}^{n-1}$):*

$$|V^*_{\mathcal{T}^-} - V^*_{\mathcal{T}}| \leq \frac{a_n(s_{max} + \gamma d V_{mid})}{\gamma(1-\gamma)} \Big| \sum_{i=1}^{n-1} \frac{a_i \mathcal{T}_i}{1-a_n} - \mathcal{T}_n \Big|_\infty.$$

The special case when $\sum_{i=1}^{n-1} \frac{a_i \mathcal{T}_i}{1-a_n} = \mathcal{T}_n$ for the linear dynamics formulation when an agent n can in principle be rendered redundant if the rest of the agents in the team can effectively provide a perfect substitute. In fact, this holds true as long as capacity $\mathcal{T}_n$ can be formed from a convex combination of the capabilities $\mathcal{T}_i, i \in \{1..n-1\}$. The latter case however requires using the corresponding convex coefficients instead of re-1normalization. A similar bound can be easily constructed for reusing the policy after an agent eliminated to give the corresponding transfer bound along the lines of Theorem 2.

Corollary 3.1 (Population increase bound)**.** *For the team composition $\mathcal{T} \in \mathcal{C}^n$ with influence weights $a \in \Delta_{n-1}$. If agent $n+1$ is added with capability $\mathcal{T}_{n+1}$ and weight a_{n+1} (other weights scaled down by $\lambda = 1 - a_{n+1}$) we have that for the new team ($\mathcal{T}^+ \triangleq (\mathcal{T}_1..\mathcal{T}_n, \mathcal{T}_{n+1})$):*

$$|V^*_{\mathcal{T}^+} - V^*_{\mathcal{T}}| \leq \frac{a_{n+1}(s_{max} + \gamma d V_{mid})}{\gamma(1-\gamma)} \Big| \sum_{i=1}^{n} a_i \mathcal{T}_i - \mathcal{T}_{n+1} \Big|_\infty .$$

We next extend the generalization bound Theorem 1 to include the scenario where the reward and the transition dynamics are not exactly linear but are approximately linear with deviation $\hat{\epsilon}_R$,$\hat{\epsilon}_P$ respectively.

Theorem 4 (Approximate $\hat{\epsilon}_R$,$\hat{\epsilon}_P$ dynamics)**.** *Let $\mathcal{T}^x, \mathcal{T}^y \in \mathcal{C}^n$, $a^x, a^y \in \Delta_{n-1}$ and the dynamics be only approximately linear so that $|R_{\mathcal{T}}(s) - \sum_{i=1}^{n} a_i \langle c_i \cdot W_R s \rangle| \leq \hat{\epsilon}_R$ and $|P_{\mathcal{T}}(s'|s, \mathbf{u}) - \sum_{i=1}^{n} a_i \langle c_i \cdot W_P(s', s, \mathbf{u}) \rangle| \leq \hat{\epsilon}_P$. Then:*

$$|V^*_{\mathcal{T}^x} - V^*_{\mathcal{T}^y}| \leq \frac{s_{max} + \gamma d V_{mid}}{\gamma(1-\gamma)} \Psi + \frac{2(\hat{\epsilon}_R + \gamma \hat{\epsilon}_P V_{mid})}{\gamma(1-\gamma)},$$

where Ψ is defined as in Eq. (4).

The other bounds for transfer and population change can similarly be obtained for the approximate dynamics case.

We now consider the scenario when the capabilities are not directly observed but inferred using an approximator which in turn introduces some errors in their estimation (this could happen due to noise in observations, inaccurate implicit or explicit communication protocols, etc.).

Theorem 5 (Error from estimation of capabilities)**.** *For the team composition $\mathcal{T} \in \mathcal{C}^n$ with influence weights $a \in \Delta_{n-1}$. If the agent capabilities are inaccurately inferred as $\hat{\mathcal{T}}$ with $\max_i |\mathcal{T}_i - \hat{\mathcal{T}}_i|_\infty \leq \epsilon_{\mathcal{T}}$ and agents learn the inexact policy $\hat{\pi}^*$ then:*

$$|V^*_{\mathcal{T}} - V^{\hat{\pi}^*}_{\mathcal{T}}| \leq \frac{2\epsilon_{\mathcal{T}}(s_{max} + \gamma d V_{mid})}{\gamma(1-\gamma)},$$

where $V_{mid} = \frac{1}{2} \max_s V^*_{\hat{\mathcal{T}}}(s)$.

All our results can be easily extended to the setting where rewards $R_{\mathcal{T}}(s) = \langle f(\mathcal{T}) \cdot W_R s \rangle$, $f(\mathcal{T})$ is not linear in capabilities as in Eq. (2) but is Lipschitz with coefficient L_i for $i \in \mathcal{A}$. For example, Theorem 1 becomes:

Theorem 6. *For rewards L_i Lipschitz in the capabilities with respect to $|\cdot|_\infty$ norm, the difference in optimal values between team compositions $\mathcal{T}^x, \mathcal{T}^y$ satisfy:*

$$|V^*_{\mathcal{T}^x} - V^*_{\mathcal{T}^y}| \leq \frac{s_{max} \sum_{i=1}^{n} L_i |\mathcal{T}^x_i - \mathcal{T}^y_i|_\infty}{\gamma(1-\gamma)}.$$

See Appendix A.6 for the proof, which also provides a method for extending the other results in a similar fashion. For the case of general dependence of f on $\mathcal{T}$ (as is common for dense capability embeddings), see Appendix A.7. We also present an insight as to why generalization becomes harder in this setting. We provide experiments elucidating the bounds stated above in Section 5.1.

4 Experimental Setup

We evaluate the ability of existing MARL algorithms to generalize to novel settings where the capabilities of teammates change during the training. We are interested in evaluating the gap between settings encountered during training and held-out agent configurations reserved for testing. Furthermore, we aim to study how well algorithms ground privileged information about teammate capabilities and use that during unseen settings at test time. Lastly, we evaluate the bounds derived in Section 3 on a simple multi-agent problem. Code for the setup is provided in supplementary material.

4.1 Environments

4.1.1 Fruit Forage

We use the fruit forage task on a grid world to empirically demonstrate the generalisation bounds in Section 3. On a 8×8 grid world we have n agents and d types of fruit trees. For each agent i, $\mathcal{T}_i(j), j \in \{1..d\}$ represents the utility of fruit j for agent i. The state vector is appended with the d dimensional binary vector representing whether each of the tree types has foraged at a given time step. The details for the team compositions can be found in Appendix B.1.1.

4.1.2 Predator Prey

We consider the grid-world version of the multi-agent Predator Prey task where 4 agents have to hunt 4 prey in an 8×8 grid. Here, both predators and prey have certain capabilities. Specifically, each predator has a parameter describing the hit point damage it can cause the prey. Similarly, the prey comes with variations in health. For example, a prey with a capability of 5 can only be caught if the total capability of agents taking the capture action simultaneously on it have capabilities ≥ 5 (such as [1,1,1,2]), otherwise, the whole team receives a penalty p. Here, we test for generalization to novel team composition where test tasks contain a team composition which has not been encountered during training (PP Unseen Team in Figure 4), and additionally test tasks where novel team compositions can also have agent types with capabilities not encountered during training (PP Unseen Team, Agent in Figure 4). More details are provided in the Appendix B.1.2.

4.1.3 StarCraft II

To assess the generalization capabilities of modern MARL approaches, we make use of a modified version of StarCraft II unit micromanagement tasks of the SMAC benchmark Samvelyan et al. (2019). Particularly, we consider novel scenarios featuring three unit types from each race of the game where the team composition changes during training and testing, unlike standard SMAC which is static. The opponent's team is always identical to the ally team which ensures that we can directly compare the joint policy with the game AI policy. In the simple cases (`10_Protoss`, `10_Zerg`, and `10_Terran`), agents are trained on various team formations of 10 units that feature all combinations of one, two, and all three unit types, and is later tested on held out team formations.

In the hard cases (`10_Protoss_Hard`, `10_Zerg_Hard`, and `10_Terran_Hard`), agents are exposed to various team formations including two unit types during training. During testing, however, the agents encounter held-out scenarios featuring scenarios with using all three unit types (see Appendix B.1.3 for more details). Fig. 2 illustrates three episodes from the `10_Protoss_Hard` environment. In these tasks, agent capabilities are described as a one-hot encoding of agent types.

To test performance on continuously varying capabilities, we also use variants of the environment where either the health or attack accuracy of certain units are reduced. We randomize these configurations for the allied units during training and later test on held-out team configurations. We evaluate baselines on the `3m`, `2s3z`, `8m` scenarios from the original benchmark with these modifications. The varying team size also helps understand how grounding the capabilities becomes harder as team size increases. Here agent capabilities are described as their accuracy or health coefficients. Further details are provided in the Appendix B.1.3.

4.2 Baselines

Our empirical evaluation is based on various types of MARL algorithms. We use two popular value-based approaches, QMIX (Rashid et al., 2020) and VDN (Sunehag et al., 2017) that train fully decentralized policies in a centralized fashion. We also use thus spacepolicy gradient method PPO (Schulman et al., 2017) that has recently shown good results on various MARL domains, both with decentralised (Independent PPO) (de Witt et al., 2020) and centralised critics (MAPPO) Yu et al. (2021). We assess the performance of all baselines when the information about teammates capabilities are provided as observation (denoted with a 'C' in parentheses) and when it is not. **The evaluation procedure, architectures and training details are presented in Appendix B.2.**

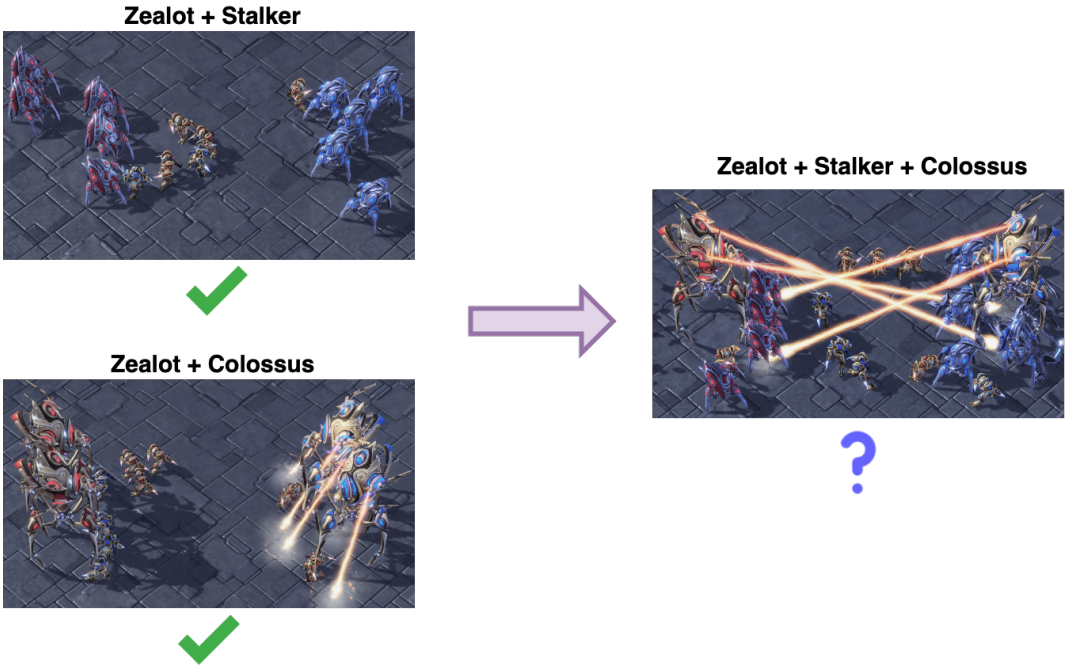
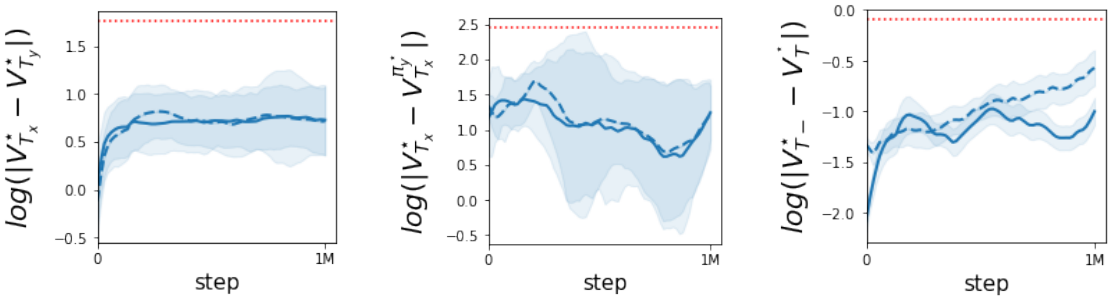


Figure 2: Three episodes from the `10_Protoss_Hard` task (a) Top-left featuring only Zealot and Stalkers during training. (b) Bottom-left featuring only Zealot and Colossus during training. (c) Right: A held-out episode featuring Zealot, Stalker, and Colossus encountered during testing.



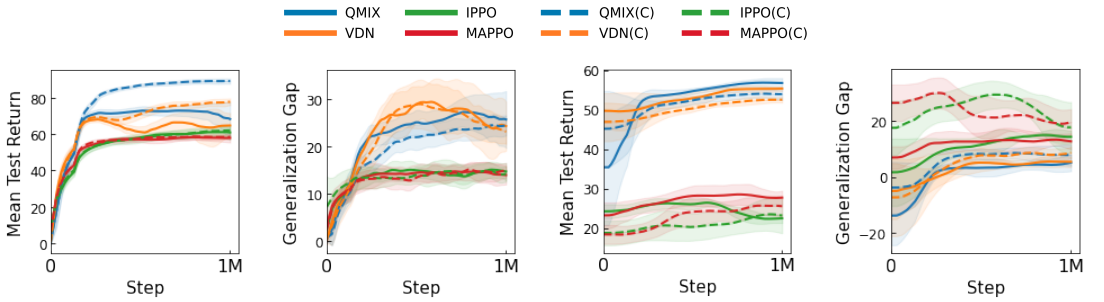
(a) Theorem 1 (b) Theorem 2 (c) Theorem 3

Figure 3: Evaluating the bounds for QMIX on Fruit Forage domain. Dashed blue line indicates the setting where agent capabilities are observable. The red dotted line indicates the corresponding upper bound for each theorem.

5 Results and Discussion

5.1 Generalization Bounds

Fig. 3 provides empirical evaluation of bounds presented in Section 3 in the Fruit Forage domain. We present the plots for training the agents for one million steps of training using QMIX. Fig. 3(a) shows that the policies in both the domains converge quickly leading to a stable difference in performance thus comfortably satisfying Theorem 1. Fig. 3(b) shows the gap between optimal and transferred policy and reveals interesting variations as training proceeds (we posit this happens because the transferred policy becomes steadily specialized thus getting less useful for the target task); the bound in Theorem 2 gives a tight fit despite these variations. Finally, we see similarly good fit for the agent elimination scenario in Theorem 3 in Fig. 3(c).



(a) PP Unseen Team (b) PP Unseen Team (c) PP Unseen Team, Agent (d) PP Unseen Team, Agent

Figure 4: Experimental results for the Predator Prey domain. Standard deviation is shaded.

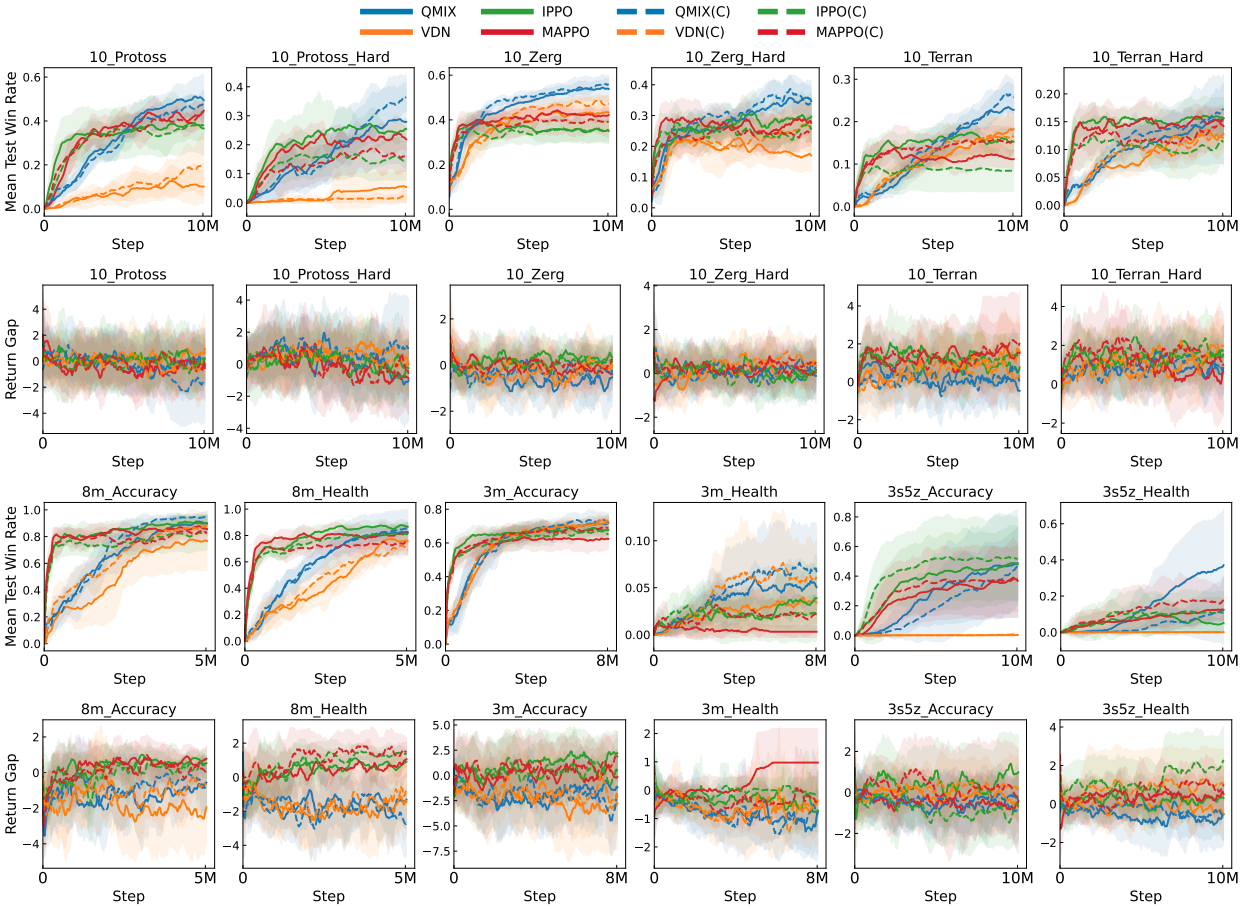


Figure 5: Experimental results on the SMAC benchmark. Standard deviation is shaded. Rows show win rates and generalization gaps.

5.2 Utilizing Information of Agent Capabilities

Fig. 4 presents the results of the baselines on Predator Prey domain. Fig. 4(a) shows that providing additional information on agent capabilities improves the test-time performance of the baselines with the maximal effect seen on QMIX and VDN. Furthermore, Fig. 4(b) shows that when capabilities are observable to the agents, baselines are able to generalize to new team compositions, thus successfully grounding the additional information. This hypothesis is additionally supported by the fact that knowing agent capabilities results in a lower generalization gap. Finally, the gap between the settings with known vs. unknown capabilities (dashed

vs. solid) indicates that agents have likely not come up with any appropriate protocol to communicate their capabilities during test time. Furthermore, the PPO variants do not perform as well as the value-based approaches. Therefore, their low generalization gap Fig. 4(b) is unlikely representative of good grounding of capability. We posit that this is just because PPO agents are ignoring the privileged information when available.

For a harder scenario, where both new team composition and agent types appear during evaluation, Fig. 4(c) shows that the situation is reversed from the previous setting as the agents that do not have access to each other's capabilities now perform slightly better. This is strongly indicative of insufficient grounding of the privileged information given to them, which highlights the need for better grounding mechanisms to obtain CG. We see a similar pattern on generalization gap in Fig. 4(d) where privileged information hurts the performance and is likely perceived as observation noise.

On the more challenging domain of StarCraft, we see that for easier capability variations of health and accuracy (as they are continuous and more readily usable for an agent's immediate actions), knowing the capabilities is advantageous to the agent during test time. Moreover, the relative gains of knowing the privileged information go down as the task difficulty increases. The accuracy variations tend to be easier as typical joint policies like 'focus fire' where a group of units to attack a single target together remain unchanged. Moreover, health variations on smaller teams make the task much harder than on bigger teams due to relative loss in team hit points. In this regard, `8m, 3s5z` accuracy versions show good grounding and generalization. This changes as tasks get harder. On the harder tasks that involve swapping unit types within `Protoss, Zerg, Terran` races, we observe that knowing the capabilities of other agents gives little advantage. This is especially noticeable on the `Hard` versions where all unit types are never within a single team during training. Furthermore, with win-rate performances on these maps being low, we hypothesise that the agents do not successfully utilize the capability information. Thus, it is highly unlikely that they learn any meaningful communication protocols for exchanging capability information. **For full StarCraft II results, including `8m_vs_9m` & `10m_vs_11m` scenarios, see Appendix C**.

Compared to the relatively simple Predator Prey task, generalization in StarCraft proved to be more difficult for the baselines. Although static versions of SMAC environments are comfortably solved by them Rashid et al. (2020); de Witt et al. (2020); Yu et al. (2021), changing unit formations or unit health/accuracy makes the tasks significantly difficult, even for configurations seen during the training. As observed in Fig. 5, providing the capability information does not consistently improve the test-time performance. This suggests the poor grounding abilities of the baseline algorithms, which reinforces the need for better grounding mechanisms in MARL algorithms (e.g., forward dynamics prediction as in Jaderberg et al. (2016)). The failure to generalize on index-based privileged information regarding agent types suggests using mechanisms such as latent embeddings to compose and reason about capabilities. Finally, a low test performance gap between agents having privileged information vs. those that do not, coupled with a low generalization gap, suggests that these methods do not facilitate information sharing between the agents, which is another desideratum towards attaining CG.

6 Related Work

Multi-agent systems (Claus & Boutilier, 1998; Busoniu et al., 2008) offer means to overcome theoretical barriers like exponential blow up in state-action space and compute resource requirements for large problems. MARL is a promising approach for training cooperative MAS. Recent progress in cooperative MARL (Lowe et al., 2017; Sunehag et al., 2017; Rashid et al., 2020; Mahajan et al., 2021) has demonstrated impressive applications in solving complex tasks in games such as StarCraft II (Samvelyan et al., 2019). Specialized methods which improve exploration in MARL have been proposed using hierarchical learning Mahajan et al. (2019) and successor features Gupta et al. (2021). Methods for factorizing the action space (Wang et al., 2020) have shown improvement in sample complexity. Iqbal et al. (2021) regularize value functions to share factors comprised of sub-groups of entities, in order to transfer knowledge across cooperative tasks. In the competitive/general sum MARL space (Lowe et al., 2017; OpenAI et al., 2019) have shown impressive performance on complex tasks. Vezhnevets et al. (2020) use an options framework to learn agents which generalize against different opponents. Czarnecki et al. (2020); Tuyls et al. (2020); Piliouras et al. (2021) explore the structural and theoretical properties of general payoff games. Mehta et al. (2023) provide domains

for social generalization in MARL, similarly Ellis et al. (2022) provide scenarios for procedural generation in StarCraft.

Ad-hoc coordination was first formalised by Stone et al. (2010) by modelling the multi-agent problem as a single-agent task and using competency scores to measure agent compatibility. Methods for using explicit hard-coded protocols for adaptations were explored in Tambe (1997); Grosz & Kraus (1996). Opponent modelling for general game was explored in Stone et al. (2000); Markovitch & Reger (2005); Ledezma et al. (2004). Several approaches to the ad-hoc cooperation problem assume that the behaviour of other agents in the ensemble are fixed Bowling & McCracken (2005). Planning methods like Monte Carlo tree search are used for finding optimal adaptation policy from a fixed set of choices (Barrett et al., 2011; Albrecht et al., 2016; Albrecht & Stone, 2019). Nikolaidis et al. (2014) develop over this by enabling learning a set of behaviours for the adapting agent while performing the task with human agents instead of assuming that it is given beforehand. Recent methods allow a change in the behaviour of the other agents to ones picked from a fixed set and account for the possible non-stationarities using change point detection Hernandez-Leal et al. (2017); Ravula (2019). However, these methods do not consider arbitrary learning for other agents. Furthermore, they do not focus on generalization to unseen agent capabilities.

Generalization in RL aims to develop approaches that generalize well to the novel, unseen scenarios after training (Kirk et al., 2022). Such methods avoid overfitting to seen tasks and can produce robust behaviour when deployed to novel settings. Recent work on generalization in single-agent RL make use of techniques such as data augmentation (Raileanu et al., 2021; Kostrikov et al., 2021), environment generation (Team et al., 2021; Leibo et al., 2021), encoding inductive biases (Higgins et al., 2017), and regularization (Cobbe et al., 2019). Methods in contextual MDPs Hallak et al. (2015); Zhang et al. (2020); Mahajan & Zhang (2023) also provide generalization with guarantees. Recent work also elucidate some of the fundamental bounds arising from computational complexity which prevents sample efficient generalization (Du et al., 2020; Ghosh et al., 2021; Malik et al., 2021).

7 Conclusion and Future work

In this work, we studied the generalization properties in multi-agent systems (MAS) following Markovian dynamics with a linear dependence of dynamics on the agent capabilities. We showed how the framework extends the successor feature setting to MAS. We explored performance bounds for various interesting scenarios arising in MAS including generalization, transfer, agent substitutions, approximate inference of capabilities and deviations in environment dynamics. Furthermore, we showed how the bounds can be extended to the Lipschitz reward setting and elucidated the most general form of rewards and how they make generalization difficult. Finally, we extensively tested the popular MARL algorithms on domains presenting a wide spectrum of hardness for CG. We saw that while some algorithms demonstrated preliminary CG on easier domains, all of the algorithms are insufficient towards ensuring CG on the challenging domains. We further highlighted how the first step towards ensuring CG should be ensuring proper grounding of agent capabilities. For future work, we aim to provide tighter bounds for CG for more general dynamics and create methods for better grounding of agent capabilities.

References

Albrecht, S. V. and Stone, P. Reasoning about hypothetical agent behaviours and their parameters. *arXiv preprint arXiv:1906.11064*, 2019.

Albrecht, S. V., Crandall, J. W., and Ramamoorthy, S. Belief and truth in hypothesised behaviours. *Artificial Intelligence*, 235:63–94, 2016.

Anderson, C. and McMillan, E. Of ants and men: Self-organized teams in human and insect organizations. *Emergence*, 5(2):29–41, 2003.

Barreto, A., Dabney, W., Munos, R., Hunt, J. J., Schaul, T., Van Hasselt, H., and Silver, D. Successor features for transfer in reinforcement learning. *arXiv preprint arXiv:1606.05312*, 2016.

Barreto, A., Borsa, D., Quan, J., Schaul, T., Silver, D., Hessel, M., Mankowitz, D., Zidek, A., and Munos, R. Transfer in deep reinforcement learning using successor features and generalised policy improvement. In *International Conference on Machine Learning*, pp. 501–510. PMLR, 2018.

Barreto, A., Hou, S., Borsa, D., Silver, D., and Precup, D. Fast reinforcement learning with generalized policy updates. *Proceedings of the National Academy of Sciences*, 117(48):30079–30087, 2020.

Barrett, S., Stone, P., and Kraus, S. Empirical evaluation of ad hoc teamwork in the pursuit domain. In *AAMAS*, pp. 567–574, 2011.

Bowling, M. and McCracken, P. Coordination and adaptation in impromptu teams. In *AAAI*, volume 5, pp. 53–58, 2005.

Busoniu, L., Babuska, R., and De Schutter, B. A Comprehensive Survey of Multiagent Reinforcement Learning. *IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews)*, 38 (2):156–172, 2008.

Claus, C. and Boutilier, C. The dynamics of reinforcement learning in cooperative multiagent systems. *AAAI/IAAI*, 1998(746-752):2, 1998.

Cobbe, K., Klimov, O., Hesse, C., Kim, T., and Schulman, J. Quantifying generalization in reinforcement learning. In Chaudhuri, K. and Salakhutdinov, R. (eds.), *Proceedings of the 36th International Conference on Machine Learning*, volume 97 of *Proceedings of Machine Learning Research*, pp. 1282–1289. PMLR, 09–15 Jun 2019. URL `https://proceedings.mlr.press/v97/cobbe19a.html`.

Crozier, R. H., Newey, P. S., Schluens, E. A., Robson, S. K., et al. A masterpiece of evolution–oecophylla weaver ants (hymenoptera: Formicidae). *Myrmecological News*, 13(5), 2010.

Czarnecki, W. M., Gidel, G., Tracey, B., Tuyls, K., Omidshafiei, S., Balduzzi, D., and Jaderberg, M. Real world games look like spinning tops, 2020.

Dayan, P. Improving generalization for temporal difference learning: The successor representation. *Neural Computation*, 5(4):613–624, 1993.

de Witt, C. S., Gupta, T., Makoviichuk, D., Makoviychuk, V., Torr, P. H., Sun, M., and Whiteson, S. Is independent learning all you need in the starcraft multi-agent challenge? *arXiv preprint arXiv:2011.09533*, 2020.

Du, S. S., Kakade, S. M., Wang, R., and Yang, L. F. Is a good representation sufficient for sample efficient reinforcement learning? In *8th International Conference on Learning Representations, ICLR 2020, Addis Ababa, Ethiopia, April 26-30, 2020*. OpenReview.net, 2020. URL `https://openreview.net/forum?id=r1genAVKPB`.

Ellis, B., Moalla, S., Samvelyan, M., Sun, M., Mahajan, A., Foerster, J. N., and Whiteson, S. Smacv2: An improved benchmark for cooperative multi-agent reinforcement learning. *arXiv preprint arXiv:2212.07489*, 2022.

Ghosh, D., Rahme, J., Kumar, A., Zhang, A., Adams, R. P., and Levine, S. Why Generalization in RL is Difficult: Epistemic POMDPs and Implicit Partial Observability. *arXiv:2107.06277 [cs, stat]*, 2021. URL `http://arxiv.org/abs/2107.06277`.

Grosz, B. and Kraus, S. Collaborative plans for complex group action. *Artificial Intelligence*, 1996.

Gupta, T., Mahajan, A., Peng, B., Böhmer, W., and Whiteson, S. Uneven: Universal value exploration for multi-agent reinforcement learning. In *International Conference on Machine Learning*, pp. 3930–3941. PMLR, 2021.

Hallak, A., Castro, D. D., and Mannor, S. Contextual markov decision processes, 2015.

Hausknecht, M. and Stone, P. Deep Recurrent Q-Learning for Partially Observable MDPs. In *AAAI Fall Symposium on Sequential Decision Making for Intelligent Agents*, 2015.

Hernandez-Leal, P., Zhan, Y., Taylor, M. E., Sucar, L. E., and De Cote, E. M. Efficiently detecting switches against non-stationary opponents. *Autonomous Agents and Multi-Agent Systems*, 31(4):767–789, 2017.

Higgins, I., Pal, A., Rusu, A. A., Matthey, L., Burgess, C., Pritzel, A., Botvinick, M., Blundell, C., and Lerchner, A. DARLA: improving zero-shot transfer in reinforcement learning. In Precup, D. and Teh, Y. W. (eds.), *Proceedings of the 34th International Conference on Machine Learning, ICML 2017, Sydney, NSW, Australia, 6-11 August 2017*, volume 70 of *Proceedings of Machine Learning Research*, pp. 1480–1490. PMLR, 2017. URL `http://proceedings.mlr.press/v70/higgins17a.html`.

Iqbal, S., de Witt, C. A. S., Peng, B., Böhmer, W., Whiteson, S., and Sha, F. Randomized entity-wise factorization for multi-agent reinforcement learning, 2021.

Jaderberg, M., Mnih, V., Czarnecki, W. M., Schaul, T., Leibo, J. Z., Silver, D., and Kavukcuoglu, K. Reinforcement learning with unsupervised auxiliary tasks, 2016.

Kaelbling, L. P., Littman, M. L., and Cassandra, A. R. Planning and acting in partially observable stochastic domains. *Artificial intelligence*, 101(1-2):99–134, 1998.

Kirk, R., Zhang, A., Grefenstette, E., and Rocktäschel, T. A survey of generalisation in deep reinforcement learning, 2022.

Kostrikov, I., Yarats, D., and Fergus, R. Image augmentation is all you need: Regularizing deep reinforcement learning from pixels, 2021.

Ledezma, A., Aler, R., Sanchis, A., and Borrajo, D. Predicting opponent actions by observation. In *Robot Soccer World Cup*, pp. 286–296. Springer, 2004.

Leibo, J. Z., Dueñez-Guzman, E. A., Vezhnevets, A., Agapiou, J. P., Sunehag, P., Koster, R., Matyas, J., Beattie, C., Mordatch, I., and Graepel, T. Scalable evaluation of multi-agent reinforcement learning with melting pot. In *Proceedings of the 38th International Conference on Machine Learning*, pp. 6187–6199. PMLR, 2021.

Lowe, R., Wu, Y., Tamar, A., Harb, J., Abbeel, O. P., and Mordatch, I. Multi-agent actor-critic for mixed cooperative-competitive environments. In *Advances in Neural Information Processing Systems*, pp. 6382–6393, 2017.

Mahajan, A. and Tulabandhula, T. Symmetry learning for function approximation in reinforcement learning. *arXiv preprint arXiv:1706.02999*, 2017.

Mahajan, A. and Zhang, A. Generalization across observation shifts in reinforcement learning, 2023.

Mahajan, A., Rashid, T., Samvelyan, M., and Whiteson, S. Maven: Multi-agent variational exploration. In *Advances in Neural Information Processing Systems*, pp. 7613–7624, 2019.

Mahajan, A., Samvelyan, M., Mao, L., Makoviychuk, V., Garg, A., Kossaifi, J., Whiteson, S., Zhu, Y., and Anandkumar, A. Tesseract: Tensorised actors for multi-agent reinforcement learning. In *Proceedings of the 38th International Conference on Machine Learning*, volume 139, pp. 7301–7312. PMLR, 2021. URL `https://proceedings.mlr.press/v139/mahajan21a.html`.

Malik, D., Li, Y., and Ravikumar, P. When Is Generalizable Reinforcement Learning Tractable? *arXiv:2101.00300 [cs, stat]*, 2021. URL `http://arxiv.org/abs/2101.00300`.

Markovitch, S. and Reger, R. Learning and exploiting relative weaknesses of opponent agents. *Autonomous Agents and Multi-Agent Systems*, 10(2):103–130, 2005.

Mehta, K., Mahajan, A., and Kumar, P. marl-jax: Multi-agent reinforcement leaning framework for social generalization. *arXiv preprint arXiv:2303.13808*, 2023.

Nikolaidis, S., Gu, K., Ramakrishnan, R., and Shah, J. Efficient model learning for human-robot collaborative tasks. arxiv, 2014.

Nouyan, S., Groß, R., Bonani, M., Mondada, F., and Dorigo, M. Teamwork in self-organized robot colonies. *IEEE Transactions on Evolutionary Computation*, 13(4):695–711, 2009.

Oliehoek, F. A. and Amato, C. *A Concise Introduction to Decentralized POMDPs*. SpringerBriefs in Intelligent Systems. Springer, 2016.

OpenAI, Berner, C., Brockman, G., Chan, B., Cheung, V., Dębiak, P., Dennison, C., Farhi, D., Fischer, Q., Hashme, S., Hesse, C., Józefowicz, R., Gray, S., Olsson, C., Pachocki, J., Petrov, M., Pinto, H. P. d. O., Raiman, J., Salimans, T., Schlatter, J., Schneider, J., Sidor, S., Sutskever, I., Tang, J., Wolski, F., and Zhang, S. Dota 2 with Large Scale Deep Reinforcement Learning. *arXiv:1912.06680 [cs, stat]*, 2019. URL `http://arxiv.org/abs/1912.06680`.

Piliouras, G., Rowland, M., Omidshafiei, S., Elie, R., Hennes, D., Connor, J., and Tuyls, K. Evolutionary dynamics and ϕ-regret minimization in games, 2021.

Raileanu, R., Goldstein, M., Yarats, D., Kostrikov, I., and Fergus, R. Automatic data augmentation for generalization in deep reinforcement learning, 2021.

Rashid, T., Samvelyan, M., de Witt, C. S., Farquhar, G., Foerster, J., and Whiteson, S. Monotonic value function factorisation for deep multi-agent reinforcement learning. *Journal of Machine Learning Research*, 21(178):1–51, 2020. URL `http://jmlr.org/papers/v21/20-081.html`.

Ravula, M. C. R. *Ad-hoc teamwork with behavior-switching agents*. PhD thesis, 2019.

Ryu, H., Shin, H., and Park, J. Multi-agent actor-critic with hierarchical graph attention network. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 34, pp. 7236–7243, 2020.

Samvelyan, M., Rashid, T., de Witt, C. S., Farquhar, G., Nardelli, N., Rudner, T. G., Hung, C.-M., Torr, P. H., Foerster, J., and Whiteson, S. The StarCraft Multi-Agent Challenge. In *Proceedings of the 18th International Conference on Autonomous Agents and MultiAgent Systems*, 2019.

Schulman, J., Wolski, F., Dhariwal, P., Radford, A., and Klimov, O. Proximal Policy Optimization Algorithms. *arXiv:1707.06347 [cs]*, 2017. URL `http://arxiv.org/abs/1707.06347`.

Stone, P., Riley, P., and Veloso, M. Defining and using ideal teammate and opponent agent models: A case study in robotic soccer. In *Proceedings Fourth International Conference on MultiAgent Systems*, pp. 441–442. IEEE, 2000.

Stone, P., Kaminka, G. A., Kraus, S., and Rosenschein, J. S. Ad hoc autonomous agent teams: Collaboration without pre-coordination. In *Twenty-Fourth AAAI Conference on Artificial Intelligence*, 2010.

Sunehag, P., Lever, G., Gruslys, A., Czarnecki, W. M., Zambaldi, V., Jaderberg, M., Lanctot, M., Sonnerat, N., Leibo, J. Z., Tuyls, K., et al. Value-decomposition networks for cooperative multi-agent learning. *arXiv preprint arXiv:1706.05296*, 2017.

Tambe, M. Towards flexible teamwork. *Journal of artificial intelligence research*, 7:83–124, 1997.

Team, O. E. L., Stooke, A., Mahajan, A., Barros, C., Deck, C., Bauer, J., Sygnowski, J., Trebacz, M., Jaderberg, M., Mathieu, M., McAleese, N., Bradley-Schmieg, N., Wong, N., Porcel, N., Raileanu, R., Hughes-Fitt, S., Dalibard, V., and Czarnecki, W. M. Open-ended learning leads to generally capable agents, 2021.

Tuyls, K., Perolat, J., Lanctot, M., Hughes, E., Everett, R., Leibo, J. Z., Szepesvári, C., and Graepel, T. Bounds and dynamics for empirical game theoretic analysis. *Autonomous Agents and Multi-Agent Systems*, 34(1):1–30, 2020.

Vezhnevets, A., Wu, Y., Eckstein, M., Leblond, R., and Leibo, J. Z. OPtions as REsponses: Grounding behavioural hierarchies in multi-agent reinforcement learning. In *Proceedings of the 37th International Conference on Machine Learning*, Proceedings of Machine Learning Research, pp. 9733–9742. PMLR, 2020.

Vinyals, O., Babuschkin, I., Czarnecki, W. M., Mathieu, M., Dudzik, A., Chung, J., Choi, D. H., Powell, R., Ewalds, T., Georgiev, P., Oh, J., Horgan, D., Kroiss, M., Danihelka, I., Huang, A., Sifre, L., Cai, T., Agapiou, J. P., Jaderberg, M., Vezhnevets, A. S., Leblond, R., Pohlen, T., Dalibard, V., Budden, D., Sulsky, Y., Molloy, J., Paine, T. L., Gulcehre, C., Wang, Z., Pfaff, T., Wu, Y., Ring, R., Yogatama, D., Wünsch, D., McKinney, K., Smith, O., Schaul, T., Lillicrap, T., Kavukcuoglu, K., Hassabis, D., Apps, C., and Silver, D. Grandmaster level in StarCraft II using multi-agent reinforcement learning. *Nature*, 575(7782):350–354, 2019. ISSN 1476-4687. doi: 10.1038/s41586-019-1724-z. URL `https://www.nature.com/articles/s41586-019-1724-z`.

Wang, T., Gupta, T., Mahajan, A., Peng, B., Whiteson, S., and Zhang, C. Rode: Learning roles to decompose multi-agent tasks. *arXiv preprint arXiv:2010.01523*, 2020.

Yu, C., Velu, A., Vinitsky, E., Wang, Y., Bayen, A., and Wu, Y. The surprising effectiveness of ppo in cooperative, multi-agent games, 2021.

Zhang, A., Sodhani, S., Khetarpal, K., and Pineau, J. Multi-task reinforcement learning as a hidden-parameter block mdp. *arXiv e-prints*, pp. arXiv–2007, 2020.

A Proofs

A.1 Generalisation between team compositions

Theorem 1 (Generalisation between team compositions)**.** *Let team compositions $\mathcal{T}^x, \mathcal{T}^y \in \mathcal{C}^n$ with influence weights $a^x, a^y \in \Delta_{n-1}$, $s_{max} = \max_s ||W_R s||_1$, $V_{mid} = \frac{1}{2}\max_s V^*_{\mathcal{T}^y}(s)$, Then*[5]*:*

$$|V^*_{\mathcal{T}^x} - V^*_{\mathcal{T}^y}| \leq \frac{s_{max} + \gamma d V_{mid}}{\gamma(1-\gamma)}\Psi, \text{ where}$$

$$\Psi = \Big[|\sum_i a^x_i(\mathcal{T}^x_i - \mathcal{T}^y_i)|_\infty + |\sum_i (a^x_i - a^y_i)\mathcal{T}^y_i|_\infty\Big]$$

Proof. Let $\epsilon_R = \max_s |R_{\mathcal{T}^x}(s) - R_{\mathcal{T}^y}(s)|$ and $\epsilon_P = \max_{s,\mathbf{u}} 2 \cdot D_{TV}\Big(P_{\mathcal{T}^x}(\cdot|s,\mathbf{u}), P_{\mathcal{T}^y}(\cdot|s,\mathbf{u})\Big)$ where D_{TV} is the total variation distance. We have that:

$$\begin{aligned}
&|Q^*_{\mathcal{T}^x}(s,\mathbf{u}) - Q^*_{\mathcal{T}^y}(s,\mathbf{u})| \\
&= |R_{\mathcal{T}^x}(s) - R_{\mathcal{T}^y}(s) + \gamma\Big(\sum_{s'} P_{\mathcal{T}^x}(s'|s,\mathbf{u}) \max_{u'} Q^*_{\mathcal{T}^x}(s',\mathbf{u}') - \sum_{s'} P_{\mathcal{T}^y}(s'|s,\mathbf{u}) \max_{u'} Q^*_{\mathcal{T}^y}(s',\mathbf{u}'))\Big)| \\
&\leq |R_{\mathcal{T}^x}(s) - R_{\mathcal{T}^y}(s)| + \gamma\Big\{|\sum_{s'} P_{\mathcal{T}^x}(s'|s,\mathbf{u})\Big[\max_{u'} Q^*_{\mathcal{T}^x}(s',\mathbf{u}') - \max_{u'} Q^*_{\mathcal{T}^y}(s',\mathbf{u}')\Big]| \\
&\quad + |\sum_{s'}\Big[P_{\mathcal{T}^x}(s'|s,\mathbf{u}) - P_{\mathcal{T}^y}(s'|s,\mathbf{u})\Big](\max_{u'} Q^*_{\mathcal{T}^y}(s',\mathbf{u}') - V_{mid})|\Big\} \\
&\leq \epsilon_R + \gamma\Big\{\sum_{s'} P_{\mathcal{T}^x}(s'|s,\mathbf{u})|\max_{u'} Q^*_{\mathcal{T}^x}(s',\mathbf{u}') - \max_{u'} Q^*_{\mathcal{T}^y}(s',\mathbf{u}')| + \sum_{s'} |P_{\mathcal{T}^x}(s'|s,\mathbf{u}) - P_{\mathcal{T}^y}(s'|s,\mathbf{u})||\max_{u'} Q^*_{\mathcal{T}^y}(s',\mathbf{u}') - V_{mid}|\Big\} \\
&\leq \epsilon_R + \gamma\Big\{\sum_{s'} P_{\mathcal{T}^x}(s'|s,\mathbf{u}) \max_{u'} |Q^*_{\mathcal{T}^x}(s',\mathbf{u}') - Q^*_{\mathcal{T}^y}(s',\mathbf{u}')| + 2 \cdot D_{TV}\Big(P_{\mathcal{T}^x}(s'|s,\mathbf{u}), P_{\mathcal{T}^y}(s'|s,\mathbf{u})\Big)V_{mid}\Big\} \\
&\leq \epsilon_R + \gamma\Big\{\max_{s',u'} |Q^*_{\mathcal{T}^x}(s',\mathbf{u}') - Q^*_{\mathcal{T}^y}(s',\mathbf{u}')| + \epsilon_P V_{mid}\Big\}
\end{aligned}$$

Next taking max w.r.t. s, u of the above we get:

$$\max_{s,u} |Q^*_{\mathcal{T}^x}(s,\mathbf{u}) - Q^*_{\mathcal{T}^y}(s,\mathbf{u})| \leq \frac{\epsilon_R + \gamma\epsilon_P V_{mid}}{1-\gamma}$$

We now bound the deviation quantities appearing above:

$$\begin{aligned}
\epsilon_R &= \max_s |R_{\mathcal{T}^x}(s) - R_{\mathcal{T}^y}(s)| \\
&= \max_s |\sum_{i=1}^n a^x_i \langle \mathcal{T}^x_i \cdot W_R s\rangle - \sum_{i=1}^n a^y_i \langle \mathcal{T}^y_i \cdot W_R s\rangle| \\
&\leq \max_s \Big[|\sum_{i=1}^n a^x_i \langle (\mathcal{T}^x_i - \mathcal{T}^y_i) \cdot W_R s\rangle| + |\sum_{i=1}^n (a^x_i - a^y_i)\langle \mathcal{T}^y_i \cdot W_R s\rangle|\Big] \\
&\leq \max_s \Big[|\sum_i a^x_i(\mathcal{T}^x_i - \mathcal{T}^y_i)|_\infty |W_R s|_1 + |\sum_i (a^x_i - a^y_i)\mathcal{T}^y_i|_\infty |W_R s|_1\Big] \\
&= s_{max}\Big[|\sum_i a^x_i(\mathcal{T}^x_i - \mathcal{T}^y_i)|_\infty + |\sum_i (a^x_i - a^y_i)\mathcal{T}^y_i|_\infty\Big]
\end{aligned}$$

Similarly,

$$\epsilon_P = \max_{s,\mathbf{u}} 2 \cdot D_{TV}\Big(P_{\mathcal{T}^x}(\cdot|s,\mathbf{u}), P_{\mathcal{T}^y}(\cdot|s,\mathbf{u})\Big)$$

[5] for $\gamma \in (0, \frac{\sqrt{5}-1}{2})$ we can replace $\frac{1}{\gamma(1-\gamma)}$ by $\frac{1+\gamma}{1-\gamma}$

$$
\begin{aligned}
&= \max_{s,\mathbf{u}} \sum_{s'} |P_{\mathcal{T}^x}(s'|s,\mathbf{u}) - P_{\mathcal{T}^y}(s'|s,\mathbf{u})| \\
&= \max_{s,\mathbf{u}} \sum_{s'} |\sum_{i=1}^{n} a_i^x \langle \mathcal{T}_i^x \cdot W_P(s',s,\mathbf{u})\rangle - \sum_{i=1}^{n} a_i^y \langle \mathcal{T}_i^y \cdot W_P(s',s,\mathbf{u})\rangle| \\
&\leq \max_{s,\mathbf{u}} \sum_{s'} \Big[|\sum_{i=1}^{n} a_i^x \langle (\mathcal{T}_i^x - \mathcal{T}_i^y) \cdot W_P(s',s,\mathbf{u})\rangle| + |\sum_{i=1}^{n} (a_i^x - a_i^y) \langle \mathcal{T}_i^y \cdot W_P(s',s,\mathbf{u})\rangle| \Big] \\
&\leq \max_{s,\mathbf{u}} \sum_{s'} \Big[|\sum_{i} a_i^x (\mathcal{T}_i^x - \mathcal{T}_i^y)|_\infty |W_P(s',s,\mathbf{u})|_1 + |\sum_{i} (a_i^x - a_i^y) \mathcal{T}_i^y|_\infty |W_P(s',s,\mathbf{u})|_1 \Big] \\
&= \Big[|\sum_{i} a_i^x (\mathcal{T}_i^x - \mathcal{T}_i^y)|_\infty + |\sum_{i} (a_i^x - a_i^y) \mathcal{T}_i^y|_\infty \Big] \max_{s,\mathbf{u}} \sum_{s'} |W_P(s',s,\mathbf{u})|_1 \\
&= d \Big[|\sum_{i} a_i^x (\mathcal{T}_i^x - \mathcal{T}_i^y)|_\infty + |\sum_{i} (a_i^x - a_i^y) \mathcal{T}_i^y|_\infty \Big]
\end{aligned}
$$

Thus, we get:

$$
|Q^*_{\mathcal{T}^x}(s,\mathbf{u}) - Q^*_{\mathcal{T}^y}(s,\mathbf{u})| \leq \frac{s_{max} + \gamma d V_{mid}}{1-\gamma} \Big[|\sum_{i} a_i^x (\mathcal{T}_i^x - \mathcal{T}_i^y)|_\infty + |\sum_{i} (a_i^x - a_i^y) \mathcal{T}_i^y|_\infty \Big]
$$

Finally we get the value difference bound by considering a dummy state $s^{\#}$ which always transitions according to ρ and then using the Bellman equation. (Note that for $\gamma \in (0, \frac{\sqrt{5}-1}{2})$ we can replace $\frac{1}{\gamma(1-\gamma)}$ by $\frac{1+\gamma}{1-\gamma}$ for a tighter bound without considering a dummy start state) □

Corollary 1.1 (Change in optimal value as a result of agent substitution)**.** *Let $\mathcal{T} \in \mathcal{C}^n$ be a team composition with influence weights $a \in \Delta_{n-1}$. If agent i is substituted with i' keeping a_i unchanged such that $|\mathcal{T}_{i'} - \mathcal{T}_i|_\infty \leq \epsilon_C$ then the new team ($\mathcal{T}'$) optimal value follows:*

$$
|V^*_{\mathcal{T}'} - V^*_{\mathcal{T}}| \leq \frac{(s_{max} + \gamma d V_{mid}) a_i \epsilon_C}{\gamma(1-\gamma)}
$$

Proof. Applying Theorem 1 on original task and a new task with same influence weights and agent i capability replaced with $\mathcal{T}_{i'}$ immediately gives the result. □

A.2 Transfer of optimal policy

Theorem 2 (Transfer of optimal policy)**.** *Let $\mathcal{T}^x, \mathcal{T}^y \in \mathcal{C}^n$, $a^x, a^y \in \Delta_{n-1}$, $s_{max} = \max_s ||W_R s||_1$, $V_{mid} = \frac{1}{2}\max_s V^*_{\mathcal{T}^y}(s)$. Let π^*_y be the optimal policy for the team composed of agents with capabilities $\mathcal{T}^y$ and influence weights a^y. Then:*

$$
V^*_{\mathcal{T}^x} - V^{\pi^*_y}_{\mathcal{T}^x} \leq 2\frac{s_{max} + \gamma d V_{mid}}{\gamma(1-\gamma)} \Psi,
$$

where Ψ is defined as in Eq. (4)*.*

Proof. We have that:

$$
Q^*_{\mathcal{T}^x}(s,\mathbf{u}) - Q^{\pi^*_y}_{\mathcal{T}^x}(s,\mathbf{u}) \leq |Q^*_{\mathcal{T}^x}(s,\mathbf{u}) - Q^*_{\mathcal{T}^y}(s,\mathbf{u})| + |Q^*_{\mathcal{T}^y}(s,\mathbf{u}) - Q^{\pi^*_y}_{\mathcal{T}^x}(s,\mathbf{u})| \tag{5}
$$

The first term on the RHS of Eq. (5) is taken care of by Theorem 1. We now focus on the second term:

$$
\begin{aligned}
&|Q^*_{\mathcal{T}^y}(s,\mathbf{u}) - Q^{\pi^*_y}_{\mathcal{T}^x}(s,\mathbf{u})| \\
&= |R_{\mathcal{T}^y}(s) - R_{\mathcal{T}^x}(s) + \gamma \Big(\sum_{s'} P_{\mathcal{T}^y}(s'|s,\mathbf{u}) \max_{u'} Q^*_{\mathcal{T}^y}(s',\mathbf{u}') - \sum_{s'} P_{\mathcal{T}^x}(s'|s,\mathbf{u}) Q^{\pi^*_y}_{\mathcal{T}^x}(s',\pi^*_y(\mathbf{u}')) \Big)| \\
&\leq \epsilon_R + \gamma \Big\{ |\sum_{s'} P_{\mathcal{T}^x}(s'|s,\mathbf{u}) \Big[\max_{u'} Q^*_{\mathcal{T}^y}(s',\mathbf{u}') - Q^{\pi^*_y}_{\mathcal{T}^x}(s',\pi^*_y(\mathbf{u}') \Big]|
\end{aligned}
$$

$$+ |\sum_{s'} \Big[P_{\mathcal{T}^y}(s'|s,\mathbf{u}) - P_{\mathcal{T}^x}(s'|s,\mathbf{u})\Big](\max_{u'} Q^*_{\mathcal{T}^y}(s',\mathbf{u}') - V_{mid})|\Big\}$$
$$\leq \epsilon_R + \gamma\Big\{\max_{s',u'} |Q^*_{\mathcal{T}^y}(s',\mathbf{u}') - Q^{\pi^*_y}_{\mathcal{T}^x}(s',\pi^*_y(\mathbf{u}')| + \epsilon_P V_{mid}\Big\}$$

Once again, taking max w.r.t. $s, \mathbf{u}$ of the above we get:

$$\max_{s,u} |Q^*_{\mathcal{T}^y}(s,\mathbf{u}) - Q^{\pi^*_y}_{\mathcal{T}^x}(s,\mathbf{u})| \leq \frac{\epsilon_R + \gamma\epsilon_P V_{mid}}{1-\gamma}$$

Substituting for deviation expressions and using Theorem 1 in Eq. (5) we get:

$$|Q^*_{\mathcal{T}^x}(s,\mathbf{u}) - Q^{\pi^*_y}_{\mathcal{T}^x}(s,\mathbf{u})| \leq 2\frac{s_{max} + \gamma d V_{mid}}{1-\gamma}\Big[|\sum_i a^x_i(\mathcal{T}^x_i - \mathcal{T}^y_i)|_\infty + |\sum_i (a^x_i - a^y_i)\mathcal{T}^y_i|_\infty\Big]$$

Note the absolute on LHS above can be dropped as $Q^*_{\mathcal{T}^x}$ is optimal. Finally using the same technique as above for Theorem 1 we get the statement of the theorem. □

Corollary 2.1 (Out of distribution performance)**.** *Let $\mathcal{T} \notin Sup(\mathcal{M})$ be an out of distribution task, we then have that the performance of the absolute oracle policy on $\mathcal{T}$ satisfies:*

$$V^*_{\mathcal{T}} - V^{\pi^*_{\mathcal{M}}}_{\mathcal{T}} \leq 2\frac{s_{max} + \gamma d V_{mid}}{\gamma(1-\gamma)} d_a(\mathcal{T}, Sup(\mathcal{M})),$$

Proof. For any task that belongs to $\arg\min_{\mathcal{T}^l \in Sup(\mathcal{M})} d_a(\mathcal{T}^l, \mathcal{T})$, we have by application of Theorem 2 that the result immediately holds given definition of $\pi^*_{\mathcal{M}}$. □

A.3 Population decrease

Theorem 3 (Population decrease bound)**.** *For the team composition $\mathcal{T} \in \mathcal{C}^n$ with influence weights $a \in \Delta_{n-1}$. If agent n is eliminated followed by a re-normalization of influence weights, we have that for the remaining team ($\mathcal{T}^- \triangleq (\mathcal{T})_{i=1}^{n-1}$):*

$$|V^*_{\mathcal{T}^-} - V^*_{\mathcal{T}}| \leq \frac{a_n(s_{max} + \gamma d V_{mid})}{\gamma(1-\gamma)}\Big|\sum_{i=1}^{n-1} \frac{a_i \mathcal{T}_i}{1-a_n} - \mathcal{T}_n\Big|_\infty$$

Proof. We use Theorem 1 with influence weights $(a_i)_1^n$ and $(\lambda \cdot a_i : i = 1..n-1, a_n = 0)$ where $\lambda = \frac{1}{1-a_n}$ □

Corollary 3.1 (Population increase bound)**.** *For the team composition $\mathcal{T} \in \mathcal{C}^n$ with influence weights $a \in \Delta_{n-1}$. If agent $n+1$ is added with capability $\mathcal{T}_{n+1}$ and weight a_{n+1} (other weights scaled down by $\lambda = 1 - a_{n+1}$) we have that for the new team ($\mathcal{T}^+ \triangleq (\mathcal{T}_1..\mathcal{T}_n, \mathcal{T}_{n+1})$):*

$$|V^*_{\mathcal{T}^+} - V^*_{\mathcal{T}}| \leq \frac{a_{n+1}(s_{max} + \gamma d V_{mid})}{\gamma(1-\gamma)}\Big|\sum_{i=1}^{n} a_i \mathcal{T}_i - \mathcal{T}_{n+1}\Big|_\infty$$

Proof. Consider the team compositions $\mathcal{T}^x = (\mathcal{T}_1..\mathcal{T}_n, 0)$ with influence weights $= (a_1..a_n, 0)$ and $\mathcal{T}^y = (\mathcal{T}_1..\mathcal{T}_n, \mathcal{T}_{n+1})$ with influence weights $= (\lambda a_1..\lambda a_n, a_{n+1})$ where $\lambda = 1 - a_{n+1}$, we have that:

$$\Psi = \Big[|\sum_i a^x_i(\mathcal{T}^x_i - \mathcal{T}^y_i)|_\infty + |\sum_i (a^x_i - a^y_i)\mathcal{T}^y_i|_\infty\Big]$$
$$= |\sum_{i=1}^{n}(1-\lambda)a_i\mathcal{T}^y_i - a_{n+1}\mathcal{T}^y_{n+1}|_\infty$$
$$= a_{n+1}|\sum_{i=1}^{n} a_i\mathcal{T}^y_i - \mathcal{T}^y_{n+1}|_\infty$$

which on applying Theorem 1 yields the result. □

A.4 Approximate $\hat{\epsilon}_R$,$\hat{\epsilon}_P$ dynamics

Theorem 4 (Approximate $\hat{\epsilon}_R$,$\hat{\epsilon}_P$ dynamics)**.** *Let $\mathcal{T}^x, \mathcal{T}^y \in \mathcal{C}^n$, $a^x, a^y \in \Delta_{n-1}$ and the dynamics be only approximately linear so that $|R_{\mathcal{T}}(s) - \sum_{i=1}^n a_i \langle c_i \cdot W_R s\rangle| \leq \hat{\epsilon}_R$ and $|P_{\mathcal{T}}(s'|s,\mathbf{u}) - \sum_{i=1}^n a_i \langle c_i \cdot W_P(s',s,\mathbf{u})\rangle| \leq \hat{\epsilon}_P$. Then:*

$$|V^*_{\mathcal{T}^x} - V^*_{\mathcal{T}^y}| \leq \frac{s_{max} + \gamma d V_{mid}}{\gamma(1-\gamma)}\Psi + \frac{2(\hat{\epsilon}_R + \gamma \hat{\epsilon}_P V_{mid})}{\gamma(1-\gamma)},$$

where Ψ is defined as in Eq. (4).

Proof. We begin as in proof of Theorem 1 to get:

$$\max_{s,u} |Q^*_{\mathcal{T}^x}(s,\mathbf{u}) - Q^*_{\mathcal{T}^y}(s,\mathbf{u})| \leq \frac{\epsilon_R + \gamma \epsilon_P V_{mid}}{1-\gamma}$$

Next we apply the corrections to the relative differences:

$$\begin{aligned}
\epsilon_R &= \max_s |R_{\mathcal{T}^x}(s) - R_{\mathcal{T}^y}(s)| \\
&\leq \max_s \Big[|R_{\mathcal{T}^x}(s) - \sum_{i=1}^n a_i^x \langle \mathcal{T}_i^x \cdot W_R s\rangle| + |\sum_{i=1}^n a_i^x \langle \mathcal{T}_i^x \cdot W_R s\rangle - \sum_{i=1}^n a_i^y \langle \mathcal{T}_i^y \cdot W_R s\rangle| + |R_{\mathcal{T}^y}(s) - \sum_{i=1}^n a_i^y \langle \mathcal{T}_i^y \cdot W_R s\rangle| \Big] \\
&\leq 2\hat{\epsilon}_R + \max_s \Big[|\sum_{i=1}^n a_i^x \langle (\mathcal{T}_i^x - \mathcal{T}_i^y) \cdot W_R s\rangle| + |\sum_{i=1}^n (a_i^x - a_i^y) \langle \mathcal{T}_i^y \cdot W_R s\rangle| \Big] \\
&\leq 2\hat{\epsilon}_R + \max_s \Big[|\sum_i a_i^x (\mathcal{T}_i^x - \mathcal{T}_i^y)|_\infty |W_R s|_1 + |\sum_i (a_i^x - a_i^y) \mathcal{T}_i^y|_\infty |W_R s|_1 \Big] \\
&= 2\hat{\epsilon}_R + s_{max} \Big[|\sum_i a_i^x (\mathcal{T}_i^x - \mathcal{T}_i^y)|_\infty + |\sum_i (a_i^x - a_i^y) \mathcal{T}_i^y|_\infty \Big]
\end{aligned}$$

Proceeding similarly with the transition probabilities we get the desired result. □

A.5 Error from estimation of capabilities

Theorem 5 (Error from estimation of capabilities)**.** *For the team composition $\mathcal{T} \in \mathcal{C}^n$ with influence weights $a \in \Delta_{n-1}$. If the agent capabilities are inaccurately inferred as $\hat{\mathcal{T}}$ with $\max_i |\mathcal{T}_i - \hat{\mathcal{T}}_i|_\infty \leq \epsilon_{\mathcal{T}}$ and agents learn the inexact policy $\hat{\pi}^*$ then:*

$$|V^*_{\mathcal{T}} - V^{\hat{\pi}^*}_{\mathcal{T}}| \leq \frac{2\epsilon_{\mathcal{T}}(s_{max} + \gamma d V_{mid})}{\gamma(1-\gamma)}$$

where $V_{mid} = \frac{1}{2}\max_s V^*_{\hat{\mathcal{T}}}(s)$

Proof. We have that for the actual and inferred team compositions with same influence weights:

$$\begin{aligned}
\Psi &= \Big[|\sum_i a_i (\mathcal{T}_i - \hat{\mathcal{T}}_i)|_\infty + |\sum_i (a_i - a_i) \hat{\mathcal{T}}_i|_\infty \Big] \\
&= |\sum_i a_i (\mathcal{T}_i - \hat{\mathcal{T}}_i)|_\infty \\
&\leq \sum_i a_i |\mathcal{T}_i - \hat{\mathcal{T}}_i|_\infty \\
&\leq \sum_i a_i \epsilon_{\mathcal{T}} \\
&= \epsilon_{\mathcal{T}}
\end{aligned}$$

Now applying Theorem 2 gives the result □

A.6 Extending to Lipschitz rewards

We demonstrate how to extend the results in Section 3 to Lipschitz function of capabilities. For brevity we consider only the setting where the rewards vary with capabilities. Thus, for the reward function form $R_{\mathcal{T}}(s) = \langle f(\mathcal{T}) \cdot W_R s\rangle$ where $f(\mathcal{T})$ is L_i Lipschitz with respect to the capability $\mathcal{T}_i$ for $i \in \mathcal{A}$ for the $|\cdot|_\infty$ norm. We get that for two different team compositions $\mathcal{T}^x, \mathcal{T}^y$

$$\begin{aligned}
\epsilon_R &= \max_s |R_{\mathcal{T}^x}(s) - R_{\mathcal{T}^y}(s)| \\
&= \max_s |\langle f(\mathcal{T}^x) \cdot W_R s\rangle - \langle f(\mathcal{T}^y) \cdot W_R s\rangle| \\
&= \max_s |\sum_{i=1}^n \langle f(\mathcal{T}^i) \cdot W_R s\rangle - \langle f(\mathcal{T}^{i+1}) \cdot W_R s\rangle| \\
&\leq \max_s \sum_{i=1}^n |\langle f(\mathcal{T}^i) \cdot W_R s\rangle - \langle f(\mathcal{T}^{i+1}) \cdot W_R s\rangle| \\
&\leq \max_s \sum_{i=1}^n |\langle f(\mathcal{T}^i) \cdot W_R s\rangle - \langle f(\mathcal{T}^{i+1}) \cdot W_R s\rangle| \\
&\leq \max_s \sum_{i=1}^n |f(\mathcal{T}^i) - f(\mathcal{T}^{i+1})|_\infty |W_R s|_1 \\
&\leq s_{max} \sum_{i=1}^n L_i |\mathcal{T}_i^x - \mathcal{T}_i^y|_\infty
\end{aligned}$$

Where $\mathcal{T}^i$ was the sequence satisfying $\mathcal{T}^1 = \mathcal{T}^x$ and $\mathcal{T}^{n+1} = \mathcal{T}^y$ and changing $\mathcal{T}^x$ one index at a time. We have thus proved that:

Theorem 6. *For rewards L_i Lipschitz in the capabilities with respect to $|\cdot|_\infty$ norm, the difference in optimal values between team compositions $\mathcal{T}^x, \mathcal{T}^y$ satisfy:*

$$|V^*_{\mathcal{T}^x} - V^*_{\mathcal{T}^y}| \leq \frac{s_{max} \sum_{i=1}^n L_i |\mathcal{T}_i^x - \mathcal{T}_i^y|_\infty}{\gamma(1-\gamma)}$$

A.7 General dependence of rewards on capabilities:

We now consider the dependence of rewards on the capabilities in the most general form. For this, we introduce the notion of (α, k)-rewards where $\alpha \geq 0, k \in \mathbb{N}$.

$$R_{\mathcal{T}}(s) = \Big\langle \sum_{k_i \in \mathbb{N}, \sum k_i \leq k} a_{k_1..k_n} \Pi_{i=1}^n c_i^{k_i} \cdot W_R s \Big\rangle \tag{6}$$

where $\mathbb{N}$ are non negative integers, $|a_{k_1..k_n}| \leq \alpha$ and $c_i^k{}_i$ represents element-wise exponentiation. . Rewards in Eq. (2) can be seen as a special case belonging to Eq. (6) the choice $\alpha, k = 1$. Similarly the union $\cup_{\alpha \geq 0, k \in \mathbb{N}} (\alpha, k)$-rewards cover all possible reward dependencies on capabilities. We have further relaxed the assumption of influence weights belonging to a simplex here and replaced it with individual bounds on the power series coefficients here. We next see that for this scenario, even a small change in the capability of a single agent can shift the rewards massively. Let the capability of agent i be changed from $\mathcal{T}_i$ to $\mathcal{T}_{i'}$ such that $|\mathcal{T}_i - \mathcal{T}_{i'}|_\infty \leq \delta$. Then we have

Lemma 1. *For substitution $\mathcal{T}_i$ to $\mathcal{T}_{i'}$ such that $|\mathcal{T}_i - \mathcal{T}_{i'}|_\infty \leq \delta$ under the (α, k)-rewards setting we have that*

$$\begin{aligned}
\epsilon_R &= \max_{s \in S} \Big| \Big\langle \sum_{k_i \in \mathbb{N}, \sum k_i \leq k} a_{k_1..k_n} \Pi_{j \neq i} \mathcal{T}_j^{k_j} (\mathcal{T}_i^{k_i} - \mathcal{T}_{i'}^{k_i}) \cdot W_R s \Big\rangle \Big| \\
&\leq \max_{s \in S} \Big| \sum_{k_i \in \mathbb{N}, \sum k_i \leq k} a_{k_1..k_n} \Pi_{j \neq i} \mathcal{T}_j^{k_j} (\mathcal{T}_i^{k_i} - \mathcal{T}_{i'}^{k_i}) \Big|_\infty \Big| W_R s \Big|_1
\end{aligned}$$

$$\leq \alpha s_{max} \sum_{j=0}^{k} \sum_{l=1}^{j} \binom{l}{j} l |\mathcal{T}_i^{k_i} - \mathcal{T}_{i'}^{k_i}|_\infty$$

$$\leq \alpha \delta s_{max} \sum_{j=0}^{k} j 2^{j-1} = \mathcal{O}(\alpha \delta s_{max} k 2^k)$$

While this is not a lower bound, the above still suggests that even a small change in the capability of an agent can cause the rewards to change by a lot, hence it is natural to expect that generalization becomes harder as the problem start showing the needle in the haystack phenomenon where only the *right combination* of capabilities gives a large optimal value.

B Experimental Setup

B.1 Environments

B.1.1 Fruit Forage

We use the fruit forage task on a grid world to empirically demonstrate the generalisation bounds in Section 3. On a $k \times k$ grid world we have n agents and d types of fruit trees. For each agent i, $\mathcal{T}_i(j), j \in \{1..d\}$ represents the utility of fruit j for agent i. The state vector is appended with the d dimensional binary vector representing whether each of the tree types was foraged at a given time step. The details for the team compositions can be found in Appendix B.1.1. We define three team compositions as follows:

T_x: [[0.05, 0.1, 0.6, 2.8], [0.05, 0.1, 2.1, 0.8], [0.05, 0.1, 1.8, 1.2], [0.05, 0.1, 0.9, 2.4]]

T_y: [[0.7, 0.4, 0.15, 0.2], [0.2, 1.4, 0.15, 0.2], [0.3, 1.2, 0.15, 0.2], [0.6, 0.6, 0.15, 0.2]]

T_z: [[0.1, 0.3, 0.6, 0.0], [0.4, 0.1, 0.5, 0.0], [0.05, 0.06, 0.89, 0.0], [0.0, 0.0, 0.0, 1.0]]

For proving bounds on Theorem-1, we compare the mean test returns achieved on tasks T_x and T_y using $V^\star_{T_x} - V^\star_{T_y}$. For Theorem-2, we compare the mean test returns achieved on tasks T_x and optimal policies of task T_y evaluated on task T_x i.e. $V^\star_{T_x} - V^{\pi^\star_{T_y}}_{T_x}$. Finally, for Theorem-3, we compare the mean test returns achieved on tasks T_z and optimal policies of task T_z evaluated on task T_z but removing the last agent i.e. $V^\star_{T_{z-}} - V^\star_{T_z}$.

B.1.2 Predator Prey

We consider a complicated partially observable predator-prey (PP) task in an 8×8 grid involving four agents (predators) and four prey that is designed to test coordination between agents. Specifically, each predator has a parameter describing the hit point damage it can cause the prey. Similarly, the prey comes with variations in health. For example, a prey with a capability of 5 can only be caught if the total capability of agents taking the capture action simultaneously on it have capabilities ≥ 5 (such as [1,1,3]), otherwise, the whole team receives a penalty p. On successful capture, agents get a reward of $+1$. Once prey is captured, another prey is spawned at a random location. Therefore, agents have to collaborate and capture as many preys as possible within 100 time steps.

Each agent can take 6 actions i.e. move in one of the 4 directions (Up, Left, Down, Right), remain still (no-op), or try to catch (capture) any adjacent prey. The prey moves around in the grid with a probability of 0.7 and remains still at its position with the probability of 0.3. Impossible actions for both agents and prey are marked unavailable, for eg. moving into an occupied cell or trying to take a capture action with no adjacent prey.

In this domain, we test for two types of generalization: (1) novel team composition where test tasks contain a team composition which has not been encountered during training (PP Unseen Team in Figure 4), and second, (2) test tasks where novel team compositions can also have agent types with capabilities not encountered during training (PP Unseen Team, Agent in Figure 4).

For (PP Unseen Team), we train on preys with capabilities [2,2,2,3], and agents with capabilities [2,3,2,3],[1,2,1,2], thereby having agent teams with total hit points of 10 and 6 respectively. We also train on two separate penalties p for miscoordination i.e. $p \in \{0.0, -0.008\}$, this helps inject additional

stochasticity in the environment as the agents don't know the penalty value. For test tasks, we create novel team compositions not encountered during training i.e. agents with capabilities [1,1,2,3],[1,1,1,3] having total hit points of 7 and 6 respectively.

For (PP Unseen Team, Agent) we train on preys with capabilities [1,2,3,4], and agents with capabilities [1, 2, 2, 3], [1, 1, 2, 2], [1, 3, 2, 1], thereby having agent teams with total hit points of 8, 6 and 7 respectively. We also train on two separate penalties p for miscoordination i.e. $p \in \{0.0, -0.008\}$. For test tasks, we create novel team compositions with an unseen agent of capability 4 not encountered during training i.e. agents with capabilities [1, 1, 1, 4], [1, 1, 3, 4], [1, 1, 2, 4] having total hit points of 7, 9, and 8 respectively.

Experimental Setup: For (PP Unseen Team, and PP Unseen Team, Agent) oracle baseline (leftmost), we show the average difference in performance across all test tasks when capability information is included ((c) for each method.

For testing the generalization gap in (PP Unseen Team), we show the difference in returns achieved by training task [1,2,1,2] (hit point 6) and test task [1,1,1,3] (hit point 6). For testing the generalization gap in (PP Unseen Team, Agent), we show the difference in returns achieved by training task [1,3,2,1] (hit point 7) and test task [1,1,1,4] (hit point 7) with a new agent of capability 4. All PP experiments are based on 8 seeds.

B.1.3 StarCraft II

We use the standard set of actions and global state information included as part of the SMAC benchmark Samvelyan et al. (2019). The sight range of the agent units has been increased to the fully observable setting. In the oracle mode, agent capabilities are included as part of individual observations. Each agent always observes its own capabilities. Furthermore, capabilities are always included in the global state.

`10_Terran` and `10_Terran_Hard` environment includes Marine, Maradeur, and Medivac units. `10_Protoss` and `10_Protoss_Hard` environments feature Stalker, Zealot, and Colossus units. `10_Zerg` and `10_Zerg_Hard` environments include Zergling, Hydralisk and Baneling units.

In `Accuracy` and `Health` tasks, specific values reduced from full unit capabilities are chosen to be equivalent to a loss of a single teammate. For example, if there three agents, their accuracy could be set to 0.75, 0.75 and 0.5 given that $(1 - 0.5) + (1 - 0.75) + (1 - 0.75) = 1$. Consequently, the overall reduction in accuracy would be roughly equivalent to losing one ally unit. This was chosen to ensure that the difficulty of the tasks was not too high.

All SMAC experiments are based on 5 seeds.

Table 1, 2, and 3 describe the training and evaluation distributions used in unit type swapping tasks.

Table 1: Team formations in `Terran` tasks

`10_Terran`	`10_Terran_Hard`
Training	**Training**
1 marine & 9 marauders	1 marine & 9 marauders
3 marines & 7 marauders	2 marines & 8 marauders
4 marines & 6 marauders	3 marines & 7 marauders
5 marines & 5 marauders	4 marines & 6 marauders
6 marines & 4 marauders	5 marines & 5 marauders
8 marines & 2 marauders	6 marines & 4 marauders
9 marines & 1 marauder	7 marines & 3 marauders
5 marauders & 5 medivacs	8 marines & 2 marauders
7 marauders & 3 medivacs	9 marines & 1 marauder
9 marauders & 1 medivac	5 marauders & 5 medivacs
7 marines & 3 medivacs	6 marauders & 4 medivacs
8 marines & 2 medivacs	7 marauders & 3 medivacs
9 marines & 1 medivac	8 marauders & 2 medivacs
10 marines	9 marauders & 1 medivac
10 marauders	7 marines & 3 medivacs
8 marines & 1 marauder & 1 medivac	8 marines & 2 medivacs
1 marine & 8 marauders & 1 medivac	9 marines & 1 medivac
5 marines & 3 marauders & 2 medivacs	**Testing**
2 marines & 7 marauders & 1 medivac	10 marines
6 marines & 2 marauders & 2 medivacs	10 marauders
2 marines & 6 marauders & 2 medivacs	8 marines & 1 marauder & 1 medivac
4 marines & 4 marauders & 2 medivacs	1 marine & 8 marauders & 1 medivac
Testing	5 marines & 3 marauders & 2 medivacs
2 marines & 8 marauders	3 marines & 5 marauders & 2 medivacs
7 marines & 3 marauders	4 marines & 3 marauders & 3 medivacs
6 marauders & 4 medivacs	3 marines & 4 marauders & 3 medivacs
8 marauders & 2 medivacs	7 marines & 2 marauders & 1 medivac
3 marines & 5 marauders & 2 medivacs	2 marines & 7 marauders & 1 medivac
4 marines & 3 marauders & 3 medivacs	6 marines & 2 marauders & 2 medivacs
3 marines & 4 marauders & 3 medivacs	2 marines & 6 marauders & 2 medivacs
7 marines & 2 marauders & 1 medivac	4 marines & 4 marauders & 2 medivacs

Table 2: Team formations in `Zerg` tasks

`10_Zerg`	`10_Zerg_Hard`
Training	**Training**
1 zergling & 9 hydralisks	1 zergling & 9 hydralisks
2 zerglings & 8 hydralisks	2 zerglings & 8 hydralisks
4 zerglings & 6 hydralisks	3 zerglings & 7 hydralisks
5 zerglings & 5 hydralisks	4 zerglings & 6 hydralisks
6 zerglings & 4 hydralisks	5 zerglings & 5 hydralisks
7 zerglings & 3 hydralisks	6 zerglings & 4 hydralisks
9 zerglings & 1 hydralisk	7 zerglings & 3 hydralisks
4 hydralisks & 6 banelings	8 zerglings & 2 hydralisks
5 hydralisks & 5 banelings	9 zerglings & 1 hydralisk
6 hydralisks & 4 banelings	4 hydralisks & 6 banelings
8 hydralisks & 2 banelings	5 hydralisks & 5 banelings
9 hydralisks & 1 baneling	6 hydralisks & 4 banelings
4 zerglings & 6 banelings	7 hydralisks & 3 banelings
6 zerglings & 4 banelings	8 hydralisks & 2 banelings
7 zerglings & 3 banelings	9 hydralisks & 1 baneling
8 zerglings & 2 banelings	4 zerglings & 6 banelings
10 zerglings	5 zerglings & 5 banelings
10 hydralisks	6 zerglings & 4 banelings
10 banelings	7 zerglings & 3 banelings
8 zerglings & 1 hydralisk & 1 baneling	8 zerglings & 2 banelings
1 zergling & 8 hydralisks & 1 baneling	9 zerglings & 1 baneling
7 zerglings & 2 hydralisks & 1 baneling	**Testing**
2 zerglings & 7 hydralisks & 1 baneling	10 zerglings
5 zerglings & 3 hydralisks & 2 banelings	10 hydralisks
3 zerglings & 5 hydralisks & 2 banelings	10 banelings
4 zerglings & 4 hydralisks & 2 banelings	8 zerglings & 1 hydralisk & 1 baneling
3 zerglings & 4 hydralisks & 3 banelings	1 zergling & 8 hydralisks & 1 baneling
Testing	7 zerglings & 2 hydralisks & 1 baneling
3 zerglings & 7 hydralisks	2 zerglings & 7 hydralisks & 1 baneling
8 zerglings & 2 hydralisks	6 zerglings & 2 hydralisks & 2 banelings
7 hydralisks & 3 banelings	2 zerglings & 6 hydralisks & 2 banelings
5 zerglings & 5 banelings	5 zerglings & 3 hydralisks & 2 banelings
9 zerglings & 1 baneling	3 zerglings & 5 hydralisks & 2 banelings
6 zerglings & 2 hydralisks & 2 banelings	4 zerglings & 4 hydralisks & 2 banelings
4 zerglings & 3 hydralisks & 3 banelings	4 zerglings & 3 hydralisks & 3 banelings
2 zerglings & 6 hydralisks & 2 banelings	3 zerglings & 4 hydralisks & 3 banelings

Table 3: Team formations in `Protoss` tasks

`10_Protoss`	`10_Protoss_Hard`
Training	**Training**
1 stalker & 9 zealots	1 stalker & 9 zealots
3 stalkers & 7 zealots	2 stalkers & 8 zealots
4 stalkers & 6 zealots	3 stalkers & 7 zealots
5 stalkers & 5 zealots	4 stalkers & 6 zealots
6 stalkers & 4 zealots	5 stalkers & 5 zealots
8 stalkers & 2 zealots	6 stalkers & 4 zealots
9 stalkers & 1 zealot	7 stalkers & 3 zealots
4 zealots & 6 colossi	8 stalkers & 2 zealots
5 zealots & 5 colossi	9 stalkers & 1 zealot
7 zealots & 3 colossi	4 zealots & 6 colossi
8 zealots & 2 colossi	5 zealots & 5 colossi
9 zealots & 1 colossus	6 zealots & 4 colossi
4 stalkers & 6 colossi	7 zealots & 3 colossi
5 stalkers & 5 colossi	8 zealots & 2 colossi
7 stalkers & 3 colossi	9 zealots & 1 colossus
8 stalkers & 2 colossi	4 stalkers & 6 colossi
10 stalkers	5 stalkers & 5 colossi
10 zealots	6 stalkers & 4 colossi
10 colossi	7 stalkers & 3 colossi
8 stalkers & 1 zealot & 1 colossus	8 stalkers & 2 colossi
1 stalker & 8 zealots & 1 colossus	9 stalkers & 1 colossus
2 stalkers & 7 zealots & 1 colossus	**Testing**
6 stalkers & 2 zealots & 2 colossi	10 stalkers
5 stalkers & 3 zealots & 2 colossi	10 zealots
3 stalkers & 5 zealots & 2 colossi	10 colossi
4 stalkers & 4 zealots & 2 colossi	8 stalkers & 1 zealot & 1 colossus
4 stalkers & 3 zealots & 3 colossi	1 stalker & 8 zealots & 1 colossus
Testing	7 stalkers & 2 zealots & 1 colossus
2 stalkers & 8 zealots	2 stalkers & 7 zealots & 1 colossus
7 stalkers & 3 zealots	6 stalkers & 2 zealots & 2 colossi
6 zealots & 4 colossi	2 stalkers & 6 zealots & 2 colossi
6 stalkers & 4 colossi	5 stalkers & 3 zealots & 2 colossi
9 stalkers & 1 colossus	3 stalkers & 5 zealots & 2 colossi
7 stalkers & 2 zealots & 1 colossus	4 stalkers & 4 zealots & 2 colossi
3 stalkers & 4 zealots & 3 colossi	4 stalkers & 3 zealots & 3 colossi
2 stalkers & 6 zealots & 2 colossi	3 stalkers & 4 zealots & 3 colossi

B.2 Architecture, Training and Evaluation

The evaluation procedure is similar to the one in (Rashid et al., 2020). The training is paused after every 30k timesteps during which 16 test episodes are run with agents performing action selection greedily in a decentralised fashion. The percentage of episodes where the agents defeat all enemy units within the permitted time limit is referred to as the test win rate.

To speed up the learning, the agent networks are parameters are shared across all agents. A one-hot encoding of the `agent_id` is concatenated onto each agent's observations. All neural networks are trained using RMSprop without weight decay or momentum.

Value-based baselines

The architecture of all agent networks is a DRQN Hausknecht & Stone (2015) with a recurrent layer comprised of a GRU with a 64-dimensional hidden state, with a fully-connected layer before and after. We sample batches of 32 episodes uniformly from the replay buffer, and train on fully unrolled episodes, performing a single gradient descent step after 8 episodes.

Table 4: Hyperparameters of QMIX and VDN

Method	Name	Value
QMIX & VDN	learning rate	5×10^{-4}
	RMSprop α	0.99
	replay buffer size	5000 episodes
	target network update interval	200 episodes
	γ	0.99
	double DQN target	True
	initial ϵ	1
	final ϵ	0.05
	ϵ anneal period	50000 steps
	ϵ anneal rule	linear
QMIX	mixing network hidden layers	1
	mixing network hidden layer units	32
	mixing network non-linearity	ELU
	hypernetwork hidden layers	2
	hypernetwork hidden layer units	64
	hypernetwork non-linearity	ReLU

PPO baselines

We parameterize the actor and critic with two independent recurrent neural networks, each of which is comprised of a GRU with a 64-dimensional hidden state, with a fully-connected layer as the input and output.

Table 5: Hyperparameters of IPPO and MAPPO

Method	Name	Value
IPPO & MAPPO	critic learning rate	0.001
	actor learning rate	0.99
	γ	0.99
	λ	0.95
	ϵ	0.2
	clip range	0.1
	normalize advantage	True
	normalize inputs	True
	grad norm	0.5
	number of actors	8
	critic coefficient	2
	entropy coefficient	0
	mini epochs for actor update	10
	mini epochs for critic update	10
	mini batch size	64

C Full StarCraft II Results

Complete results for StarCraft II are as shown in Fig. 6, Fig. 7, Fig. 8.

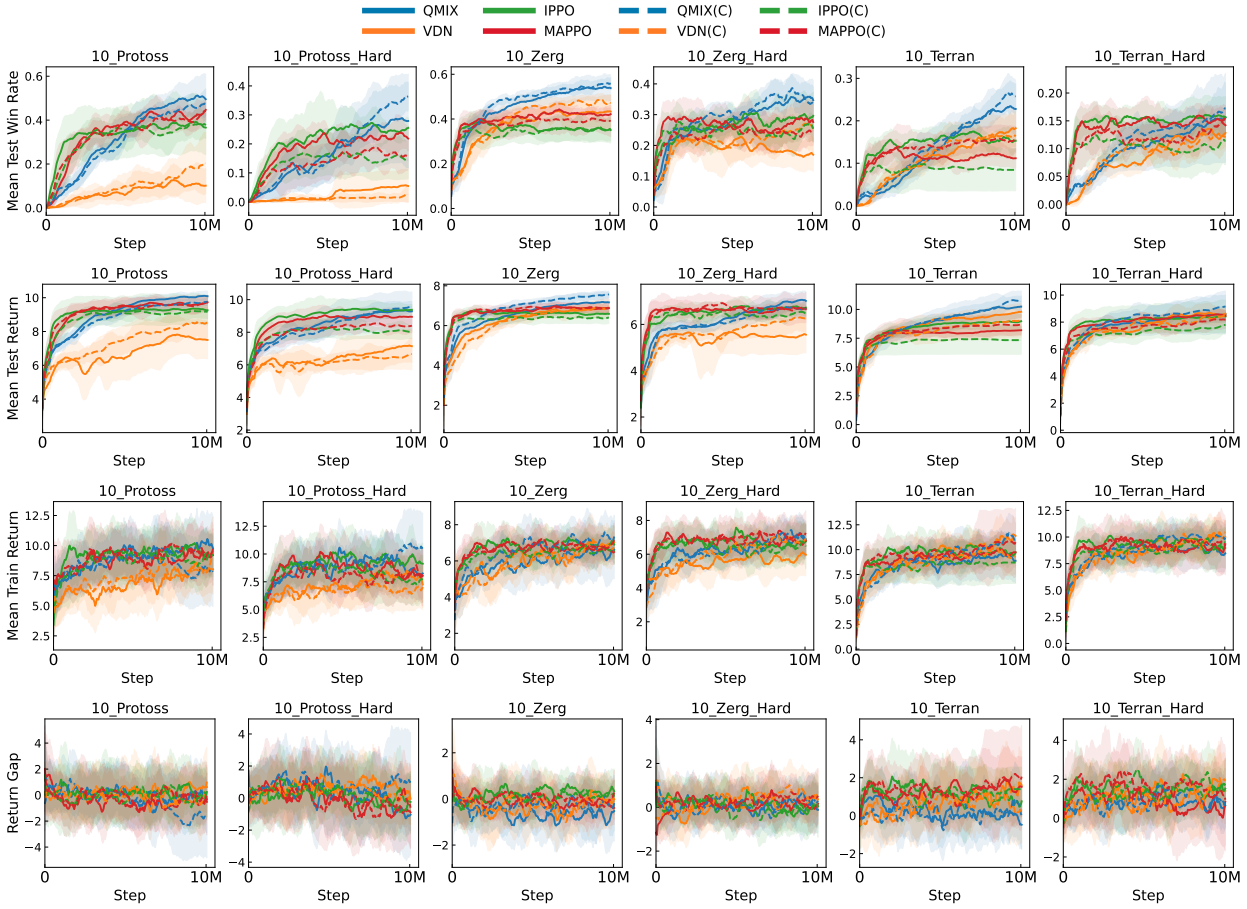


Figure 6: Experimental results on SMAC unit swapping tasks. Dashed lines indicate the inclusion of information on capabilities as part of the agent observations. Standard deviation is shaded.

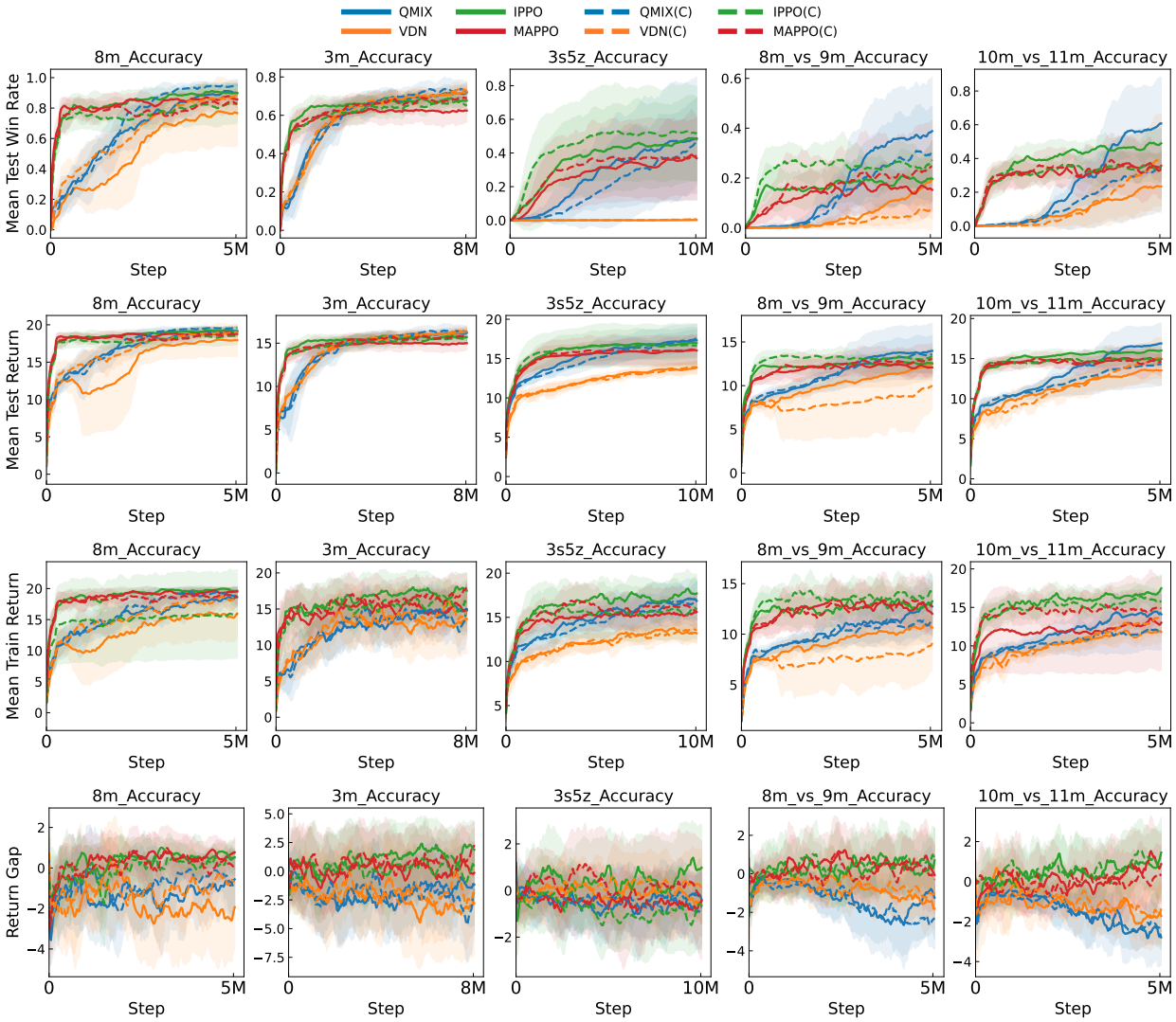


Figure 7: Experimental results on SMAC unit accuracy tasks. Dashed lines indicate the inclusion of information on capabilities as part of the agent observations. Standard deviation is shaded.

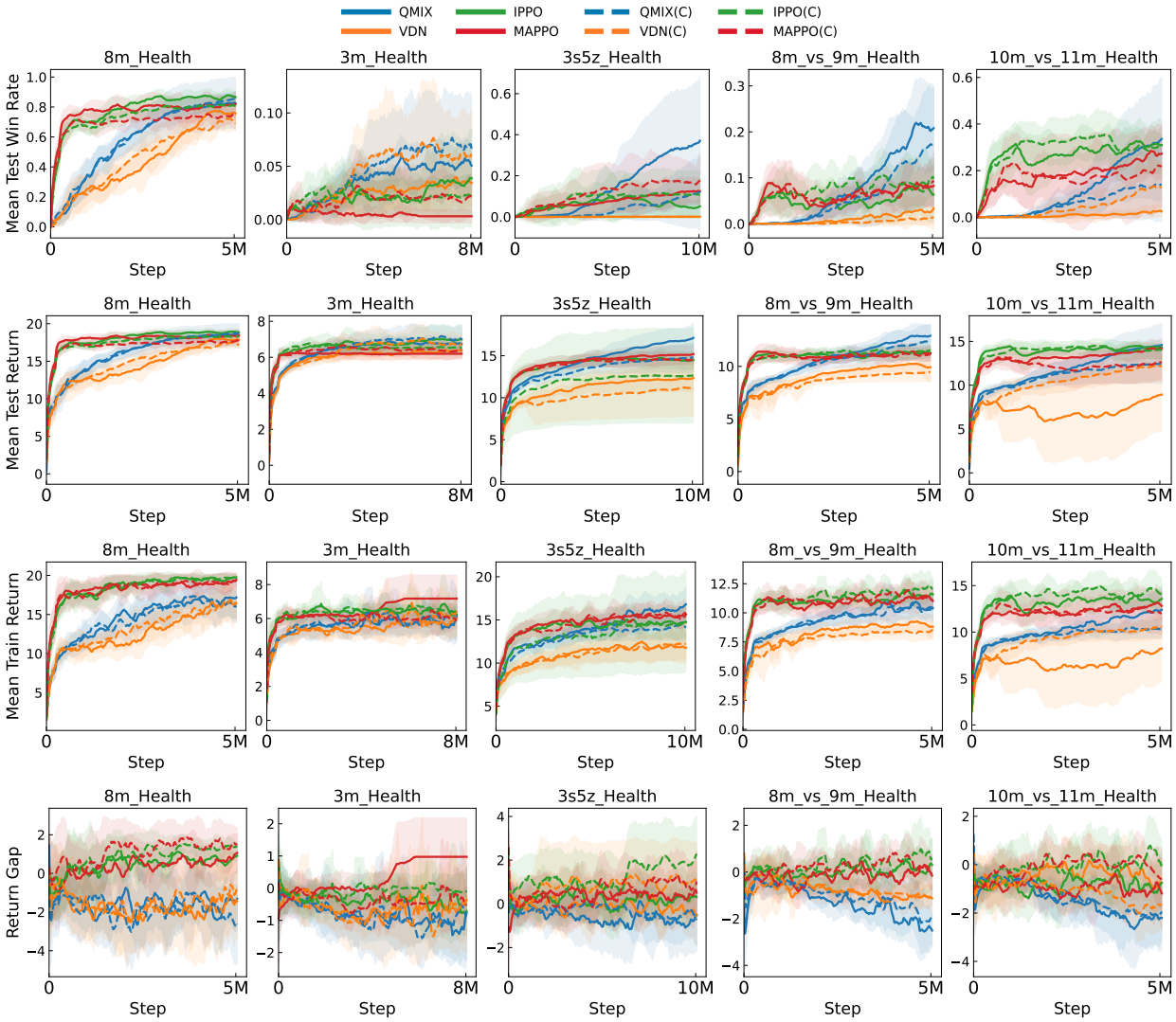


Figure 8: Experimental results on SMAC unit health tasks. Dashed lines indicate the inclusion of information on capabilities as part of the agent observations. Standard deviation is shaded.