A SIMPLE UNIFIED INFORMATION REGULARIZATION FRAMEWORK FOR MULTI-SOURCE DOMAIN ADAPTATION

Anonymous authors
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ABSTRACT

Adversarial learning strategy has demonstrated remarkable performance in dealing with single-source unsupervised Domain Adaptation (DA) problems, and it has recently been applied to multi-source DA problems. A potential pitfall of the existing DA methods that use multiple domain discriminators for each source domain is that domain-discriminative information is inevitably distributed across multiple discriminators. Despite this issue, the effect of using multiple discriminators on the quality of latent space representations has been poorly understood. To fully address this issue, we situate adversarial DA in the context of information regularization. First, we present a unified information regularization framework for multi-source DA. Our framework shows that the information shared across domains cannot be gleaned with multiple discriminators. It further provides a theoretical justification for using a single and unified domain discriminator to encourage the synergistic integration of the information gleaned from each domain. Second, this motivates us to implement a novel neural architecture called a Multi-source Information-regularized Adaptation Networks (MIAN). The proposed model significantly reduces the variance of stochastic gradients and increases computational-efficiency. Large-scale simulations on various multi-source DA scenarios demonstrate that MIAN, despite its structural simplicity, reliably outperforms other state-of-the-art methods by a large margin. Our work offers the possibility of garnering fundamental insights from multiple domains into the development of highly generalizable algorithms.

1 INTRODUCTION

Although a large number of studies have demonstrated the ability of deep neural networks to solve challenging tasks, the tasks solved by networks are mostly confined to a similar type or a single domain. One remaining challenge is the problem known as domain shift (Gretton et al. (2009)), where a direct transfer of information gleaned from a single source domain to unseen target domains may lead to significant performance impairment. Domain adaptation (DA) approaches aim to mitigate this problem by learning to map data of both domains onto a common feature space. Whereas several theoretical results (Ben-David et al. (2007); Blitzer et al. (2008); Zhao et al. (2019a)) and algorithms for DA (Long et al. (2015; 2017); Ganin et al. (2016)) have focused on the case in which only a single-source domain dataset is given, we consider a more challenging and generalized problem of knowledge transfer, referred to as Multi-source unsupervised DA (MDA). Following a seminal theoretical result on MDA (Blitzer et al. (2008); Ben-David et al. (2010)), technical advances have been made (Chattopadhyay et al. (2012); Xu et al. (2018); Peng et al. (2019)).

However, most earlier works did not provide a theoretical guarantee that the unnecessary domain-specific information is fully filtered out. Under the condition of limited channel capacity, domain-specific information is transmitted at the expense of task performance because it does not contain reliable context information for the task. This necessitates filtering out domain-specific information, which is performed by exploiting a combination of multi-domain datasets. To fully accelerate such distillation, domain-specific and domain-shared information should be separately estimated. However, several existing adversarial MDA methods independently feed each domain feature to the corresponding discriminator (Xu et al. (2018); Zhao et al. (2018); Li et al. (2018); Zhao et al. (2019b)), often making it difficult to estimate the domain-shared information.

To overcome such issues, we propose a novel MDA method, called Multi-source Information-regularized Adaptation Networks (**MIAN**), that constrains the mutual information between latent representations and domain labels. First, we illustrate that such mutual information regularization is closely related to the explicit optimization of the $\mathcal{H}$-divergence between the source and target domains. This affords the theoretical insight that the conventional adversarial DA can be translated into an information-theoretic-regularization problem. Second, based on our findings, we propose a new optimization problem for MDA: minimizing adversarial loss over multiple domains with a single domain discriminator. In our framework, the encoder extracts domain-independent features by minimizing domain-specific information while maximizing domain-shared information which was not measurable in earlier works.

Despite its structural simplicity, we found that **MIAN** works efficiently across a wide variety of MDA scenarios, including the DIGITS-Five (Peng et al. (2019)), Office-31 (Saenko et al. (2010)), and Office-Home datasets (Venkateswara et al. (2017)). Intriguingly, **MIAN** reliably and significantly outperformed several state-of-the-art methods that either employ a domain discriminator separately for each source domain (Xu et al. (2018)) or align the moments of deep feature distribution for every pairwise domain (Peng et al. (2019)).

2 RELATED WORKS

Several DA methods have been used in attempt to learn domain-invariant representations. Along with the increasing use of deep neural networks, contemporary work focuses on matching deep latent representations from the source domain with those from the target domain. Several measures have been introduced to handle domain shift, such as maximum mean discrepancy (MMD) (Long et al. (2014; 2015)), correlation distance (Sun et al. (2016); Sun & Saenko (2016)), and Wasserstein distance (Courty et al. (2017)). Recently, adversarial DA methods (Ganin et al. (2016); Tzeng et al. (2017); Hoffman et al. (2017); Saito et al. (2018; 2017)) have become mainstream approaches owing to the development of generative adversarial networks (Goodfellow et al. (2014)). However, the abovementioned single-source DA approaches inevitably sacrifice performance for the sake of multi-source DA.

Some MDA studies (Blitzer et al. (2008); Ben-David et al. (2010); Mansour et al. (2009); Hoffman et al. (2018)) have provided a theoretical background for algorithm-level solutions. (Blitzer et al. (2008); Ben-David et al. (2010)) explore an extended upper bound of true risk on unlabeled samples from the target domain with respect to a weighted combination of multiple source domains. Following such theoretical studies, MDA studies with shallow models (Duan et al. (2012b;a); Chattopadhyay et al. (2012)) as well as with deep neural networks (Mancini et al. (2018); Peng et al. (2019); Li et al. (2018)) have been proposed. Recently, some adversarial MDA methods have also been proposed. Xu et al. (2018) implements a k-way domain discriminator and classifier to battle both domain and category shift. Zhao et al. (2018) also uses multiple discriminators to optimize the average case generalization bounds. Zhao et al. (2019b) choose the relevant source training samples for DA by minimizing empirical Wasserstein distance between source and target domain. Instead of using separate encoders, domain discriminators or classifiers for each source domain as earlier works, our approach uses unified networks, making it resource-efficient and scalable. Also, one of our contributions is revealing that the variance of stochastic gradients increases in the process of using multiple discriminators through analyzing existing studies in terms of information regularization.

Several existing MDA works proposed methods to estimate source domain weights following (Blitzer et al. (2008); Ben-David et al. (2010)). Mansour et al. (2009) assumed that the target hypothesis can be approximated by a convex combination of the source hypotheses. (Peng et al. (2019); Zhao et al. (2018)) suggest an ad-hoc scheme for domain weights based on the empirical risk of each source domain. Li et al. (2018) computes a softmax transformed weight vector by using empirical Wasserstein-like measure instead of empirical risks. Compared to proposed methods without robust theoretical justification, our analysis does not require any assumption or estimation for such domain coefficients. In our framework, representations are distilled in a way to be independent to each domain, rendering the performance relatively insensitive to the explicit weighting strategy.

3 THEORETICAL INSIGHTS

We first introduce the notations for the MDA problem in classification. A set of source domains and the target domain are denoted by $\{D_{S_i}\}_{i=1}^{N}$ and D_T, respectively. Let $X_{S_i} = \left\{\mathbf{x}_{S_i}^j\right\}_{j=1}^{m}$ and $Y_{S_i} = \left\{\mathbf{y}_{S_i}^j\right\}_{j=1}^{m}$ be a set of m i.i.d. samples from D_{S_i}. Let $X_T = \left\{\mathbf{x_T}^j\right\}_{j=1}^{m} \sim (D_T^X)^m$ be the set of m i.i.d. samples generated from the marginal distribution D_T^X. The domain label and its probability distribution are denoted by V and $P_V(\mathbf{v})$, where $\mathbf{v} \in \mathcal{V}$ and $\mathcal{V}$ is the set of domain labels. In line with prior works (Hoffman et al. (2012); Gong et al. (2013); Mancini et al. (2018); Gong et al. (2019)), domain label can be generally treated as a stochastic latent random variable in our framework. However, for simplicity, we take the empirical version of the true distributions with given samples assuming that the domain labels for all samples are known. The latent representation of the sample is given by Z, and the encoder is defined as $F : \mathcal{X} \to \mathcal{Z}$. Accordingly, Z_{S_i} and Z_T refer to the outputs of the encoder $F(X_{S_i})$ and $F(X_T)$, respectively. For notational simplicity, we will omit the index i from D_{S_i}, X_{S_i} and Z_{S_i} when $N = 1$. A classifier $C : \mathcal{Z} \to \mathcal{Y}$ is defined on the latent space $\mathcal{Z}$ as where $\mathcal{Y}$ is the class label space.

3.1 PROBLEM FORMULATION

For comparison with our formulation, we recast single-source DA as a constrained optimization problem. The true risk $\epsilon_T(h)$ on unlabeled samples from the target domain is bounded above the sum of three terms (Ben-David et al. (2010)): (1) true risk $\epsilon_S(h)$ of hypothesis h on the source domain; (2) $\mathcal{H}$-divergence $d_{\mathcal{H}}(D_S, D_T)$ between a source and a target domain distribution; and (3) the optimal joint risk λ^*.

Theorem 1 (Ben-David et al. (2010))**.** *Let hypothesis class $\mathcal{H}$ be a set of binary classifiers $h : \mathcal{X} \to \{0, 1\}$. Then for the given domain distributions D_S and D_T,*

$$\forall h \in \mathcal{H}, \epsilon_T(h) \leq \epsilon_S(h) + d_{\mathcal{H}}(D_S, D_T) + \lambda^*, \tag{1}$$

where $d_{\mathcal{H}}(D_S, D_T) = 2\,sup_{h\in\mathcal{H}}\Big|\mathop{\mathbb{E}}_{\mathbf{x}\sim D_S^X}\big[\mathbb{I}(h(\mathbf{x}) = 1)\big] - \mathop{\mathbb{E}}_{\mathbf{x}\sim D_T^X}\big[\mathbb{I}(h(\mathbf{x}) = 1)\big]\Big|$ and $\mathbb{I}(a)$ is an indicator function whose value is 1 if a is true, and 0 otherwise.

The empirical $\mathcal{H}$-divergence $\hat{d}_{\mathcal{H}}(X_S, X_T)$ can be computed as follows (Ben-David et al. (2010)):

Lemma 1.

$$\hat{d}_{\mathcal{H}}(X_S, X_T) = 2\Big(1 - \min_{h\in\mathcal{H}}\Big[\frac{1}{m}\sum_{\mathbf{x}\in X_S}\mathbb{I}[h(\mathbf{x}) = 1] + \frac{1}{m}\sum_{\mathbf{x}\in X_T}\mathbb{I}[h(\mathbf{x}) = 0]\Big]\Big) \tag{2}$$

Following Lemma 1, a domain classifier $h : \mathcal{Z} \to \mathcal{V}$ can be used to compute the empirical $\mathcal{H}$-divergence. Suppose the optimal joint risk λ^* is sufficiently small as assumed in most adversarial DA studies (Saito et al. (2017); Chen et al. (2019)). Thus, one can obtain the ideal encoder and classifier minimizing the upper bound of $\epsilon_T(h)$ by solving the following min-max problem:

$$\begin{aligned} F^*, C^* &= \arg\min_{F,C}\ L(F, C) + \beta\hat{d}_{\mathcal{H}}(Z_S, Z_T) \\ &= \arg\min_{F,C}\max_{h\in\mathcal{H}}\ L(F, C) + \beta\frac{1}{m}\Big(\sum_{i:\mathbf{z}_i\in Z_S}\mathbb{I}[h(\mathbf{z}_i) = 1] + \sum_{j:\mathbf{z}_j\in Z_T}\mathbb{I}[h(\mathbf{z}_j) = 0]\Big), \end{aligned} \tag{3}$$

where $L(F, C)$ is the loss function on samples from the source domain, β is a Lagrangian multiplier, $\mathcal{V} = \{0, 1\}$ such that each source instance and target instance are labeled as 1 and 0, respectively, and h is the binary domain classifier. Note that the latter min–max problem is obtained by converting $-min$ into max and removing the constant term from Lemma 1.

3.2 INFORMATION-REGULARIZED MIN–MAX PROBLEM FOR MDA

Intuitively, it is not highly desirable to adapt the learned representation in the given domain to the other domains, particularly when the representation itself is not sufficiently domain-independent.

This motivates us to explore ways to learn representations independent of domains. Inspired by a contemporary fair model training study (Roh et al. (2020)), the mutual information between the latent representation and the domain label $I(Z;V)$ can be expressed as follows:

Theorem 2. *Let $P_Z(\mathbf{z})$ be the distribution of Z where $\mathbf{z} \in \mathcal{Z}$. Let h be a domain classifier $h : \mathcal{Z} \to \mathcal{V}$, where $\mathcal{Z}$ is the feature space and $\mathcal{V}$ is the set of domain labels. Let $h_{\mathbf{v}}(\mathbf{z})$ be a conditional probability of V where $\mathbf{v} \in \mathcal{V}$ given $Z = \mathbf{z}$, defined by h. Then the following holds:*

$$I(Z;V) = \max_{h_{\mathbf{v}}(\mathbf{z}):\sum_{\mathbf{v}\in\mathcal{V}} h_{\mathbf{v}}(\mathbf{z})=1,\forall\mathbf{z}} \sum_{\mathbf{v}\in\mathcal{V}} P_V(\mathbf{v})\mathbb{E}_{\mathbf{z}\sim P_{Z|\mathbf{v}}}\big[\log h_{\mathbf{v}}(\mathbf{z})\big] + H(V) \tag{4}$$

The detailed proof is provided in the Roh et al. (2020) and Supplementary Material. As done in Roh et al. (2020), we can derive the empirical version of Theorem 2 as follows:

$$\hat{I}(Z;V) = \max_{h_{\mathbf{v}}(\mathbf{z}):\sum_{\mathbf{v}\in\mathcal{V}} h_{\mathbf{v}}(\mathbf{z})=1,\forall\mathbf{z}} \frac{1}{M}\sum_{\mathbf{v}\in\mathcal{V}}\sum_{i:\mathbf{v}_i=\mathbf{v}} \log h_{\mathbf{v}_i}(\mathbf{z}_i) + H(V), \tag{5}$$

where M is the number of total representation samples, i is the sample index, and $\mathbf{v}_i$ is the corresponding domain label of the ith sample. Using this equation, we combine our information-constrained objective function and the results of Lemma 1. For binary classification $\mathcal{V} = \{0, 1\}$ with Z_S and Z_T of equal size $M/2$, we propose the following information-regularized minimax problem:

$$\begin{aligned} F^*, C^* &= \arg\min_{F,C} \; L(F,C) + \beta\hat{I}(Z;V) \\ &= \arg\min_{F,C}\max_{h\in\mathcal{H}} \; L(F,C) + \beta\frac{1}{M}\Big[\sum_{i:\mathbf{z}_i\in Z_S}\log h(\mathbf{z}_i) + \sum_{j:\mathbf{z}_j\in Z_T}\log(1-h(\mathbf{z}_j))\Big], \end{aligned} \tag{6}$$

where β is a Lagrangian multiplier, $h(\mathbf{z}_i) \triangleq h_{\mathbf{v}_i=1}(\mathbf{z}_i)$ and $1 - h(\mathbf{z}_i) \triangleq h_{\mathbf{v}_i=0}(\mathbf{z}_i)$, with $h(\mathbf{z}_i)$ representing the probability that $\mathbf{z}_i$ belongs to the source domain. This setting automatically dismisses the condition $\sum_{\mathbf{v}\in\mathcal{V}} h_{\mathbf{v}}(\mathbf{z}) = 1, \forall\mathbf{z}$. Note that we have accommodated a simple situation in which the entropy $H(V)$ remains constant.

The relationship between (3) and (6) provides us a mathematical guarantee that the problem of minimizing mutual information between the latent representation and the domain label is equivalent to minimizing the $\mathcal{H}$-divergence using the adversarial learning scheme. This equivalence clearly underlines the significance of information regularization for MDA. Compared to the existing MDA approaches (Xu et al. (2018); Zhao et al. (2018)), which inevitably distribute domain-discriminative knowledge over N different domain classifiers, the above objective function (6) enables us to seamlessly integrate such information with the single-domain classifier h.

3.3 Advantages over other MDA methods

Our information constraint can be reformulated as:

$$I(Z;V) = \sum_{i=1}^{N+1} I(Z;V_i \mid V_1, ..., V_{i-1}), \tag{7}$$

where V_i indicates whether the representation is from the ith domain with $V = (V_1, V_2, \ldots, V_{N+1})$ representing the one-hot encoded domain label vector, $i = N + 1$ represents the target domain. Our constraint measures the dependence with each feature and the domain label value considering the synergistic contributions of other domain label values. It thus implicitly includes interaction information terms; For example, $I(Z;V) = I(Z;V_1) + I(Z;V_2) - I(Z;V_1;V_2)$ where $I(Z;V_1;V_2)$ represents interaction information, given $N = 1$.

By excluding the weighted source combination strategy, we can also simplify the optimization constraint of existing adversarial MDA methods as sum of information constraints:

$$\sum_{k=1}^{N} I(Z_k;U_k) = \sum_{k=1}^{N}\max_{h^k_{\mathbf{u}}(\mathbf{z}):\sum_{\mathbf{u}\in\mathcal{U}} h^k_{\mathbf{u}}(\mathbf{z})=1,\forall\mathbf{z}} \sum_{\mathbf{u}\in\mathcal{U}} P_{U_k}(\mathbf{u})\mathbb{E}_{\mathbf{z}_k\sim P_{Z_k|\mathbf{u}}}\big[\log h^k_{\mathbf{u}}(\mathbf{z}_k)\big] + \sum_{k=1}^{N} H(U_k), \tag{8}$$

where U_k is the kth domain label with $\mathcal{U} = \{0, 1\}$, $P_{Z_k|\mathbf{u}=0}(\cdot) = P_{Z|\mathbf{v}=N+1}(\cdot)$ corresponds to the target domain, $P_{Z_k|\mathbf{u}=1}(\cdot) = P_{Z|\mathbf{v}=k}(\cdot)$ corresponds to the kth source domain, and $h^k_{\mathbf{u}}(\mathbf{z}_k)$ is the conditional probability of $\mathbf{u} \in \mathcal{U}$ given $\mathbf{z}_k$ defined by the kth discriminator indicating whether the sample is generated from the kth source domain or the target domain. Again we treat the entropy $H(U_k)$ as a constant. Note that the interaction information cannot be measured with (8).

Since the existing approaches use multiple discriminators to discriminate each source and target domain, domain-discriminative knowledge from other domains is not fully leveraged by each discriminator, making discriminators difficult to grasp optimal domain-specific information. For example, a discriminator may rely not only on the domain-specific but also on the domain-shared information. It is because data from other source domains has never been provided to the discriminator. Therefore, it is not guaranteed that domain-shared information is preserved during adaptation. It bias the representation towards the cue which is not generalizable but overfitted to the specific domain. It implies that the domain-interaction information, which earlier works cannot estimate, should be increased to encode the representation which is informative to given domains in common. Our framework enables filtering out domain-specific information while preserving the amount of domain-shared information.

Using a single domain discriminator also helps reduce the variance of gradient. The large variance of stochastic gradients slows down the convergence which leads to poor performance (Johnson & Zhang (2013)). Here we analyze the variance of stochastic gradients of the existing optimization constraint. Given $M = m(N + 1)$ samples with m representing the number of samples per domain, an empirical version of (8) is:

$$\sum_{k=1}^{N} \hat{I}(Z_k; U_k) = \frac{1}{M} \sum_{k=1}^{N} \max_{h^k_{\mathbf{u}}(\mathbf{z}):\sum_{\mathbf{u}\in\mathcal{U}} h^k_{\mathbf{u}}(\mathbf{z})=1,\forall \mathbf{z}} \sum_{\mathbf{u}\in\mathcal{U}} \sum_{i:\mathbf{u}^i=\mathbf{u}} \log h^k_{\mathbf{u}}(\mathbf{z}^i_k) + \sum_{k=1}^{N} H(U_k). \tag{9}$$

Let I_k be a shorthand for the kth term inside the first summation. Without loss of generality we make simplifying assumptions that all $Var[I_k]$ are the same for all k and so are $Cov[I_k, I_j]$ for all pairs, and that $Var[I(Z; V)]$ is comparable to $\frac{1}{M^2} Var[I_k]$. Then the variance of (9) is given by:

$$\begin{aligned} Var[\sum_{k=1}^{N} \hat{I}(Z_k; U_k)] &= \frac{1}{M^2} \Big(\sum_{k=1}^{N} Var[I_k] + 2 \sum_{k=1}^{N} \sum_{j=k}^{N} Cov[I_k, I_j] \Big) \\ &= \frac{1}{m^2} \Big(\frac{N}{(N+1)^2} Var[I_k] + \frac{N(N-1)}{(N+1)^2} Cov[I_k, I_j] \Big). \end{aligned} \tag{10}$$

As earlier works solve N adversarial minimax problems, the covariance term is additionally included and its contribution to the variance does not decrease with increasing N. In other words, the covariance term may dominate the variance of the gradients as the number of domain increases. In contrast, the variance of our constraint (5) is inversely proportional to $(N + 1)^2$. It implies that our framework can significantly improve the stability of stochastic gradient optimization compared to existing approaches, especially when the model is deemed to learn from a large number of domains.

3.4 Situating domain adaptation in context of information bottleneck theory

In this Section, we bridge the gap between the existing adversarial DA method and the information bottleneck (IB) theory (Tishby et al. (2000); Tishby & Zaslavsky (2015); Alemi et al. (2016)). Tishby et al. (2000) examined the problem of learning an encoding Z such that it is maximally informative about the class Y while being minimally informative about the sample X:

$$\min_{P_{enc}(\mathbf{z}|\mathbf{x})} \beta I(Z; X) - I(Z; Y), \tag{11}$$

where β is a Lagrangian multiplier. Indeed, the role of the bottleneck term $I(Z; X)$ matches our mutual information $I(Z; V)$ between the latent representation and the domain label. We foster close collaboration between two information bottleneck terms by incorporating those into $I(Z; X, V)$.

Theorem 3. *Let $P_{Z|\mathbf{x},\mathbf{v}}(\mathbf{z})$ be a conditional probabilistic distribution of Z where $\mathbf{z} \in \mathcal{Z}$, defined by the encoder F, given a sample $\mathbf{x} \in \mathcal{X}$ and the domain label $\mathbf{v} \in \mathcal{V}$. Let $R_Z(\mathbf{z})$ denotes a prior marginal distribution of Z. Then the following inequality holds:*

$$\begin{aligned} I(Z; X, V) \leq & \max_{h_{\mathbf{v}}(\mathbf{z}):\sum_{\mathbf{v}\in\mathcal{V}} h_{\mathbf{v}}(\mathbf{z})=1,\forall \mathbf{z}} \sum_{\mathbf{v}\in\mathcal{V}} P_V(\mathbf{v}) \mathbb{E}_{P_{\mathbf{z}\sim Z|\mathbf{v}}} \big[\log h_{\mathbf{v}}(\mathbf{z}) \big] + H(V) \\ & + \mathbb{E}_{\mathbf{x},\mathbf{v}\sim P_{X,V}} \big[D_{KL}[P_{Z|\mathbf{x},\mathbf{v}} \parallel R_Z] \big] \end{aligned} \tag{12}$$

The proof of Theorem 3 uses the chain rule: $I(Z;X,V) = I(Z;V) + I(Z;X \mid V)$. The detailed proof is provided in the Supplementary Material. Whereas the role of $I(Z;X \mid V)$ is to purify the latent representation generated from the given domain, $I(Z;V)$ serves as a proxy for regularization that aligns the purified representations across different domains. Thus, the existing DA approaches (Luo et al. (2019); Song et al. (2019)) using variational information bottleneck (Alemi et al. (2016)) can be reviewed as special cases for Theorem 3 with a single-source domain.

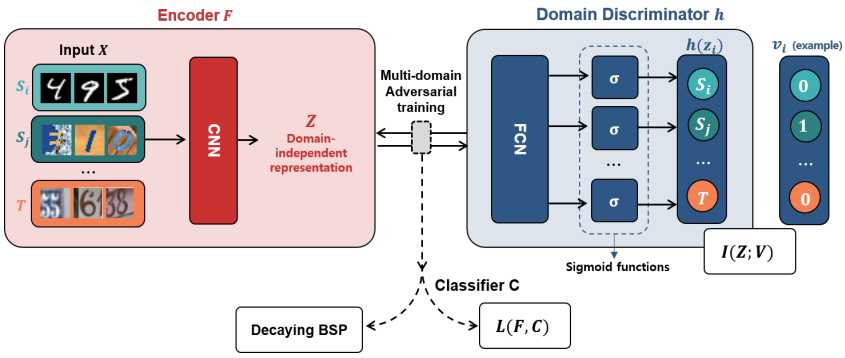


Figure 1: Proposed neural architecture for multi-source domain adaptation: Multi-source Information-regularized Adaptation Network (MIAN). Multi-source and target domain input data are fed into the encoder. We denote arbitrary source domains as S_i and S_j without loss of generality. The domain discriminator outputs a logit vector, where each dimension corresponds to each domain.

4 Multi-source Information-regularized Adaptation Networks

In this Section, we provide the details of our proposed architecture, referred to as a multi-source information-regularized adaptation network (**MIAN**). **MIAN** addresses the information-constrained min–max problem for MDA (Section 3.2) using the three subcomponents depicted in Figure 1: information regularization, source classification, and Decaying Batch Spectral Pe nalization (DBSP).

Information regularization. In this study, we optimize every pairwise domain discrepancy through independent adversarial learning processes of a given domain versus all other domains. Let $\mathcal{V} = \{1, 2, ..., N+1\}$ and denote $h(\mathbf{z})$ as $N+1$ dimensional vector of the conditional probability for each domain given the sample $\mathbf{z}$. Let $\mathbb{1}$ be a $N+1$ dimensional vector of all ones, and $\mathbb{1}_{[k=\mathbf{v}]}$ be a $N+1$ dimensional vector whose $\mathbf{v}$th value is 1 and 0 otherwise. $\mathbb{1}_{[k\neq\mathbf{v}]}$ is flipped version of $\mathbb{1}_{[k=\mathbf{v}]}$. Then the objective function for the domain discriminator is:

$$\min_{h} -\frac{1}{M} \sum_{\mathbf{v}\in\mathcal{V}} \sum_{i:\mathbf{v}_i=\mathbf{v}} \left[\mathbb{1}_{[k=\mathbf{v}_i]}^T \log h(\mathbf{z}_i) + \mathbb{1}_{[k\neq\mathbf{v}_i]}^T \log(\mathbb{1} - h(\mathbf{z}_i))\right], \tag{13}$$

where the objective function for encoder training is to maximize (13). Our objective function is closely related to that of GAN (Goodfellow et al. (2014)), and we experimentally found that using the variant objective function of GAN (Mao et al. (2017)) works slightly better.

Source classification. Along with learning domain-independent latent representations illustrated in the above, we train the classifier with the labeled source domain datasets that can be directly applied to the target domain representations in practice. To minimize the empirical risk on source domain, we use a generic softmax cross-entropy loss function with labeled source domain samples as $L(F, C)$

Decaying batch spectral penalization. Applying above information-theoretic insights, we further describe a potential side effect of existing adversarial DA methods. Information regularization may lead to overriding implicit entropy minimization, particularly in the early stages of the training, impairing the richness of latent feature representations. To prevent such a pathological phenomenon, we introduce a new technique called Decaying Batch Spectral Penalization (DBSP), which is intended to control the SVD entropy of the feature space. Our version improves training efficiency compared to original Batch Spectral Penalization (Chen et al. (2019)). We refer to this version of our model as **MIAN-γ**. As vanila **MIAN** is sufficient to outperform other state-of-the-art methods (Section 5), **MIAN-γ** is further discussed in the Supplementary Material.

5 EXPERIMENTS

To assess the performance of **MIAN**, we ran a large-scale simulation using the following benchmark datasets: Digits-Five, Office-31 and Office-Home. For a fair comparison, we reproduced all the other baseline results using the same backbone architecture and optimizer settings as the proposed method. For the source-only and single-source DA standards, we introduce two MDA approaches (Xu et al. (2018); Peng et al. (2019)): (1) source-combined, i.e., all source-domains are incorporated into a single source domain; (2) single-best, i.e., the best adaptation performance on the target domain is reported. Owing to limited space, details about simulation settings and datasets are presented in the Supplementary Material.

Table 1: Accuracy (%) on Digits-Five dataset. SYNTH denotes Synthetic Digits (Ganin & Lempitsky (2014)). The baseline results for the Digits-Five dataset were taken from (Peng et al. (2019)).

Standards	Models	MNIST-M	MNIST	USPS	SVHN	SYNTH	Avg
Source-combined	Source Only	63.70	92.30	90.71	71.51	83.44	80.33
	DAN	67.87	97.50	93.49	67.80	86.93	82.72
	DANN	70.81	97.90	93.47	68.50	87.37	83.61
Single-best	Source Only	63.37	90.50	88.71	63.54	82.44	77.71
	DAN	63.78	96.31	94.24	62.45	85.43	80.44
	CORAL	62.53	97.21	93.45	64.40	82.77	80.07
	DANN	71.30	97.60	92.33	63.48	85.34	82.01
	JAN	65.88	97.21	95.42	75.27	86.55	84.07
	ADDA	71.57	97.89	92.83	75.48	86.45	84.84
	MEDA	71.31	96.47	97.01	78.45	84.62	85.60
	MCD	72.50	96.21	95.33	78.89	87.47	86.10
Multi-source	DCTN	70.53	96.23	92.81	77.61	86.77	84.79
	M^3SDA	69.76	**98.58**	95.23	78.56	87.56	86.13
	M^3SDA-β	72.82	98.43	96.14	81.32	89.58	87.65
	MIAN	**84.36**	97.91	**96.49**	**88.18**	**93.23**	**92.03**

Table 2: Accuracy (%) on Office-31 dataset.

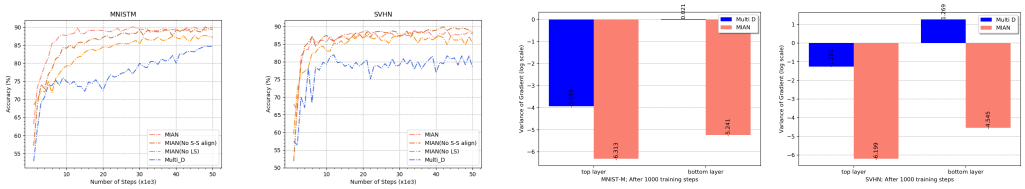
Standards	Models	Amazon	DSLR	Webcam	Avg
Single-best	Source Only	55.23±0.72	95.59±1.37	87.06±1.50	79.29
	DAN	64.19±0.56	**100.00±0.00**	97.45±0.44	87.21
	JAN	69.57±0.27	99.80±0.00	97.4±0.26	88.92
Source-combined	Source Only	60.80±2.00	92.68±0.31	86.91±2.37	80.13
	JAN	70.15±0.19	95.20±0.36	95.15±0.23	86.83
	DANN	68.15±0.42	97.59±0.60	96.77±0.26	87.50
	DAN	65.77±0.74	99.26±0.23	97.51±0.41	87.51
	DANN+BSP	71.13±0.44	96.65±0.30	98.32±0.26	88.70
	MCD	68.57±1.06	99.49±0.25	**99.30±0.38**	89.12
Multi-source	DCTN	62.74±0.50	99.44±0.25	97.92±0.29	86.70
	M^3SDA	67.19±0.22	99.34±0.19	98.04±0.21	88.19
	M^3SDA-β	69.41±0.82	99.64±0.19	**99.30±0.31**	89.45
	MIAN	**74.65±0.48**	99.48±0.35	98.49±0.59	**90.87**

5.1 SIMULATION RESULTS

The classification accuracy for Digits-Five, Office-31, and Office-Home are summarized in Tables 1, 2, and 3, respectively. We found that **MIAN** outperforms most of other state-of-the-art single-source and multi-source DA methods by a large margin. Note that our method demonstrated a significant improvement in challenging domains, such as MNIST-M, Amazon or Clipart.

Table 3: Accuracy (%) on Office-Home dataset.

Standards	Models	Art	Clipart	Product	Realworld	Avg
Source-combined	Source Only	64.58±0.68	52.32±0.63	77.63±0.23	80.70±0.81	68.81
	DANN	64.26±0.59	58.01±1.55	76.44±0.47	78.80±0.49	69.38
	DANN+BSP	66.10±0.27	61.03±0.39	78.13±0.31	79.92±0.13	71.29
	DAN	68.28±0.45	57.92±0.65	78.45±0.05	**81.93±0.35**	71.64
	MCD	67.84±0.38	59.91±0.55	79.21±0.61	80.93±0.18	71.97
Multi-source	M^3SDA	66.22±0.52	58.55±0.62	79.45±0.52	81.35±0.19	71.39
	DCTN	66.92±0.60	61.82±0.46	79.20±0.58	77.78±0.59	71.43
	MIAN	**69.39±0.50**	**63.05±0.61**	**79.62±0.16**	80.44±0.24	**73.12**



(a) (b) (c) (d)

Figure 2: **(a)**∼**(b)**: Test accuracies for **(a)** MNIST-M and **(b)** SVHN as target domains. **(c)**∼**(d)**: Variance of stochastic gradients after 1000 steps for **(c)** MNIST-M and **(d)** SVHN as target domains in log scale. Less is better.

5.2 Qualitative and Quantitative Analyses

Design of domain discriminator. To quantify the extent to which performance improvement is caused by unifying domain discriminators, we compared the performance of the four different versions of **MIAN** (Figure 2a, 2b). *No S-S align* is the same as **MIAN** except that only the target domain and each source domains are aligned with. *No LS* uses the objective function exactly like (13), not like (Mao et al. (2017)). *Multi D* employs as many discriminators as the number of source domains which is analogous to existing approaches. For a fair comparison, all the other experimental settings are fixed. Results illustrate that all the versions with unified discriminator reliably outperform *Multi D* in terms of both accuracy and reliability. This suggests that the unification of the domain discriminators per se significantly improves the performance of the task.

Variance of stochastic gradients. Related to above analysis, we compared the variance of stochastic gradients computed with different available domain discriminators. We trained **MIAN** and *Multi D* using mini-batches of samples. The number of samples in a batch is fixed as 128 per domain. After early stages of training, we computed the gradients for the weights and biases of both top and bottom layer of encoder on the full training set. Figure 2c, 2d show that **MIAN** with the unified discriminator yields exponentially lower variance of the gradients compared to *Multi D*. Thus it is more feasible to train with the unified discriminator when a large number of domains are given.

Visualization of learned latent representations. We visualized domain-independent representations extracted by the input layer of the classifier with t-SNE (Figure 3 in Supplementary Material). Before the adaptation process, the representations from the target domain were isolated from the representations from each source domain. However, after adaptation, the representations were well-aligned with respect to the class of digits, as opposed to the domain.

6 Conclusion

In this paper, we have presented a unified information-regularization framework for MDA. The proposed framework allows us to examine the existing adversarial DA methods and also motivates us to implement a novel neural architecture for MDA. The proposed model improves resource-efficiency, training stability and preserve interaction information over domains. Our framework also establishes a bridge between adversarial domain adaptation and Information Bottleneck theory.

REFERENCES

Alexander A Alemi, Ian Fischer, Joshua V Dillon, and Kevin Murphy. Deep variational information bottleneck. *arXiv preprint arXiv:1612.00410*, 2016.

Orly Alter, Patrick O Brown, and David Botstein. Singular value decomposition for genome-wide expression data processing and modeling. *Proceedings of the National Academy of Sciences*, 97 (18):10101–10106, 2000.

Shai Ben-David, John Blitzer, Koby Crammer, and Fernando Pereira. Analysis of representations for domain adaptation. In *Advances in neural information processing systems*, pp. 137–144, 2007.

Shai Ben-David, John Blitzer, Koby Crammer, Alex Kulesza, Fernando Pereira, and Jennifer Wortman Vaughan. A theory of learning from different domains. *Machine learning*, 79(1-2):151–175, 2010.

John Blitzer, Koby Crammer, Alex Kulesza, Fernando Pereira, and Jennifer Wortman. Learning bounds for domain adaptation. In *Advances in neural information processing systems*, pp. 129–136, 2008.

Rita Chattopadhyay, Qian Sun, Wei Fan, Ian Davidson, Sethuraman Panchanathan, and Jieping Ye. Multisource domain adaptation and its application to early detection of fatigue. *ACM Transactions on Knowledge Discovery from Data (TKDD)*, 6(4):1–26, 2012.

Xinyang Chen, Sinan Wang, Mingsheng Long, and Jianmin Wang. Transferability vs. discriminability: Batch spectral penalization for adversarial domain adaptation. In *International Conference on Machine Learning*, pp. 1081–1090, 2019.

Nicolas Courty, Rémi Flamary, Amaury Habrard, and Alain Rakotomamonjy. Joint distribution optimal transportation for domain adaptation. In *Advances in Neural Information Processing Systems*, pp. 3730–3739, 2017.

Lixin Duan, Dong Xu, and Shih-Fu Chang. Exploiting web images for event recognition in consumer videos: A multiple source domain adaptation approach. In *2012 IEEE Conference on Computer Vision and Pattern Recognition*, pp. 1338–1345. IEEE, 2012a.

Lixin Duan, Dong Xu, and Ivor Wai-Hung Tsang. Domain adaptation from multiple sources: A domain-dependent regularization approach. *IEEE Transactions on neural networks and learning systems*, 23(3):504–518, 2012b.

Yaroslav Ganin and Victor Lempitsky. Unsupervised domain adaptation by backpropagation. *arXiv preprint arXiv:1409.7495*, 2014.

Yaroslav Ganin, Evgeniya Ustinova, Hana Ajakan, Pascal Germain, Hugo Larochelle, François Laviolette, Mario Marchand, and Victor Lempitsky. Domain-adversarial training of neural networks. *The Journal of Machine Learning Research*, 17(1):2096–2030, 2016.

Boqing Gong, Kristen Grauman, and Fei Sha. Reshaping visual datasets for domain adaptation. In *Advances in Neural Information Processing Systems*, pp. 1286–1294, 2013.

Rui Gong, Wen Li, Yuhua Chen, and Luc Van Gool. Dlow: Domain flow for adaptation and generalization. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pp. 2477–2486, 2019.

Ian Goodfellow, Jean Pouget-Abadie, Mehdi Mirza, Bing Xu, David Warde-Farley, Sherjil Ozair, Aaron Courville, and Yoshua Bengio. Generative adversarial nets. In *Advances in neural information processing systems*, pp. 2672–2680, 2014.

Arthur Gretton, Alex Smola, Jiayuan Huang, Marcel Schmittfull, Karsten Borgwardt, and Bernhard Schölkopf. Covariate shift by kernel mean matching. *Dataset shift in machine learning*, 3(4):5, 2009.

Judy Hoffman, Brian Kulis, Trevor Darrell, and Kate Saenko. Discovering latent domains for multisource domain adaptation. In *European Conference on Computer Vision*, pp. 702–715. Springer, 2012.

Judy Hoffman, Eric Tzeng, Taesung Park, Jun-Yan Zhu, Phillip Isola, Kate Saenko, Alexei A Efros, and Trevor Darrell. Cycada: Cycle-consistent adversarial domain adaptation. *arXiv preprint arXiv:1711.03213*, 2017.

Judy Hoffman, Mehryar Mohri, and Ningshan Zhang. Algorithms and theory for multiple-source adaptation. In *Advances in Neural Information Processing Systems*, pp. 8246–8256, 2018.

Sergey Ioffe and Christian Szegedy. Batch normalization: Accelerating deep network training by reducing internal covariate shift. *arXiv preprint arXiv:1502.03167*, 2015.

Rie Johnson and Tong Zhang. Accelerating stochastic gradient descent using predictive variance reduction. In *Advances in neural information processing systems*, pp. 315–323, 2013.

Diederik P Kingma and Jimmy Ba. Adam: A method for stochastic optimization. *arXiv preprint arXiv:1412.6980*, 2014.

Yann LeCun, Léon Bottou, Yoshua Bengio, and Patrick Haffner. Gradient-based learning applied to document recognition. *Proceedings of the IEEE*, 86(11):2278–2324, 1998.

Yitong Li, David E Carlson, et al. Extracting relationships by multi-domain matching. In *Advances in Neural Information Processing Systems*, pp. 6798–6809, 2018.

Hong Liu, Mingsheng Long, Jianmin Wang, and Michael Jordan. Transferable adversarial training: A general approach to adapting deep classifiers. In *International Conference on Machine Learning*, pp. 4013–4022, 2019.

Mingsheng Long, Jianmin Wang, Guiguang Ding, Jiaguang Sun, and Philip S Yu. Transfer joint matching for unsupervised domain adaptation. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pp. 1410–1417, 2014.

Mingsheng Long, Yue Cao, Jianmin Wang, and Michael I Jordan. Learning transferable features with deep adaptation networks. *arXiv preprint arXiv:1502.02791*, 2015.

Mingsheng Long, Han Zhu, Jianmin Wang, and Michael I Jordan. Deep transfer learning with joint adaptation networks. In *Proceedings of the 34th International Conference on Machine Learning-Volume 70*, pp. 2208–2217. JMLR. org, 2017.

Yawei Luo, Ping Liu, Tao Guan, Junqing Yu, and Yi Yang. Significance-aware information bottleneck for domain adaptive semantic segmentation. In *Proceedings of the IEEE International Conference on Computer Vision*, pp. 6778–6787, 2019.

Massimiliano Mancini, Lorenzo Porzi, Samuel Rota Bulò, Barbara Caputo, and Elisa Ricci. Boosting domain adaptation by discovering latent domains. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pp. 3771–3780, 2018.

Yishay Mansour, Mehryar Mohri, and Afshin Rostamizadeh. Domain adaptation with multiple sources. In *Advances in neural information processing systems*, pp. 1041–1048, 2009.

Xudong Mao, Qing Li, Haoran Xie, Raymond YK Lau, Zhen Wang, and Stephen Paul Smolley. Least squares generative adversarial networks. In *Proceedings of the IEEE international conference on computer vision*, pp. 2794–2802, 2017.

Zak Murez, Soheil Kolouri, David Kriegman, Ravi Ramamoorthi, and Kyungnam Kim. Image to image translation for domain adaptation. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pp. 4500–4509, 2018.

Paul K Newton and Stephen A DeSalvo. The shannon entropy of sudoku matrices. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 466(2119):1957–1975, 2010.

Xingchao Peng, Qinxun Bai, Xide Xia, Zijun Huang, Kate Saenko, and Bo Wang. Moment matching for multi-source domain adaptation. In *Proceedings of the IEEE International Conference on Computer Vision*, pp. 1406–1415, 2019.

Yuji Roh, Kangwook Lee, Steven Euijong Whang, and Changho Suh. Fr-train: A mutual information-based approach to fair and robust training. *arXiv preprint arXiv:2002.10234*, 2020.

Kate Saenko, Brian Kulis, Mario Fritz, and Trevor Darrell. Adapting visual category models to new domains. In *European conference on computer vision*, pp. 213–226. Springer, 2010.

Kuniaki Saito, Yoshitaka Ushiku, Tatsuya Harada, and Kate Saenko. Adversarial dropout regularization. *arXiv preprint arXiv:1711.01575*, 2017.

Kuniaki Saito, Kohei Watanabe, Yoshitaka Ushiku, and Tatsuya Harada. Maximum classifier discrepancy for unsupervised domain adaptation. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pp. 3723–3732, 2018.

Swami Sankaranarayanan, Yogesh Balaji, Carlos D Castillo, and Rama Chellappa. Generate to adapt: Aligning domains using generative adversarial networks. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pp. 8503–8512, 2018a.

Swami Sankaranarayanan, Yogesh Balaji, Arpit Jain, Ser Nam Lim, and Rama Chellappa. Learning from synthetic data: Addressing domain shift for semantic segmentation. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pp. 3752–3761, 2018b.

Yuxuan Song, Lantao Yu, Zhangjie Cao, Zhiming Zhou, Jian Shen, Shuo Shao, Weinan Zhang, and Yong Yu. Improving unsupervised domain adaptation with variational information bottleneck. *arXiv preprint arXiv:1911.09310*, 2019.

Baochen Sun and Kate Saenko. Deep coral: Correlation alignment for deep domain adaptation. In *European conference on computer vision*, pp. 443–450. Springer, 2016.

Baochen Sun, Jiashi Feng, and Kate Saenko. Return of frustratingly easy domain adaptation. In *Thirtieth AAAI Conference on Artificial Intelligence*, 2016.

Naftali Tishby and Noga Zaslavsky. Deep learning and the information bottleneck principle. In *2015 IEEE Information Theory Workshop (ITW)*, pp. 1–5. IEEE, 2015.

Naftali Tishby, Fernando C Pereira, and William Bialek. The information bottleneck method. *arXiv preprint physics/0004057*, 2000.

Yi-Hsuan Tsai, Wei-Chih Hung, Samuel Schulter, Kihyuk Sohn, Ming-Hsuan Yang, and Manmohan Chandraker. Learning to adapt structured output space for semantic segmentation. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pp. 7472–7481, 2018.

Eric Tzeng, Judy Hoffman, Kate Saenko, and Trevor Darrell. Adversarial discriminative domain adaptation. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pp. 7167–7176, 2017.

Hemanth Venkateswara, Jose Eusebio, Shayok Chakraborty, and Sethuraman Panchanathan. Deep hashing network for unsupervised domain adaptation. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pp. 5018–5027, 2017.

Jindong Wang, Wenjie Feng, Yiqiang Chen, Han Yu, Meiyu Huang, and Philip S Yu. Visual domain adaptation with manifold embedded distribution alignment. In *Proceedings of the 26th ACM international conference on Multimedia*, pp. 402–410, 2018.

Ruijia Xu, Ziliang Chen, Wangmeng Zuo, Junjie Yan, and Liang Lin. Deep cocktail network: Multi-source unsupervised domain adaptation with category shift. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pp. 3964–3973, 2018.

Han Zhao, Shanghang Zhang, Guanhang Wu, José MF Moura, Joao P Costeira, and Geoffrey J Gordon. Adversarial multiple source domain adaptation. In *Advances in neural information processing systems*, pp. 8559–8570, 2018.

Han Zhao, Remi Tachet des Combes, Kun Zhang, and Geoffrey J Gordon. On learning invariant representation for domain adaptation. *arXiv preprint arXiv:1901.09453*, 2019a.

Sicheng Zhao, Guangzhi Wang, Shanghang Zhang, Yang Gu, Yaxian Li, Zhichao Song, Pengfei Xu, Runbo Hu, Hua Chai, and Kurt Keutzer. Multi-source distilling domain adaptation. *arXiv preprint arXiv:1911.11554*, 2019b.

A PROOFS

In this Section, we present the detailed proofs for Theorems 2 and 3, explained in the main paper. Following (Roh et al. (2020)), we provide a proof of Theorem 2 below for the sake of completeness.

A.1 PROOF OF THEOREM 2

Theorem 2. *Let $P_Z(\mathbf{z})$ be the distribution of Z where $\mathbf{z} \in \mathcal{Z}$. Let h be a domain classifier $h : \mathcal{Z} \to \mathcal{V}$, where $\mathcal{Z}$ is the feature space and $\mathcal{V}$ is the set of domain labels. Let $h_{\mathbf{v}}(Z)$ be a conditional probability of V where $\mathbf{v} \in \mathcal{V}$ given $Z = \mathbf{z}$, defined by h. Then the following holds:*

$$I(Z;V) = \max_{h_{\mathbf{v}}(\mathbf{z}):\sum_{\mathbf{v}\in\mathcal{V}} h_{\mathbf{v}}(\mathbf{z})=1,\forall \mathbf{z}} \sum_{\mathbf{v}\in\mathcal{V}} P_V(\mathbf{v})\mathbb{E}_{\mathbf{z}\sim P_{Z|\mathbf{v}}}\big[\log h_{\mathbf{v}}(\mathbf{z})\big] + H(V) \tag{14}$$

Proof. By definition,

$$\begin{aligned} I(Z;V) &= D_{KL}\big(P(Z,V) \parallel P(Z)P(V)\big) \\ &= \sum_{\mathbf{v}\in\mathcal{V}} P_V(\mathbf{v})\mathbb{E}_{\mathbf{z}\sim P_{Z|\mathbf{v}}}\Big[\log \frac{P_{Z,V}(\mathbf{z},\mathbf{v})}{P_Z(\mathbf{z})}\Big] + H(V) \end{aligned} \tag{15}$$

Let us constrain the term inside the log by $h_{\mathbf{v}}(\mathbf{z}) = \frac{P_{Z,V}(\mathbf{z},\mathbf{v})}{P_Z(\mathbf{z})}$ where $h_{\mathbf{v}}(\mathbf{z})$ represents the conditional probability of $V = \mathbf{v}$ for any $\mathbf{v} \in \mathcal{V}$ given $Z = \mathbf{z}$. Then we have: $\sum_{\mathbf{v}\in\mathcal{V}} h_{\mathbf{v}}(\mathbf{z}) = 1$ for all possible values of $\mathbf{z}$ according to the law of total probability. Let $\mathbf{h}$ denote the collection of $h_{\mathbf{v}}(\mathbf{z})$ for all possible values of $\mathbf{v}$ and $\mathbf{z}$, and $\boldsymbol{\lambda}$ be the collection of $\lambda_{\mathbf{z}}$ for all values of $\mathbf{z}$. Then, we can construct the Lagrangian function by incorporating the constraint $\sum_{\mathbf{v}\in\mathcal{V}} h_{\mathbf{v}}(\mathbf{z}) = 1$ as follows:

$$L(\mathbf{h},\boldsymbol{\lambda}) = \sum_{\mathbf{v}\in\mathcal{V}} P_V(\mathbf{v})\mathbb{E}_{\mathbf{z}\sim P_{Z|\mathbf{v}}}\Big[log\big(h_{\mathbf{v}}(\mathbf{z})\big)\Big] + H(V) + \sum_{\mathbf{z}\in\mathcal{Z}} \lambda_{\mathbf{z}}\Big(1 - \sum_{\mathbf{v}\in\mathcal{V}} h_{\mathbf{v}}(\mathbf{z})\Big) \tag{16}$$

We can use the following KKT conditions:

$$\frac{\partial L(\mathbf{h},\boldsymbol{\lambda})}{\partial h_{\mathbf{v}}(\mathbf{z})} = P_V(\mathbf{v})\frac{P_{Z|\mathbf{v}}(\mathbf{z})}{h^*_{\mathbf{v}}(\mathbf{z})} - \lambda^*_{\mathbf{z}} = 0, \quad \forall(\mathbf{z},\mathbf{v}) \in \mathcal{Z}\times\mathcal{V} \tag{17}$$

$$1 - \sum_{\mathbf{v}\in\mathcal{V}} h^*_{\mathbf{v}}(\mathbf{z}) = 0, \quad \forall \mathbf{z} \in \mathcal{Z} \tag{18}$$

Solving the two equations, we have $1 - \sum_{\mathbf{v}\in\mathcal{V}} \frac{P_V(\mathbf{v})P_{Z|\mathbf{v}}(\mathbf{z})}{\lambda^*_{\mathbf{z}}} = 0$ such that $\lambda^*_{\mathbf{z}} = P_Z(\mathbf{z})$ for all $\mathbf{z}$. Then for all the possible values of $\mathbf{z}$,

$$\begin{aligned} h^*_{\mathbf{v}}(\mathbf{z}) &= \frac{P_{Z,V}(\mathbf{z},\mathbf{v})}{P_Z(\mathbf{z})} \\ &= P_{V|\mathbf{z}}(\mathbf{v}), \end{aligned} \tag{19}$$

where the given $h^*_{\mathbf{v}}(\mathbf{z})$ is same as the term inside log in (15). Thus, the optimal solution of concave Lagrangian function (16) obtained by $h^*_{\mathbf{v}}(\mathbf{z})$ is equal to the mutual information in (15). The substitution of $h^*_{\mathbf{v}}(\mathbf{z})$ into (15) completes the proof. □

Our framework can further be applied to segmentation problems because it provides a new perspective on pixel space (Sankaranarayanan et al. (2018a;b); Murez et al. (2018)) and segmentation space (Tsai et al. (2018)) adaptation. The generator in pixel space and segmentation space adaptation learns to transform images or segmentation results from one domain to another. In the context of information regularization, we can view these approaches as limiting information $I(\hat{X};V)$ between the generated output $\hat{X}$ and the domain label V, which is accomplished by involving the encoder for pixel-level generation. This alleviates the domain shift in a raw pixel level. Note that one can choose between limiting the feature-level or pixel-level mutual information. These different regularization terms may be complementary to each other depending on the given task.

A.2 PROOF OF THEOREM 3

Theorem 3. *Let $P_{Z|\mathbf{x},\mathbf{v}}(\mathbf{z})$ be a conditional probabilistic distribution of Z where $\mathbf{z} \in \mathcal{Z}$, defined by the encoder F, given a sample $\mathbf{x} \in \mathcal{X}$ and the domain label $\mathbf{v} \in \mathcal{V}$. Let $R_Z(\mathbf{z})$ denotes a prior marginal distribution of Z. Then the following inequality holds:*

$$\begin{aligned} I(Z;X,V) \leq & \max_{h_{\mathbf{v}}(\mathbf{z}):\sum_{\mathbf{v}\in\mathcal{V}} h_{\mathbf{v}}(\mathbf{z})=1,\forall \mathbf{z}} \sum_{\mathbf{v}\in\mathcal{V}} P_V(\mathbf{v}) \mathbb{E}_{P_{\mathbf{z}\sim Z|\mathbf{v}}}\big[\log h_{\mathbf{v}}(\mathbf{z})\big] + H(V) \\ & + \mathbb{E}_{\mathbf{x},\mathbf{v}\sim P_{X,V}}\big[D_{KL}[P_{Z|\mathbf{x},\mathbf{v}} \parallel R_Z]\big] \end{aligned} \tag{20}$$

Proof. Based on the chain rule for mutual information,

$$\begin{aligned} I(Z;X,V) &= I(Z;V) + I(Z;X \mid V) \\ &= \max_{h_{\mathbf{v}}(\mathbf{z}):\sum_{\mathbf{v}\in\mathcal{V}} h_{\mathbf{v}}(\mathbf{z})=1,\forall \mathbf{z}} \sum_{\mathbf{v}\in\mathcal{V}} P_V(\mathbf{v}) \mathbb{E}_{\mathbf{z}\sim P_{Z|\mathbf{v}}}\big[\log h_{\mathbf{v}}(\mathbf{z})\big] + H(V) + I(Z;X \mid V), \end{aligned} \tag{21}$$

where the latter equality is given by Theorem 2. Considering $I(Z;X \mid V)$,

$$\begin{aligned} I(Z;X \mid V) &= \mathbb{E}_{\mathbf{v}\sim P_V}\Big[\mathbb{E}_{\mathbf{z},\mathbf{x}\sim P_{Z,X|\mathbf{v}}}\Big[\log \frac{P_{Z,X|\mathbf{v}}(\mathbf{z},\mathbf{x})}{P_{Z|\mathbf{v}}(\mathbf{z})P_{X|\mathbf{v}}(\mathbf{x})}\Big]\Big] \\ &= \mathbb{E}_{\mathbf{x},\mathbf{v}\sim P_{X,V}}\Big[\mathbb{E}_{\mathbf{z}\sim P_{Z|\mathbf{x},\mathbf{v}}}\Big[\log \frac{P_{Z|\mathbf{x},\mathbf{v}}(\mathbf{z})}{P_{Z|\mathbf{v}}(\mathbf{z})}\Big]\Big] \\ &= \mathbb{E}_{\mathbf{x},\mathbf{v}\sim P_{X,V}}\Big[\mathbb{E}_{\mathbf{z}\sim P_{Z|\mathbf{x},\mathbf{v}}}\big[\log P_{Z|\mathbf{x},\mathbf{v}}(\mathbf{z})\big]\Big] - \mathbb{E}_{\mathbf{v}\sim P_V}\Big[\mathbb{E}_{\mathbf{z}\sim P_{Z|\mathbf{v}}}\big[\log P_{Z|\mathbf{v}}(\mathbf{z})\big]\Big] \\ &\leq \mathbb{E}_{\mathbf{x},\mathbf{v}\sim P_{X,V}}\Big[\mathbb{E}_{\mathbf{z}\sim P_{Z|\mathbf{x},\mathbf{v}}}\big[\log P_{Z|\mathbf{x},\mathbf{v}}(\mathbf{z})\big]\Big] - \mathbb{E}_{\mathbf{v}\sim P_V}\Big[\mathbb{E}_{\mathbf{z}\sim P_{Z|\mathbf{v}}}\big[\log R_Z(\mathbf{z})\big]\Big] \\ &= \mathbb{E}_{\mathbf{x},\mathbf{v}\sim P_{X,V}}\Big[\mathbb{E}_{\mathbf{z}\sim P_{Z|\mathbf{x},\mathbf{v}}}\Big[\log \frac{P_{Z|\mathbf{x},\mathbf{v}}(\mathbf{z})}{R_Z(\mathbf{z})}\Big]\Big] \\ &= \mathbb{E}_{\mathbf{x},\mathbf{v}\sim P_{X,V}}\Big[D_{KL}\big[P_{Z|\mathbf{x},\mathbf{v}} \parallel R_Z\big]\Big] \end{aligned} \tag{22}$$

The second equality is obtained by using $P_{Z,X|\mathbf{v}}(\mathbf{z},\mathbf{x}) = P_{X|\mathbf{v}}(\mathbf{x})P_{Z|\mathbf{x},\mathbf{v}}(\mathbf{z})$. The inequality is obtained by using $D_{KL}[P_{Z|\mathbf{v}} \parallel R_Z] = \mathbb{E}_{\mathbf{z}\sim P_{Z|\mathbf{v}}}\big[\log P_{Z|\mathbf{v}}(\mathbf{z}) - \log R_Z(\mathbf{z})\big] \geq 0$, where $R_Z(\mathbf{z})$ is a variational approximation of the prior marginal distribution of Z. The last equality is obtained from the definition of KL-divergence. The substitution of (22) into (21) completes the proof. □

The existing DA work on semantic segmentation tasks (Luo et al. (2019); Song et al. (2019)) can be explained as the process of fostering close collaboration between the aforementioned information bottleneck terms. The only difference between Theorem 3 for $\mathcal{V} = \{0, 1\}$ and the objective function in (Luo et al. (2019)) is that (Luo et al. (2019)) employed the shared encoding $P_{Z|\mathbf{x}}(\mathbf{z})$ instead of $P_{Z|\mathbf{x},\mathbf{v}}(\mathbf{z})$, whereas some adversarial DA approaches use the unshared one (Tzeng et al. (2017)).

B EXPERIMENTAL SETUP

In this Section, we describe the datasets, network architecture and hyperparameter configuration.

B.1 DATASETS

We validate the Multi-source Information-regularized Adaptation Networks (**MIAN**) with the following benchmark datasets: Digits-Five, Office-31 and Office-Home. Every experiment is repeated four times and the average accuracy in target domain is reported.

Digits-Five (Peng et al. (2019)) dataset is a unified dataset including five different digit datasets: MNIST (LeCun et al. (1998)), MNIST-M (Ganin & Lempitsky (2014)), Synthetic Digits (Ganin & Lempitsky (2014)), SVHN, and USPS. Following the standard protocols of unsupervised MDA (Xu

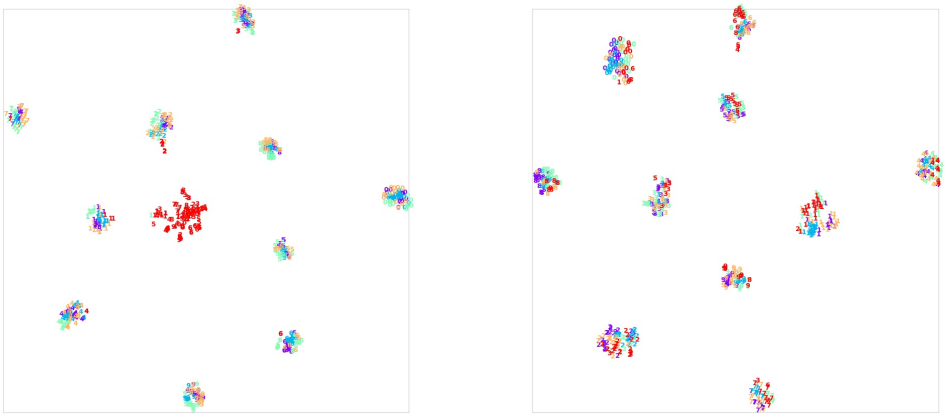
et al. (2018); Peng et al. (2019)), we used 25000 training images and 9000 test images sampled from a training and a testing subset for each of MNIST, MNIST-M, SVHN, and Synthetic Digits. For USPS, all the data is used owing to the small sample size. All the images are bilinearly interpolated to 32×32.

Office-31 (Saenko et al. (2010)) is a popular benchmark dataset including 31 categories of objects in an office environment. Note that it is a more difficult problem than Digits-Five, which includes 4652 images in total from the three domains: Amazon, DSLR, and Webcam. All the images are interpolated to 224×224 using bicubic filters.

Office-Home (Venkateswara et al. (2017)) is a challenging dataset that includes 65 categories of objects in office and home environments. It includes 15,500 images in total from the four domains: Artistic images (Art), Clip Art(Clipart), Product images (Product), and Real-World images (Realworld). All the images are interpolated to 224×224 using bicubic filters.

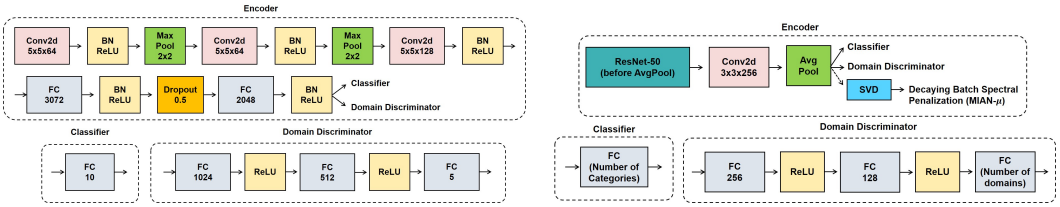
B.2 ARCHITECTURES

For the Digits-Five dataset, we use the same network architecture and optimizer setting as in (Peng et al. (2019)). For all the other experiments, the results are based on ResNet-50, which is pre-trained on ImageNet. The domain discriminator is implemented as a three-layer neural network. Detailed architecture is shown in Figure 4.



(a) (b)

Figure 3: tSNE visualization (a) before and (b) after adaptation. Representations from target domain (SVHN) are shown in red. Digit class labels are shown with corresponding numbers.



(a) Encoder, domain discriminator, and classifier used in Digits-Five experiments

(b) Encoder, domain discriminator, and classifier used in Office-31 and Office-Home experiments

Figure 4: Network architectures. BN denotes Batch Normalization (Ioffe & Szegedy (2015)) and SVD denotes differentiable SVD in PyTorch for **MIAN-γ** (Section C)

We compare our method with the following state-of-the-art domain adaptation methods: Deep Adaptation Network (**DAN**, Long et al. (2015)), Joint Adaptation Network (**JAN**, Long et al. (2017)), Manifold Embedded Distribution Alignment (**MEDA**, Wang et al. (2018)), Correlation Alignment (**CORAL**, Sun et al. (2016)), Domain Adversarial Neural Network (**DANN**, Ganin et al. (2016)), Batch Spectral Penalization (**BSP**, Chen et al. (2019)), Adversarial Discriminative Domain Adaptation (**ADDA**, Tzeng et al. (2017)), Maximum Classifier Discrepancy (**MCD**, Saito et al. (2018)), Deep Cocktail Network (**DCTN**, Xu et al. (2018)), and Moment Matching for Multi-Source Domain Adaptation (**M^{3}SDA**, Peng et al. (2019)).

Hyperparameters Details of the experimental setup are summarized in Table 4. Other state-of-the-art adaptation models are trained based on the same setup except for these cases: **DCTN** show poor performance with the learning rate shown in Table 4 for both Office-31 and Office-Home datasets. Following the suggestion of the original authors, $1e^{-5}$ is used as a learning rate with the Adam optimizer (Kingma & Ba (2014)); **MCD** show poor performance for the Office-Home dataset with the learning rate shown in Table 4. $1e^{-4}$ is selected as a learning rate. For both the proposed and other baseline models, the learning rate of the classifier or domain discriminator trained from the scratch is set to be 10 times of those of ImageNet-pretrained weights, in Office-31 and Office-Home datasets. More hyperparameter configurations are summarized in Table 5 (Section C)

Table 4: Experimental setup. The batch size for each domain is reported.

Dataset	Optimization method	Learning rate	Momentum	Batch size	Iteration
Digits-Five	Adam	$2e^{-4}$	(0.9, 0.99)	128	50000
Office-31	mini-batch SGD	$1e^{-3}$	0.9	16	25000
Office-Home	mini-batch SGD	$1e^{-3}$	0.9	16	25000

C DECAYING BATCH SPECTRAL PENALIZATION

In this Section, we provides details on the Decaying Batch Spectral Penalization (DBSP) which expands **MIAN** into **MIAN-γ**.

C.1 BACKGROUNDS

There is little motivation for models to control the complex mutual dependence to domains if reducing the entropy of representations is sufficient to optimize the value of $I(Z;V) = H(Z) - H(Z \mid V)$. If so, such implicit entropy minimization substantially reduce the upper bound of $I(Z;Y)$, leading to a increase in optimal joint risk λ^*. In other words, the decrease in the entropy of representations may occur as the side effect of $I(Z;V)$ regularization. Such unexpected side effect of information regularization is highly intertwined with the hidden deterioration of discriminability through adversarial training (Chen et al. (2019); Liu et al. (2019)).

Based on these insights, we employ the SVD-entropy $H_{SVD}(\mathbf{Z})$ (Alter et al. (2000)) of a representation matrix $\mathbf{Z}$ to assess the richness of the latent representations during adaptation, since it is difficult to compute $H(Z)$. Note that while $H_{SVD}(\mathbf{Z})$ is not precisely equivalent to $H(Z)$, $H_{SVD}(\mathbf{Z})$ can be used as a proxy of the level of disorder of the given matrix (Newton & DeSalvo (2010)). In future works, it would be interesting to evaluate the temporal change in entropy with other metrics. We found that $H_{SVD}(\mathbf{Z})$ indeed decreases significantly during adversarial adaptation, suggesting that some eigenfeatures (or eigensamples) become redundant and, thus, the inherent feature-richness diminishes. (Figure 5a) To preclude such deterioration, we employ Batch Spectral Penalization (BSP) (Chen et al. (2019)), which imposes a constraint on the largest singular value to solicit the contribution of other eigenfeatures. The overall objective function in the multi-domain setting is defined as:

$$\min_{F,C} \; L(F,C) + \beta \hat{I}(Z;V) + \gamma \sum_{i=1}^{N+1} \sum_{j=1}^{k} s_{i,j}^2, \tag{23}$$

where β and γ are Lagrangian multipliers and $s_{i,j}$ is the jth singular value from the ith domain. We found that SVD entropy of representations is severely deteriorated especially in the early stages of

training, suggesting the possibility of over-regularization. The noisy domain discriminative signals in the initial phase (Ganin et al. (2016)) may distort and simplify the representations. To circumvent the impaired discriminability in the early stages of the training, the discriminability should be prioritized first with high γ and low β, followed by a gradual decaying and annealing in γ and β, respectively, so that a sufficient level of domain transferability is guaranteed. Based on our temporal analysis, we introduce the training-dependent scaling of β and γ by modifying the progressive training schedule (Ganin et al. (2016)):

$$\begin{aligned} \beta_p &= \beta_0 \cdot 2\big(1 - \frac{1}{1 + exp(-\sigma \cdot p)}\big) \\ \gamma_p &= \gamma_0 \cdot \big(\frac{2}{1 + exp(-\sigma \cdot p)} - 1\big), \end{aligned} \tag{24}$$

where β_0 and γ_0 are initial values, σ is a decaying parameter, and p is the training progress from 0 to 1. We refer to this version of our model as **MIAN-γ**. Note that **MIAN** only includes annealing-β, excluding DBSP.

Table 5: Hyper parameters configuration. Annealing-β is not adopted in the Digits-Five experiment. Decaying batch spectral penalization is not adopted in the **MIAN**.

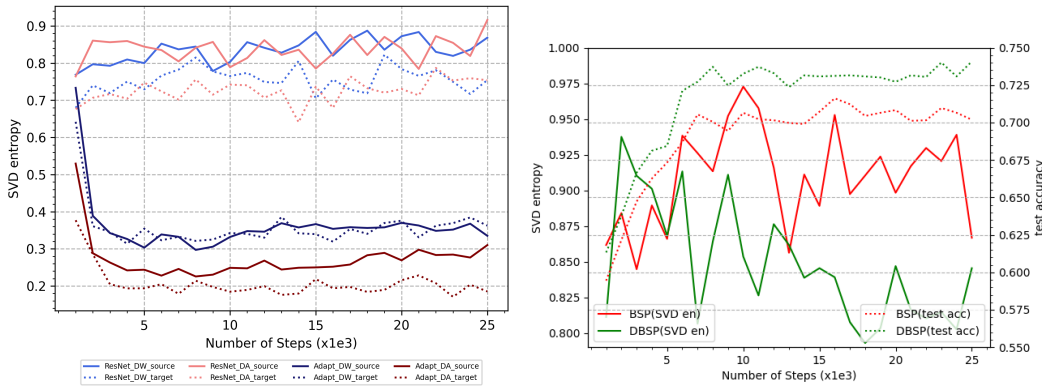
Dataset(Model)	β_0	γ_0	σ	k
Digits-Five (**MIAN**)	1.0	N/A	N/A	N/A
Office-31 (**MIAN**)	0.1	N/A	10.0	N/A
Office-31 (**MIAN-γ**)	0.1	0.0001	10.0	1
Office-Home (**MIAN**)	0.2	N/A	10.0	N/A
Office-Home (**MIAN-γ**)	0.2	0.0001	10.0	1

C.2 EXPERIMENTS

SVD-entropy. We evaluated the degree of compromise of SVD-entropy owing to transfer learning. For this, DSLR was fixed as the source domain, and each Webcam and Amazon target domain was used to simulate low (DSLR→Webcam; *DW*) and high domain (DSLR→Amazon; *DA*) shift conditions, respectively. SVD-entropy was applied to the representation matrix extracted from ResNet-50 and **MIAN** (denoted as *Adapt* in Figure 5a) with constant $\beta = 0.1$. For accurate assessment, we avoided using spectral penalization. As depicted in the Figure 5a, adversarial adaptation, or information regularization, significantly decreases the SVD-entropy of both the source and target domain representations, especially in the early stages of training, indicating that the representations are simplified in terms of feature-richness. Moreover, when comparing the *Adapt_DA_source* and *Adapt_DW_source* conditions, we found that SVD-entropy decreases significantly as the degree of domain shift increases.

We additionally conducted analyses on temporal changes of SVD entropy by comparing BSP and decaying BSP (Figure 5b). SVD entropy gradually decreases as the degree of compensation decreases in DBSP which leads to improved transferability and accuracy. Thus DBSP can control the trade-off between the richness of the feature representations and adversarial adaptation as the training proceeds.

Ablation study. We performed an ablation study to assess the contribution of the decaying spectral penalization and annealing information regularization to DA performance (Table 6, 7). We found that the prioritization of feature-richness in early stages (by controlling β and γ) significantly improves the performance. We also found that the constant penalization schedule (Chen et al. (2019)) is not reliable and sometimes impedes transferability in the low domain shift condition (Webcam, DSLR in Table 6). This implies that the conventional BSP may over-regularize the transferability when the degree of domain shift and SVD-entropy decline are relatively small.



(a) (b)

Figure 5: **(a)**: SVD-entropy analysis. (Office-31; Source domain: DSLR) **(b)**: Comparisons between BSP and DBSP. (Office-31; DSLR $\rightarrow$ Amazon)

Table 6: Ablation study of decaying batch spectral penalization and annealing information regularization (Office-31). For accurate assessment of extent to which performance improvement is caused by each strategies, γ is fixed as 0 in *Annealing*-β, and β is fixed as 0.1 in *Decaying*-γ. Results from *Annealing*-β are reported in main paper.

Standards	Hyper parameters	Amazon	DSLR	Webcam	Avg
Baseline	$\beta = 0.1$ as a constant	69.98	99.48	98.13	89.20
Annealing-β	$\beta_0 = 0.1, \sigma = 10$	**74.65**	**99.48**	**98.49**	**90.87**
Decaying-γ	$\gamma = 1e^{-4}$ as a constant	74.73	98.65	96.24	89.87
	$\gamma_0 = 1e^{-4}, \sigma = 10$	**75.01**	**99.68**	98.10	**90.93**
Full version	$\beta_0 = 0.1, \gamma_0 = 1e^{-4}, \sigma = 10$	**76.15**	99.22	**98.39**	**91.26**

Table 7: Accuracy (%) on Office-Home dataset. Results from **MIAN** are reported in main paper.

Standards	Art	Clipart	Product	Realworld	Avg
MIAN	69.39±0.50	63.05±0.61	79.62±0.16	80.44±0.24	73.12
MIAN-γ	**69.88±0.35**	**64.20±0.68**	**80.87±0.37**	**81.49±0.24**	**74.11**