DeepSITH: Efficient Learning via Decomposition of What and When Across Time Scales

Abstract

Extracting temporal relationships over a range of scales is a hallmark of human perception and cognition—and thus it is a critical feature of machine learning applied to real-world problems. Neural networks are either plagued by the exploding/vanishing gradient problem in recurrent neural networks (RNNs) or must adjust their parameters to learn the relevant time scales (e.g., in LSTMs). This paper introduces DeepSITH, a network comprising biologically-inspired Scale-Invariant Temporal History (SITH) modules in series with dense connections between layers. SITH modules respond to their inputs with a geometrically-spaced set of time constants, enabling the DeepSITH network to learn problems along a continuum of time-scales. We compare DeepSITH to LSTMs and other recent RNNs on several time series prediction and decoding tasks. DeepSITH achieves results comparible to state-of-the-art performance on these problems.

1 Introduction

The natural world contains structure at many different time scales. Natural learners can spontaneously extract meaningful information from a range of nested time scales allowing a listener of music to appreciate the structure of a concerto over time scales ranging from milliseconds to thousands of seconds. Recurrent neural networks (RNNs) enable information to persist over time and have been proposed as models of natural memory in brain circuits [1]. For decades, long-range temporal dependencies have been recognized as a serious challenge for RNNs [2]. This problem with RNNs is fundamental, arising from the exploding/vanishing gradient problem [3, 4]. The importance of long-range dependencies coupled with the difficulties with RNNs has led to a resurgence of interest in long short-term memory networks (LSTMs) over the last several years.

LSTMs, however, are neurobiologically implausible and, as a practical matter, tend to fail when time scales are very large [5, 6]. More recent approaches attempt to solve the problem of learning across multiple scales by constructing a scale-invariant memory. For example, Legendre Memory Units (LMU) are RNNs that utilize a specialized weight initialization technique that theoretically guarantees the construction of long time-scale associations [7]. LMUs construct a memory for the recent past using Legendre polynomials as basis functions. In a different approach, the Coupled oscillatory Recurrent Neural Network (coRNN, [8]) treats each internal node as a series of coupled oscillators which has the benefit of orthogonalizing every discrete moment in time. In this paper, we introduce a novel approach to machine learning problems that depend on long-range dependencies inspired by recent advances in the neuroscience of memory.

Submitted to 35th Conference on Neural Information Processing Systems (NeurIPS 2021). Do not distribute.

Figure 1: *Structure of the DeepSITH network.* **A**: A diagram of the DeepSITH network, depicting an example with two layers. **B**: The input signal $f(t)$ is convolved with the precalculated temporal filters to produce an output of the $\overset{*}{\tau}$ nodes. **C**: A table outlining the parameters that need to be specified for each DeepSITH layer. Typically, we recommend that N, $\overset{*}{\tau}_{min}$, and hidden size remain constant across layers, but $\overset{*}{\tau}_{max}$ should increase logarithmically, and k is calculated via the formula outlined in the text. The $\#hidden$ parameter dictates the output of the dense layers, constant across layers, combining both what and when information into new features.

In neuroscience, it has become increasingly clear that the brain makes use of a memory of the recent past that 1) contains information about both the time and identity of past events 2) represents the time of past events with decreasing accuracy for events further in the past and 3) represents the past over many different time scales. So-called "time cells" with these properties have been observed in many different brain regions in multiple mammalian species in a variety of behavioral tasks (see [9], and [10] for reviews). These neuroscience findings were anticipated by computational work that describes a method for computing a scale-invariant temporal history [11] and consistent with a long tradition in human memory research noting the scale-invariance of behavioral memory [12]. Here, we introduce a Deep Scale-Invariant Temporal History (DeepSITH) network. This network consists of a series of layers. Each layer includes a biologically-inspired scale-invariant temporal history (SITH) that projects its inputs over time onto a logarithmically-compressed axis. A dense layer with learnable weights completes a single SITH layer (see Figure 1), transforming temporal relationships into new features for the next layer. We compare this network to an LSTM, LMU and coRNN on a set of time series prediction and decoding tasks designed to tax the networks' ability to learn and exploit long-range dependencies in supervised learning situations.

1.1 Scale-invariant temporal history

Each DeepSITH layer includes a scale-invariant temporal history (SITH) layer. Given a time series input $f(t)$, the state of the memory at time t is denoted $\tilde{f}(t, \overset{*}{\tau})$. This memory approximates the past in that $\tilde{f}(t, \overset{*}{\tau})$ is an approximation of $f(t - \overset{*}{\tau})$. At each moment t, $\tilde{f}(t, \overset{*}{\tau})$ is given by

$$\tilde{f}(t, \overset{*}{\tau}) = \overset{*}{\tau}^{-1} \int_{t'=t}^{-\infty} \Phi_k \left(\frac{t'}{\overset{*}{\tau}} \right) f\left(t - t'\right) dt', \tag{1}$$

where the scale-invariant filter $\Phi(x)$ is given by

$$\Phi_k(x) \propto (x)^k \, e^{-kx}. \tag{2}$$

The function $\Phi_k(x)$ defined in Eq. 2 describes a unimodal impulse response function that peaks at $\overset{*}{\tau}$. The width of the impulse response function is controlled by k; higher values of k result in sharper peaks (Fig. 2). Because the filter in Eq. 1 is a function of $t/\overset{*}{\tau}$, $\tilde{f}(t, \overset{*}{\tau})$ samples different parts of $f(t' < t)$ with the same *relative* resolution [11]. Figure 1B shows examples of $\Phi_k(t'\overset{*}{\tau})$ with the lines given by different values of t and $-\overset{*}{\tau}$ on the x axis.

78 In implementing this formal approach, the vari-
79 able $\overset{*}{\tau}$ must be mapped onto a population of
80 units. Noting the scale-invariance of $\tilde{f}(t, \overset{*}{\tau})$ one
81 obtains constant resolution per unit if the $\overset{*}{\tau}$ of
82 the ith unit is chosen as

$$\overset{*}{\tau}_i = \overset{*}{\tau}_{\min}(1+c)^i$$

83 This means that $\overset{*}{\tau}$ values are sampled more
84 densely for values close to the present $\overset{*}{\tau} = 0$
85 and less densely for time points further in the
86 past. This property is shared with populations
87 of time cells in the brain.

88 Equation 1 requires access to the entire temporal
89 history to construct the representation $\tilde{f}(t, \overset{*}{\tau})$;
90 in the experiments described below, we used
91 numerical convolution of $f(t' < t)$ and Φ_k to
92 construct $\tilde{f}(t, \overset{*}{\tau})$. In principle it is possible to
93 construct $\tilde{f}(t, \overset{*}{\tau})$ more efficiently by first con-
94 structing the real Laplace transform of $f(t' < t)$,
95 $F(s)$, using a time-local differential equation as

$$\frac{dF(t,s)}{dt} = -sF(t,s) + f(t). \qquad (3)$$

96 The filter Φ_k described above is the analytic
97 result for the Post approximation for the inverse
98 Laplace transform, which can be approximated
99 with a linear operator $\mathbf{L}_k^{-1}$:

$$\tilde{f}(t, \overset{*}{\tau}) = \mathbf{L}_k^{-1} F(t,s) \equiv C_k s^{k+1} \frac{d^k}{ds^k} F(t,s) \qquad (4)$$

100 with the mapping $s = k/\overset{*}{\tau}$. Defining $\tilde{f}$ in this
101 way gives Eq. 1 as a solution.

102 Note that Eq. 4 does not require storing the his-
103 tory of f to construct the real Laplace transform
104 of that history. It is possible to sample s logarith-

Figure 2: *SITH temporal filters and selection of
k.* Curves in panels **A**, **B**, and **C** show impulse
responses of 10 SITH filters, with each panel corre-
sponding to a different value of parameter k, which
controls the coefficient of variation of the filters
(larger k results in narrower filters). Note that the
filters are scale-invariant: the coefficient of varia-
tion is proportional to the peak time. The filters
in this figure have been normalized for easier vi-
sualization and for our calculation of the optimal
k. In the experiments the amplitude will decay as
a power-law function of time, such that the area
under each filter is the same. In panel **C** we applied
an automated method to optimize the selection of
k (here, $k = 16$), such that the filters were overlap-
ping, but not so much that there was redundant in-
formation. **D** illustrates the optimization approach
minimizing the ratio between the standard devia-
tion across time of the sum of all of the filters and
the standard deviation of the sums of every other
filter (Please see the supplemental information for
more detail.)

105 mically as well as $\overset{*}{\tau}$, resulting in exponential memory savings. However, in the experiments presented
106 here we used numerical convolution to avoid errors that arise from approximating derivatives with
107 large values of k [13]. To distinguish this from prior machine learning work that used a direct
108 implementation of $\mathbf{L}_k^{-1}$, we refer to this implementation of scale-invariant temporal history as iSITH.
109 The geometric spacing of nodes in $\tilde{f}(t, \overset{*}{\tau})$ nonetheless provides a large savings in the number of
110 weights that must be trained.

111 It is perhaps worth noting that the Laplace transform in Eq. 3 can be understood as an RNN with a
112 diagonal connectivity matrix that does not change with learning. Similarly, $\tilde{f}$ can also be understood
113 as an RNN with fixed weights, although the form of the weights is more complex [14].

1.2 DeepSITH: A Deep Neural Network Using Neurally Plausible Representations of Time

115 A DeepSITH network consists of a series of DeepSITH layers. At the input stage of the ith DeepSITH
116 layer, a SITH representation is constructed for each of the n_i input features. Informally, one can
117 understand the SITH representation as a conjunctive representation of what happened when. The
118 number of units in this SITH representation is equal to the number of input features n_i times the
119 number of $\overset{*}{\tau}$s, N_i. At the output stage of each DeepSITH layer, the SITH representation is fed

through a dense layer with modifiable weights $\mathbf{W}^{(i)}$ and a ReLU activation function $g(.)$:

$$\mathbf{o}(t) = g\left[\mathbf{W}^{(i)} \tilde{f}(t, \overset{*}{\tau})\right].\tag{5}$$

The weight matrix $\mathbf{W}^{(i)}$ connects the output of layer i to the input of layer $i + 1$. $\mathbf{W}^{(i)}$ has $n_{i+1} \times (n_i \times N_i)$ entries. In the experiments examined here, the input to the first layer is one- or two-dimensional, but the number of features is greater for subsequent layers.

One final dense linear layer converts the output from the final DeepSITH layer to the dimensionality required for the specific problem. A diagram depicting this network is shown in Figure 1.A.

1.3 Parameterization of DeepSITH

There are a few hyper-parameters that need to be specified to optimize the performance of a DeepSITH network. Regarding the number of layers, a network with three layers proved successful for most of the chosen problems, but four layers were used for Mackey-Glass and Hateful-8 below. Layer-specific parameters are $\overset{*}{\tau}_{\max}$ and number of $\overset{*}{\tau}$s, N_i. We set $\overset{*}{\tau}_{max}$ to increase from layer to layer, and $\overset{*}{\tau}_{min}$ was set to 1 for all problems. The number of $\overset{*}{\tau}$s, N_i was constant across layers; we found that values from 10-30 were roughly equivalent.

The value of k was chosen to be dependent on the values of $\overset{*}{\tau}_{max}$ and N_i (see Figure 2.D). The density of the centers of the temporal filters are controlled by the value c, where $c = \left(\overset{*}{\tau}_{max}/\overset{*}{\tau}_{min}\right)^{1/N} + 1$. The rate in which width of the filters increase as a function of $\overset{*}{\tau}$ is dependent on k. We choose k in order to minimize the ratio of the standard deviation of the sum of all the filters, std_{all}, to the standard deviation of the sum of alternating filters, std_{alt}. This minimization is so that, for a given c, we do not over represent the past in our iSITH representation, nor do we construct temporal filters in iSITH that have too much overlap. More detail on this minimization is provided in the supplemental sections.

The final key hyper-parameters when setting up a DeepSITH network are the sizes of the dense layers. This decision depends largely on the number of input features and the expected complexity of the input. The number of output features from each dense layer, or the hidden size, is an estimate of how many unique temporal associations can be extracted at each moment from the history of input features decomposed via the SITH layer. In order to keep the network size small, this hidden size should be kept relatively small (less than 100 nodes), and we have found in practice that it can be consistent across layers after the first layer. In the cases where an input signal is sufficiently complex in the breadth of temporal dynamics that need to be encoded, a larger hidden size may be required.

All of the hyperparameters used in the following experiments can be seen in Table 1.

2 Experiments

We compared the DeepSITH network to previous RNNs—LSTM, LMU, and coRNN—in several experiments that rely on long-range dependencies. Each of the networks is based on a recurrent architecture, but with different approaches to computing the weights and combining the information across the temporal scales. Specifically, SITH builds a log-compressed representation of the input signal and DeepSITH learns a relevant set of features at every layer, which become the input to the next layer. Where practical, we explicitly manipulate the characteristic time scale of the experiment and evaluate the performance of the network as the scale is increased. We ran all of the experiments presented in this work with the PyTorch machine learning framework [15]. In Table 1 we show the hyperparameters chosen for each experiment. We attempted to keep the number of learnable parameters for each network to be close to those presented in the current state of the art [16]. For the following experiments, we applied a 20% dropout on the output of each DeepSITH layer, except for the last one, during training. We utilized the Adam optimization algorithm [17] for training all of the networks.

Table 1: Parameter values of the DeepSITH network in each task.

EXPERIMENT	# LAYERS	τ_{max}	k	N_i	#HIDDEN	TOT. WTS.
P/SMNIST	3	30, 150, 750	125, 61, 35	20	60	146350
ADDING PROBLEM	4	20, 120, 720, 4320	75, 27, 14, 8	13	25	25151
MACKEY-GLASS	3	25, 50, 150	15, 8, 4	8	25	10301
HATEFUL-8	4	25, 100, 400, 1200	35, 16, 9, 6	10	35	37808

Figure 3: *The DeepSITH network achieves results comparable to state-of-the-art on the sequential MNIST tasks.* **Top**: Stacked in order are 10 examples of each of the 10 digits, flattened to be 1-dimensional time series, from sMNIST and psMNIST items. **Middle**: Plots the accuracy across epochs on sMNIST and psMNIST. DeepSITH achieves a classification accuracy of 99.32% on sMNIST, and 97.36% on psMNIST with identically configured networks. **Bottom**: Plots the loss across epochs on sMNIST and psMNIST.

2.1 Permuted/Sequential MNIST

In the MNIST task [18], handwritten numerical digits can be identified by neural networks with almost 100% accuracy utilizing a convolutional neural network (CNN). This task is transformed into a more difficult, memory intensive task by presenting each pixel in the image one at a time, creating the time series classification task known as sequential MNIST (sMNIST). An even harder task, permuted sequential MNIST (psMNIST), is constructed by randomizing the order of pixels such that each image is shuffled in the same way and then presented to the networks for classification. Here, we train a DeepSITH network to classify digits in the sequential and permuted sequential MNIST tasks by learning to recognize multiple scales of patterns in each time series.

The DeepSITH network is trained with a batch size of 64, with a cross–entropy loss function. In between each layer we applied batch normalization, and applied a step learning rate annealing after every third of the training epochs (2e-3, 2e-4, 2e-5). It should be noted that the permutation that we applied in our tests was the same as in the [8] study examining the coRNN, making the results directly comparable. Test set accuracy was queried after every training epoch for visualization purposes. While we did not attempt to minimize the number of learnable weights in this network, DeepSITH had 146k weights, compared to 134k, 102k, and 165k weights for the coRNN, LMU, and LSTM networks, respectively.

Table 2: Results from DeepSITH on psMNIST and sMNIST compared to other networks. Best performances are in bold.

Network	psMNIST	sMNIST
DeepSITH	**97.36%**	99.32%
LSTM	90.20%	98.90%
LMU	97.15%	NA
coRNN	97.34%	**99.40%**

The performance of DeepSITH on the standard (left) and permuted (right) sequential MNIST task is shown in Figure 3, along with comparisons to other networks in Table 2. DeepSITH was able to achieve performance comparable to state-of-the-art on permuted sequential MNIST with a test accuracy of 97.36%[8], and comparable performance to the best performing networks at 99.32% for normal sequential MNIST [6, 7]. This result displays the expressivity of DeepSITH on this difficult time series classification task.

2.2 Adding Problem

We examined the Adding Problem [5] as a way to measure the ability of different networks to process and encode long-range temporal associations. In this implementation of the task, taken directly from [8], a 2-dimensional time series of length T is generated. The first dimension contains a series of uniformly distributed values between 0 and 1, and the second dimension contains all zeros except two indexes set to 1. These two indexes are chosen at random such that one occurs within the first $T/2$ indexes and the other within the second $T/2$ indexes. The goal of this task is to maintain the random values from the first dimension presented at the same time as the 1's in the second dimension until the end of the sequence and then add them together. The evaluation criterion is mean square error (MSE). The experiments were ran with a batch size of 50 items for all tests, and the loss was logged for every other batch. In terms of complexity, DeepSITH had 25k learnable weights for all sequence lengths, the LMU ranged from 2k to 11k , LSTM had 67k across all lengths, and the coRNN had 33k across all lengths.

Figure 4: *DeepSITH learns the adding problem within the first 2500 training steps, faster than any other tested network.* Plotted here are the running average mean squared error losses over the previous 100 items trained with each network on signal lengths of $T = 100$, $T = 500$, $T = 2000$, and $T = 5000$. As T gets larger, the signals become much more difficult to learn, as they require the networks to make associations between larger temporal distances.

Figure 4 shows the results of the four networks across different signal lengths, T. The DeepSITH network learned the task quickly. The LSTM and LMU struggled to perform the task in a reasonable amount of time. The coRNN also struggled, only able to learn quickly in the cases where $T <= 500$. It should be noted that coRNN has been shown in a previous study to learn the adding problem very well for all signal lengths tested here, but we were unable to replicate the results in any of the conditions other than $T = 100$. Therefore, we are not claiming that these three networks are incapable of learning the adding problem, but the results shown here indicate that the DeepSITH network was able to solve the adding problem quickly and consistently regardless of signal length.

2.3 Mackey-Glass Prediction of Chaotic Time Series

The Mackey-Glass equations are a series of delay differential equations originally applied to describe both healthy and abnormal variations in blood cell counts and other biological systems [19]. Here we generated a one-dimensional time series with the differential equations with different lag values, tau, which controls a time scale of the dynamics. The task was then to predict values some number of time steps into the future as the time series is fed into the network.

The starting parameters for this experiment were taken from previous work, with tau of 17 and the prediction distance into the future set to 15 time steps [7]. To test prediction accuracy for increasing levels of complexity in the signal, we then generated signals with increasing multiples of tau, while also ensuring the ratio of complexity to the prediction duration was kept constant, giving rise to tau/prediction distance values of 17/15, 34/30, 51/45, 68/60, 85/75. To illustrate the effect of tau on the complexity of the signal, we plot examples for $tau = 17$ and 85 in the top of Figure 5. Different values of tau introduce correlations at different temporal scales into the chaotic dynamics, allowing us to test the networks' ability to predict and encode multiple time-scales of information simultaneously.

Figure 5: *DeepSITH learns to predict multiple levels of Mackey-Glass complexity.* **Top** These plots contain examples of Mackey-Glass time series with different values for tau, which controls the complexity of the signal. **Bottom** This plot contains prediction results in terms of normalized root mean squared error for different levels of signal complexity and number of timesteps into the future the networks were predicting.

We generated 128 continuous one-dimensional signals for each value of tau, and split these signals half for training and half for testing. Each network was trained and tested separately on each tau/prediction distance combination. Training was with a batch size of 32, and the networks were evaluated on the testing set with normed root mean squared error (NRMSE) calculated over predictions made at each time step, as in previous work [7]. The parameterizations for each network did not change across values of tau. For the LSTM, LMU, and DeepSITH, we kept the number of weights to around 18k, while the coRNN had 32k weights.

The bottom of figure 5 shows the NRMSE for each network as a function of tau and prediction distance. DeepSITH outperforms the other architectures. It is worth noting that all four networks are able to learn the underlying Mackey-Glass dynamics, and that all four networks are able to use that understanding to predict even complex signals far out into the future.

2.4 The Hateful-8

When learning to decode a time series after a delay, there may be noise during the delay that will hinder the network's ability to maintain the signal through time. Noise can be even more of an issue if it is similar to the signal, such that it is difficult to separate signal from noise. In such cases the network must learn the key features of the signal that can enable successful classification, while ignoring similar features until the end of the time series. Here, we introduce a novel time series classification task based on Morse code, which we call the Hateful 8. In Morse code, all letters are defined by a unique one-dimensional pattern of dots (represented by activation lasting for one time step) and dashes (represented by activation lasting for three time steps), each separated by one inactivated time step, with three inactive time steps indicating the end of a letter. In the Hateful 8 task there are 8 unique patterns of dots and dashes making up the signals to decode, followed by noise made up of semi-random dots and dashes similar to the signal.

Figure 6: *DeepSITH consistently and quickly classifies long, noisy time series signals known as the Hateful 8.* **Left** 8 example Hateful-8 signals. The useful signal occurs in the first 17 time steps, and after a short pause, the rest is semi-random, signal-similar noise. **Center and Right** Plotted are the performance on a held out test set after training for various noise lengths. The darker color indicates longer noise durations. The LSTM was the only network that was unable to learn to decode these 8 signals after a reasonable amount of training once the duration of noise became too large. The DeepSITH and LMU networks were able to learn this task the fastest, approximately 10 times faster than the coRNN network.

The total decodable signal is 17 time steps long (including the 3 time steps long pause), followed by signal-similar noise. We trained and tested the four networks over increasingly longer durations of noise added to the end of the decodable signal. Thus, to exhibit high performance a network must maintain decoded information in the face of noise that resembles the signal. We generated the signal-similar noise pseudo randomly such that there is an even mix of dots and dashes, along with pauses similar to the one to indicate the end of the decodable signal. Figure 6 plots examples of all 8 different decodable signals followed by example signal-similar noise.

For training, we generated 32 different versions of each of the Hateful 8, each with a different randomization of the noise. For the testing set, we again generated randomized noise, and generated 10 noise patterns for each of the 8 different decodable portions. In total, we had 256 training signals and 80 test signals for each amount of noise. Note, each duration of noise was trained and tested in independent experiments. We trained networks with a batch size of 32. DeepSITH had 38k weights, LSTM had 30k weights, LMU had 30k weights, and the coRNN 32k weights.

In the center and right of Figure 6 we see the performance of the four networks on the testing set as a function of epoch and amount of noise. All networks exhibited some degree of success with the lower amounts of noise, and only the LSTM was unable to learn to 100% accuracy at the higher noise lengths. In terms of training time, the LMU and DeepSITH networks learned quickly and were more stable than the coRNN and LSTM networks.

3 Discussion

DeepSITH is a brain-inspired approach to solving time series prediction and decoding tasks in ML [20], especially those that require sensitivity to temporal relationships among events. DeepSITH was able to achieve near state-of-the-art performance in all the examples examined in this paper. While we tried to select the best fitting hyperparameters for each of the networks, we did not do an exhaustive search and can not be confident that some other set of hyperparameters would not provide better results, either for DeepSITH or for other networks we tested.

For comparison within the p/sMNIST framework, we did not have to select any hyperparameters for other networks since their results were published in previous studies. With the p/sMNIST tasks,

DeepSITH was able to achieve near state-of-the-art performance on permuted sMNIST and sMNIST proper. For Mackey-Glass prediction DeepSITH achieved state of the art prediction accuracy. For the other networks, we used parameter values directly from their source papers in the cases where they ran the same experiments, otherwise we made an educated guess based on the parameters in the published work. DeepSITH was able to learn the Adding Problem with essentially the same amount of training time regardless of the length of the input, and did so seemingly faster than the other networks. It was also able to achieve 100% accuracy on the Hateful-8 problem regardless of how much noise we presented, as was the LMU network. We attribute the successes of DeepSITH to the scale-invariance of the memory representation and the decomposition of what information from when information.

We compared DeepSITH to LMU [7] and coRNN [8], which are both recent approaches based on modified RNNs. They both demonstrated remarkable success in time series prediction on similar problems, paving the road for new approaches that tackle the problem differently from most common RNNs, such as LSTM. While similar in spirit, LMU and coRNN use different memory representations than the one proposed here.

The compression introduced by $\Phi_k(t/\overset{*}{\tau})$ results in a loss of information about timing of events as they recede into the past. On its face, this may seem like a disadvantage of this approach. However, the ubiquity of logarithmic scales in perception and psychophysics suggest that this form of compression is adopted widely by the brain [10, 21, 22]. Viewed from one perspective, this gradual loss of information is a positive good. The blur induced by Φ_k forces the system to generalize over a range of time points. The width of that range scales up linearly with the time point in the past at which the observation is made. The deep architecture with SITH representations enables the network to store fine temporal relationships at early layers and turn them into features that are sent to all time scales at subsequent layers.

Future work can build on the neurally-inspired scale invariant representation at the core of DeepSITH. While DeepSITH is essentially a series of RNNs, the analytically computed weights give rise to an internal representation in the inverse-Laplace space that reflects the temporal structure of the input through neurons with sequentially activated scale-invariant bell-shaped tuning curves. Approaches that exploit the structural organization of the temporal data, such as the temporal convolution network (TCN) have seen great success in running convolutions over the time dimension to encode activity patterns. We postulate that, given the properties of inverse-Laplace space, a convolution over $\log \overset{*}{\tau}$ space would allow a neural network to be scale invariant. By that, we mean that if the same input pattern is presented at multiple timescales, i.e., $f(t) \to f(at)$, it would appear translated by some number of units equivalent to $\log a$ [23]. As such, a SITH-based TCN would be largely immune to changes in scale even after training, a hallmark of human perceptual processing.

4   Societal Impact

Given that we are introducing a methodology that is attempting to function as a drop-in replacement for RNNs, we also inherit the fundamental risks associated with improving the accuracy of these types of network architectures. Action-detection, voice recognition, and other time-series decoding tasks run the risk of being heavily biased, and by introducing a human inspired method we run the risk of introducing more human-like biases. Furthermore, in the realm of natural language processing we run the risk of creating an A.I. that better captures long-range dependencies and is, consequently, able to generate extremely human-like text, which could potentially add to the already problematic fake-news epidemic. Extra caution should be taken that these methods are steered away from applications that could be used maliciously, but we believe that, ultimately, these AI advances will do more good than harm.

5   References

1. Rajan K, Harvey CD, Tank DW (2016) Recurrent Network Models of Sequence Generation and Memory. Neuron 90:128–142. https://doi.org/10.1016/j.neuron.2016.02.009

2. Mozer MC (1991) Induction of multiscale temporal structure. In: Proceedings of the 4th International Conference on Neural Information Processing Systems. Morgan Kaufmann Publishers Inc., San Francisco, CA, USA, pp 275–282

3. Bengio Y, Simard P, Frasconi P (1994) Learning long-term dependencies with gradient descent is difficult. IEEE Transactions on Neural Networks 5:157–166. https://doi.org/10.1109/72.279181

4. Pascanu R, Mikolov T, Bengio Y (2012) On the difficulty of training Recurrent Neural Networks. arXiv:12115063 [cs]

5. Hochreiter S, Schmidhuber J (1997) Long Short-Term Memory. Neural Computation 9:1735–1780. https://doi.org/10.1162/neco.1997.9.8.1735

6. Li S, Li W, Cook C, et al (2019) Deep Independently Recurrent Neural Network (IndRNN). arXiv:191006251 [cs]

7. Voelker A, Kajic I, Eliasmith C (2019) Legendre Memory Units: Continuous-Time Representation in Recurrent Neural Networks

8. Rusch TK, Mishra S (2020) Coupled Oscillatory Recurrent Neural Network (coRNN): An accurate and (gradient) stable architecture for learning long time dependencies. arXiv:201000951 [cs, stat]

9. Eichenbaum H (2017) On the Integration of Space, Time, and Memory. Neuron 95:1007–1018. https://doi.org/10.1016/j.neuron.2017.06.036

10. Howard MW (2018) Memory as perception of the past: Compressed time in mind and brain. Trends in cognitive sciences 22:124–136. https://doi.org/10.1016/j.tics.2017.11.004

11. Shankar KH, Howard MW (2012) Optimally fuzzy temporal memory. J Mach Learn Res 14:3785–3812

12. Pashler H, Cepeda N, Lindsey RV, et al (2009) Predicting the optimal spacing of study: A multiscale context model of memory. In: Bengio Y, Schuurmans D, Lafferty J, et al (eds) Advances in neural information processing systems. Curran Associates, Inc., pp 1321–1329

13. Gosmann J (2018) An Integrated Model of Contex, Short-Term, and Long-Term Memory

14. Liu Y, Howard M (2019) Generation of scale-invariant sequential activity in recurrent neural circuits. R67.014

15. Paszke A, Gross S, Chintala S, et al (2017) Automatic Differentiation in PyTorch. In: NIPS Autodiff Workshop

16. Stojnic R, Taylor R, Kardas M, et al (2021) Papers with Code - The latest in Machine Learning. Papers With Code

17. Kingma DP, Ba J (2014) Adam: A Method for Stochastic Optimization. arXiv:14126980 [cs]

18. Schwabbauer ML (1975) Use of the latent image technique to develop and evaluate problem-solving skills. The American Journal of Medical Technology 41:457–462

19. Mackey M, Glass L (1977) Oscillation and Chaos in Physiological Control Systems. Science (New York, NY) 197:287–9. https://doi.org/10.1126/science.267326

20. Fan J, Fang L, Wu J, et al (2020) From Brain Science to Artificial Intelligence. Engineering 6:248–252. https://doi.org/10.1016/j.eng.2019.11.012

21. Smith MA, Sommer MA (2013) Spatial and Temporal Scales of Neuronal Correlation in Visual Area V4. Journal of Neuroscience 33:5422–5432. https://doi.org/10.1523/JNEUROSCI.4782-12.2013

22. Lindeberg T, Fagerström D (1996) Scale-space with casual time direction. In: Buxton B, Cipolla R (eds) Computer Vision ECCV '96. Springer, Berlin, Heidelberg, pp 229–240

23. Jansson Y, Lindeberg T (2021) Exploring the ability of CNNs to generalise to previously unseen scales over wide scale ranges. arXiv:200401536 [cs]