Multiplayer Combinatorial Bandits Under Information Asymmetry

Anonymous authors
Paper under double-blind review

Abstract

In this paper, we extend linear combinatorial bandits Gai et al. (2012) to a multiplayer setting with information asymmetry Chang et al. (2022); Chang and Lu (2024), where each player controls an arm and independently decides whether to pull it, with coordination allowed only before rounds begin. We analyze three scenarios: action asymmetry (players can't observe others' actions but receive identical rewards per iteration), reward asymmetry (players observe actions but receive private i.i.d. rewards), and combined asymmetry. We derive near-optimal, gap-independent regret bounds for all scenarios: For action or reward asymmetry, we achieve $\tilde{\mathcal{O}}(\sqrt{T})$, which improves significantly from Gai et al. (2010); for both action and reward asymmetry, we achieve near-optimal bounds similar to that of Chang et al. (2022). We finally generalize our results to settings where players decide either not to pull or to pull one out of multiple arms, and achieve similar bounds in similar settings as above.

Keywords: multi-agent learning, combinatorial bandits, UCB algorithm, information asymmetry, regret analysis.

1 Introduction

Multi-Armed Bandit (MAB) problems are a foundational model involving sequential decision-making under uncertainty, where an agent selects from K arms to maximize rewards over time. A common approach to solve the problem is using the upper confidence bound (UCB) Lai and Robbins (1985), which selects the arm with the highest upper bound on each round.

Combinatorial MAB extends traditional MAB by allowing actions that comprise multiple arms, with agents observing both individual and total rewards - a framework that efficiently handles exponential action spaces by leveraging information overlap. This has direct applications in online advertising, where an advertiser iteratively selects webpage subsets for ad placement, learning user click probabilities across different page combinations to maximize overall engagement. Similarly, viral marketing presents an analogous challenge: marketers must select optimal seed nodes in social networks while learning influence probabilities between users to maximize campaign spread.

On another branch of MAB, information asymmetric MAB algorithms have been extensively covered by Chang et al. (2022); Chang and Lu (2024; 2025); Chang and Karthik (2025). Three particular sub-problems arise: action asymmetry (where players cannot observe other players' actions but receive identical rewards per iteration), reward asymmetry (where players observe actions but receive private i.i.d. rewards), and combined asymmetry (both action and reward asymmetry). As mentioned above, information asymmetry has covered multiple settings, but no extension towards combinatorial work has been discussed at all.

Many modern distributed systems involve multiple agents making coordinated decisions under limited information. A prominent example arises in wireless communication networks, where devices must dynamically select channels or allocate transmission resources based on locally observed feedback. In cognitive radio networks, for instance, multiple users attempt to access available spectrum while learning channel qualities and avoiding interference with one another Gai et al. (2010; 2012). Similar decentralized learning problems appear in dynamic spectrum access systems, where agents independently sense channels and update transmission

strategies without full visibility into other agents' actions or rewards Chen and Tong (2013). Such environments naturally exhibit *information asymmetry*: each agent observes only partial feedback about the joint system state while attempting to coordinate implicitly with other agents. These challenges are widely studied in modern wireless systems and multi-agent learning frameworks, including reinforcement learning approaches for distributed spectrum allocation and cognitive radio networks Tan et al. (2022); Kaur et al. (2023); Liu et al. (2022); Wang et al. (2022). More broadly, decentralized learning and coordination under limited information have been recognized as fundamental challenges in distributed optimization and multi-agent systems Awerbuch and Kleinberg (2008). These applications motivate the study of multiplayer bandit models with asymmetric information, where agents must learn optimal decisions despite observing only partial feedback about the actions or rewards of others.

As we approach this intersection, we bring up a crucial question: *Can we extend the information asymmetric MAB system to accommodate for combinatorial MAB?* We show that the answer is affirmative, by presenting two novel formulations: the 0-1 problem (each player decides whether to pull their assigned arm) and the joint arm problem (each player selects from their private set of arms).

Our Contribution We extend combinatorial bandits to multiplayer settings with information asymmetry, considering two formulations: the 0-1 problem (binary pull/pull or no-pull decisions) and its generalization to joint arm selection. Our main contributions are:

(1) Near-optimal algorithms for three asymmetry levels: We develop UCB-based algorithms that achieve $\tilde{\mathcal{O}}(\sqrt{T})$ regret for action- or reward-only asymmetry (Problems A and B), significantly improving upon existing $\mathcal{O}(\log T/\Delta_{\min}^2)$ gap-dependent bounds. For full asymmetry (Problem C), we adapt deterministic exploration to achieve regrets similar to Chang et al. (2022).

(2) Unified framework from binary to multi-arm: Our algorithms naturally extend from the 0-1 setting to the general K-arm setting, with the dependence on K appearing only as a multiplicative factor, implying that multi-arm complexity doesn't fundamentally change the problem structure.

(3) Coordination without communication: We extend state-of-the-art mechanisms Chang et al. (2022); Chang and Lu (2024) that enable decentralized coordination despite information constraints, with only additive $O(|\mathcal{F}|)$ overhead for signaling.

2 Related Works

Combinatorial Bandits The combinatorial bandit setting has been widely studied under various structural assumptions. Chen et al. (2013) propose a general CMAB framework with the CUCB algorithm achieving tight regret bounds. Gai et al. (2010) model multiuser channel allocation as a combinatorial bandit with the MLPS algorithm, extended in Gai et al. (2012) to linear rewards via LLR. Wang and Zhu (2022) introduce TS-Explore for combinatorial pure exploration, improving over UCB methods. In adversarial settings, Cesa-Bianchi and Lugosi (2012) develop ComBand with optimal regret bounds, while Combes et al. (2015) propose ESCB and COMBEXP. Chen et al. (2014) and Liu et al. (2024) contribute to pure exploration and multivariant extensions, respectively.

Cooperative Multiplayer Bandits In cooperative multiplayer settings, players collaborate to identify optimal arms from a common pool with communication constrained by graph structure Awerbuch and Kleinberg (2008). Proposed algorithms include ϵ-greedy approaches Szorenyi et al. (2013), gossip-based UCB methods Landgren et al. (2016); Martínez-Rubio et al. (2019), and leader-based strategies Wang et al. (2020). In adversarial settings, Bar-On and Mansour (2019) introduced leader coordination with EXP3. Other models consider neighbor reward observation Cesa-Bianchi et al. (2016) and asynchronous participation Bonnefoi et al. (2017); Cesa-Bianchi et al. (2020). Proutiere and Wang (2019) extended collision-based frameworks to Lipschitz settings with the DPE algorithm.

Competing Bandits The competing bandits framework Liu et al. (2020) incorporates player preferences in collision models: only the highest-priority player receives rewards when multiple players select the same arm. The centralized CUB solution has players report UCB values to a coordinator. Cen and Shah (2022)

showed optimal logarithmic regret with platform-managed reassignments, while Jagadeesan et al. (2021) introduced equilibrium concepts via negotiated transfers. An Explore-Then-Commit algorithm Liu et al. (2020) attains optimal regret assuming known gaps, with Sankararaman et al. (2021) later removing this assumption. Liu et al. (2021) developed decentralized UCB with collision avoidance.

Game Theory Several works bridge bandits and game theory. Mao (2024) proposes decentralized algorithms for equilibria in Markov games. Bistritz and Leshem (2018) introduce Game of Thrones for fully decentralized multiplayer bandits without communication. Brânzei and Peres (2021) characterize utility alignment effects in multiplayer stochastic bandits, while Bravo et al. (2018) analyze Nash equilibria convergence using no-regret learning. Ontanón (2013; 2017) develop methods for combinatorial bandits in Monte Carlo Tree Search. Heliou et al. (2017) show exponential weights achieve Nash equilibria in potential games under bandit feedback. Scheid et al. (2024) propose IPA algorithms for principal-agent settings with misaligned objectives, and Kang et al. (2021) study behavioral aspects through energy-based measures.

3 Preliminaries

We extend the combinatorial bandit framework Cesa-Bianchi and Lugosi (2012); Gai et al. (2012) to a multiplayer setting with information asymmetry Chang et al. (2022). Consider M players interacting with the environment over a time horizon of T rounds. Each player $m \in [M] := \{1, \dots, M\}$ controls a set of K arms and must decide which arm to pull at each round, or alternatively abstain from pulling any arm. Players cannot communicate during the learning process and must coordinate only through pre-agreed strategies and environmental feedback.

Problem Structure Let $\mathcal{A}_m = \{0, 1, \dots, K\}$ denote the action set of player m, where 0 represents the null action (abstaining). A *combinatorial arm* is a joint action vector

$$\vec{a} = (a_1, a_2, \dots, a_M) \in \mathcal{A}_1 \times \cdots \times \mathcal{A}_M.$$

Each non-null individual arm $a_m \in [K]$ produces stochastic rewards drawn from an unknown distribution $\mathcal{D}_{a_m}$ with mean μ_{a_m}. We assume that the reward distributions are 1-subGaussian. The reward of a combinatorial arm is defined as the sum of rewards of its active components:

$$X_{\vec{a}}(t) = \sum_{m=1}^{M} X_{a_m}(t),$$

where $X_{a_m}(t)$ is the reward generated by arm a_m at round t (with $X_0(t) = 0$).

The expected reward of a combinatorial arm is therefore

$$\mu_{\vec{a}} = \mathbb{E}[X_{\vec{a}}(t)] = \sum_{m=1}^{M} \mu_{a_m}.$$

Not all joint actions are feasible. Let

$$\mathcal{F} \subseteq \mathcal{A}_1 \times \cdots \times \mathcal{A}_M$$

denote the set of feasible combinatorial arms known to all players. If a joint action $\vec{a} \notin \mathcal{F}$ is selected, the system produces zero reward. Let

$$\mu^* = \max_{\vec{a} \in \mathcal{F}} \mu_{\vec{a}}$$

denote the optimal expected reward. For any suboptimal feasible action $\vec{a} \in \mathcal{F}$, the *combinatorial arm gap* is defined as

$$\Delta_{\vec{a}} := \mu^* - \mu_{\vec{a}}, \qquad \Delta_{\min} := \min_{\vec{a} \in \mathcal{F} \colon \Delta_{\vec{a}} > 0} \Delta_{\vec{a}}.$$

Information Asymmetry The main challenge arises from the limited observations available to each player. Following Chang et al. (2022), we consider three feedback models:

- **Problem A (Action Asymmetry).** Players cannot observe the actions chosen by other players, but they observe the same realized reward. When joint action $\vec{a}$ is selected at round t, all players observe the global reward $X_{\vec{a}}(t)$.
- **Problem B (Reward Asymmetry).** Players observe the joint action $\vec{a}$ but receive private stochastic rewards. When arm a_m is selected by player m, each player j observes an independent sample
$$X_{a_m}^{(j)}(t) \sim \mathcal{D}_{a_m}.$$
- **Problem C (Full Asymmetry).** Players observe neither the actions nor the rewards of other players. Each player m only observes the arm they selected and the reward generated by that arm.

Coordination Mechanism To enable coordination without communication, we adopt a shared exploration structure inspired by Chang et al. (2022). Before learning begins, players agree on a *predetermined ordering* of the feasible combinatorial arms in $\mathcal{F}$: a fixed, deterministic sequence in which all feasible joint actions are enumerated and cycled through during exploration. This ordering ensures that all players know which joint action is being played at any given round without requiring real-time communication. During execution, players follow this ordering to ensure consistent updates of empirical statistics.

Remark 1 *For Problems A and C, where players cannot observe others' actions, the predetermined ordering allows players to align reward observations with arm indices. Even though the identity of other players' actions is hidden, the ordered exploration ensures that all players maintain consistent confidence intervals. For Problem B, where players receive private rewards, we introduce* signaling rounds *in which players deliberately deviate from the agreed ordering to signal arm elimination events. The choice of ordering is critical: it should be cyclic over the player indices, ensuring that no arm is neglected for more than M rounds. Without this, some players' arms may remain unsampled for exponentially many rounds, causing their confidence intervals to stay wide and leading to much worse regret (see the analysis following Theorem 4).*

Problem Variants We study two variants of the multiplayer combinatorial bandit problem:

1. **0–1 Problem.** Each player controls a single arm ($K = 1$) and decides whether to pull the arm or abstain.
2. **Joint Arm Problem.** Each player chooses one arm from a private set of K arms or abstains.

Regret Let $\vec{a}_t$ denote the combinatorial arm played at round t. The performance of a learning algorithm is measured by the cumulative regret
$$R_T = T\mu^* - \sum_{t=1}^{T} \mathbb{E}[X_{\vec{a}_t}],$$
which quantifies the expected loss relative to always playing the optimal combinatorial arm.

4 The 0–1 Problem

In the 0–1 problem, each player $m \in [M]$ controls a single binary action: either pull their arm or abstain. Thus each player's action set is $\mathcal{A}_m = \{0, 1\}$, where 1 denotes pulling and 0 denotes abstaining. A joint action is a binary vector
$$\vec{a} = (a_1, \ldots, a_M) \in \{0, 1\}^M,$$

and only joint actions in the feasible set $\mathcal{F} \subseteq \{0,1\}^M$ produce nonzero reward. As in the preliminaries, for any feasible joint action $\vec{a} \in \mathcal{F}$, the reward is

$$X_{\vec{a}}(t) = \sum_{m=1}^{M} X_{a_m}(t), \qquad \mu_{\vec{a}} = \mathbb{E}[X_{\vec{a}}(t)].$$

For any suboptimal feasible action $\vec{a} \in \mathcal{F}$, define its gap by

$$\Delta_{\vec{a}} := \mu^* - \mu_{\vec{a}}, \qquad \Delta_{\min} := \min_{\vec{a} \in \mathcal{F}:\, \Delta_{\vec{a}} > 0} \Delta_{\vec{a}}.$$

4.1 Problem A: Information Asymmetry in Actions

In Problem A, players cannot observe the actions of the other players, but all players observe the same realized global reward $X_{\vec{a}_t}(t)$. The main difficulty is therefore not reward estimation itself, but determining which individual components of the joint action should be updated when the other players' actions are hidden.

To address this, we use a UCB-style elimination scheme based on a shared predetermined ordering of feasible joint actions. For each player-arm indicator $a \in [M]$, let

$$\epsilon_a(t) := \sqrt{\frac{4 \log T}{n_a(t)}}, \tag{1}$$

where $n_a(t)$ is the number of rounds before time t in which player a pulled their arm. The empirical mean reward of arm a at time t is defined as

$$\widehat{\mu}_a(t) := \frac{1}{n_a(t)} \sum_{s \le t:\, a_s = a} X_a(s).$$

Using these quantities, we define

$$\mathrm{UCB}_a(t) = \widehat{\mu}_a(t) + \epsilon_a(t), \qquad \mathrm{LCB}_a(t) = \widehat{\mu}_a(t) - \epsilon_a(t). \tag{2}$$

For a feasible joint action $\vec{a} = (a_1, \ldots, a_M)$, define the corresponding combinatorial indices by

$$\mathrm{UCB}_{\vec{a}}(t) = \sum_{m=1}^{M} \mathrm{UCB}_{a_m}(t), \qquad \mathrm{LCB}_{\vec{a}}(t) = \sum_{m=1}^{M} \mathrm{LCB}_{a_m}(t). \tag{3}$$

Hence the confidence interval for $\mu_{\vec{a}}$ is

$$I_{\vec{a}}(t) := \left[\mathrm{LCB}_{\vec{a}}(t),\, \mathrm{UCB}_{\vec{a}}(t)\right]. \tag{4}$$

We use the following good event:

$$G := \bigcap_{t=1}^{T} \bigcap_{a=1}^{M} \left\{ |\widehat{\mu}_a(t-1) - \mu_a| \le \sqrt{\frac{4 \log T}{n_a(t-1)}} \right\}. \tag{5}$$

Lemma 2 *The bad event in* (5) *satisfies*

$$\mathbb{P}(G^c) \le \frac{M}{T}.$$

The proof is standard and deferred to the appendix.

4.1.1 Algorithmic idea

As noted in Remark 1, before learning begins, the players agree on a fixed ordering of the feasible set $\mathcal{F}$. For each player $m \in [M]$, define

$$S_m := \{\vec{a} \in \mathcal{F} : a_m = 1\}, \tag{6}$$

namely, the feasible joint actions in which player m pulls.

At each round, the players cycle through the sets $S_1, \ldots, S_M$ and execute the next surviving action in the current set. If some feasible action $\vec{a}$ is dominated by another feasible action $\vec{a}'$, in the sense that

$$\mathrm{UCB}_{\vec{a}}(t-1) < \mathrm{LCB}_{\vec{a}'}(t-1),$$

then $\vec{a}$ is eliminated from every set S_m that contains it. The predetermined ordering is chosen so that each individual arm is pulled at least once every M rounds; this prevents starvation of any player's estimate.

Lemma 3 *For every player-arm indicator $a \in [M]$ and every round t, all players maintain the same values of $n_a(t)$, $\widehat{\mu}_a(t)$, $\mathrm{UCB}_a(t)$, and $\mathrm{LCB}_a(t)$.*

Proof: The proof is by induction on t. Initially all players have identical statistics. At round t, they execute the same feasible joint action under the agreed ordering and observe the same global reward. Therefore they update the same components using the same observation, so all empirical means and confidence bounds remain synchronized. □

Algorithm 1 `mCombinatorialUCB-A` for the 0–1 problem

Require: Horizon T, feasible set $\mathcal{F}$, ordered sets $S_1, \ldots, S_M$

```
Initialize m ← 1
for each player-arm indicator a ∈ [M] do
    n_a ← 0, μ̂_a ← 0, UCB_a ← +∞, LCB_a ← −∞
end for
for t = 1 to T do
    Choose the next surviving action a⃗_t ∈ S_m
    while ∃a⃗′ ∈ F such that UCB_{a⃗_t}(t − 1) < LCB_{a⃗′}(t − 1) do
        Remove a⃗_t from all sets S_j containing a⃗_t
        Replace a⃗_t by the next surviving action in S_m
    end while
    for each a ∈ [M] with (a⃗_t)_a = 1 do
        Observe reward contribution X_a(t) and update n_a, μ̂_a
        Update UCB_a(t) and LCB_a(t) using (1)
    end for
    for each a⃗ ∈ S_m do
        Update UCB_a⃗(t) and LCB_a⃗(t) via (3)
    end for
    m ← (m mod M) + 1
end for
```

The regret guarantee is as follows.

Theorem 4 *Algorithm 1 satisfies*

$$R_T \leq 8M(M+1)\sqrt{T \log T} + M.$$

The proof is deferred to the appendix; we sketch the main ideas here. We decompose the regret into contributions from rounds where the good event G holds and rounds where it fails. Under G, every suboptimal feasible action is eliminated once its confidence interval separates from that of the optimal action, which occurs after $O(M^2 \log T/\Delta_{\vec{a}}^2)$ pulls. Summing over all feasible actions and converting to a gap-independent

bound via Cauchy–Schwarz yields the $\sqrt{T \log T}$ dependence. The bad-event contribution is bounded by Lemma 2. Relative, which gives a gap-dependent bound of order $O(\log T/\Delta_{\min}^2)$, our analysis yields a directly gap-independent regret bound. This is important because a naive conversion from a gap-dependent bound may introduce large factors of the form $\Delta_{\vec{a}}/\Delta_{\min}^2$.

As noted in Remark 1, the cyclic ordering ensures that no arm is neglected for more than M rounds, preventing starvation of any player's estimate.

4.2 Problem B: Information Asymmetry in Rewards

In Problem B, the players observe the selected joint action $\vec{a}_t$, but the reward observations are private. More precisely, if player m pulls at round t, then player j receives an independent sample

$$X_{a_m}^{(j)}(t) \sim \mathcal{D}_{a_m},$$

with mean μ_{a_m}. Thus players agree on which joint action was played, but their empirical reward estimates need not coincide.

This breaks the synchronized-update argument used in Problem A. Even if all players follow the same action sequence, their local confidence intervals may differ, so one player may wish to eliminate a feasible action that another still considers plausible. Consequently, elimination must be coordinated explicitly.

4.2.1 Signaling-based coordination

To restore coordination, we augment the protocol with *signaling rounds*, following the signaling idea of Chang and Lu (2024). Each player maintains private confidence intervals

$$\mathrm{LCB}_a^j(t) = \widehat{\mu}_a^j(t) - \epsilon_a(t), \qquad \mathrm{UCB}_a^j(t) = \widehat{\mu}_a^j(t) + \epsilon_a(t),$$

where $\epsilon_a(t)$ is shared because it depends only on the public pull counts. When player j concludes that a feasible action $\vec{a}$ should be removed, they intentionally deviate from the agreed action. Since actions are publicly observable in Problem B, the other players detect this deviation and remove $\vec{a}$ from their active sets as well.

Algorithm 2 `mCombinatorialUCB-B` for the 0–1 problem

Require: Horizon T, feasible set $\mathcal{F}$, ordered sets $S_1, \ldots, S_M$
Initialize $m \leftarrow 1$ and private statistics for each player
for $t = 1$ to T **do**
  Choose the next surviving action $\vec{a}_t \in S_m$
  if some player detects $\vec{a}_t$ is dominated **then**
    That player signals elimination by deviating from $\vec{a}_t$
    All players remove $\vec{a}_t$ from every set S_j containing it
    Continue with the next surviving action in S_m
  else
    Execute $\vec{a}_t$ and let each player update their private estimates
  end if
  Update combinatorial confidence bounds for actions in S_m
  $m \leftarrow (m \bmod M) + 1$
end for

Theorem 5 *Algorithm 2 satisfies*

$$R_T \leq |\mathcal{F}| + 8M(M+1)\sqrt{T \log T} + M.$$

The proof follows from Theorem 4 together with the observation that each signaling event costs at most one unit of regret, and each feasible action is signaled at most once. Hence the total additional cost is at most $|\mathcal{F}|$. Despite this additive term, the bound remains better than applying UCB directly over the entire feasible set, which yields regret of order $O(\sqrt{|\mathcal{F}|T})$.

4.3 Problem C: Information Asymmetry in Actions and Rewards

Problem C is the most difficult setting: players observe neither the actions of others nor their rewards. Each player sees only their own local action and its realized reward. Thus neither the synchronized-update mechanism of Problem A nor the signaling mechanism of Problem B is available.

We therefore adapt the deterministic exploration strategy of Chang et al. (2022). Time is divided into epochs indexed by λ. In epoch λ, the players first perform an exploration phase, in which each representative action from the ordered subsets (6) is sampled exactly $f(\lambda)$ times, where f is a nondecreasing exploration schedule satisfying $f(\lambda) \to \infty$. Larger values of $f(\lambda)$ improve estimation accuracy but also increase exploration regret. After exploration, each player forms local empirical estimates of feasible actions using only their own observed rewards, and then commits to their empirically best feasible action until the next epoch boundary.

Algorithm 3 `mDSEE` for the 0–1 problem

Require: Feasible set $\mathcal{F}$, exploration schedule $f(\lambda)$

$\lambda \leftarrow 1$, $t \leftarrow 1$
while $t \leq T$ **do**
  Exploration: sample each ordered representative action from the sets $S_1, \ldots, S_M$ exactly $f(\lambda)$ times
  Update local empirical means using only each player's own observed rewards
  Commit: each player selects its empirically best feasible action
  Play the resulting joint action until the next epoch boundary at time $2^r > t$
  $\lambda \leftarrow \lambda + 1$
end while

The role of f is now explicit: in epoch λ, the exploration phase has length proportional to $Mf(\lambda)$, which directly contributes to the regret, while also reducing the probability of committing to a suboptimal action in later epochs.

Theorem 6 *Algorithm 3 satisfies*

$$R_T = O(\lfloor \log_2 T \rfloor \, f(\lfloor \log_2 T \rfloor) \, M) \, .$$

The key insight is that in epoch λ, each of the M player groups contributes $f(\lambda)$ exploration rounds, so the total exploration cost per epoch is $O(Mf(\lambda))$. During the commit phase, the probability of committing to a suboptimal action decreases with $f(\lambda)$ via standard concentration arguments. Balancing these two sources of regret and summing over $\lfloor \log_2 T \rfloor$ epochs yields the stated bound. The full proof is in the appendix.

This the exploration schedule f must be chosen to balance exploration cost against estimation accuracy. In particular, if f grows too slowly, the players may continue to miscoordinate for too long; if it grows too quickly, exploration itself dominates the regret. The appendix makes this trade-off explicit and shows that for any fixed reward gaps, the regret remains logarithmic in T.

5 The Joint Arm Problem

We now generalize the 0–1 setting to the case where each player has multiple available arms. For each player $m \in [M]$, let

$$\mathcal{A}_m = \{0, 1, \ldots, K\},$$

where 0 denotes abstaining and $k \in [K]$ denotes pulling the k-th arm of player m. A joint action is therefore a vector

$$\vec{a} = (a_1, \ldots, a_M) \in \mathcal{A}_1 \times \cdots \times \mathcal{A}_M.$$

As in the preliminaries, only actions in the feasible set

$$\mathcal{F} \subseteq \mathcal{A}_1 \times \cdots \times \mathcal{A}_M$$

yield nonzero reward. If player m selects arm k, the corresponding mean reward is denoted by μ_m^k. For a feasible joint action $\vec{a} \in \mathcal{F}$, the reward is additive:

$$X_{\vec{a}}(t) = \sum_{m=1}^{M} X_{m,a_m}(t), \qquad \mu_{\vec{a}} = \mathbb{E}[X_{\vec{a}}(t)] = \sum_{m=1}^{M} \mu_{m,a_m},$$

with the convention that $\mu_{m,0} = 0$. Thus, unlike in the 0–1 problem, the quality of a feasible joint action depends not only on which players act, but also on which specific arm each active player selects.

5.1 Problem A: Information Asymmetry in Actions

We first consider the analogue of Problem A in the joint-arm setting, where players receive the same realized reward but cannot observe the actions of the others. A naive extension of the 0–1 algorithm would cycle through all possible joint actions, leading to complexity exponential in M. To avoid this, we separate elimination at the *individual-arm level* and at the *combinatorial level*.

As in the 0–1 setting, let $S_1, \ldots, S_M$ denote the ordered collections of feasible joint actions. In addition, for each player m, we maintain a set of surviving individual arms

$$D_m \subseteq [K], \tag{7}$$

where D_m contains the arms of player m that have not yet been eliminated. Initially, $D_m = [K]$ for every player m.

The key difference from the 0–1 case is the following. When the current feasible action lies in S_m, we do not execute it only once. Instead, we repeat it for $|D_m|$ rounds while player m cycles through the currently surviving arms in D_m. This guarantees that every arm still considered plausible for player m continues to receive samples.

For each individual arm $k \in D_m$, player m maintains the empirical mean $\widehat{\mu}_m^k(t)$ and the confidence bounds

$$\mathrm{UCB}_m^k(t) = \widehat{\mu}_m^k(t) + \sqrt{\frac{4\log T}{n_m^k(t)}}, \qquad \mathrm{LCB}_m^k(t) = \widehat{\mu}_m^k(t) - \sqrt{\frac{4\log T}{n_m^k(t)}}.$$

To compare feasible joint actions when the exact arm choice of an active player may still vary over D_m, we define the interval

$$I_{\vec{a}}(t) = \left[\sum_{m:\, a_m \neq 0} \min_{k \in D_m} \mathrm{LCB}_m^k(t), \sum_{m:\, a_m \neq 0} \max_{k \in D_m} \mathrm{UCB}_m^k(t) \right]. \tag{8}$$

This interval is conservative: it upper- and lower-bounds the reward of $\vec{a}$ over all surviving choices of individual arms for the active players.

Elimination now occurs at two levels:

1. **Individual-arm elimination:** if, for some player m, an arm $k \in D_m$ is dominated by another arm $k' \in D_m$, then k is removed from D_m.
2. **Combinatorial-arm elimination:** if the interval of some feasible action $\vec{a}$ is strictly below that of another feasible action $\vec{a}'$, then $\vec{a}$ is removed from the relevant sets S_j.

Because all players receive the same reward realizations and follow the same predetermined ordering, their statistics remain synchronized exactly as in the 0–1 case. Hence they eliminate the same individual arms and the same feasible joint actions at the same times.

Theorem 7 *`Joint-mCombinatorialUCB-A` satisfies*

$$R_T \leq 4K^2M^2\sqrt{T\log T}.$$

The proof is deferred to the appendix. The argument mirrors the 0–1 case, but the two-level elimination (individual arms and combinatorial arms) introduces the additional K^2 factor: each player must cycle through up to K surviving arms before a single combinatorial-level confidence interval update can be made. Relative, the additional factor in the regret bound comes from the need to explore up to K surviving arms per player while still preserving coordinated elimination across players.

5.2 Problem B: Information Asymmetry in Rewards

We next consider the joint-arm analogue of Problem B, where players observe the selected joint action but receive private reward samples. As in the 0–1 case, synchronized elimination is no longer automatic, because different players may form different empirical estimates from their private observations.

The signaling mechanism from Section 4.2.1 extends to this setting, but one additional distinction is needed. Since elimination may now occur either at the level of an individual arm in D_m or at the level of a feasible joint action in S_m, the signaling rule must make clear which object is being eliminated. We therefore separate the two types of signaling:

- **Individual-arm signaling:** only player m may signal elimination of an arm in their own surviving set D_m.
- **Combinatorial-arm signaling:** only players other than m may signal elimination of a feasible action currently being explored from S_m.

This convention makes the deviation unambiguous: when player m deviates, the other players interpret it as elimination within D_m; when another player deviates, they interpret it as elimination of the current feasible action.

Theorem 8 *`Joint-mCombinatorialUCB-B` satisfies*

$$R_T \leq MK + |\mathcal{F}| + 4K^2M^2\sqrt{T\log T}.$$

The additive MK term accounts for the possibility of eliminating up to K individual arms for each of the M players, while the additive $|\mathcal{F}|$ term accounts for signaling rounds used to eliminate feasible joint actions. Despite these costs, the bound remains better than applying UCB directly to the full feasible action space, which would incur regret of order $O(\sqrt{|\mathcal{F}|T})$. The proof and pseudocode are deferred to the appendix.

5.3 Problem C: Information Asymmetry in Actions and Rewards

Finally, we extend the `mDSEE` approach from Section 4.3 to the joint-arm setting. The main new issue is that each player must now estimate multiple local arms rather than a single binary action. Accordingly, during each exploration epoch, every representative action in the ordered sets $S_1, \ldots, S_M$ is sampled repeatedly so that player m cycles through all currently relevant arms in D_m.

More precisely, in each exploration epoch λ, the algorithm allocates $Kf(\lambda)$ exploration plays per player-group rather than $f(\lambda)$. This increases the exploration cost by a factor of K, but ensures that each player collects enough local samples to compare the arms in D_m. After the exploration phase, each player forms local empirical estimates of feasible actions using only their own observed rewards and then commits to their empirically best feasible action until the next epoch boundary.

Theorem 9 *The regret of `Joint-mDSEE` satisfies*

$$R_T = O(\lfloor \log_2 T \rfloor\, f(\lfloor \log_2 T \rfloor)\, KM)\,.$$

The multiplicative factor K reflects the additional exploration required to distinguish among the K local arms of each player. As in the binary case, the dependence on the reward gaps is absorbed into the choice of the exploration schedule f. For any fixed set of gaps, the regret remains logarithmic in T. The proof is deferred to the appendix.

6 Experimental Results

We validate our theoretical bounds through experiments on both $0-1$ and joint arm combinatorial bandits, comparing `mCombinatorialUCB-A` (action asymmetry), `mCombinatorialUCB-B` (reward asymmetry with signaling), and `mDSEE` (full asymmetry).

Setup We consider M players where each player has K arms plus a null action (0). The total action space contains $(K+1)^M - 1$ joint actions (excluding the all-zero action). The feasible set $\mathcal{F}$ is a random subset of this space, with $|\mathcal{F}|$ denoting its cardinality. Per-player true arm rewards are drawn i.i.d. from $\text{Unif}[0.2, 0.8]$, and stochastic rewards follow $\text{Beta}(\alpha, \beta)$ distributions with concentration $\kappa = 10$, where $\alpha = \mu\kappa$ and $\beta = (1-\mu)\kappa$ for mean μ.[1] We also set $f(x) = x$. We report averages over 10 independent runs with ± 1 standard deviation bands.

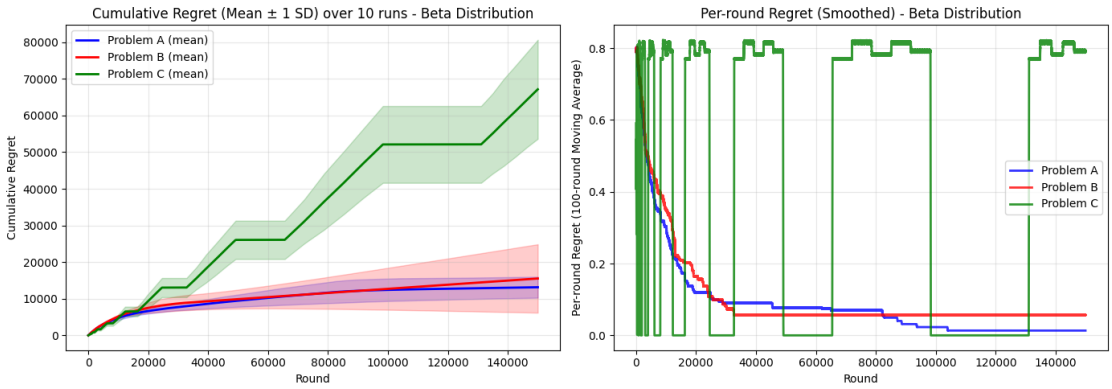


Figure 1: Binary setting: $M = 5$, $K = 1$, $|\mathcal{F}| = 19$, $T = 150{,}000$.

Binary Combinatorial Bandits With $M = 5$, $K = 1$, $|\mathcal{F}| = 19$ (60% of 31 possible actions), and $T = 150{,}000$ (Figure 1), `mCombinatorialUCB-A` attains lowest cumulative regret (≈15k), leveraging synchronized confidence bounds. `mCombinatorialUCB-B` shows higher variance due to signaling events that temporarily eliminate feasible actions, consistent with the additive $|\mathcal{F}|$ term in Theorem 5. `mDSEE` displays periodic exploration phases at each power of 2, creating its characteristic staircase pattern in cumulative regret with 0 regret during later commit phases.

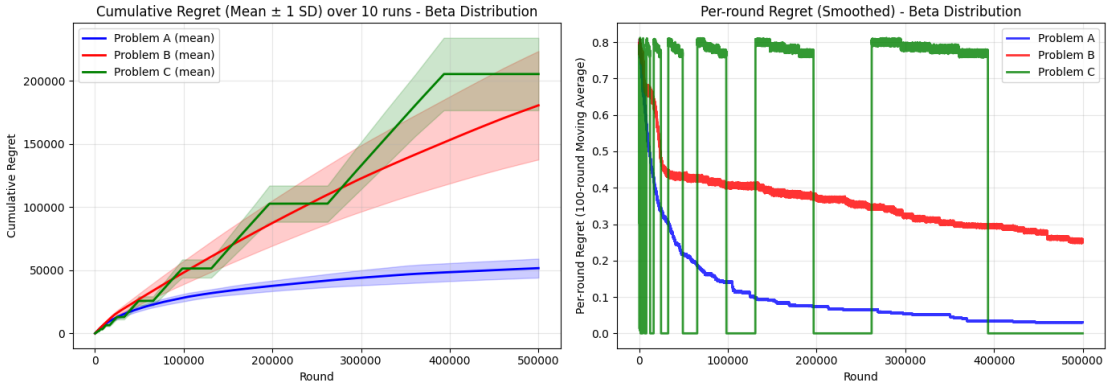


Figure 2: Multi-arm setting: $M = 4$, $K = 3$, $|\mathcal{F}| = 51$, $T = 500{,}000$.

[1] Note that any value drawn from Beta is in $[0, 1]$ which is $(1/4)$-subGaussian by Hoeffding's Lemma. See detailed proof in Appendix.

Multi-Arm Combinatorial Bandits With $M = 4$, $K = 3$, $|\mathcal{F}| = 51$ (20% of 255 possible actions), and $T = 500{,}000$, cumulative regret increases roughly K^2-fold compared to the binary case (Figure 2). `mCombinatorialUCB-A` maintains its advantage (≈50k regret), while `mCombinatorialUCB-B` reaches ≈180k with increased signaling complexity and more difficulty eliminating arms. `mDSEE` shows the steepest scaling (≈220k), requiring K times more exploration per epoch as predicted by the KM factor in Theorem 6.

Code Reproducibility Complete implementation and experimental code to reproduce all results are available at `https://anonymous.4open.science/status/combinatorial_bandits_code-FE60`.

7 Conclusions and Future Work

We studied combinatorial bandits under multiplayer information asymmetry and showed that coordination without communication is possible with only polynomial overhead in the number of players and arms. Using pre-agreed exploration orders (Problem A) or observable deviations (Problem B), we achieve near-optimal $\tilde{\mathcal{O}}(\sqrt{T})$ regret, matching single-player rates up to problem-dependent factors.

Moving from single-arm ($K = 1$) to multi-arm selection ($K > 1$) introduces only a K^2 multiplicative regret factor, indicating that coordination—not combinatorial complexity—is the primary challenge. Our gap-independent bounds also eliminate the exponential dependence on $1/\Delta^2_{\min}$ present in prior work.

Open questions include the tightness of the M^2 dependence, whether signaling-based coordination in Problem B is inherently costly, and whether Problem C admits gap-independent polylogarithmic regret without the exploration function. Extensions to time-varying feasible sets or intermediate observability models are promising future directions.

References

Baruch Awerbuch and Robert Kleinberg. Competitive collaborative learning. *Journal of Computer and System Sciences*, 74(8):1271–1288, 2008.

Yogev Bar-On and Yishay Mansour. Individual regret in cooperative nonstochastic multi-armed bandits. *Advances in Neural Information Processing Systems*, 32, 2019.

Ilai Bistritz and Amir Leshem. Distributed multi-player bandits-a game of thrones approach. *Advances in Neural Information Processing Systems*, 31, 2018.

Rémi Bonnefoi, Lilian Besson, Christophe Moy, Emilie Kaufmann, and Jacques Palicot. Multi-armed bandit learning in iot networks: Learning helps even in non-stationary settings. In *International Conference on Cognitive Radio Oriented Wireless Networks*, pages 173–185. Springer, 2017.

Stéphane Boucheron, Gábor Lugosi, and Pascal Massart. *Concentration Inequalities: A Nonasymptotic Theory of Independence*. Oxford University Press, 2013.

Simina Brânzei and Yuval Peres. Multiplayer bandit learning, from competition to cooperation. In *Conference on Learning Theory*, pages 679–723. PMLR, 2021.

Mario Bravo, David Leslie, and Panayotis Mertikopoulos. Bandit learning in concave n-person games. *Advances in Neural Information Processing Systems*, 31, 2018.

Sarah H Cen and Devavrat Shah. Regret, stability & fairness in matching markets with bandit learners. In *International Conference on Artificial Intelligence and Statistics*, pages 8938–8968. PMLR, 2022.

Nicolo Cesa-Bianchi and Gábor Lugosi. Combinatorial bandits. *Journal of Computer and System Sciences*, 78(5):1404–1422, 2012.

Nicolò Cesa-Bianchi, Tommaso Cesari, and Claire Monteleoni. Cooperative online learning: Keeping your neighbors updated. In *Algorithmic learning theory*, pages 234–250. PMLR, 2020.

Nicol'o Cesa-Bianchi, Claudio Gentile, Yishay Mansour, and Alberto Minora. Delay and cooperation in nonstochastic bandits. In *Conference on Learning Theory*, pages 605–622. PMLR, 2016.

William Chang and Aditi Karthik. Multiplayer information asymmetric bandits in metric spaces. *arXiv preprint arXiv:2503.08004*, 2025.

William Chang and Yuanhao Lu. Optimal cooperative multiplayer learning bandits with noisy rewards and no communication. In *2024 IEEE 63rd Conference on Decision and Control (CDC)*, pages 823–830. IEEE, 2024.

William Chang and Yuanhao Lu. Multiplayer information asymmetric contextual bandits. *arXiv preprint arXiv:2503.08961*, 2025.

William Chang, Mehdi Jafarnia-Jahromi, and Rahul Jain. Online learning for cooperative multi-player multi-armed bandits. In *2022 IEEE 61st Conference on Decision and Control (CDC)*, pages 7248–7253. IEEE, 2022.

Shiyao Chen and Lang Tong. Distributed learning and multiaccess of on-off channels. *IEEE Journal of Selected Topics in Signal Processing*, 7(5):837–845, 2013.

Shouyuan Chen, Tian Lin, Irwin King, Michael R Lyu, and Wei Chen. Combinatorial pure exploration of multi-armed bandits. *Advances in neural information processing systems*, 27, 2014.

Wei Chen, Yajun Wang, and Yang Yuan. Combinatorial multi-armed bandit: General framework and applications. In *International conference on machine learning*, pages 151–159. PMLR, 2013.

Richard Combes, Mohammad Sadegh Talebi Mazraeh Shahi, Alexandre Proutiere, et al. Combinatorial bandits revisited. *Advances in neural information processing systems*, 28, 2015.

Yi Gai, Bhaskar Krishnamachari, and Rahul Jain. Learning multiuser channel allocations in cognitive radio networks: A combinatorial multi-armed bandit formulation. In *2010 IEEE Symposium on New Frontiers in Dynamic Spectrum (DySPAN)*, pages 1–9. IEEE, 2010.

Yi Gai, Bhaskar Krishnamachari, and Rahul Jain. Combinatorial network optimization with unknown variables: Multi-armed bandits with linear rewards and individual observations. *IEEE/ACM Transactions on Networking*, 20(5):1466–1478, 2012.

Amélie Heliou, Johanne Cohen, and Panayotis Mertikopoulos. Learning with bandit feedback in potential games. *Advances in Neural Information Processing Systems*, 30, 2017.

Meena Jagadeesan, Alexander Wei, Yixin Wang, Michael Jordan, and Jacob Steinhardt. Learning equilibria in matching markets from bandit feedback. *Advances in Neural Information Processing Systems*, 34:3323–3335, 2021.

Xiaohan Kang, Hong Ri, Mohd Nor Akmal Khalid, and Hiroyuki Iida. Addictive games: Case study on multi-armed bandit game. *Information*, 12(12):521, 2021.

Amandeep Kaur, Jaismin Thakur, Mukul Thakur, Krishan Kumar, Arun Prakash, and Rajeev Tripathi. Deep recurrent reinforcement learning-based distributed dynamic spectrum access in multichannel wireless networks with imperfect feedback. *IEEE Transactions on Cognitive Communications and Networking*, 9(2): 281–292, 2023.

Tze Leung Lai and Herbert Robbins. Asymptotically efficient adaptive allocation rules. *Advances in applied mathematics*, 6(1):4–22, 1985.

Peter Landgren, Vaibhav Srivastava, and Naomi Ehrich Leonard. On distributed cooperative decision-making in multiarmed bandits. In *2016 European Control Conference (ECC)*, pages 243–248. IEEE, 2016.

Lydia T Liu, Horia Mania, and Michael Jordan. Competing bandits in matching markets. In *International Conference on Artificial Intelligence and Statistics*, pages 1618–1628. PMLR, 2020.

Lydia T Liu, Feng Ruan, Horia Mania, and Michael I Jordan. Bandit learning in decentralized matching markets. *Journal of Machine Learning Research*, 22(211):1–34, 2021.

Xin Liu, Can Sun, Kok-Lim Alvin Yau, and Celimuge Wu. Joint collaborative big spectrum data sensing and reinforcement learning based dynamic spectrum access for cognitive internet of vehicles. *IEEE Transactions on Intelligent Transportation Systems*, 25(1):805–815, 2022.

Xutong Liu, Siwei Wang, Jinhang Zuo, Han Zhong, Xuchuang Wang, Zhiyong Wang, Shuai Li, Mohammad Hajiesmaili, John Lui, and Wei Chen. Combinatorial multivariant multi-armed bandits with applications to episodic reinforcement learning and beyond. *arXiv preprint arXiv:2406.01386*, 2024.

Weichao Mao. *Multi-agent reinforcement learning for nonzero-sum Markov games*. PhD thesis, University of Illinois at Urbana-Champaign, 2024.

David Martínez-Rubio, Varun Kanade, and Patrick Rebeschini. Decentralized cooperative stochastic bandits. 2019.

Santiago Ontanón. The combinatorial multi-armed bandit problem and its application to real-time strategy games. In *Proceedings of the AAAI Conference on Artificial Intelligence and Interactive Digital Entertainment*, volume 9, pages 58–64, 2013.

Santiago Ontanón. Combinatorial multi-armed bandits for real-time strategy games. *Journal of Artificial Intelligence Research*, 58:665–702, 2017.

Alexandre Proutiere and Po-An Wang. An optimal algorithm for multiplayer multi-armed bandits. *arXiv preprint arXiv:1909.13079*, 2019.

Abishek Sankararaman, Soumya Basu, and Karthik Abinav Sankararaman. Dominate or delete: Decentralized competing bandits in serial dictatorship. In *International Conference on Artificial Intelligence and Statistics*, pages 1252–1260. PMLR, 2021.

Antoine Scheid, Daniil Tiapkin, Etienne Boursier, Aymeric Capitaine, El Mahdi El Mhamdi, Éric Moulines, Michael I Jordan, and Alain Durmus. Incentivized learning in principal-agent bandit games. *arXiv preprint arXiv:2403.03811*, 2024.

Balazs Szorenyi, Róbert Busa-Fekete, István Hegedus, Róbert Ormándi, Márk Jelasity, and Balázs Kégl. Gossip-based distributed stochastic bandit algorithms. In *International Conference on Machine Learning*, pages 19–27. PMLR, 2013.

Xiang Tan, Li Zhou, Haijun Wang, Yuli Sun, Haitao Zhao, Boon-Chong Seet, Jibo Wei, and Victor CM Leung. Cooperative multi-agent reinforcement-learning-based distributed dynamic spectrum access in cognitive radio networks. *IEEE Internet of Things Journal*, 9(19):19477–19488, 2022.

Po-An Wang, Alexandre Proutiere, Kaito Ariu, Yassir Jedra, and Alessio Russo. Optimal algorithms for multiplayer multi-armed bandits. In *International Conference on Artificial Intelligence and Statistics*, pages 4120–4129. PMLR, 2020.

Siwei Wang and Jun Zhu. Thompson sampling for (combinatorial) pure exploration. In *International Conference on Machine Learning*, pages 23470–23483. PMLR, 2022.

Zhiyang Wang, Mark Eisen, and Alejandro Ribeiro. Learning decentralized wireless resource allocations with graph neural networks. *IEEE Transactions on Signal Processing*, 70:1850–1863, 2022.

A Beta Distributions are 1-subGaussian via Hoeffding's Lemma

Definition 10 (σ^2-subGaussian) *A centered random variable X is σ^2-subGaussian if*

$$\mathbb{E}\, e^{\lambda(X-\mathbb{E}X)} \leq \exp\left(\frac{\lambda^2\sigma^2}{2}\right) \quad \textit{for all } \lambda \in \mathbb{R}.$$

Equivalently, X has tails dominated by a Gaussian with variance proxy σ^2.

Theorem 11 (Hoeffding's Lemma) *Let X be any real-valued random variable with $a \leq X \leq b$ almost surely. Then*

$$\mathbb{E}\, e^{\lambda(X-\mathbb{E}X)} \leq \exp\left(\frac{\lambda^2(b-a)^2}{8}\right) \quad \textit{for all } \lambda \in \mathbb{R}.$$

In particular, $X - \mathbb{E}X$ is sub-Gaussian with variance proxy $\sigma^2 = (b-a)^2/4$; see (Boucheron et al., 2013, Ch. 2).

Proposition 12 (Beta is 1-subGaussian) *If $X \sim \mathrm{Beta}(\alpha, \beta)$, then $X - \mathbb{E}X$ is sub-Gaussian with variance proxy $\sigma^2 \leq \frac{1}{4}$.*

Proof: If $X \sim \mathrm{Beta}(\alpha, \beta)$ with $\alpha, \beta > 0$, then X is supported on the interval $[0, 1]$.

Applying Theorem 11 with $(a, b) = (0, 1)$ gives

$$\mathbb{E}\, e^{\lambda(X-\mathbb{E}X)} \leq \exp\left(\frac{\lambda^2}{8}\right) = \exp\left(\frac{\lambda^2}{2} \cdot \frac{1}{4}\right),$$

so $X - \mathbb{E}X$ is $\frac{1}{4}$-subGaussian. Since sub-Gaussianity is monotone in the variance proxy, $X - \mathbb{E}X$ is also 1-subGaussian. □

B Important Lemmas

We present the proof of Lemma 2.

Proof: By De Morgan's law,

$$G^c = \left(\bigcap_{t=1}^{T} \bigcap_{a=1}^{M} G_a(t)\right)^c = \bigcup_{t=1}^{T} \bigcup_{a=1}^{M} G_a(t)^c. \tag{9}$$

Thus, at least one arm at some time violates the confidence bound. For each arm a and round t,

$$G_a(t)^c = \left\{ |\widehat{\mu}_a(t-1) - \mu_a| > \sqrt{\frac{4 \log T}{n_a(t-1)}} \right\}.$$

Using the concentration bound with $\delta = T^{-2}$ gives

$$\Pr(G_a(t)^c) \leq \frac{1}{T^2}.$$

Applying the union bound,

$$\Pr(G^c) = \Pr\left(\bigcup_{t=1}^{T} \bigcup_{a=1}^{M} G_a(t)^c\right) \leq \sum_{t=1}^{T} \sum_{a=1}^{M} \Pr(G_a(t)^c).$$

Hence

$$\Pr(G^c) \leq T \cdot M \cdot \frac{1}{T^2} = \frac{M}{T}. \tag{10}$$

Therefore,

$$\Pr(G^c) \leq \min\left(\frac{M}{T}, 1\right).$$

□

C Proofs for the 0–1 Problem

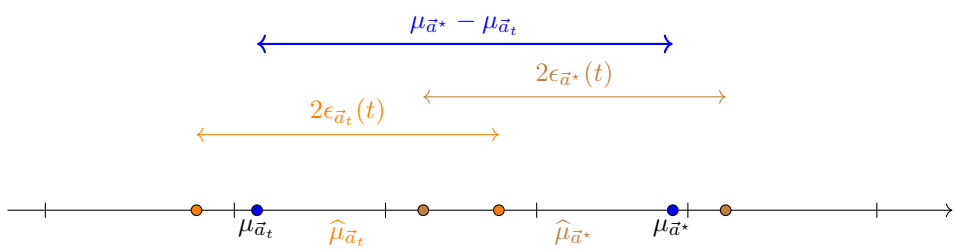
C.1 The Interval Approach for the 0–1 Problem (Problem A and Problem B)

We present the proof of Theorem 4.

Proof: Let $\mu^\star = \mu_{\vec{a}^\star}$ denote the mean reward of the optimal feasible action, and let $X_{\vec{a}_t}$ be the reward collected at round t.

$$
\begin{aligned}
R_T &\le \sum_{t=1}^{T} \mathbb{E}[\mu^\star - X_{\vec{a}_t}] && (11)\\
&= \sum_{t=1}^{T} \mathbb{E}[\mu^\star - X_{\vec{a}_t} \mid G]P(G) + \sum_{t=1}^{T} \mathbb{E}[\mu^\star - X_{\vec{a}_t} \mid G^c]P(G^c) && (12)\\
&= P(G)\sum_{t=1}^{T} \mathbb{E}[\mu^\star - X_{\vec{a}_t} \mid G] + P(G^c)\sum_{t=1}^{T} \mathbb{E}[\mu^\star - X_{\vec{a}_t} \mid G^c] && (13)\\
&\le \sum_{t=1}^{T} \mathbb{E}[\mu^\star - \mu_{\vec{a}_t} \mid G] + M. && (14)
\end{aligned}
$$

Here G is the good event from (5). The last inequality uses: (i) $P(G) \le 1$, (ii) $P(G^c) \le M/T$ by Lemma 2, and (iii) per-round regret is at most 1.



Denote the confidence interval for $\mu_{\vec{a}}$ by $I_{\vec{a}}(t)$ as in (4). At round t, the played action $\vec{a}_t$ is not dominated by $\vec{a}^\star$, so their intervals overlap under the good event. Hence

$$\mu_{\vec{a}_t} \in I_{\vec{a}_t}(t), \qquad \mu^\star \in I_{\vec{a}^\star}(t),$$

which implies

$$\mathbb{E}[\mu^\star - X_{\vec{a}_t} \mid G] \le 2\epsilon_{\vec{a}_t}(t) + 2\epsilon_{\vec{a}^\star}(t). \tag{15}$$

Summing the first term over t,

$$\sum_{t=1}^{T} 2\epsilon_{\vec{a}_t}(t) = \sum_{t=1}^{T}\sum_{a_i \in \vec{a}_t} 2\epsilon_{a_i}(t) \le \sum_{j=1}^{M} 2\sqrt{4\log T}\sum_{t=1}^{T}\frac{1}{\sqrt{t}} = 2M\sqrt{4\log T}\sum_{t=1}^{T}\frac{1}{\sqrt{t}}.$$

For the second term,

$$\sum_{t=1}^{T} 2\epsilon_{\vec{a}^\star}(t) \le 2\sum_{i=1}^{M} M \sum_{t=1}^{T}\sqrt{\frac{4\log T}{t}} = 2M^2\sqrt{4\log T}\sum_{t=1}^{T}\frac{1}{\sqrt{t}}.$$

Since the algorithm cycles through the ordered sets $S_1, \ldots, S_M$, each component of $\vec{a}^\star$ is sampled at least once every M rounds.

Finally,

$$\sum_{t=1}^{T} \frac{1}{\sqrt{t}} \leq \int_0^T \frac{1}{\sqrt{t}}\, dt = 2\sqrt{T}.$$

Substituting into (15) yields

$$R_T \leq 2M\sqrt{4\log T} \cdot 2\sqrt{T} + 2M^2\sqrt{4\log T} \cdot 2\sqrt{T} + M = 8M(M+1)\sqrt{T\log T} + M.$$

□

C.2 `mDSEE` for Problem C

We present the proof of Theorem 6.

Proof: We divide the regret into exploration and commitment phases. During exploration,

$$R_{\text{explore}} \leq \lfloor \log_2(T) \rfloor f(\lfloor \log_2(T) \rfloor) M.$$

For each feasible action $\vec{a} \in \mathcal{F}$, define

$$G_{\vec{a}}(t) = \{|\widehat{\mu}_{\vec{a}} - \mu_{\vec{a}}| < \epsilon\}, \qquad \epsilon = \frac{1}{2} \min_{\vec{a} \in \mathcal{F}} \Delta_{\vec{a}}.$$

For the commitment phase,

$$\begin{aligned}
R_{\text{commit}} &\leq \sum_{t=1}^{T} \mathbb{E}\left[\mathbb{I}\left\{\left(\bigcap_{\vec{a} \in \mathcal{F}} G_{\vec{a}}(t)\right)^c\right\}\right] && (16)\\
&= \sum_{t=1}^{T} \Pr\left(\bigcup_{\vec{a} \in \mathcal{F}} G_{\vec{a}}(t)^c\right) && (17)\\
&\leq \sum_{t=1}^{T} \sum_{\vec{a} \in \mathcal{F}} \Pr(G_{\vec{a}}(t)^c) && (18)\\
&\leq \sum_{t=1}^{T} \sum_{\vec{a} \in \mathcal{F}} e^{-n_{\vec{a}}(T)\epsilon^2/2} && (19)\\
&\leq \sum_{t=1}^{T} M e^{-M_0(t)\log_2(t)\epsilon^2/2} && (20)\\
&= \sum_{t=1}^{T} M t^{-M_0(t)\epsilon^2/(2\log 2)}. && (21)
\end{aligned}$$

Here $M_0(t)$ is a lower bound on the number of times each base arm in the support of any feasible action has been sampled during exploration. Since $\log_2(t)$ grows with t, this term decays super-polynomially. Hence

$$R_{\text{commit}} \leq \sum_{t=1}^{T} M t^{-c} = O(1)$$

for some constant $c > 0$. Therefore,

$$\begin{aligned}
R_T &\leq R_{\text{explore}} + R_{\text{commit}}\\
&\leq \lfloor \log_2(T) \rfloor f(\lfloor \log_2(T) \rfloor) M + O(1)\\
&= O(\lfloor \log_2(T) \rfloor f(\lfloor \log_2(T) \rfloor) M).
\end{aligned}$$

□

D The Joint Arm Problem Pseudocode

The pseudocode for Joint Arm Problem A is given in Algorithm 4.

Algorithm 4 `mCombinatorialUCB-Joint-A`

Require: Horizon T; player sets $S_1, \ldots, S_M$ with fixed internal orders; per-player choice lists $D_1, \ldots, D_M$
Ensure: Cyclic elimination and UCB updates for individual and combinatorial arms

Initialize: for each individual arm a_i^j, set $n_{a_i^j}(0) \leftarrow 0$, $\hat{\mu}_{a_i^j}(0) \leftarrow 0$, $\mathrm{UCB}_{a_i^j}(0) \leftarrow +\infty$, $\mathrm{LCB}_{a_i^j}(0) \leftarrow -\infty$
$i \leftarrow 1$
for $t = 1$ **to** T **do**
  Let S_i be the current set and let $\vec{a}$ be the next combinatorial arm in (6)
  Let a_i^k be the current arm in D_i
  $\vec{a}_t \leftarrow \vec{a}$
  while exists $a' \in D_i$ **such that** $\mathrm{UCB}_{a_i^k}(t-1) < \mathrm{LCB}_{a'}(t-1)$ **do**
    Remove a_i^k from D_i
    Set a_i^k to the next element of D_i
  end while
  while exists $\vec{a}' \in S_i$ **such that** $\mathrm{UCB}_{\vec{a}_t}(t-1) < \mathrm{LCB}_{\vec{a}'}(t-1)$ **do**
    Remove $\vec{a}_t$ from all S_j that contain it
    Set $\vec{a}_t$ to the next arm in S_i
  end while
  for all $a_i^j \in \vec{a}_t$ **do**
    Observe reward $X_{a_i^j}(t)$
    Update $n_{a_i^j}(t)$, $\hat{\mu}_{a_i^j}(t)$, $\mathrm{UCB}_{a_i^j}(t)$, $\mathrm{LCB}_{a_i^j}(t)$
  end for
  for all $\vec{b} \in S_i$ **do**
    Update $\mathrm{UCB}_{\vec{b}}(t)$ and $\mathrm{LCB}_{\vec{b}}(t)$
  end for
  $i \leftarrow \big(i \bmod M\big) + 1$
end for

The pseudocode for Joint Arm Problem B is given in Algorithm 5.

The pseudocode for Joint Arm Problem C is given in Algorithm 6.

D.1 Proofs for the Interval Approach for Problem A and Problem B

We present the proof of Theorem 7.

Proof: Recall that the upper endpoint of the interval for a combinatorial action is obtained by maximizing over the surviving individual arms for each player. Let this maximizing action be denoted by $\vec{a}_t^\star$. Then under the good event,

$$\mathbb{E}[\mu^\star - X_{\vec{a}_t} \mid G] \leq 2 \sum_{a \in \vec{a}_t} \epsilon_a(t) + 2 \sum_{a \in \vec{a}_t^\star} \epsilon_a(t) + 2 \sum_{a \in \vec{a}^\star} \epsilon_a(t).$$

For the first term,

$$2 \sum_{t=1}^{T} \sum_{a \in \vec{a}_t} \epsilon_a(t) = 2 \sum_{k=1}^{K} \sum_{m=1}^{M} \sum_{s=1}^{n_{a_m^k}(T)} \sqrt{\frac{4 \log T}{s}} \leq 4KM \sqrt{\log T} \sum_{s=1}^{T} \frac{1}{\sqrt{s}} \leq 8KM \sqrt{T \log T}.$$

For the second term,

$$2 \sum_{t=1}^{T} \sum_{a \in \vec{a}_t^\star} \epsilon_a(t) \leq 2K \sum_{k=1}^{K} \sum_{m=1}^{M} \sum_{s=1}^{n_{a_m^k}(T)} \sqrt{\frac{4 \log T}{s}} \leq 8K^2 M \sqrt{T \log T}.$$

Algorithm 5 `mCombinatorialUCB-Joint-B`

Require: Horizon T; player sets $S_1, \ldots, S_M$ with fixed internal orders; per-player choice lists $D_1, \ldots, D_M$
Ensure: Joint elimination via player-triggered sabotage and UCB updates

Initialize: for each individual arm a_i^j, set $n_{a_i^j}(0) \leftarrow 0$, $\hat{\mu}_{a_i^j}(0) \leftarrow 0$, $\mathrm{UCB}_{a_i^j}(0) \leftarrow +\infty$, $\mathrm{LCB}_{a_i^j}(0) \leftarrow -\infty$
$i \leftarrow 1$
for $t = 1$ **to** T **do**
  Let S_i be the current set and let $\vec{a}$ be the next combinatorial arm in (6)
  Let a_i^k be the current arm in D_i
  $\vec{a}_t \leftarrow \vec{a}$
  if exists $a' \in D_i$ **such that** $\mathrm{UCB}_{a_i^k}(t-1) < \mathrm{LCB}_{a'}(t-1)$ **then**
    Player i deviates to signal elimination of a_i^k
    All players remove a_i^k from D_i
    Set a_i^k to the next element of D_i
  end if
  if exists player $j \neq i$ and **exists** $\vec{a}' \in S_i$ **such that** $\mathrm{UCB}_{\vec{a}_t}(t-1) < \mathrm{LCB}_{\vec{a}'}(t-1)$ **then**
    Player j deviates so that the resulting joint action is not $\vec{a}_t$
    All players remove $\vec{a}_t$ from every S_r that contains it
    Set $\vec{a}_t$ to the next arm in S_i
  end if
  for all $a_i^j \in \vec{a}_t$ **do**
    Observe reward $X_{a_i^j}(t)$
    Update $n_{a_i^j}(t)$, $\hat{\mu}_{a_i^j}(t)$, $\mathrm{UCB}_{a_i^j}(t)$, $\mathrm{LCB}_{a_i^j}(t)$
  end for
  for all $\vec{b} \in S_i$ **do**
    Update $\mathrm{UCB}_{\vec{b}}(t)$ and $\mathrm{LCB}_{\vec{b}}(t)$
  end for
  $i \leftarrow \big(i \bmod M\big) + 1$
end for

Algorithm 6 `JointArm-mDSEE` for Problem C

Require: Number of players M, number of individual arms K, feasible set $\mathcal{F}$, monotone function $f(\lambda)$ with $f(\lambda) \to \infty$

Ensure: Identification of high-reward joint arms

```
t ← 1, λ ← 1
while t ≤ T do
    Exploration phase:
    for m = 1 to M do
        for k = 1 to K do
            for p = 1 to M do
                Pull the representative joint arm a⃗ ∈ S_p
                if p = m then
                    Pull arm a_k^p from D_p
                else
                    Pull the arm in D_p with largest empirical mean
                end if
                Observe reward X_{a_k^p}^{(p)}(t)
            end for
            Update empirical means for player m
        end for
    end for
    Commit phase:
    for m = 1 to M do
        Compute local empirical means for all a⃗ ∈ F
        a⃗_commit^m ← arg max_{a⃗∈F} μ̂_a⃗^m
    end for
    λ ← λ + 1, t ← t + 1
end while
```

For the third term,

$$2\sum_{t=1}^{T}\sum_{a\in\vec{a}^\star}\epsilon_a(t) \le 2KM\sum_{k=1}^{K}\sum_{m=1}^{M}\sum_{s=1}^{n_{a_m^k}(T)}\sqrt{\frac{4\log T}{s}} \le 8K^2M^2\sqrt{T\log T}.$$

Combining the three terms gives the stated bound up to constants. □

D.2 Joint mDSEE algorithm proof

We present the proof of Theorem 9.

Proof: As before, we split regret into exploration and commitment phases. The exploration phase contributes

$$R_{\text{explore}} \le \lfloor \log_2(T)\rfloor f(\lfloor \log_2(T)\rfloor)KM.$$

Define

$$G_{\vec{a}}(t) = \{|\widehat{\mu}_{\vec{a}} - \mu_{\vec{a}}| < \epsilon\}, \qquad \epsilon = \frac{1}{2}\min_{\vec{a}\in\mathcal{F}}\Delta_{\vec{a}}.$$

Then

$$\begin{aligned}
R_{\text{commit}} &\le \sum_{t=1}^{T}\mathbb{E}\left[\mathbb{I}\left\{\left(\bigcap_{\vec{a}\in\mathcal{F}} G_{\vec{a}}(t)\right)^c\right\}\right] && (22)\\
&= \sum_{t=1}^{T}\Pr\left(\bigcup_{\vec{a}\in\mathcal{F}} G_{\vec{a}}(t)^c\right) && (23)\\
&\le \sum_{t=1}^{T}\sum_{\vec{a}\in\mathcal{F}}\Pr(G_{\vec{a}}(t)^c) && (24)\\
&\le \sum_{t=1}^{T} Me^{-M_0(t)\log_2(t)\epsilon^2/2}. && (25)
\end{aligned}$$

Thus $R_{\text{commit}} = O(1)$, and therefore

$$R_T \le R_{\text{explore}} + R_{\text{commit}} = O(\lfloor \log_2(T)\rfloor f(\lfloor \log_2(T)\rfloor)KM)\,.$$

□