Out-of-Distribution Detection with An Adaptive Likelihood Ratio on Informative Hierarchical VAE

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Abstract

Unsupervised out-of-distribution (OOD) detection is essential for the reliability of machine learning. In the literature, existing work has shown that higher-level semantics captured by hierarchical VAEs can be used to detect OOD instances. However, we empirically show that, the inherent issue of hierarchical VAEs, *i.e.*, "*posterior collapse*", would seriously limit their capacity for OOD detection. Based on a thorough analysis for "*posterior collapse*", we propose a novel informative hierarchical VAE to alleviate this phenomenon through enhancing the connections between the data sample and its multi-layer stochastic latent representations during training. Furthermore, we propose a novel score function for unsupervised OOD detection, referred to as Adaptive Likelihood Ratio, which can selectively aggregate the semantic information on multiple hidden layers of hierarchical VAEs, leading to a strong separability between in-distribution and OOD samples. Experimental results demonstrate that our method can significantly outperform existing state-of-the-art unsupervised OOD detection approaches.

1 Introduction

Despite achieving great success in real-world applications recently, existing machine learning (ML) systems are still designed to be tested on the in-distribution dataset, whose statistics are similar to those of the training set [1]. However, when applied to deal with the dataset consisting of out-of-distribution (OOD) samples, whose statistics are quite different from those of the training set, these ML systems would produce some serious incorrect results [2, 3]. Considering the fact that the OOD data is very common in the real world, OOD detection before execution is increasingly important for these systems to make sure their reliability and safety. Although several supervised OOD detection methods [4, 5, 6, 7, 8] have achieved great success in this field, the unsupervised methods are more practical and challenging if the labels of in-distribution samples are not given.

Without labels, likelihood-based generative models could be a promising way for OOD detection, such as Flow-based models [9], autogressive models [10, 11], and variational autoencoders (VAEs) [12, 13]. However, some recent studies have shown that, in some cases, these generative models tend to achieve higher likelihoods on certain types of OOD samples [1, 14, 15, 16, 17], which makes the OOD detection rules based on likelihood threshold problematic. To address this issue, based on the prior knowledge collected from OOD samples, some unsupervised OOD methods are developed, *i.e.*, Ren et al. [1] take additional datasets as the OOD validation sets for choosing the best hyperparameters; Hendrycks et al. [18] introduce an auxiliary dataset to teach the network to learn better representations for OOD detection; Serrà et al. [19] incorporate the input complexity to detect certain types of OOD data. However, the behaviour of borrowing the prior knowledge of OOD data is unreasonable in practice, because we will never know what kind of OOD data to be dealt with.

To conduct purely unsupervised OOD detection without the labels or prior assumptions, deep ensemble method named WAIC [14] is developed by making full use of the difference between the

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density estimations of multiple independent models trained on the in-distribution data. Recently, through capturing the semantic information with multi-layer latent variables, Maaløe et al. [20] and Havtorn et al. [17] develop various score functions based on Likelihood-Ratio, which help hierarchical VAEs achieve competitive performance in unsupervised OOD detection. However, in our implementation, we find that the phenomenon of "*posterior collapse*" in hierarchical VAEs still limits their performance on OOD detection, where the main reason could be that "*posterior collapse*" will make the high-level latent variables meaningless and cannot provide faithful summaries for the data.

With this insight in hand, in this paper, we start from rethinking the cause of "*posterior collapse*" in hierarchical VAEs, and then theoretically explain why "*posterior collapse*" will limit the OOD detection performance of these Likelihood-Ratio based methods. Further, we develop an informative hierarchical VAE to alleviate "*posterior collapse*" and a novel Adaptive Likelihood Ratio score function for unsupervised OOD detection. The major contributions of this work include:

- With a thorough analysis of "*posterior collapse*" in hierarchical VAEs, we enhance the connections between the data sample and its multiple latent representations in the expected log-likelihood term of evidence lower bound (ELBO) for training, and develop a novel informative hierarchical VAE to extract more expressive hierarchical latent representations.
- We theoretically explain why alleviating "*posterior collapse*" in hierarchical VAEs can help the performance of Likelihood Ratio on OOD detection, and then develop a novel score function for fully unsupervised OOD detection, termed Adaptive Likelihood Ratio, which owns fewer hyperparameters to be tuned and can make full use of the semantic divergences between in-distribution and OOD samples across all hidden layers of hierarchical VAEs.
- Combing the informative hierarchical VAE with the Adaptive Likelihood Ratio, we demonstrate that our method can achieve state-of-the-art OOD detection performance across a wide range of benchmarks in an unsupervised manner.

2 Background and Related Works

2.1 Hierarchical Variational Autoencoder

Preliminary: Extending the basic VAE [12], a hierarchical VAE [21, 22] is defined by the observation $\boldsymbol{x}$ that depends on a hierarchy of stochastic latent variables $\boldsymbol{z} = \boldsymbol{z}_1, ..., \boldsymbol{z}_L$, where the generative model is defined with a top-down structure, formulated as $p_\theta(\boldsymbol{x}, \boldsymbol{z}) = p_\theta(\boldsymbol{x}|\boldsymbol{z}_1) \prod_{l=1}^{L-1} p_\theta(\boldsymbol{z}_l|\boldsymbol{z}_{l+1}) p_\theta(\boldsymbol{z}_L)$; the inference model is designed to approximate the posterior over the latent variables, commonly factorized with a top-down structure as $q_\phi(\boldsymbol{z}|\boldsymbol{x}) = \prod_{l=1}^{L-1} q_\phi(\boldsymbol{z}_l|\boldsymbol{z}_{l+1}) q_\phi(\boldsymbol{z}_L|\boldsymbol{x})$ or a bottom-up structure as $q_\phi(\boldsymbol{z}|\boldsymbol{x}) = \prod_{l=1}^{L-1} q_\phi(\boldsymbol{z}_{l+1}|\boldsymbol{z}_l) q_\phi(\boldsymbol{z}_1|\boldsymbol{x})$. The demonstration of these structures could be seen in Fig. 1. The parameters of the generative and infernce models, denoted as θ and ϕ respectively, can be jointly optimized by maximizing the evidence lower bound (ELBO) expressed as

$$\mathcal{L} = \mathbb{E}_{p(\boldsymbol{x})} \left[\mathcal{L}_x(\, \boldsymbol{x}; \theta, \phi) \right], \tag{1}$$

where $\mathcal{L}_x$ is denoted as

$$\mathcal{L}_x = \log p_\theta(\boldsymbol{x}) - D_{\mathrm{KL}}(q_\phi(\boldsymbol{z}|\boldsymbol{x}) || p_\theta(\boldsymbol{z}|\boldsymbol{x})) = \mathbb{E}_{q_\phi(\boldsymbol{z}|\boldsymbol{x})} \left[\log p_\theta(\boldsymbol{x}|\boldsymbol{z}) \right] - D_{\mathrm{KL}}(q_\phi(\boldsymbol{z}|\boldsymbol{x}) || p_\theta(\boldsymbol{z})), \tag{2}$$

where $D_{\mathrm{KL}}(\cdot||\cdot)$ denotes the KL divergence and maximizing ELBO is equivalent to minimize the divergence between the varaitional distribution $q_\phi(\boldsymbol{z}|\boldsymbol{x})$ and true posterior $p_\theta(\boldsymbol{z}|\boldsymbol{x})$.

Related Works: While the hierarchy of latent stochastic variables can improve the generation capability of standard VAEs, in practice, the posterior of higher-level stochastic latent variables have a tendency to *collapse* into the prior, called "*posterior collapse*". To address this issue, Sønderby et al. [21] propose a Ladder VAE (LVAE) to change the bottom up inference process into a top-down one; Vahdat and Kautz [22] develop a variant of the LVAE, which carefully designs a sophisticated network architecture to achieve better generation quality; Maaløe et al. [20] combine the bottom-up inference with the top-down inference by proposing a bidirectional inference scheme. Despite obtaining performance improvements with more flexible inference networks, these hierarchical VAEs still lack a theoretical guide to alleviate the phenomenon of "*posterior collapse*".

2.2 Out-of-distribution Detection with Variational Autoencdoer

Problem Formulation: Suppose that there are a set of N training samples $\{\boldsymbol{x}_i\}_{i=1}^N$ drawn from the data distribution $\boldsymbol{x}_i \sim p(\boldsymbol{x})$, after training a VAE to model the generation of these data samples,

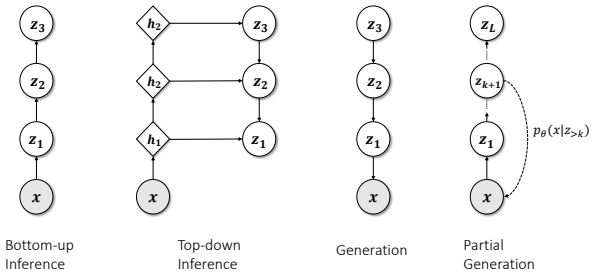


Figure 1: Illustration of the usual structures of both inference network and generative model in hierarchical VAEs.

the generative model $p_\theta(\boldsymbol{x})$ is supposed to detect whether a testing sample $\boldsymbol{x}$ is an outlier, where the outlier should have a low density estimation under the true data distribution $p(\boldsymbol{x})$. However, in contrary, former likelihood-based methods found the generative model always assign higher $p_\theta(\boldsymbol{x})$ for OOD data than in-distribution data [1, 14, 15, 16]. Luckily, although the likelihood based methods with VAE are rarely investigated, they have revealed the potential to better address this problem without the help of labels or assumptions about the prior.

Related Works: A pioneering VAE-based OOD detection method is Likelihood Regret (LRe) [23], which exploits the difference of the pretrained model and refitted model for each test data sample, and achieves competitive performance in an unsupervised manner, at the cost of bringing heavy burdens. Maaløe et al. [20] reveal the potential of hierarchical VAE for detecting OOD. Havtorn et al. [17] further propose a score function based on it to achieve better performance, which needs to specify the backbone model and the layer index k. The backbone models in the likelihood based methods [1, 14, 19] can be directly replaced with VAEs, but these methods usually underperform the Flow models like Glow [9] or autogressive models like PixcelCNN [11].

3 From Informative Hierarchical VAE to Adaptive Likelihood Ratio

3.1 Rethinking of "*posterior collapse*" in Hierarchical VAEs

Firstly, let's understand the cause of "*posterior collapse*" in hierarchical VAEs theoretically. Taking an L-layer hierarchical VAE with a top-down inference network as an example, the set of latent variables can be separated as the lower-level variables $\boldsymbol{z}_{\leq k} = \{\boldsymbol{z}_1, ..., \boldsymbol{z}_k\}$ and the higher-level ones $\boldsymbol{z}_{>k} = \{\boldsymbol{z}_{k+1}, ..., \boldsymbol{z}_L\}$, where $k \in \{0, ..., L-1\}$, then the ELBO in Eq. (1) can be reformulated as

$$\mathcal{L} = \mathbb{E}_{p(\boldsymbol{x})}\left[\mathbb{E}_{q_\phi(\boldsymbol{z}_{\leq k}|\boldsymbol{z}_{>k})q_\phi(\boldsymbol{z}_{>k}|\boldsymbol{x})}\left[\log p_\theta(\boldsymbol{x}|\boldsymbol{z}_1)\right] - \sum\nolimits_{l=1}^{L} D_{\mathrm{KL}}(q_\phi(\boldsymbol{z}_l|\boldsymbol{z}_{l+1})||p_\theta(\boldsymbol{z}_l|\boldsymbol{z}_{l+1}))\right], \tag{3}$$

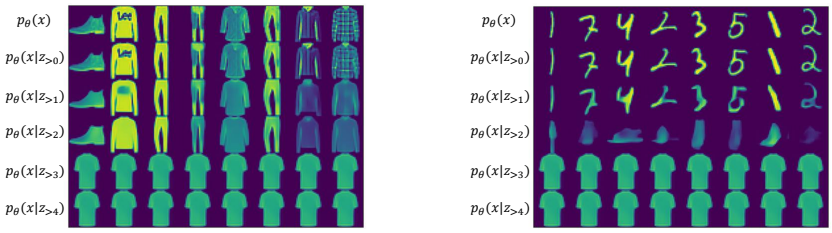
where the main contribution to the expected log-likelihood term is coming from the lower-level latent variables $\boldsymbol{z}_{\leq k}$ before the kth layer [20]. When the generation capacity of the generative model $p_\theta(\boldsymbol{x}|\boldsymbol{z}_{\leq k})$ is powerful enough to reconstruct the observation $\boldsymbol{x}$ well, the variational posteriors of higher-level latent variables $\boldsymbol{z}_{>k}$ will be merely optimized to be close to their priors, specifically $q_\phi(\boldsymbol{z}_{>k}|\boldsymbol{x}) \approx p_\theta(\boldsymbol{z}_{>k})$, leading to that the representations learned by VAE at higher layers tend to be meaningless and cannot provide faithful summaries for $\boldsymbol{x}$, which is well known as the phenomenon of "*posterior collapse*" or "*latent variable collapse*" [21, 24].

To find the potential solutions to alleviating "*posterior collapse*", in the following, we reinterpret this phenomenon from the perspective of information theory [25] by extending the findings in [24] to a hierarchical VAE scenario. For ease of undertanding, we define the mutual information between the data $\boldsymbol{x}$ and the higher-level latent variables $\boldsymbol{z}_{>k}$ as

$$\mathcal{I}_q(\boldsymbol{x}, \boldsymbol{z}_{>k}) = -\mathcal{H}_q(\boldsymbol{z}_{>k}|\boldsymbol{x}) + \mathcal{H}_q(\boldsymbol{z}_{>k}) = \mathbb{E}_{p(\boldsymbol{x})q_\phi(\boldsymbol{z}_{>k}|\boldsymbol{x})} \log q_\phi(\boldsymbol{z}_{>k}|\boldsymbol{x}) - \mathbb{E}_{q_\phi(\boldsymbol{z}_{>k})} \log q_\phi(\boldsymbol{z}_{>k}),$$

which is induced by the variational posterior $q_\phi(\boldsymbol{z}_{>k}|\boldsymbol{x})$. Then KL term in Eq. (3) can be rewritten as

$$\begin{aligned}
&\mathbb{E}_{p(\boldsymbol{x})}\left[\sum\nolimits_{l=1}^{L} D_{\mathrm{KL}}(q_\phi(\boldsymbol{z}_l|\boldsymbol{z}_{l+1})||p_\theta(\boldsymbol{z}_l|\boldsymbol{z}_{l+1}))\right] \\
=&\mathbb{E}_{p(\boldsymbol{x})}\left[\sum\nolimits_{l=1}^{k} D_{\mathrm{KL}}(q_\phi(\boldsymbol{z}_l|\boldsymbol{z}_{l+1})||p_\theta(\boldsymbol{z}_l|\boldsymbol{z}_{l+1}))\right] + \mathbb{E}_{p(\boldsymbol{x})}\left[D_{\mathrm{KL}}(q_\phi(\boldsymbol{z}_{>k}|\boldsymbol{x})||p_\theta(\boldsymbol{z}_{>k}))\right] \\
=&\mathbb{E}_{p(\boldsymbol{x})}\left[\sum\nolimits_{l=1}^{k} D_{\mathrm{KL}}(q_\phi(\boldsymbol{z}_l|\boldsymbol{z}_{l+1})||p_\theta(\boldsymbol{z}_l|\boldsymbol{z}_{l+1}))\right] + \mathcal{I}_q(\boldsymbol{x}, \boldsymbol{z}_{>k}) + D_{\mathrm{KL}}(q_\phi(\boldsymbol{z}_{>k})||p_\theta(\boldsymbol{z}_{>k})),
\end{aligned} \tag{4}$$



(a) FashionMNIST (in) (b) MNIST (out)

Figure 2: Illustration of "*posterior collapse*" in a 5-layer hierarchical VAE trained on FashionMNIST (in) by visualizing its reconstructions conditioned on various $\boldsymbol{z}_{>k}$ for both in-distribution and MNIST (out) samples. The first row is the input $\boldsymbol{x}$, and the other rows are generated from the partial generative model $p_\theta(\boldsymbol{x}|\boldsymbol{z}_{>k})$ by taking $\boldsymbol{z}_{>k}$ drawn from $q_\phi(\boldsymbol{z}_{>k}|\boldsymbol{x})$ as input, where $k \in \{0, ..., 4\}$.

where $q_\phi(\boldsymbol{z}_{>k}) = \mathbb{E}_{p(\boldsymbol{x})}\left[q_\phi(\boldsymbol{z}_{>k}|\boldsymbol{x})\right]$ and the detailed derivation can be found in Appendix A. By substituting Eq. (4) into Eq. (3), due to the non-negativity of mutual information and KL divergence, we can find that maximizing the ELBO is opposite to maximizing the mutual information $\mathcal{I}_q(\boldsymbol{x}, \boldsymbol{z}_{>k})$. When $\mathcal{I}_q(\boldsymbol{x}, \boldsymbol{z}_{>k})$ is minimized to zero, the variational posterior $q_\phi(\boldsymbol{z}_{>k}|\boldsymbol{x})$ will be independent of the data $\boldsymbol{x}$, which leads to the phenomenon of "*posterior collapse*".

3.2 Why "*posterior collapse*" limits Likelihood-Ratio for OOD Detection

OOD detection has become one of the most important applications of VAEs, which can be applied to filter OOD samples by setting a threshold on the score of log-likelihood term. However, some recent studies have shown that, in some cases, VAEs tend to achieve higher likelihoods on certain types of OOD samples [26], which makes the OOD detection rules based on likelihood threshold problematic. Recently, Havtorn et al. [17] reinterpreted this problematic behavior of generative model by out-of-distribution data having similar low-level features with in-distribution, and developed a likelihood-ratio score $\mathcal{LLR}^{>k}$ based on hierarahical VAEs, formulated as

$$\mathcal{LLR}^{>k} = D_{\mathrm{KL}}(p_\theta(\boldsymbol{z}_{\leq k}|\boldsymbol{z}_{>k})q_\phi(\boldsymbol{z}_{>k}|\boldsymbol{x})||p_\theta(\boldsymbol{z}|\boldsymbol{x})) - D_{\mathrm{KL}}(q_\phi(\boldsymbol{z}_{\leq k}|\boldsymbol{z}_{>k})q_\phi(\boldsymbol{z}_{>k}|\boldsymbol{x})||p_\theta(\boldsymbol{z}|\boldsymbol{x})), \tag{5}$$

which achieves state-of-the-art unsupervised OOD detection performance.

To intuitively understand the nature of success in the likelihood-ratio score and illustrate why alleviating *"posterior collapse"* in hierarchical VAEs can improve its performance on OOD detection, we provide an insightful analysis on $\mathcal{LLR}^{>k}$ in Eq. (5) by reformulating it as follows:

$$\begin{aligned}\mathcal{LLR}^{>k} &= \mathbb{E}_{q_\phi(\boldsymbol{z}_{>k}|\boldsymbol{x})}\left[D_{\mathrm{KL}}(p_\theta(\boldsymbol{z}_{\leq k}|\boldsymbol{z}_{>k})||p_\theta(\boldsymbol{z}_{\leq k}|\boldsymbol{z}_{>k}, \boldsymbol{x})) - D_{\mathrm{KL}}(q_\phi(\boldsymbol{z}_{\leq k}|\boldsymbol{z}_{>k})||p_\theta(\boldsymbol{z}_{\leq k}|\boldsymbol{z}_{>k}, \boldsymbol{x})\right] \\ &\approx \mathbb{E}_{q_\phi(\boldsymbol{z}_{>k}|\boldsymbol{x})}\left[D_{\mathrm{KL}}(p_\theta(\boldsymbol{z}_{\leq k}|\boldsymbol{z}_{>k})||q_\phi(\boldsymbol{z}_{\leq k}|\boldsymbol{z}_{>k}))\right],\end{aligned} \tag{6}$$

where the detailed derivations can be found in Appendix B. Eq. (6) shows that, when the inference network can approximate the true posterior $p_\theta(\boldsymbol{z}_{\leq k}|\boldsymbol{z}_{>k}, \boldsymbol{x})$ very well, $\mathcal{LLR}^{>k}$ will approach the expected KL divergence between the prior $p_\theta(\boldsymbol{z}_{\leq k}|\boldsymbol{z}_{>k})$ and variational posterior $q_\phi(\boldsymbol{z}_{\leq k}|\boldsymbol{z}_{>k})$. More specifically, conditioned on the expectation of $\boldsymbol{z}_{>k}$ drawn from its variational posterior $q_\phi(\boldsymbol{z}_{>k}|\boldsymbol{x})$, $\mathcal{LLR}^{>k}$ is developed to calculate the summation of k KL divergence terms, measuring the distance between the lower-level variables $\boldsymbol{z}_{\leq k}$ drawn from the generative model $p_\theta(\boldsymbol{z}_{\leq k}|\boldsymbol{z}_{>k})$ and those from the variational inference network $q_\phi(\boldsymbol{z}_{\leq k}|\boldsymbol{z}_{>k}, \boldsymbol{x})$.

The premise of applying $\mathcal{LLR}^{>k}$ to OOD detection is that, after training a hierarahical VAE on in-distribution samples, for each OOD sample, the latent variables $\boldsymbol{z}_{\leq k}$ generated from the generative model $p_\theta(\boldsymbol{z}_{\leq k}|\boldsymbol{z}_{>k})$ will be clearly distinct from those drawn from the variational inference network $q_\phi(\boldsymbol{z}_{\leq k}|\boldsymbol{z}_{>k})$. For ease of understanding the principle, after training a 5-layer hierarchical VAE on FashionMNIST, for each hidden layer, we exhibit the reconstructions of both in-distribution (FashionMNIST) and OOD (MNIST) data samples with the partial generative model $p_\theta(\boldsymbol{x}|\boldsymbol{z}_{>k})$ conditioned on the latent variables $\boldsymbol{z}_{>k}$ drawn from $q_\phi(\boldsymbol{z}_{>k}|\boldsymbol{x})$ as shown in Fig. 2. From the results, we can find that, when setting $k = 2$, the reconstructions of MNIST samples tend to reflect the high-level semantic structures learned from FashionMNIST, indicating that the generation mechanism of $p_\theta(\boldsymbol{x}|\boldsymbol{z}_{>2})$ seems to prevent accurate reconstruction of out-of-distribution data, which implies that a score function based on the distance between $p_\theta(\boldsymbol{z}_{\leq 2}|\boldsymbol{z}_{>2})$ and $q_\phi(\boldsymbol{z}_{\leq 2}|\boldsymbol{z}_{>2})$, like $\mathcal{LLR}^{>2}$ in Eq. (6), could be a promising metric for OOD detection.

However, when the phenomenon of "*posterior collapse*" occurs, the variational posterior $q_\phi(\boldsymbol{z}_{>k}|\boldsymbol{x})$ will be independent of the data $\boldsymbol{x}$, resulting in $q_\phi(\boldsymbol{z}_{>k}|\boldsymbol{x}) \approx p_\theta(\boldsymbol{z}_{>k})$, and the reconstructions of in-distribution and OOD samples, which are generated from the partial generative model $p_\theta(\boldsymbol{x}|\boldsymbol{z}_{>k})$, will be almost the same, such as the visualization examples shown in Fig. 2 by setting $k = 3$ or $k = 4$. In that case, for each in-distribution sample, the latent variables $\boldsymbol{z}_{\leq k}$ generated $p_\theta(\boldsymbol{z}_{\leq k}|\boldsymbol{z}_{>k})$ will be clearly distinct from those drawn from $q_\theta(\boldsymbol{z}_{\leq k}|\boldsymbol{z}_{>k}, \boldsymbol{x})$, and similar conclusions can also be achieved by OOD samples, which will reduce the variance of $\mathcal{LLR}^{>k}$ scores between in-distribution and OOD samples and further bring troubles for OOD detection with $\mathcal{LLR}^{>k}$.

3.3 Informative Hierarchical VAE to Alleviate "*posterior collapse*"

Recall to the conflict between the ELBO objective and $\mathcal{I}_q(\boldsymbol{x}, \boldsymbol{z}_{>k})$ as discussed in Sec. 3.1, which causes "*posterior collapse*" in hierarchical VAEs, there could be two main approaches to alleviate this phenomenon, including: 1) downweight the KL term, like applying a warm-up scheme on it [27], which is the most common heuristic in practice but still cannot essentially address this issue; 2) enhance the connections between the observation and its multi-layer stochastic latent representations in the expected log-likelihood term, like modifying the generative process described by $p_\theta(\boldsymbol{x}|\boldsymbol{z})$ [20].

In this paper, focused on exploring the potential of alleviating "*posterior collapse*" with the second approach, we try to introduce skip-connection-liked structures into expected log-likelihood term to enhance the connections between the data $\boldsymbol{x}$ and the latent variables $\boldsymbol{z} = \boldsymbol{z}_1, ..., \boldsymbol{z}_L$. However, constrained by the layer-by-layer generation process of hierarchical VAE, there remains a great challenge to introduce physical skip connections into the generative model $p_\theta(\boldsymbol{x}|\boldsymbol{z})$, because arbitrarily adding or concatenating the stochastic hidden layers at different semantic levels will hurt the hierarchy of these multi-layer latent representations. We emphasize that the skip connections between the single stochastic layer and multiple deterministic layers [24] cannot be extended for hierarchical VAEs with multiple stochastic hidden layers, but our developed method below can be applied to any existing hierarchical VAE, which is one of the main contributions of this paper.

Generally speaking, moving beyond downweighting $\mathcal{I}_q(\boldsymbol{x}, \boldsymbol{z}_{>k})$ included in the KL term or modifying the structure of generative model $p_\theta(\boldsymbol{x}|\boldsymbol{z})$, our main idea is to upweight the mutual information between the data $\boldsymbol{x}$ and the higher-level variables $\boldsymbol{z}_{>k}$, which is denoted as

$$\mathcal{I}_p(\boldsymbol{x}, \boldsymbol{z}_{>k}) = -\mathcal{H}_p(\boldsymbol{x}|\boldsymbol{z}_{>k}) + \mathcal{H}_p(\boldsymbol{x}) = \mathbb{E}_{p(\boldsymbol{x})p_\theta(\boldsymbol{z}_{>k}|\boldsymbol{x})} \log p_\theta(\boldsymbol{x}|\boldsymbol{z}_{>k}) - \mathbb{E}_{p(\boldsymbol{x})} \log p(\boldsymbol{x}), \quad (7)$$

in the objective function of hierarchical VAEs. In Eq. (7), the first item can be approximated by $\mathcal{H}_{p,q}(\boldsymbol{x}|\boldsymbol{z}_{>k}) = \mathbb{E}_{p(\boldsymbol{x})q_\phi(\boldsymbol{z}_{>k}|\boldsymbol{x})} \log p_\theta(\boldsymbol{x}|\boldsymbol{z}_{>k})$ and the second item $\mathcal{H}_p(\boldsymbol{x})$ is a positive constant, leading to $\mathcal{I}_p(\boldsymbol{x}, \boldsymbol{z}_{>k}) \geq \mathcal{H}_{p,q}(\boldsymbol{x}|\boldsymbol{z}_{>k})$. Thus, targeted at directly maximizing multiple $\mathcal{H}_{p,q}(\boldsymbol{x}|\boldsymbol{z}_{>k})$ in the objective function, we develop an informative loss for training hierarchical VAEs, denoted as

$$\mathcal{L}^{in} = \mathbb{E}_{p(\boldsymbol{x})} \left[\frac{1}{L} \sum\nolimits_{k=0}^{L-1} \mathbb{E}_{q_\phi(\boldsymbol{z}_{>k}|\boldsymbol{x})} \left[\log p_\theta(\boldsymbol{x}|\boldsymbol{z}_{>k})\right] - \sum\nolimits_{l=1}^{L} D_{\mathrm{KL}}(q_\phi(\boldsymbol{z}_l|\boldsymbol{z}_{l+1})||p_\theta(\boldsymbol{z}_l|\boldsymbol{z}_{l+1})) \right] \quad (8)$$

where $p_\theta(\boldsymbol{x}|\boldsymbol{z}_{>k}) = \mathbb{E}_{p_\theta(\boldsymbol{z}_{\leq k}|\boldsymbol{z}_{>k})} \left[p_\theta(\boldsymbol{x}|\boldsymbol{z}_{\leq k})\right]$ describes a partial generative model to reconstruct the observation $\boldsymbol{x}$ taking $\boldsymbol{z}_{>k}$ drawn from the variational inference network $q_\phi(\boldsymbol{z}_{>k}|\boldsymbol{x})$ as input; the weight $1/L$ before each expected term is introduced to keep numerical stability.

Inspired by [17, 20], in practice, the informative loss in Eq. (8) can be optimized by maximizing its lower bound $\hat{\mathcal{L}}^{in}$ as

$$\begin{aligned} \mathcal{L}^{in} \geq \hat{\mathcal{L}}^{in} =& \mathbb{E}_{p(\boldsymbol{x})} \left[\frac{1}{L} \sum\nolimits_{k=0}^{L-1} \mathbb{E}_{p_\theta(\boldsymbol{z}_{\leq k}|\boldsymbol{z}_{>k})q_\phi(\boldsymbol{z}_{>k}|\boldsymbol{x})} \left[\log p_\theta(\boldsymbol{x}|\boldsymbol{z}_{\leq k})\right] \right] \\ & - \mathbb{E}_{p(\boldsymbol{x})} \left[\sum\nolimits_{l=1}^{L} D_{\mathrm{KL}}(q_\phi(\boldsymbol{z}_l|\boldsymbol{z}_{l+1})||p_\theta(\boldsymbol{z}_l|\boldsymbol{z}_{l+1})) \right], \end{aligned} \quad (9)$$

where the expected log-likelihood term is the summation of L components and each component can be obtained by replacing the inference network $q_\phi(\boldsymbol{z}_{\leq k}|\boldsymbol{z}_{>k})$ in the original log-likelihood term as described in Eq. (3) with the generative model $p_\theta(\boldsymbol{z}_{\leq k}|\boldsymbol{z}_{>k})$. Intuitively, for each expected log-likelihood term $\mathcal{LL}^{>k} = \mathbb{E}_{p_\theta(\boldsymbol{z}_{\leq k}|\boldsymbol{z}_{>k})q_\phi(\boldsymbol{z}_{>k}|\boldsymbol{x})} \left[\log p_\theta(\boldsymbol{x}|\boldsymbol{z}_{\leq k})\right]$, after sampling the top variables $\boldsymbol{z}_{>k}$ from the variational posterior $q_\phi(\boldsymbol{z}_{>k}|\boldsymbol{x})$, these variables $\boldsymbol{z}_{>k}$ will be forced to reconstruct the observation $\boldsymbol{x}$ with the partial generative model $p_\theta(\boldsymbol{x}|\boldsymbol{z}_{>k})$, which builds the straightforward

connections between $\boldsymbol{x}$ and $\boldsymbol{z}_{>k}$ to alleviate "*posterior collapse*". Moreover, the $1/L$ original expected log-likelihood term ($\mathcal{LL}^{>0}$) included in $\hat{\mathcal{L}}^{in}$ ensures the quality of data reconstruction.

We refer to the hierarchical VAE trained with the lower bound of informative loss in Eq. (9) as informative hierarchical VAE. We note that this method is applicable for hierarchical VAEs with either top-down or bottom-up inference network to explicitly utilize the generative hierarchy of the multi-layered stochastic variables during training, and can be flexibly extended in future works.

3.4 Adaptive Likelihood Ratio for OOD detection

Besides "*posterior collapse*", an inappropriate choice of k in the likelihood-ration score function, denoted as $\mathcal{LLR}^{>k}$ in Eq. (5), will also bring negative impact on the performance of applying hierarchical VAEs for OOD detection. Recall to the visualization examples in Fig. 2, when setting $k = 0$ or $k = 1$, the reconstruction quality of either in-distribution or OOD samples is surprisingly high, leading to the $\mathcal{LLR}^{>k}$ scores of both in-distribution and OOD samples are relatively small and further making it difficult to distinguish whether the data sample is OOD or not. Moving beyond cherry picking the hyperparameter k on testing OOD samples, which is unreasonable for unsupervised OOD detection, we develop a novel adaptive log-likelihood ration score function, referred to as $\mathcal{LLR}^{ada}$ formulated as

$$\mathcal{LLR}^{ada} = \sum\nolimits_{k=0}^{L-1} \frac{\mathcal{R}(\boldsymbol{x}, \boldsymbol{z}_{>k-1})}{\mathcal{R}(\boldsymbol{x}, \boldsymbol{z}_{>k})} (\mathcal{LLR}^{>k} - \mathcal{LLR}^{>k-1}), \tag{10}$$

where $\mathcal{R}(\boldsymbol{x}, \boldsymbol{z}_{>k})$ is designed to measure the relevance between the data sample $\boldsymbol{x}$ and its latent variables $\boldsymbol{z}_{>k}$ sampled from the variational posterior $q_\phi(\boldsymbol{z}_{>k}|\boldsymbol{x})$, specifically defining $\mathcal{LLR}^{>-1} := 0$ and $\mathcal{R}(\boldsymbol{x}, \boldsymbol{z}_{>-1}) := \mathcal{R}(\boldsymbol{x}, \boldsymbol{z}_{>0})$. More specifically, there are many choices for the definition of $\mathcal{R}(\boldsymbol{x}, \boldsymbol{z}_{>k})$, but in the following experiments, we only use the log-likelihood score for brevity, by setting $\mathcal{R}(\boldsymbol{x}, \boldsymbol{z}_{>k}) = \log p_\theta(\boldsymbol{x}|\boldsymbol{z}_{>k})$.

Compared to the previous score functions for OOD detection [17, 23], the developed $\mathcal{LLR}^{ada}$ in Eq. (10) owns less hyperparameters to be tuned, making its performance more stable on various benchmarks. Moreover, the adaptive weight $\frac{\mathcal{R}(\boldsymbol{x}, \boldsymbol{z}_{>k-1})}{\mathcal{R}(\boldsymbol{x}, \boldsymbol{z}_{>k})}$ in $\mathcal{LLR}^{ada}$ will be relatively large when the data information drop rapidly at the current hidden layer, and thus introducing those adaptive weights can enlarge the gap between the metric scores of in-distribution and OOD samples at some discriminative hidden layers, where the reconstruction quality of the in-distribution and OOD samples is extremely different, such as $k = 2$ in Fig. 2, leading to further performance improvements. More discussions about $\mathcal{LLR}^{ada}$ can be found in Appendix C.

4 Experiments

4.1 Experimental setup

Datasets: Following [1, 17, 23], we compare our method with previous works on two dataset pairs, including: FashionMNIST [28] (in) / MNIST [29] (out) and CIFAR10 [30] (in) / SVHN [31] (out), where the suffix "in" and "out" denote the in-distribution dataset and OOD dataset, respectively. To better evaluate the generalization ability of these methods, we introduce additional OOD datasets: for FashionMNIST/MNIST pair, we add KMNIST [32], notMNIST [33], Omniglot [34] and SmallNORB [35] datasets; for CIFAR10/SVHN pair, we add CelebA [36], Places365 [37], Flower102 [38] and LFWPeople [39] datasets. More details about datasets can be found in Appendix D.

Evaluation and Metrics: We follow the evaluation procedure in Havtorn et al. [17], where all methods are trained on the training split of the in-distribution dataset, and their OOD detection performance is evaluated on both the testing split of the in-distribution dataset and OOD dataset. To conduct an effective quantitative evaluation, following previous works [4, 5, 18], we adopt the recent popular threshold-independent evaluation metrics, including: Area Under the Receiver Operator Characteristic (AUROC↑), Area Under the Precision Recall Curve (AUPRC↑) and False Positive Rate at 80% true positive rate (FPR80↓), where the arrow indicates the direction of improvement.

Baselines: The comparisons in our experiments mainly include two categories: **i**) the comparisons with previous OOD detection methods to see whether our method can achieve competitive performance; **ii**) the comparisons with several hierarchical VAEs to see whether the new training objective of our method can lead to better performance. For the comparisons in **i**, the baselines can be divided

Table 1: The comparisons of the 5-layer informative hierarchical VAE with $\mathcal{LLR}^{ada}$ and other OOD detection methods. The state-of-the-art results achieved by the methods of the category "Not ensembles" of "Unsupervised" have been bold.

FashinMNIST(in)/MNIST(out)						**CIFAR10(in)/SVHN(out)**					
Labels		**Prior**		**Unsupervised**		**Labels**		**Prior**		**Unsupervised**	
Method	AUROC↑	Mehod	AUROC↑	Method	AUROC↑	Method	AUROC↑	Mehod	AUROC↑	Method	AUROC↑
CP	73.4	LR(PC)	99.4	*-Ensembles*		MD	99.7	LR(PC)	93.0	*-Ensembles*	
CP(Ent)	74.6	LR(BC)	45.5	WAIC(5VAE)	76.6	LMD	27.9	LR(VAE)	26.5	WAIC(5Glow)	99.0
ODIN	75.2	CP(OOD)	87.7	WAIC(5PC)	22.1			OE	98.4	WAIC(5PC)	62.8
VIB	94.1	CP(Cal)	90.4	*-Not Ensembles*				IC(Glow)	95.0	*-Not Ensembles*	
MD(CNN)	94.2	IC(Glow)	99.8	LRe	**98.8**			IC(PC++)	92.9	LRe	87.5
MD(DN)	98.6	IC(PC++)	96.7	HVK	98.4			IC(HVAE)	83.3	HVK	89.1
DE	85.7			$\mathcal{LLR}^{ada}$(Ours)	98.0					$\mathcal{LLR}^{ada}$(Ours)	**94.2**

Table 2: The comparisons of the 3-layer informative hierarchical VAEs with various score functions and other unsupervised OOD detection methods.

FashinMNIST(in)/SVHN(out)				**CIFAR10(in)/SVHN(out)**			
Method	AUROC↑	AUPRC↑	FPR80↓	Method	AUROC↑	AUPRC↑	FPR80↓
WAIC(5PC)	22.1	40.1	91.1	WAIC(5PC)	62.8	61.6	65.7
HVK	**98.4**	**98.4**	1.3	HVK	89.1	87.5	17.2
-Ours:				***-Ours:***			
$\mathcal{L}$	55.3	51.8	67.9	$\mathcal{L}$	49.9	51.0	79.4
$\mathcal{LLR}^{>1}$	97.5	97.0	2.8	$\mathcal{LLR}^{>1}$	68.4	71.3	61.8
$\mathcal{LLR}^{>2}$	97.4	97.7	**1.2**	$\mathcal{LLR}^{>2}$	**93.0**	**92.5**	**10.8**
$\mathcal{LLR}^{ada}$	97.0	97.6	**0.9**	$\mathcal{LLR}^{ada}$	**92.6**	**91.8**	**11.1**

into three categories: "**Labels**": methods using in-distribution data labels [4, 5, 7, 8, 40, 41], "**Prior**": methods using the prior knowledge collected from OOD data [1, 18, 19], and "**Unsupervised**": methods without any OOD-specific assumptions [14, 17, 23]. For the comparisons in **ii**, we compare our method with a normal bottom-up inference hierarchical VAE (HVAE) [12], which is also the backbone of our method, and its two major variants: a top-down inference hierarchical VAE named Ladder VAE (LVAE) [21] and a bidirectional inference hierarchical VAE (BIVA) [20]. More details about these categories and the baselines in Tab. 1 can be found in Appendix E.

Implementation Details: For the comparisons on FashionMNIST(in)/MNIST(out), we set the network structure of hierarchical VAEs as [16, 8, 4] and [32, 24, 16, 8, 4] from shallow to deep, respectively. For CIFAR10(in)/SVHN(out), we set the network structure as [128, 64, 32] and [128, 64, 32, 28, 24], respectively. For optimization, we adopt the same Adam optimizer [42] with a learning rate of 3e-4. We train all models in comparison by setting the batch size as 128 and the max epoch as 1000. All experiments are performed on a PC with an NVIDIA RTX 3090 GPU and the our code is implemented with PyTorch [43]. More implementation details can be found in Appendix F.

4.2 Quantitative Comparisons

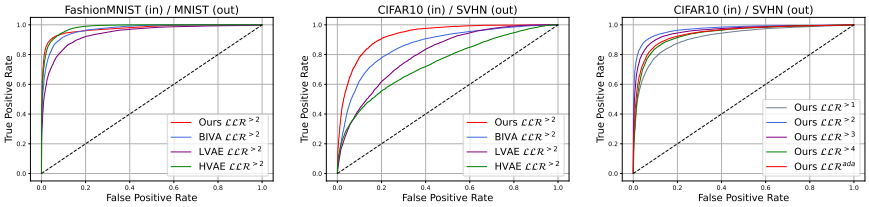
Overall Comparisons: Following the experimental settings in Sec. 4.1, we exhibit the experimental results in in Tab. 1. From the results, we can find that our method with the $\mathcal{LLR}^{ada}$ score function is comparable with those non-ensemble completely unsupervised methods in FashionMNIST/MNIST, and significantly outperform them in CIFAR10/SVHN. We emphasize that, without utilizing the labels of in-distribution samples [4, 5, 7, 8, 40, 41] or the prior knowledge collected from OOD samples [1, 18, 19], our method can still achieve competitive performance with these methods.

Effectiveness of $\mathcal{LLR}^{ada}$: Focused on the comparison between $\mathcal{LLR}^{>k}$ and $\mathcal{LLR}^{ada}$ exhibited in Tab. 2 and Fig. 3(c), when the performance of $\mathcal{LLR}^{>k}$ is sensitive to the selection of k, we can find that the performance of $\mathcal{LLR}^{ada}$ can approach the best performance achieved by $\mathcal{LLR}^{>k}$ with the optimal k, as shown in the right part of Tab. 2 (CIFAR/SVHN). Furthermore, when the performance of $\mathcal{LLR}^{>k}$ is stable, the developed $\mathcal{LLR}^{ada}$ can still achieve comparable OOD detection performance, as shown in the left part of Tab. 2 (FashionMNIST/MNIST) and Fig. 3(c). The experimental results above demonstrate the adaptability of our developed $\mathcal{LLR}^{ada}$.

Effectiveness of Informative Hierarchical VAE: Taking the same $\mathcal{LLR}^{ada}$ as the score function, we compare the performance of informative hierarchical VAE with other hierarchical VAEs in Tab. 3. From the results, we can find that our model outperforms others, indicating the effectiveness of alleviating "*posterior collapse*" with informative hierarchical VAE for OOD detection.

Table 3: The comparisons of the OOD detection performance of various 3-layer hierarchical VAEs with the same $\mathcal{LLR}^{ada}$ score function. “M1” refers to the metric AUROC↑, “M2” refers to the metric AUPRC↑, and “M3” refers to the metric "FPR80↓".

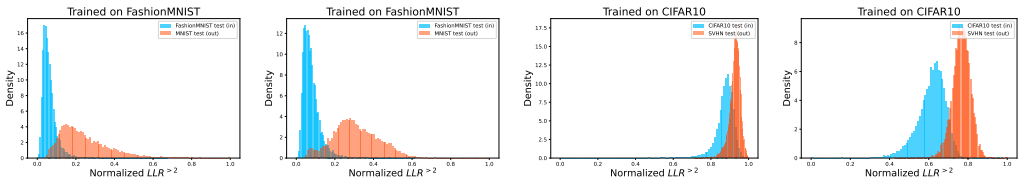
	Trained on FashionMNIST.												**Trained on CIFAR10.**											
OOD	KMNIST			Omniglot			notMNIST			SmallNORB			CelebA			Places365			Flower102			LFWPeople		
Model	M1	M2	M3	M1	M2	M3	M1	M2	M3	M1	M2	M3	M1	M2	M3	M1	M2	M3	M1	M2	M3	M1	M2	M3
HVAE	86.4	89.7	29.0	99.8	99.9	**0.00**	85.3	88.1	29.2	**1.00**	**1.00**	**0.00**	39.8	44.7	90.0	40.1	46.6	94.0	45.2	51.7	92.0	42.5	48.2	92.5
LVAE	85.9	87.7	24.8	93.1	96.0	0.5	94.0	93.6	6.0	97.3	97.7	0.8	53.1	54.2	80.5	56.2	53.7	74.4	56.5	52.3	70.9	63.0	65.8	61.6
BIVA	86.5	87.0	27.0	**1.00**	**1.00**	**0.00**	96.4	97.0	2.4	98.7	98.6	1.9	70.5	67.8	53.2	60.1	**63.0**	74.6	61.9	69.2	84.4	75.2	74.0	44.6
Ours	**95.0**	**95.1**	**7.1**	**1.00**	**1.00**	**0.00**	**99.7**	**99.8**	**0.00**	**1.00**	**1.00**	0.1	**72.1**	**70.5**	**49.0**	**63.3**	62.1	**62.6**	**63.4**	**70.1**	**71.2**	**83.0**	**83.4**	**29.0**



(a) (b) (c)

Figure 3: Plots of the ROC curves on the OOD detection performance of various hierarchical VAEs. (a)∼(b): ROC curves on FashionMNIST (in) / MNIST (out) and CIFAR10 (in) / SVHN (out) with the same score function $\mathcal{LLR}^{>2}$. (c): ROC curves on CIFAR10 (in) / SVHN (out) with various score functions ($\mathcal{LLR}^{>k}$ and $\mathcal{LLR}^{ada}$).

To evaluate the degree of “*posterior collapse*”, we use the ROC curves in Fig. 3 and the empirical densities in Fig. 4 to compare the performance of $\mathcal{LLR}^{>2}$ scores based on the top-level latent variables $\boldsymbol{z}_2$ of these VAEs. The ROC results in Fig. 3 demonstrate the superiority of the informative hierarchical VAE, confirming that our model can provide more expressive higher-level latent representations for OOD detection. The empirical densities in Fig. 4 show that $\boldsymbol{z}_2$ learned by our model on the CIFAR10 (in)/SVHN (out) pairs owns better separability than those learned by BIVA.

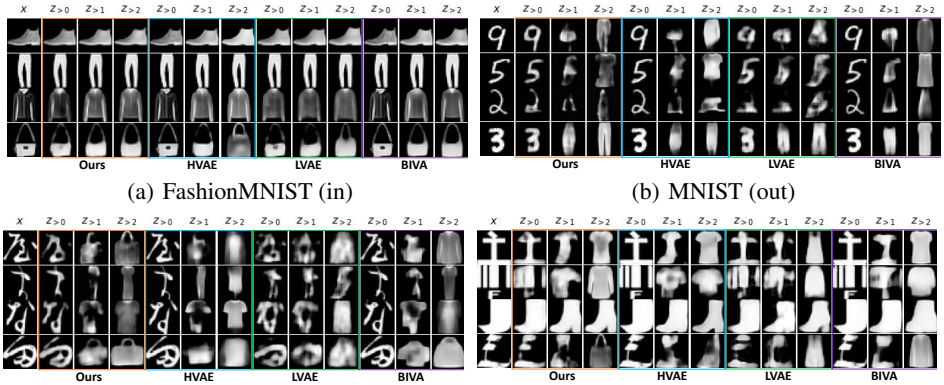


(a) BIVA (b) Ours (c) BIVA (d) Ours

Figure 4: Empirical densities under the score function $\mathcal{LLR}^{>2}$ on FashionMNIST (in)/MNIST (out) and CIFAR10 (in)/SVHN (out) dataset pairs.

4.3 Qualitative Analysis

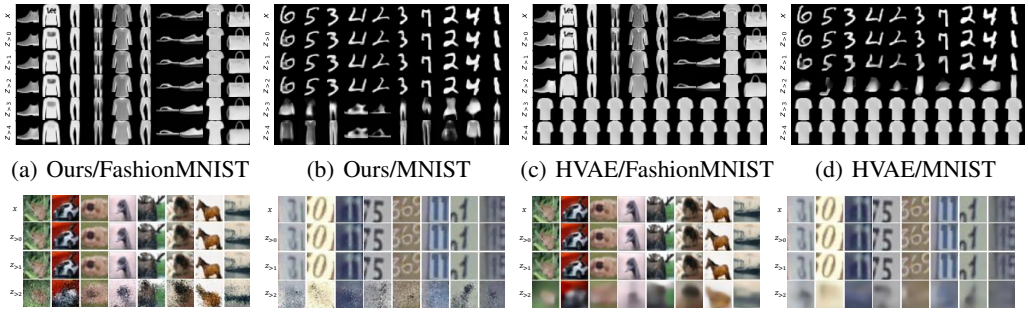
Meaningful Semantic Space Learned by Informative Hierarchical VAE: Following the same procedure as Fig. 2, we visualize more reconstructed samples on various benchmarks in Fig. 5. From the visualized results, for both in-distribution and OOD samples, we can find that the quality of the reconstructions generated from $p_\theta(\boldsymbol{x}|\boldsymbol{z}_{>0})$ is surprisedly high, indicating that these reconstructions are almost dominated by the low-level features, which potentially explains the previous problematic phenomenon that these methods based on single-layer likelihood will assign higher likelihood scores for OOD samples and fail on OOD detection. Focused on the reconstructions generated by $p_\theta(\boldsymbol{x}|\boldsymbol{z}_{>2})$, we can find that the developed informative hierarchical VAE can provide more realistic and clear reconstructed samples for both in-distribution and OOD inputs, indicating that our model can learn a more meaningful high-level latent semantic space than other models. Furthermore, based on providing higher-quality reconstruction for OOD samples, the gap between the metric scores of in-distribution and OOD samples in informative hierarchical VAE tend to be larger than other models, leading to a better OOD performance as shown in Tab. 3.



(a) FashionMNIST (in) (b) MNIST (out)

(c) KMNIST (out) (d) notMNIST (out)

Figure 5: Comparisons of reconstructions with 3-layer hierarchical VAEs trained on FashionMNIST, where the leftmost column in each subfigure is the input $\boldsymbol{x}$ and the column noted with $\boldsymbol{z}_{>k}$ means the generation from the partial generative model $p_\theta(\boldsymbol{x}|\boldsymbol{z}_{>k})$ with $k \in \{0, 1, 2\}$.



(a) Ours/FashionMNIST (b) Ours/MNIST (c) HVAE/FashionMNIST (d) HVAE/MNIST

(e) Ours/CIFAR10 (in) (f) Ours/SVHN (out) (g) HVAE/CIFAR10 (in) (h) HVAE/SVHN (out)

Figure 6: Comparisons of the degree of "*posterior collapse*" in 5-layer hierarchical VAEs. For each subfigure, the 1st row is the input image $\boldsymbol{x}$ and the ith row is generated from $p_\theta(\boldsymbol{x}|\boldsymbol{z}_{>k})$.

Alleviating "*Posterior Collapse*" with Informative Hierarchical VAE: As discussed in Sec. 3.1, "*posterior collapse*" will cause the higher-level latent variables to become uninformative. Considering the developed informative hierarchical VAE shares the same network structure with the basic HVAE, in this part, we focus on evaluating whether our model can alleviate "*posterior collapse*" in higher layers when the network depth becomes deeper. As shown in Fig. 6, for both in-distribution data and OOD data samples, the overall quality of the reconstructions generated by our informative hierarchical VAE is significantly higher than those generated by the basic HVAE. Specifically, for FashionMNIST (in)/MNIST (out), the reconstructions generated by $p_\theta(\boldsymbol{x}|\boldsymbol{z}_{>3})$ and $p_\theta(\boldsymbol{x}|\boldsymbol{z}_{>4})$ of HVAE are almost the same, indicating that its posterior described by $q_\phi(\boldsymbol{z}_{>3}|\boldsymbol{x})$ or $q_\phi(\boldsymbol{z}_{>4}|\boldsymbol{x})$ tend to collapse to a prior distribution about T-shirts. On the contrary, the reconstructions generated by the informative hierarchical VAE still are still realistic, where the reconstructions of MINST (out) samples generated by $q_\phi(\boldsymbol{z}_{>3}|\boldsymbol{x})$ or $q_\phi(\boldsymbol{z}_{>4}|\boldsymbol{x})$ persevere the semantic structural information learned form FashionMNIST (in), explaining the underlying reason why our model can achieve better OOD detection performance. Similar conclusions can be achieved by the experimental results on CIFAR10 (in)/SVHN (out) as shown in Fig. 6.

5 Conclusion

In this paper, after presenting a thorough analysis of "*posterior collapse*", we develop a novel informative hierarchical VAE to extract more expressive hierarchical latent representations by alleviating "*posterior collapse*". Then we theoretically explain why "*posterior collapse*" will limit the performance of existing hierarchical VAEs, and develop a novel Adaptive Likelihood Ratio score function for unsupervised OOD detection. Experiments demonstrate the effectiveness of our method, whose main thought can be borrowed other hierarchical VAEs to improving their performance on downstream tasks relied on the hierarchy of latent representations.

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Checklist

1. For all authors...
   (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [Yes] See Abstract
   (b) Did you describe the limitations of your work? [Yes] See Appendix H
   (c) Did you discuss any potential negative societal impacts of your work? [N/A]
   (d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [Yes]
2. If you are including theoretical results...
   (a) Did you state the full set of assumptions of all theoretical results? [Yes]
   (b) Did you include complete proofs of all theoretical results? [Yes] See Appendix A and Appendix B
3. If you ran experiments...
   (a) Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? [Yes] See supplemental material
   (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? [Yes] See Section 4.1 and Appendix F
   (c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? [Yes] See Appendix G
   (d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? [Yes] See Section 4.1 and Appendix F
4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...
   (a) If your work uses existing assets, did you cite the creators? [Yes] See supplemental material
   (b) Did you mention the license of the assets? [Yes] See supplemental material
   (c) Did you include any new assets either in the supplemental material or as a URL? [Yes] See supplemental material
   (d) Did you discuss whether and how consent was obtained from people whose data you're using/curating? [N/A]
   (e) Did you discuss whether the data you are using/curating contains personally identifiable information or offensive content? [N/A]
5. If you used crowdsourcing or conducted research with human subjects...
   (a) Did you include the full text of instructions given to participants and screenshots, if applicable? [N/A]
   (b) Did you describe any potential participant risks, with links to Institutional Review Board (IRB) approvals, if applicable? [N/A]
   (c) Did you include the estimated hourly wage paid to participants and the total amount spent on participant compensation? [N/A]