Pessimistic Model Selection for Offline Deep Reinforcement Learning

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Abstract

Deep Reinforcement Learning (DRL) has demonstrated great potentials in solving sequential decision making problems in many applications. Despite its promising performance, practical gaps exist when deploying DRL in real-world scenarios. One main barrier is the over-fitting issue that leads to poor generalizability of the policy learned by DRL. In particular, for offline DRL with observational data, model selection is a challenging task as there is no ground truth available for performance validation, in contrast with the online setting with simulated environments. In this work, we propose a pessimistic model selection (PMS) approach for offline DRL with a theoretical guarantee, which features a provably effective framework for finding the best policy among a set of candidate models. Two refined approaches are also proposed to address the potential bias of DRL model in identifying the optimal policy. Numerical studies demonstrated the superior performance of our approach over existing methods.

1 INTRODUCTION

The success of deep reinforcement learning [Mnih et al., 2013, Henderson et al., 2018] (DRL) often leverages upon executive training data with considerable efforts to select effective neural architectures. Deploying online simulation to learn useful representations for DRL is not always realistic and feasible, especially in some high-stake environments, such as automatic navigation [Kahn et al., 2018, Hase et al., 2020], dialogue learning [Jaques et al., 2020], and clinical applications [Tang et al., 2020a]. *Offline reinforcement learning* (OffRL) [Singh and Sutton, 1996, Levine et al., 2020, Agarwal et al., 2020] has prompted strong interests [Paine et al., 2020, Kidambi et al., 2020] to empower DRL toward problem-solving involving notable costs and risks. The idea of OffRL is to train DRL models with only logged data and recorded trajectories. However, with given observational data, designing a successful neural architecture in OffRL is often expensive [Levine et al., 2020], requiring intensive experiments, time, and computing resources.

Unlike most aforementioned applications with online interaction, *offline* tasks for reinforcement learning often face the challenges of insufficient observational data from offline collection to construct a universal approximated model for fully capturing the temporal dynamics. Therefore, relatively few attempts in the literature have been presented for providing a provably effective pipeline to automate the development process for model selection and neural architecture search in OffRL settings. Here, model selection refers to selecting the best model (e.g., the policy learned by a trained neural network) among a set of candidate models (e.g., different neural network hyperparameters).

In this work, we propose a novel model selection approach to automate OffRL development process, which provides an evaluation mechanism to identify the best DRL model given offline data. Our method utilizes statistical inference to provide uncertainty quantification on the "optimal" value functions trained by different DRL models, based on which a pessimistic rule is incorporated to select the best model/policy. In addition, two refined approaches are further proposed to address the possible biases of DRL models in identifying the optimal policy. In this work, we mainly focus on deep Q-network (DQN) [Mnih et al., 2013, 2015] based architectures, while our proposed methods can be flexibly extended to other settings. Figure 1 demonstrates the superior performance of the proposed pessimistic model selection (PMS) method in identifying the best model among 70 DRL models of different algorithms on one navigation task (See Appendix C for details), compared with the model selection method by [Tang and Wiens, 2021] which uses three offline policy evaluation (OPE) estimates for validation. Specifically, based on the derived confidence interval of the OPE

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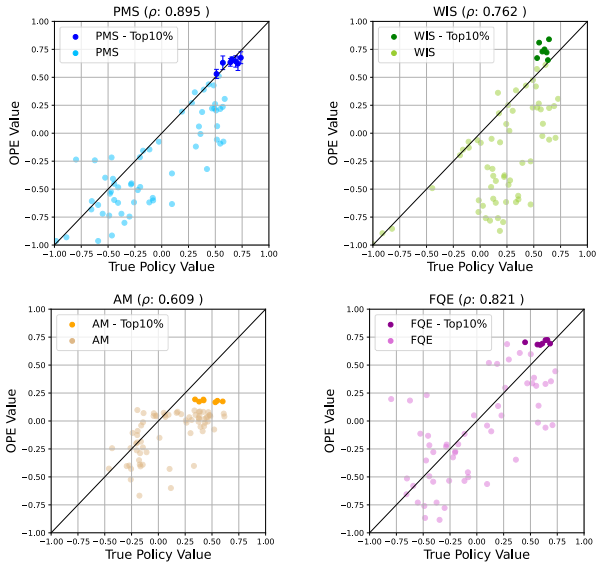

Figure 1: Comparisons of model selection algorithms for offline deep Q-network (DQN) learning: (a) proposed pessimistic model selection (PMS); (b) weighted importance sampling (WIS) [Gottesman et al., 2018]; (c) approximate model (AM) [Voloshin et al., 2019]; (d) fitted Q evaluation (FQE) [Le et al., 2019]. In this figure, the algorithms are trained and evaluated in a navigation task ($\mathbf{E}_2$) discussed in Section 7 and Appendix C. Vertical axis shows the offline policy evaluation (OPE) value. Higher Pearson's correlation coefficient ρ means better performance.

value for each candidate model, the final selected model by our PMS method is the one that has the largest lower confidence limit, which exactly has the largest true OPE value among all candidate models. In contrast, none of three OPE estimates used for model selection by Tang and Wiens [2021] can identify the best model due to the inevitable overfitting issue during the validation procedure.

To close this section, we summarize the contributions of this work as follows:

- We propose a novel PMS framework, which targets finding the best policy from given candidate models (e.g., neural architecture, hyperparameters, etc) with offline data for DQN learning. Unlike many existing methods, our approach essentially does not involve additional hyperparameter tuning except for two interpretable parameters.
- Leveraging asymptotic analysis in statistical inference, we provide uncertainty quantification on each candidate model, based on which our method can guarantee that the worst performance of finally selected model is the best among all candidate models. See Corollary 2 for our key insight.
- To address potential biases of candidate models in identifying the optimal policy, two refined approaches are proposed, one of which can be shown to have regret bounded by the smallest error bound among all candidate models under some technical conditions (See Corollary 4). To the best of our knowledge, this is the first model-selection method in offline DRL with such a guarantee.
- The numerical results demonstrate that the proposed PMS shows superior performance in six different DQN benchmark environments.

2 RELATED WORK

Model Selection for Reinforcement Learning: Model selection has been studied in online decision-making environments [Fard and Pineau, 2010, Lee and Taylor, 2014]. Searching nearly optimal online model is a critical topic for online bandits problems with limited information feedbacks. For linear contextual bandits, Abbasi-Yadkori et al. [2011], Chu et al. [2011] aim to find the best worst-case bound when the optimal model class is given. For model-based reinforcement learning, Pacchiano et al. [2020] introduces advantages of using noise augmented Markov Decision Processes (MDP) to archive a competitive regret bound to select an individual model with constraints for ensemble training. Recently, Lee et al. [2021] utilizes an online algorithm to select a low-complexity model based on a statistical test. However, most of the previous model selection approaches are focused on the online RL setting. Only few works including Farahmand and Szepesvári [2011], Paine et al. [2020], Su et al. [2020], Yang et al. [2020], Kuzborskij et al. [2021], Tang and Wiens [2021], Xie and Jiang [2021] study the offline setting. In particular, [Su et al., 2020, Yang et al., 2020, Kuzborskij et al., 2021] focus on model selection for OPE problem. [Farahmand and Szepesvári, 2011, Xie and Jiang, 2021] propose to select the best model/policy based on minimizing the Bellman error, while the first approach requires an additional tuning and latter does not. [Paine et al., 2020, Tang and Wiens, 2021] propose several criteria to perform model selection in OffRL and mainly focused on empirical studies. In this work, we provide one of the firstline model selection approaches based on statistical inference for RL tasks with offline data collection.

Offline-Policy Learning: Training a DRL agent with offline data collection often relies on batch-wise optimization. Batch-Constrained deep Q-learning [Fujimoto et al., 2019] (BCQ) is considered one OffRL benchmark that uses a generative model to minimize the distance of selected actions to the batch-wise data with a perturbation model to maximize its value function. Other popular OffRL approaches, such as behavior regularized actor-critic (BRAC) [Wu et al., 2019], and random ensemble mixture [Agarwal et al., 2020] (REM) (as an optimistic perspective on large datasets), have also been studied in RL Unplugged (RLU) [Gulcehre et al., 2020] benchmark together with behavior cloning (BC) [Bain and Sammut, 1995, Ross and Bagnell, 2010], DQN, and DQN with quantile regression (QR-DQN) [Dabney et al., 2018]. RLU suggests a naive approach based on human experience for offline policy selection, which requires independent modification with shared domain expertise (e.g., Atari

environments) for tuning each baseline. Meanwhile, how to design a model selection algorithm for OffRL remains an open question. Motivated by the benefits and the challenges as mentioned earlier of the model selection for *offline* DRL, we aim to develop a unified approach for model selection in offline DRL with theoretical guarantee and interpretable tuning parameters.

3 BACKGROUND AND NOTATIONS

Consider a time-homogeneous Markov decision process (MDP) characterized by a tuple $\mathcal{M} = (\mathcal{S}, \mathcal{A}, p, r, \gamma)$, where $\mathcal{S}$ is the state space, $\mathcal{A}$ is the finite action space, p is the transition kernel, i.e., $p(s'|s,a)$ is the probability mass (density) of transiting to s' given current state-action (s,a), r is the reward function, i.e., $\mathbb{E}(R_t|S_t = s, A_t = a) = r(s,a)$ for $t \geq 0$, and $0 \leq \gamma < 1$ is a discount factor. For ease of presentation, we assume $\mathcal{A}$ and $\mathcal{S}$ are both finite. But our method can also be applied in continuous cases. Under this MDP setting, it is sufficient to consider stationary Markovian policies for optimizing discounted sum of rewards [Puterman, 1994]. Denote π as a stationary Markovian policy mapping from the state space $\mathcal{S}$ into a probability distribution over the action space. For example, $\pi(a|s)$ denotes the probability of choosing action a given the state value s. One essential goal of RL is to learn an optimal policy that maximizes the value function. Define $V^\pi(s) = \sum_{t=0}^{+\infty} \gamma^t \mathbb{E}^\pi[R_t|S_0 = s]$ and then the optimal policy is defined as $\pi^* \in \text{argmax}_\pi\{\mathcal{V}(\pi) \triangleq (1-\gamma)\sum_{s\in\mathcal{S}} V^\pi(s)\nu(s)\}$, where ν denotes some reference distribution function over $\mathcal{S}$. In addition, we denote Q-function as $Q^\pi(s,a) = \sum_{t=0}^{+\infty} \gamma^t \mathbb{E}^\pi(R_t|A_0 = a, S_0 = s)$ for $s \in \mathcal{S}$ and $a \in \mathcal{A}$. In this work, we consider the OffRL setting. The observed data consist of N trajectories, corresponding to N independent and identically distributed copies of $\{(S_t, A_t, R_t)\}_{t\geq 0}$. For any $i \in \{1, \cdots, n\}$, data collected from the ith trajectory can be summarized by $\{(S_{i,t}, A_{i,t}, R_{i,t}, S_{i,t+1})\}_{0\leq t<T}$, where T denotes the termination time. We assume that the data are generated by some fixed stationary policy denoted by b.

Among many RL algorithms, we focus on Q-learning type of methods. The foundation is the optimal Bellman equation given below,

$$Q^*(s,a) = \mathbb{E}[R_t + \gamma \max_{a'\in\mathcal{A}} Q^*(S_{t+1}, a') \,|\, S_t = s, A_t = a] \quad (1)$$

where Q^* is called optimal Q-function, i.e., Q-function under π^*. Among others, fitted q-iteration (FQI) is one of the most popular RL algorithms [Ernst et al., 2005]. FQI leverages supervised learning techniques to iteratively solve the optimal Bellman equation (1) and shows competitive performance in OffRL.

To facilitate our model-selection algorithm, we introduce the discounted visitation probability, motivated by the marginal importance sampling estimator in [Liu et al., 2018]. For any $t \geq 0$, let $p_t^\pi(s,a)$ denote the t-step visitation probability $\Pr^\pi(S_t = s, A_t = a)$ assuming the actions are selected according to π at time $0, \cdots, t$. In particular, $p_0^\pi(s,a) = \nu(s)\pi(a|s)$. We define the discounted visitation probability function as $d^\pi(s,a) = (1-\gamma)\sum_{t\geq 0}\gamma^t p_t^\pi(s,a)$. To adjust the distribution from behavior policy to any target policy π, we use the discounted probability ratio function defined as

$$\omega^{\pi,\nu}(s,a) = \frac{d^\pi(s,a)}{\frac{1}{T}\sum_{t=0}^{T-1} p_t^b(s,a)}, \quad (2)$$

where $p_t^b(s,a)$ is the t-step visitation probability under the behavior policy b, i.e., $\Pr^b(S_t = s, A_t = a)$. The ratio function $\omega^{\pi,\nu}(s,a)$ is always assumed well defined, where ν is the distribution of the initial state in d^π. The estimation of ratio function is motivated by the observation that for every measurable function f defined over $\mathcal{S}\times\mathcal{A}$,

$$\begin{aligned}\mathbb{E}[\frac{1}{T}\sum_{t=0}^{T-1}\omega^{\pi,\nu}(S_t,A_t)(f(S_t,A_t) \quad (3)\\ -\gamma\sum_{a'\in\mathcal{A}}\pi(a' \mid S_{t+1})f(S_{t+1},a'))]\\ = (1-\gamma)\mathbb{E}_{S_0\sim\nu}[\sum_{a\in\mathcal{A}}\pi(a \mid S_0)f(a,S_0)],\end{aligned}$$

based on which several min-max estimation methods has been proposed such as [Liu et al., 2018, Nachum et al., 2019, Uehara and Jiang, 2019]. We refer to Lemma 1 of [Uehara and Jiang, 2019] for a formal proof of equation (3).

Finally, because our proposed model selection algorithm relies on an efficient evaluation of any target policy using batch data, we introduce three types of model-free offline policy evaluation estimators in the existing RL literature. The first type is called direct method via estimating Q-function, based on the relationship that $\mathcal{V}(\pi) = (1-\gamma)\sum_{s\in\mathcal{S},a\in\mathcal{A}}\pi(a|s)Q(s,a)\nu(s)$. The second type is motivated by the importance sampling [Precup, 2000]. Based on the definition of ratio function, we can see $\mathcal{V}(\pi) = \mathbb{E}[\frac{1}{T}\sum_{t=0}^{T-1}\omega^{\pi,\nu}(S_t,A_t)R_t]$, from which a plugin estimator can be constructed. The last type of OPE methods combines the first two types of methods and construct a so-called doubly robust estimator [Kallus and Uehara, 2019, Tang et al., 2020b]. This estimator is motivated by the efficient influence function of $\mathcal{V}(\pi)$ under a transition-sampling setting and the model that consists of the set of all observed data distributions given by arbitrarily varying the initial, transition, reward, and behavior policy distributions, subject to certain minimal regularity and identifiability conditions [Kallus and Uehara, 2019], i.e.,

$$\begin{aligned}&\frac{1}{T}\sum_{t=0}^{T-1}\omega^{\pi,\nu}(S_t,A_t)(R_t+\gamma\sum_{a\in\mathcal{A}}\pi(a|S_{t+1})Q^\pi(S_{t+1},a)-\\ &Q^\pi(S_t,A_t)) + (1-\gamma)\mathbb{E}_{S_0\sim\nu}[\sum_{a\in\mathcal{A}}\pi(a|S_0)Q^\pi(S_0,a)] - \mathcal{V}(\pi).\end{aligned} \quad (4)$$

A nice property of doubly robust estimators is that as long as either the Q-function Q^π or the ratio function $\omega^{\pi,\nu}$ can

be consistently estimated, the final estimator of $\mathcal{V}(\pi)$ is consistent [Robins et al., 1994, Jiang and Li, 2015, Kallus and Uehara, 2019, Tang et al., 2020b]. Furthermore, a doubly robust estimator based on (4) can achieve semi-parametric efficiency under the conditions proposed by [Kallus and Uehara, 2019], even if nuisance parameters are estimated via black box models such as deep neural networks. Therefore, such an estimator is particularly suitable under the framework of DRL. Our proposed algorithm will rely on this doubly robust type of OPE estimator.

4 PESSIMISTIC MODEL SELECTION (PMS) FOR BEST POLICY

In this section, we discuss our pessimistic model selection approach. For the ease of presentation, we focus on the framework of (deep) Q-learning, where policy optimization is performed via estimating the optimal Q-function. While this has already covered a wide range of state-of-the-art RL algorithms such as FQI [Ernst et al., 2005], DQN [Mnih et al., 2013] and QR-DQN [Dabney et al., 2018], we remark that our method is not restricted to this class of algorithms.

Suppose we have total number of L candidate models for policy optimization, where each candidate model will output an estimated policy, say $\hat{\pi}_l$ for $1 \leq l \leq L$. Our goal is to select the best policy among L policies during our training procedure. Note that these L models can be different deep neural network architectures, hyper-parameters, and various classes of functions for approximating the optimal Q-function or policy, etc.

4.1 DIFFICULTIES AND CHALLENGES

Given a candidate l among L models, we can apply for example FQI using the whole batch data $\mathcal{D}_n$ to learn an estimate of Q^* as $\widehat{Q}_l$ and an estimated optimal policy $\hat{\pi}_l$ defined as $\hat{\pi}_l(a|s) \in \text{argmax}_{a\in\mathcal{A}}\widehat{Q}_l(s,a)$, for every $s \in \mathcal{S}$. In order to select the final policy, one may use a naive greedy approach to choose some $\tilde{l}$ such that $\tilde{l} \in \text{argmax}_l \mathbb{E}_{S_0\sim\nu}[\sum_{a\in\mathcal{A}}\hat{\pi}_l(a|s)\widehat{Q}_l(S_0,a)]$, as our goal is to maximize $\mathcal{V}(\pi)$. However, using this criterion will lead to over-fitting. Specifically, due to the distributional mismatch between the behavior policy and target policies, which is regarded as a fundamental challenge in OffRL [Levine et al., 2020], we may easily overestimate Q-function, especially when some state-action pairs are not sufficiently visited in the batch data. This issue becomes more serious when we apply max-operator during our policy optimization procedure. Such observations have already been noticed in recent works, such as [Kumar et al., 2019, 2020, Paine et al., 2020, Yu et al., 2020, Tang and Wiens, 2021, Jin et al., 2021]. Therefore, it may be inappropriate to use this criterion for selecting the best policy among L models.

One may also use cross-validation procedure to address the issue of over-fitting or overestimating Q-function for model selection. For example, one can use OPE approaches on the validate dataset to evaluate the performance of estimated policies from the training data set (see Tang and Wiens [2021] for more details). However, since there is no ground truth for the value function of any policies, the OPE procedure on the validation dataset cannot avoid involving additional tuning on hyperparameters. Therefore, such a procedure may still incur a large variability due to the over-fitting issue. In addition, arbitrarily splitting the dataset for cross-validation and ignoring the Markov dependent structure will cause additional errors, which should be seriously taken care of.

4.2 SEQUENTIAL MODEL SELECTION

In the following, we propose a pessimistic model selection algorithm for finding an optimal policy among L candidate models. Our goal is to develop an approach to estimate the value function under each candidate model during our policy optimization procedure with theoretical guarantee. The proposed algorithm is motivated by recent development in statistical inference of sequential decision making [Luedtke and Van Der Laan, 2016, Shi et al., 2020]. The idea is to first estimate optimal Q-function Q^*, optimal policy π^* and the resulting ratio function based on a chunk of data, and evaluate the performance of the estimated policy on the next chunk of data using previously estimated nuisance functions. Then, we combine the first two chunks of data, perform the same estimation procedure and evaluation on the next chunk of data. The framework of MDP provides a nature way of splitting the data.

Specifically, denote the index of our batch dataset $\mathcal{D}_n$ as $J_0 = \{(i,t) : 1 \leq i \leq n, 0 \leq t < T\}$. We divide J_0 into O number of non-overlapping subsets, denoted by $J_1, \cdots, J_O$ and the corresponding data subsets are denoted by $\mathcal{D}_1, \cdots, \mathcal{D}_O$. Without loss of generality, we assume these data subsets have equal size. We require that for any $1 \leq o_1 < o_2 \leq O$, any $(i_1,t_1) \in J_{o_1}$ and $(i_2,t_2) \in J_{o_2}$, either $i_2 \neq i_1$ or $t_1 < t_2$. For $1 \leq o \leq O$, denote the aggregate chunks of data as $\bar{\mathcal{D}}_o = \left\{(S_{i,t}, A_{i,t}, R_{i,t}, S_{i,t+1}), (i,t) \in \bar{J}_o = J_1 \cup \cdots \cup J_o\right\}$.

We focus on FQI algorithm for illustrative purpose and it should be noticed that our algorithm can be applied to other RL algorithms. Starting from the first chunk of our batch data, at the o-th step ($o = 1, \cdots, O-1$), for each candidate model $l = 1, \cdots, L$, we apply FQI on $\bar{\mathcal{D}}_o$ to compute $\widehat{Q}_l^{(o)}$ as an estimate of optimal Q-function and obtain $\hat{\pi}_l^{(o)}$ correspondingly such that $\hat{\pi}_l^{(o)}(a|s) \in \text{argmax}_{a\in\mathcal{A}}\widehat{Q}_l^{(o)}(s,a)$ for every $s \in \mathcal{S}$. Additionally, we compute an estimate of ratio function $\omega^{\hat{\pi}_l^{(o)},\nu}$ using $\bar{\mathcal{D}}_o$ by many existing algorithms such as Nachum et al. [2019]. Denote the resulting estimator as $\widehat{\omega}^{\hat{\pi}_l^{(o)},\nu}$. The purpose of estimating this ratio function is to

improve the efficiency and robustness of our value function estimation for each candidate model. Then, we compute the estimated value function of $\hat{\pi}_l^{(o)}$ on $\mathcal{D}_{o+1}$ as

$$\begin{aligned}
&\hat{\mathcal{V}}_{\mathcal{D}_{o+1}}(\hat{\pi}_l^{(o)}) \\
=&(1-\gamma)\mathbb{E}_{S_0\sim\nu}[\sum_{a_0\in\mathcal{A}}\hat{\pi}_l^{(o)}(a_0|S_0)\widehat{Q}_l^{(o)}(S_0,a_0)] \\
&+\mathbb{E}_{\mathcal{D}_{o+1}}[\widehat{\omega}^{\hat{\pi}_l^{(o)},\nu}(S,A)(R+\gamma\sum_{a'\in\mathcal{A}}\hat{\pi}_l^{(o)}(a'|S')\widehat{Q}_l^{(o)}(S',a'))] \\
&-\mathbb{E}_{\mathcal{D}_{o+1}}[\widehat{\omega}^{\hat{\pi}_l^{(o)},\nu}(S,A)\widehat{Q}_l^{(o)}(S,A)], \qquad (5)
\end{aligned}$$

where $\mathbb{E}_{\mathcal{D}_{o+1}}$ denotes the empirical average over the $(o+1)$ chunk of dataset and (S,A,R,S') is one transition tuple in $\mathcal{D}_{o+1}$. While one can aggregate $\hat{\mathcal{V}}_{\mathcal{D}_{o+1}}(\hat{\pi}_l^{(o)})$ for $1\le o\le(O-1)$ to evaluate the performance of L models, the uncertainty of these estimates due to the finite sample estimation should not be ignored. Therefore, in the following, we derive an uncertainty quantification of our estimated value function for each candidate model, for performing model selection. Based on equation (4), (conditioning on $\bar{\mathcal{D}}_o$), the variance of $\hat{\mathcal{V}}_{\mathcal{D}_{o+1}}(\hat{\pi}_l^{(o)})$ is

$$\begin{aligned}
&\sigma^2(\hat{\pi}_l^{(o)}) \\
=&\mathbb{E}\left[\{\widehat{\omega}^{\hat{\pi}_l^{(o)},\nu}(S,A)(R+\gamma\sum_{a'\in\mathcal{A}}\hat{\pi}_l^{(o)}(a'|S')\widehat{Q}_l^{(o)}(S',a')\right. \\
&\left.-\widehat{Q}_l^{(o)}(S,A))\}^2\right], \qquad (6)
\end{aligned}$$

where (S,A,S') is a transition tuple with (S,A) follows some stationary distribution. See Assumption 1. Correspondingly we have an estimator defined as

$$\begin{aligned}
&\hat{\sigma}^2_{o+1}(\hat{\pi}_l^{(o)}) \\
=&\mathbb{E}_{\mathcal{D}_{o+1}}[\{\widehat{\omega}^{\hat{\pi}_l^{(o)},\nu}(S,A)(R+\gamma\sum_{a'\in\mathcal{A}}\hat{\pi}_l^{(o)}(a'|S')\widehat{Q}_l^{*(o)}(S',a') \\
&-\widehat{Q}_l^{*(o)}(S,A))\}^2]. \qquad (7)
\end{aligned}$$

The estimation procedure stops once we have used all our offline data and denote the final estimated policy as $\hat{\pi}_l$ for each $l=1,\cdots,L$. Notice that $\hat{\pi}_l=\hat{\pi}_l^{(O)}$. Finally, we compute the weighted average of all the intermediate value functions as our final evaluation of the estimated policy $\hat{\pi}_l$, i.e.,

$$\hat{\mathcal{V}}(\hat{\pi}_l)=\left(\sum_{o=1}^{O-1}\frac{1}{\hat{\sigma}_{o+1}(\hat{\pi}_l^{(o)})}\right)^{-1}\left(\sum_{o=1}^{O-1}\frac{\hat{\mathcal{V}}_{\mathcal{D}_{o+1}}(\hat{\pi}_l^{(o)})}{\hat{\sigma}_{o+1}(\hat{\pi}_l^{(o)})}\right). \qquad (8)$$

In Section 5, we show that under some technical conditions, the following asymptotic result holds:

$$\frac{\sqrt{nT(O-1)/O}\left(\hat{\mathcal{V}}(\hat{\pi}_l)-\mathcal{V}(\hat{\pi}_l)\right)}{\hat{\sigma}(l)}\Longrightarrow\mathcal{N}(0,1), \qquad (9)$$

where $\hat{\sigma}(l)=(O-1)(\sum_{o=1}^{O-1}\{\sigma_{o+1}(\hat{\pi}_l^{(o)})\}^{-1})^{-1}$, $\Longrightarrow$ refers to weak convergence when either n or T goes to infinity, and $\mathcal{N}(0,1)$ refers to the standard normal distribution. Based on the asymptotic result in (9), we can construct a confidence interval for the value function of each policy $\hat{\pi}_l$. Given a confidence level α, for each l, we can compute $U(l)=\hat{\mathcal{V}}(\hat{\pi}_l)-z_{\alpha/2}\sqrt{O/nT(O-1)}\hat{\sigma}(l)$, where $z_{\alpha/2}$ is $(1-\frac{\alpha}{2})$-quantile of the standard normal distribution. Our final selected one is $\hat{l}\in\text{argmax}_{1\le l\le L}\ U(l)$.

The use of $U(l)$ is motivated by the recent proposed pessimistic idea to address the overestimation issue of value (or Q) function in the OffRL setting. See Kumar et al. [2019, 2020], Jin et al. [2021], Xie et al. [2021], Uehara and Sun [2021], Zanette et al. [2021] for details. The final output of our algorithm is $\hat{\pi}_{\hat{l}}$ and an outline of the proposed algorithm can be found in Algorithm 1. As we can see, our algorithm is nearly tuning-free, which provides great flexibility in real-world applications. The only two adjustable parameters are O and α and they are directly interpretable. The size of O balances the computational cost and the finite-sample accuracy of evaluating each candidate model. Specifically, we can indeed show that the variance of the estimated value function by our algorithm can achieve the semi-parametric efficiency bound, which is best one can hope for. So in the asymptotic sense, the effect of O is negligible. In the finite-sample setting, it is rational to assume the performance will be discounted by a factor $\sqrt{O-1/O}$. Therefore, if O is large enough, $\sqrt{O-1/O}$ will have a mere effect on the performance. See Theorem 1 for more details. However, using large O will result in a large computational cost. As a sacrifice for the nearly tuning-free algorithm, we need to apply OffRL algorithms O times for each candidate model. The parameter α determines how worst the performance of each policy we should use to evaluate each policy. See Corollary 2 for more insights.

5 THEORETICAL RESULTS

In this section, we justify our asymptotic result given in (9). We use O_p to denote the stochastic boundedness. Before that, we make several technical assumptions:

Assumption 1 *The stochastic process $\{A_t,S_t\}_{t\ge0}$ is stationary with stationary distribution p_∞.*

Assumption 2 *For every $1\le l\le L$ and $1\le o\le O$, we have $\mathbb{E}|\mathcal{V}(\hat{\pi}_l^{(o)})-\mathcal{V}(\pi^*)|\le C_0(nT/O)^{-\kappa}$, for some constant C_0 and $\kappa>1/2$.*

Assumption 1 is standard in the existing literature such as [Kallus and Uehara, 2019]. Assumption 2 is key to our developed asymptotic results developed. This assumption essentially states that all candidate models are good enough so that eventually their value functions will converge to that of the optimal one. This implies that there is no asymptotic bias in estimating the optimal policy. While this is reasonable thanks to the capability of deep neutral networks, which has demonstrated their empirical success in many RL applications, such an assumption could still be strong. In Section 6,

Algorithm 1 Pessimistic Model Selection (PMS) for OffRL

Input: Dataset $\mathcal{D}_n$ and L candidate models for estimating optimal Q-function and policy; We divide $\mathcal{D}_n$ into non-overlapping subsets denoted by $\mathcal{D}_1, \cdots, \mathcal{D}_O$. We require that for any $1 \le o_1 < o_2 \le O$, any $(i_1, t_1) \in J_{o_1}$ and $(i_2, t_2) \in J_{o_2}$, either $i_2 \neq i_1$ or $t_1 \le t_2$.

for $l \in L$ **do**

  for $o = 1$ ***to*** $O - 1$ **do**

    For $l \in L$ models, construct the optimal $\widehat{Q}_l^{(o)}$ and $\hat{\pi}_l^{(o)}$ using $\bar{\mathcal{D}}_o$ data subset.

    Compute $\widehat{\omega}^{\hat{\pi}_l^{(o)},\nu}$ using $\bar{\mathcal{D}}_o$ by Nachum et al. [2019] and min-max solver for (3).

    Compute $\hat{\mathcal{V}}_{\mathcal{D}_{o+1}}(\hat{\pi}_l^{(o)})$ and $\hat{\sigma}^2_{o+1}(l)$ using $\mathcal{D}_{o+1}$ given in (5) and (7) respectively.

  end

  For l-th model, we compute $U(l) = \hat{\mathcal{V}}(\hat{\pi}_l) - z_{\alpha/2}\sqrt{nT(O-1)/O}\hat{\sigma}(l)$, where $\hat{\mathcal{V}}(\hat{\pi}_l)$ and $\hat{\sigma}(l)$ are given in (8) and (9) respectively.

end

Pick $\hat{l} = \arg\max_l U(l)$ as the selected model and run the algorithm on full dataset to obtain $\hat{\pi}_{\hat{l}}$.

Return $\hat{\pi}_{\hat{l}}$.

we aim to relax this assumption and provide two remedies for addressing possibly biased estimated policies. In addition, Assumption 1 also requires that the convergence rates of value functions under estimated policies are fast enough. This has been shown to hold under the margin condition on π^*, see e.g., [Hu et al., 2021] for more details.

Assumption 3 *For every $1 \le l \le L$ and $1 \le o \le O-1$, suppose $\mathbb{E}_{(S,A)\sim p_\infty}|\widehat{Q}_l^{(o)}(S,A) - Q^{\hat{\pi}_l^{(o)}}(S,A)|^2 = O_p\{(nT/O)^{-2\kappa_1}\}$ for some constant $\kappa_1 \ge 0$. In addition, $\widehat{Q}_l^{(o)}$ is uniformly bounded almost surely.*

Assumption 4 *For every $1 \le l \le L$ and $1 \le o \le O-1$, suppose $\mathbb{E}_{(S,A)\sim p_\infty}|\widehat{\omega}^{\hat{\pi}_l^{(o)},\nu}(S,A) - \omega^{\hat{\pi}_l^{(o)},\nu}(S,A)|^2 = O_p\{(nT/O)^{-2\kappa_2}\}$ for some constant $\kappa_2 \ge 0$. In addition, both $\omega^{\hat{\pi}_l^{(o)},\nu}$ and $\widehat{\omega}^{\hat{\pi}_l^{(o)},\nu}$ are uniformly bounded above and below away from 0 almost surely.*

Assumption 5 *For every $1 \le l \le L$ and $1 \le o \le O-1$, $\sigma^2(\hat{\pi}_l^{(o)})$ and $\hat{\sigma}^2_{o+1}(\hat{\pi}_l^{(o)})$ are bounded above and below from 0 almost surely.*

Assumptions 3 and 4 impose high-level conditions on two nuisance functions. Our theoretical results only require $\kappa_1 + \kappa_2 > 1/2$, which is a mild assumption. For example, if considered parametric models for both Q-function and ratio function, then $\kappa_1 = \kappa_2 = 1/2$. If considered non-parametric models for these two nuisance functions such as deep neural networks, then $1/4 < \kappa_1, \kappa_2 < 1/2$ can be obtained under some regularity conditions. See Fan et al. [2020] and Liao et al. [2020], Uehara et al. [2021] for the convergence rates of Q-function and ratio function by non-parametric models respectively. In addition, Assumption 5 is a mild assumption, mainly for theoretical justification. Then, we have the following main theorem as a foundation of our proposed algorithm.

Theorem 1 *Under Assumptions 1-5, we have*

$$(\sqrt{nT(O-1)/O}(\hat{\mathcal{V}}(\hat{\pi}_l) - \mathcal{V}(\hat{\pi}_l)))/\hat{\sigma}(l) \Longrightarrow \mathcal{N}(0,1). \quad (10)$$

Theorem 1 provides an uncertainty quantification of each candidate model used in policy optimization. Such uncertainty quantification is essential in OffRL as data are often limited. We highlight the importance of such results in Appendix A. A consequent result following Theorem 1 validates the proposed Algorithm 1:

Corollary 2 $\liminf_{nT\to\infty} \Pr(\mathcal{V}(\hat{\pi}_{\hat{l}}) \ge \max_{1\le l\le L}\mathcal{V}(\hat{\pi}_l) - 2z_{\alpha/2}\sqrt{nT(O-1)/O}\hat{\sigma}(l)) \ge 1 - L\alpha$ *under Assumptions 1-5.*

As can be seen clearly from Corollary 2 and the proposed PMS method, with a high probability (by letting α small), we consider the worst performance of each candidate model $\hat{\pi}_l$ in the sense of the lower confidence limit of the value function, and then select the best one among all models.

6 TWO REFINED APPROACHES

In this section, we relax Assumption 2 by allowing possibly non-negligible bias in estimating the optimal policy and introduce two refined approaches for addressing this issue. Instead of imposing Assumption 2, we make an alternative assumption below.

Assumption 6 *For $1 \le l \le L$, there exists $B(l)$ such that $\max_{1\le o\le(O-1)}|\mathcal{V}(\hat{\pi}_l^{(o)}) - \mathcal{V}(\pi^*)| \le B(l)$ almost surely.*

Assumption 6 is a very mild assumption. It essentially states that the biases for all our intermediate value function estimates are bounded by some constant, which is much weaker than Assumption 2. In this case, the asymptotic results in (10) may not hold in general. Correspondingly, we have the following result.

Theorem 3 *Under Assumptions 1, 3-6, for every $1 \le l \le L$, the following inequality holds:*

$$\liminf_{nT\to\infty} \Pr\Big(|\mathcal{V}(\pi^*) - \hat{\mathcal{V}}(\hat{\pi}_l)| \le z_{\alpha/2}\sqrt{O/nT(O-1)}\hat{\sigma}(l) + B(l)\Big) \ge 1 - \alpha. \quad (11)$$

Motivated by Lepski's principle [Lepski and Spokoiny, 1997] from nonparametric statistics and [Su et al., 2020] studying the model selection of OPE, we consider the following refined model-selection procedure to find the best policy. We first rank L candidate models in an non-increasing order based on the value of $\hat{\sigma}(l)$, i.e., for $1 \leq i < j \leq L$, $\hat{\sigma}(i) \geq \hat{\sigma}(j)$. Then for i-th model, we construct an interval as $I(l) = [\hat{\mathcal{V}}(\hat{\pi}_l) - 2z_{\alpha/(2L)}\sqrt{O/nT(O-1)}\hat{\sigma}(l), \hat{\mathcal{V}}(\hat{\pi}_l) + 2z_{\alpha/(2L)}\sqrt{O/nT(O-1)}\hat{\sigma}(l)]$. Finally the optimal model/policy we choose is $\hat{\pi}_{\hat{i}}$ such that $\hat{i} = \max\{i : 1 \leq i \leq L, \cap_{1\leq j\leq i} I(j) \neq \emptyset\}$. To show this procedure is valid, we need to make one additional assumption.

Assumption 7 *There exists a $\zeta < 1$ such that for $1 \leq i \leq L$, $B(i) \leq B(i+1)$ and $\zeta\hat{\sigma}(i) \leq \hat{\sigma}(i+1) \leq \hat{\sigma}(i)$ almost surely.*

While this assumption is borrowed from Su et al. [2020], we consider model selection for policy learning instead of OPE in Su et al. [2020], which is substantially more challenging. This assumption typically assumes that after model sorting based on $\hat{\sigma}(l)$, the bias of estimated policy is monotonically increasing and the standard deviation is monotonically decreasing but not too quickly. This is commonly seen when all candidate estimators exhibit some bias-variance trade-off phenomena. Define the following event

$$\mathcal{E} = \Big\{|\hat{\mathcal{V}}(\hat{\pi}_{\hat{i}}) - \mathcal{V}(\pi^*)| \leq 6(1+\zeta^{-1}) \times \min_{1\leq i\leq L}\{B(i) + z_{\alpha/(2L)}\sqrt{O/nT(O-1)}\hat{\sigma}(i)\}\Big\}.$$

Then we have the following theoretical guarantee for our refined procedure.

Corollary 4 *Under Assumptions 1, 3-5, and 7, we have $\liminf_{nT\to\infty} \Pr(\mathcal{E}) \geq 1-\alpha$. If we further assume that for any $\delta > 0$, with probability at least $1-\delta$, for every $1 \leq i \leq L$, $|\mathcal{V}(\hat{\pi}_i) - \hat{\mathcal{V}}(\hat{\pi}_i)| \leq c(\delta)\log(L)\hat{\sigma}(i)/\sqrt{NT}$ for some constant $c(\delta)$, then $\liminf_{nT\to\infty} \Pr(\bar{\mathcal{E}}) \geq 1-\alpha-\delta$, where*

$$\bar{\mathcal{E}} = \{|\mathcal{V}(\hat{\pi}_{\hat{i}}) - \mathcal{V}(\pi^*)| \leq 3(1+\zeta^{-1}) \times \min_{1\leq i\leq L}\{B(i) + (c(\delta)\log(L) + z_{\alpha/(2L)})\sqrt{O/nT(O-1)}\hat{\sigma}(i)\}\}.$$

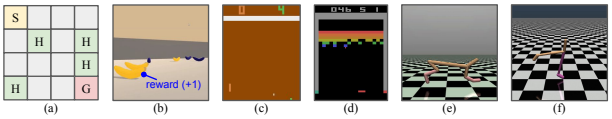

Figure 2: DQN environments in our studies: (a) $\mathbf{E}_1$: *FrozenLake-v0*; (b) $\mathbf{E}_2$: *Banana Collectors* (3D geometrical navigation task); (c) $\mathbf{E}_3$: *Pong-v0*; (d) $\mathbf{E}_4$: *Breakout-v0*; (e) $\mathbf{E}_5$: *Halfcheetah-v1*; (f) $\mathbf{E}_6$: *Walker2d-v1*.

The additional assumption (i.e., the high probability bound) in Corollary 4 can be shown to hold by the empirical process theory under some technical conditions [Van de Geer, 2000]. Hence Corollary 4 provides a strong guarantee that the regret of the final selected policy is bounded by the smallest error bound among all L candidate policies. Note that Assumption 3 imposed here could be strong.

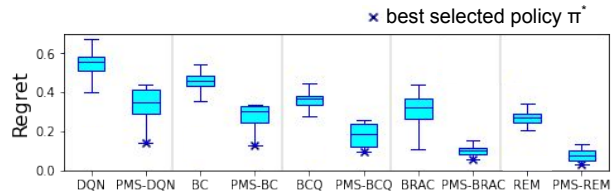

Figure 3: Box plots of model selection performance from offline learning in each DRL algorithm for $\mathbf{E}_2$.

Another refined approach: Notice that the above refined approach indeed focuses on OPE estimates to select the best policy with regret warranty. The motivation behind it is to find a policy that has the smallest estimation error to the optimal one. However, such procedure may not directly match the goal of maximizing the value function in OffRL. To relieve this issue, we can alternatively choose the final policy as $\hat{\pi}_{\hat{\hat{i}}}$ such that $\hat{\hat{i}} = \text{argmax}_{1\leq i\leq \hat{i}}\hat{\mathcal{V}}(\hat{\pi}_i) - 2z_{\alpha/2}\sqrt{nT(O-1)/O}\hat{\sigma}(i)$, where the argmax is taken over 1 to $\hat{i}$ models. This approach can be viewed as a combination of PMS and the above refined approach. By adopting this approach, candidate models with large biases are firtly removed by the truncation on $\hat{i}$. Then, we use the idea of PMS to select the best model having the best worst performance among the remaining candidates. Unfortunately, we do not have theoretical guarantee for this combined approach.

7 EXPERIMENTAL RESULTS

We select six DQN environments ($\mathbf{E}_1$ to $\mathbf{E}_6$) from open-source benchmarks [Brockman et al., 2016, Juliani et al., 2018] to conduct numerical experiments, as shown in Fig. 2. These tasks of deployed environments cover different domains that include tabular learning (Fig 2(a)); automatic navigation in a geometry environment with a physical ray-tracker (Fig 2(b)); Atari digital gaming (Fig 2(c) and (d)), and continuous control (Fig 2(e) and (f)). We provide detailed task description and targeted reward for each environment in Appendix C. We will also provide our reproducible code and implementations.

Experiment setups. To evaluate the performance of PMS with DQN models in offRL, we choose different neural network architectures under five competitive DRL algorithms including DQN by [Mnih et al., 2013, 2015], BCQ by [Fujimoto et al., 2019], BC by [Bain and Sammut, 1995, Ross and Bagnell, 2010], BRAC by [Wu et al., 2019] from RLU benchmarks, and REM by [Agarwal et al., 2020]. Within each architecture, 70 candidate models are created by as-

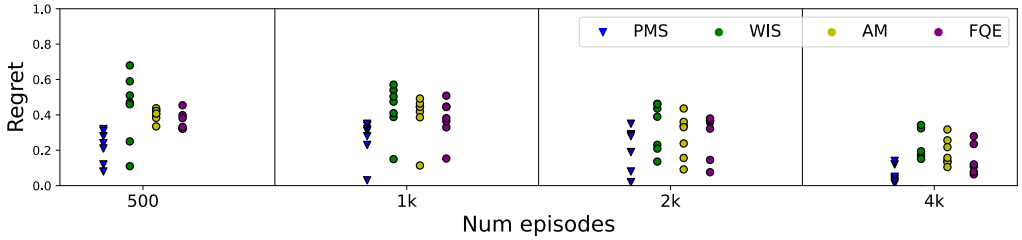

Figure 4: Sensitivity analysis for different training data size. PMS attains the best performance and has the least sensitivity.

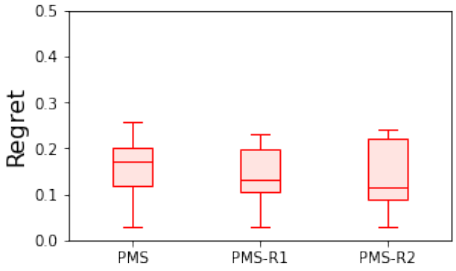

Figure 5: PMS and its refinements (R1/R2).

signing different hyperparameters and training setups. See Appendix D for details. We then conduct performance evaluation of different OffRL model selection methods on these generated candidate models.

Evaluation procedure. We utilize validation scores from OPE for each model selection algorithm, which picks the best (or a good) policy from the candidate set of size L based on its own criterion. Regret is used as the evaluation metric for each candidate. The regret for model l is defined as $\mathcal{V}(\pi_{l^*}) - \mathcal{V}(\hat{\pi}_l)$, where $l^* = \arg\max_{l'=1\ldots L} \mathcal{V}(\pi_{l'})$ corresponds to the candidate policy with the best OPE validation performance. In our implementation, we treat π_{l^*} as π^*, the oracle but unknown best possible policy. A small regret is desirable after model selection. Note the optimal regret is not zero since we can only use data to obtain $\hat{\pi}_l$ instead of π_l for each model. We provide additional top-k regret and precision results from Figures 6 to 16 in Appendix C.

Performance comparison. As highlighted in Fig. 1 in the introduction, we report estimated OPE values by different model selection approaches, i.e., PMS and three methods by [Tang and Wiens, 2021], versus the true OPE values. In this experiment, we consider 70 DQN models under the above mentioned five DRL algorithms, i.e., 14 models are considered for each architecture. We use fewer models for each DRL algorithm mainly for clear presentation. By using the confidence interval constructed by our PMS procedure, our method is able to correctly select the top models, while the other three methods fail. To further investigate the performance of PMS, we implement model selection among 70 models within each DRL algorithm separately. Fig. 3 shows the box plots of averaged regret over six environments after OPE per neural network architecture. Each subfigure contains results from one particular DRL algorithm with different hyperparameters or training setups. The left box plot refers to the regrets of all 70 models and the right one represents the regrets of top 10% models selected by the proposed PMS method. Note that the right box plot is a subset of the left one. The results show that our proposed PMS successfully helps to select models with the best policies and improve the average regret by a significant margin. In particular, PMS-REM-based models attain the lowest regrets, due to the benefit from its ensemble process. Detailed results for each environment is given in Appendix C, where $\alpha = 0.01$ and $O = 20$ are used in all experiments.

Sensitivity analysis. Fig. 4 compares different selection algorithms with varying training data size. PMS outperforms others across all scales, and larger number of episodes gives smaller variation and lower sensitivity.

PMS algorithm with refinements. We replicate our experiments in the offline navigation task in $\mathbf{E}_2$ (*Banana Collector*) for 30 times and report regrets of top 10% models selected by PMS and two refinements in Fig. 5. As we can see, while the overall performances of the proposed three model selection methods are similar, two refined approaches have better regrets than PMS in terms of median, demonstrating their potentials in identifying the best model. OPE results have been also evaluated also in DRL tasks with $\mathbf{E}_1$ and $\mathbf{E}_3$ to $\mathbf{E}_6$, where the refinement algorithms (PMS R1/R2) have only a small relative $\pm$ 0.423 % performance difference compared to its original PMS setups.

8 CONCLUSION

We propose a new theory-driven model selection framework (PMS) for offline deep reinforcement learning based on statistical inference. The proposed pessimistic mechanism warrants that the worst performance of the selected model is the best among all candidate models. Two refined approaches are further proposed to address the biases of DRL models. Extensive experimental results on six DQN environments with varying network architectures and training hyperparameters demonstrate that our proposed PMS method consistently yields improved model selection performance over existing baselines. The results suggest the effectiveness of PMS as a powerful and provably effective tool toward automating model selection in offline DRL.

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