Long-Tailed Visual Recognition with Global Contrastive Learning and Prototype Learning

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Abstract

We consider the visual recognition problem in long-tailed data in which few classes dominate the majority of the other classes. Most current methods employ contrastive learning to learn a representation for long-tailed data. In this paper, first, we investigate k-positive sampling, a popular baseline method widely used to build contrastive learning models for imbalanced data. Previous works show that k-positive learning, which only chooses k positive samples (instead of all positive images) for each query image, suffers from inferior performance in long-tailed data. In this work, we further point out that k-positive learning limits the learning capability of both head and tail classes. Based on this perspective, we propose a novel contrastive learning framework namely GloCo which improves the limitation in k-positive learning by enlarging its positive selection space, so it can help the model learn more semantic discrimination features. Second, we analyze how the temperature (the hyperparameter used for tuning a concentration of samples on feature space) affects the gradients of each class in long-tailed learning, and propose a new method that can mitigate inadequate gradients between classes, which can help model learning easier. Finally, we go on to introduce a new prototype learning framework namely ProCo based on coreset selection, which can help us create a global prototype for each cluster while keeping the computation cost within a reasonable time and show that combining GloCo with ProCo can further enhance the model learning ability on long-tailed data.

1 Introduction

Real-world data usually follows long-tail distribution, where only a few classes dominate the dataset (namely, head classes). In contrast, most other classes have a small number of samples (namely, tail classes). This long-tailed data poses a major inferential challenge to traditional deep learning models whose training is biased by the head classes and whose performance quickly deteriorates when the data is imbalanced (Wang et al., 2020; Cao et al., 2019; Zhang et al., 2021c). Various approaches have been proposed to address such issues, with typical methods including class-balanced re-sampling (Shen et al., 2016; More, 2016; Zhang et al., 2021c), class-level re-weighting (Alshammari et al., 2022; Lin et al., 2017; Zhang et al., 2021c) and ensemble learning (Zhou et al., 2020; Wang et al., 2020; Zhang et al., 2021b). However, the prior approaches rely on the classical cross-entropy losses which are sensitive to imbalanced data (Wang et al., 2020; Zhang et al., 2021c).

Many studies have shown that contrastive learning is more robust to noisy or unbalanced data (Khosla et al., 2020; Kang et al., 2020), therefore this algorithm is also widely applied to solve imbalanced data and achieves impressive results (Chen & He, 2021; Li et al., 2022b; Zhu et al., 2022; Liu et al., 2021; Yang & Xu, 2020). Its success is based on the contrastive loss function and a large number of negative samples (He et al., 2020), which will help the model learn more robust and semantic discrimination feature (Liu et al., 2021; Yang & Xu, 2020) thereby helping the model generalize well on the training data, and subsequently easily transfer to the test data or other data domains (Chen et al., 2020a; Liu et al., 2021). Moreover, unsupervised contrastive learning creates more balance feature space, even when the data is highly skewed (Kang et al., 2020), which is the main factor that results in the failure of the previous methods. However, the model trained in an unsupervised manner often does not perform well since it does not use label information; thus it fails to

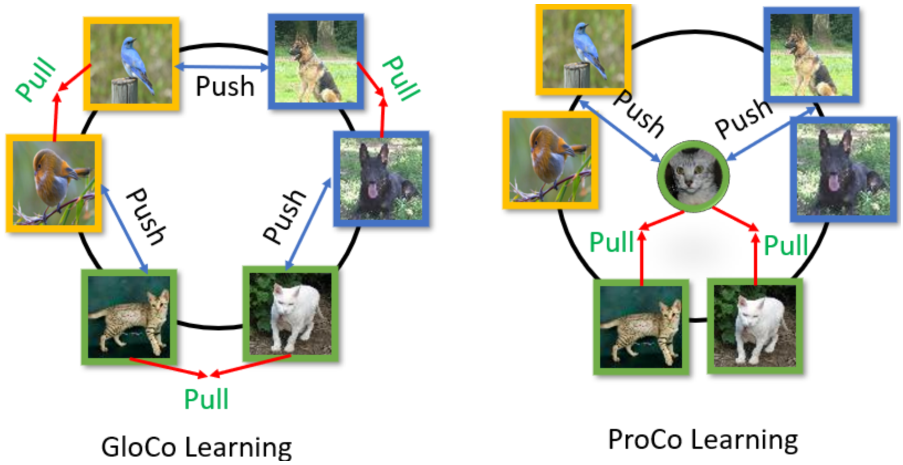

Figure 1: Overview of how GloCo and ProCo learn during training. GloCo learns to pull closer images of the same class while pushing apart images of different classes **across-batches**. Meanwhile, ProCo builds a prototype (center) for each class, by which points of the same class are pulled toward the class prototype whereas points of the other classes are pushed away.

learn rich semantic discrimination features due to lack of label information. Different from unsupervised contrastive learning, supervised contrastive learning can learn more semantic feature space (Kang et al., 2020). Nonetheless, using the label during training makes features more imbalanced as compared with their unsupervised counterpart. To overcome this problem in supervised contrastive learning, instead of using all positive samples in the batch for training, Kang et al. (2020) proposes k-positive selection, a supervised contrastive learning method in which we limit the number of positive samples to k, therefore, it can reduce the imbalance effect. However, when the data is extremely long-tailed, using k-positive alone is not enough to tackle the imbalance problem (Li et al., 2022b). Besides contrastive learning, prototype learning (Li et al., 2022b; Cui et al., 2021; Zhu et al., 2022) is also widely used to tackle long-tailed problems. This method constructs a prototype (center) for each class and pulls samples of the same class closer to the center while pushing samples of other classes away. But it remains elusive as to how to build a good prototype and efficient prototype loss function.

Motivated by the limitations of the prior approaches, in this paper, we propose a training method namely **Glo**bal-Adaptive **Co**ntrastive learning (GloCo), which constructs a more balanced feature space with better semantic discrimination features. In GloCo, we first use label information to select k-positive samples across batches for the query. This so-called k-global positive selection is in stark contrast with k-positive selection (Kang et al., 2020) which selects positive samples only within a batch. Expanding the selection space across batches helps the query connect to more positive samples; therefore the model can learn richer semantic discrimination features. Besides, we analyze the gradient of our model when training with imbalanced data, pointing out that the temperature value has an important role during the training. Based on the analysis, we propose a new method namely **Adaptive temperature**. In this method, we view temperature as a re-balanced parameter, which can help us reduce the imbalance effect between head and tail classes during the training stage by balancing their gradients. Moreover, we propose a prototype learning approach namely ProCo, which uses data distillation to efficiently construct prototypes. We empirically show that jointly training ProCo with GloCo can further improve the model performance.

Our main contributions:

- **Global-Adaptive Contrastive learning (GloCo):** We introduce a new training strategy that can adapt to any contrastive framework to improve its performance in long-tailed learning namely **k-global positive selection**. After that, we analyze the model gradient and propose re-balanced techniques namely **Adaptive temperature** to furtherly tackle the imbalance problem. We empirically evaluate with different settings and show that our model can outperform the state-of-the-art model with a large gap.

- **Prototype learning via coreset selection (ProCo):** We propose a new prototype learning method named ProCo which improves over the original prototype method from an aspect that distills coreset from training data, then determines prototypes based on coreset to help constructed prototypes efficiently and can represent global features for the original dataset without using all training samples.

2 Related work

Long-tailed recognition is one of the challenging problems in computer vision. There are many studies focusing on this problem with different solutions (Zhang et al., 2021c; Zhu et al., 2022; Shen et al., 2016; More, 2016; Alshammari et al., 2022; Lin et al., 2017; Zhu et al., 2022; Kang et al., 2020; Li et al., 2022b). The general idea is to try to re-balance the effect between head and tail classes during training from data perspective or model perspective.

Data perspective The most obvious way to solve long-tailed problems is to create a balanced dataset, where the samples of head and tail classes need to change to become more balance. There are two popular approaches including over-sampling (Shen et al., 2016; Sarafianos et al., 2018), and under-sampling (More, 2016; Drummond et al., 2003; Buda et al., 2018). While the former simply creates more data for tail classes by duplicating samples, average samples, or learning its distribution then generating more data. The latter needs to reduce the samples of head classes by random removal procedures. However, the over-sampling approaches easily lead to overfitting (Bunkhumpornpat et al., 2012; More, 2016); under-sampling loss data information (More, 2016). To overcome the limitation of the two approaches, (Park et al., 2022) introduces a new over-sampling method that creates larger tail samples by utilizing rich context features in head classes. Besides, we can use other data augmentation techniques (Li et al., 2021; Zhang et al., 2021d) which can give us a larger size for tail class, or data distillation techniques (Wang et al., 2018; Killamsetty et al., 2021; Ghadikolaei et al., 2019; Zhang et al., 2021a) to extract a smaller set for head classes while ensuring it can represent the distribution of the whole data.

Model perspective There are many techniques utilizing models to tackle long-tailed learning including re-weighting (Alshammari et al., 2022; Lin et al., 2017; Khan et al., 2017), ensemble learning (Zhou et al., 2020; Wang et al., 2020; Zhang et al., 2021b), or two stages training (Kang et al., 2019; Zhou et al., 2020). In re-weighting methods, we set weights for each class, usually reducing head class dominating by setting it small weight or enlarging the weight of tail classes to emphasize its contribution. The simplest way is to set weights inverse to a number of samples in class. Besides, normalizing the weight of network (Alshammari et al., 2022) also shows an impressive result, or extends focal loss to Equalized Focal Loss (Li et al., 2022a) for efficient one-stage object detection. Along this direction, decoupling techniques use two stages of training, where we need to learn a good representation in the first stage and fine-tune it in the second stage, this training strategy shows that without carefully designing a model, we can also achieve good performance (Kang et al., 2019). The final method usually used in long-tailed learning is ensemble (Zhou et al., 2020; Wang et al., 2020; Zhang et al., 2021b), where we can join training a multiple branches network, each branch will contribute and learn specific information during training, then we combine results of these branches together to predict the final result for long-tailed data.

Contrastive learning Contrastive learning has got attention recently because of its success in training for large unlabeled data (Chen et al., 2020a; He et al., 2020; Chen et al., 2020c; Doersch et al., 2015; Grill et al., 2020; Caron et al., 2018; Chen et al., 2020b; 2021; Tian et al., 2020), and labeled data (Khosla et al., 2020). In the decoupling method that we have mentioned above, the key problem is how can we train a strong backbone model at the first stage then use it as a pre-trained and fine-tune the whole or top layer of the model in the second stage. This is similar to the behavior of training contrastive model (He et al., 2020; Chen et al., 2020c;a; Chen & He, 2021), where we use a large amount of unlabeled data to learn invariant features from input data, then use those features to initiate for downstream tasks. Therefore, contrastive learning becomes a popular method in the first stage of training of decoupling. Recently, there are an increasing number of studies trying to understand the behavior of contrastive learning on long-tailed data. For example, (Kang et al., 2020) explore that self-supervised learning create balance but lacks semantic feature space, and they propose k-positive learning which simply limits the number of positive for each sample in batch not greater than k to ensure the balance between head and tail class. (Liu et al., 2021) hypothesize that contrastive learning works better than supervised learning because it can transfer useful features from head classes to tail classes. Then they analyze the gradient and propose a method following theoretically-principled label-distribution-aware margin loss (LDAM)(Cao et al., 2019) to re-balance feature space. (Zhu et al., 2022) show that supervised contrastive learning is not the optimal solution for long-tailed data and introduce class averaging and class complement in the loss function to make SupCon possible when training with long-tailed data. (Zhou et al., 2022; Jiang et al., 2021) utilize the memorization effect of deep neural networks to recognize tail sample and enhance it in both data and model perspective.

Prototypical contrastive learning Training on self-supervised learning with long-tailed data generates more balance feature space than the supervised counterpart. However, when the dataset is extremely skewed, it is still dominated by head class. To overcome this issue, several works use prototype learning to re-balance feature space(Cui et al., 2021; Li et al., 2022b; Zhu et al., 2022). The key idea is to generate a set of prototypes (centers) for each class, then pull all samples of the same class closer to its prototypes and push samples from other classes far away. However, how to efficiently construct such prototypes is an active research area. Previous works (Li et al., 2020; 2022b) construct these prototypes by utilizing the large queue size in MoCo to save computation times. Nonetheless, it can not adapt to another contrastive framework, and when the dataset size is too large, data in the queue size can not representation the whole dataset distribution. In this paper, we propose another approach that can generate prototypes to cover the training data distribution without using all samples within an acceptable time.

3 Method

Give the input data $X = \{x_1, x_2, ...x_N\}$ and the label $Y = \{y_1, \dots, y_N\}$ which have C classes where $y_i \in \{1, 2, ..., c\}$. The training samples $S = \{(x_j, y_j)\}_{j=1}^{N} \in P$ is the distribution over instance X and label Y. Our goal is to learn a function f that map the input data space X to a label space R_c which minimizes the misclassification error $P_{x,y}$. Assuming i and j are the largest and smallest classes, respectively. Class X_i has m samples and class X_j has n samples, $m > n$. We define:

- $imb = \frac{m}{n}$ is the imbalance factor.
- B is the batch size of the data
- $t = \frac{|S|}{|B|}$ is the ratio between total training samples and batch size, and B_i is the list samples of class i in batch B.

Inspiration by previous works (Kang et al., 2020; Li et al., 2022b; Khosla et al., 2020), we propose a general framework that can apply to contrastive learning model (e.g., MoCo (He et al., 2020), SimCLR (Chen et al., 2020a)) to help it handle better with the imbalance in long-tailed learning. Our model can create rich semantic features while reducing the dominating of head class during training. This framework includes two modules namely GloCo and ProCo. In the first section, we introduce Global-Adaptive contrastive learning (GloCo), a method that can help contrastive learning models learn easier in the long-tailed setting. Then, we further improve Global contrastive learning by prototypical learning, where we propose another efficient method for prototype learning via coreset selection (ProCo).

3.1 Global Contrastive learning

***K*-global positive selection** SupCon (Khosla et al., 2020) and it improvement version for long-tailed learning: k-positive learning (Kang et al., 2020; Li et al., 2022b) select a positive sample for each query from the positives samples in the same batch. However, this selection strategy limits the learning ability of the model, especially for tail classes, where the class size is small. In the long-tailed setting, when we choose positive samples inside the batch, it is hard to tail query samples that can meet other positives in the same batch size, therefore not utilizing the information between other tail class samples to learn, reducing the quality of learned features. To overcome this problem, we propose **k-global positive selection**, our method expands the positive selection space for each query sample from inside the batch to across batches. To implement the proposed k-global selection algorithm, we have redefined the DataLoader. Specifically, our DataLoader will include an additional variable containing information about the index of samples with the same labels in the training data. During training, for each sample x in batch B, we will randomly select k positive samples with the same labels in training data(global selection) instead of choosing k positive samples with the same labels in batch (batch selection). This simple modification helps the query sample connect with all other positive samples inside that class, therefore enlarging its selection space, and helping the model learn easier. Our formula for across-batches positive selection is defined as:

$$L_{\text{contrastive}} = -\frac{1}{N(k+1)} \sum_{i=1}^{N} \sum_{v_j^+ \in \{v_i^+\} \cup P_{i,k}^+} \ell(i,j), \tag{1}$$

where

$$\ell(i,j) = \log \frac{\exp\left(v_i \cdot v_j^+ / \tau\right)}{\exp\left(v_i \cdot v_j^+ / \tau\right) + \sum_{v_j \in V_i} \exp\left(v_i \cdot v_j / \tau\right)}$$

Here v_i^+ is the augmented view of query v_i, $P_{i,k}^+$ is the k positive samples selected by k-global positive, and V_i are examples in the current batch excluding v_i.

This selection strategy is extremely helpful for tail classes because, in long-tailed data, the total number of tail classes is usually small. Therefore if we can gather all the samples in tail classes together, each sample can utilize information from all other samples to learn, then our approach can efficiently learn the feature space of tail classes.

Selection space analysis In this section, we will provide a deeper analysis of k-positive selection, as well as introduce a new concept namely: **selection space** to clarify the benefit of k-global positive learning over the k-positive selection.

Definition 1. *k-positive selection space in class i is the combination of choosing k positives samples from the batch data B_i(in the case of k-positive selection):*

$$k_b^i = \frac{|B_i|!}{k!(|B_i| - k)!}$$

or from the whole dataset X_i (in the case of k-global positive selection).

$$k_g^i = \frac{|X_i|!}{k!(|X_i| - k)!}$$

We can see that in k-positive learning, the positive selection space of each class represents its frequency during the training. It means classes with more samples will have a larger selection space and vice versa. Based on the k-positive selection space, we propose another way that can evaluate the imbalance of the k-positive learning model.

Definition 2. *The balance ratio between class i and j ($|X_i| > |X_j|$) in batch (or across-batches) data is a ratio of positive selection space between two classes, where :*

$$Br_b^{i,j} = \frac{k_b^i}{k_b^j}$$

*is a **batch balance ratio** and*

$$Br_g^{i,j} = \frac{k_g^i}{k_g^j}$$

*is a **global balance ratio** between class i and j in the whole dataset.*

Similar to the imbalance factor, a small value of the balance ratio means more balance between class i and j during the learning.

Proposition 1. ***K-global positive selection*** *reduces the balance ratio more efficiently compared with k-positive selection. So with all pairs of (i, j) that $|X_i| \geq |X_j|$:*

$$Br_b^{i,j} \geq Br_g^{i,j} \tag{2}$$

where $|X_i|$,$|X_j|$ are the length of class i, j in the training data, and $Br_b^{i,j}, Br_g^{i,j}$ are the balance ratio between class (i, j) in the batch and on the whole data, respectively.

Proof. To prove the above theory, expanding the InEq. 2 we have:

$$\frac{\frac{|B_i|!}{k!(|B_i|-k)!}}{\frac{|B_j|!}{k!(|B_j|-k)!}} \geq \frac{\frac{|X_i|!}{k!(|X_i|-k)!}}{\frac{|X_j|!}{k!(|X_j|-k)!}} \tag{3}$$

where:

$$\frac{M!}{(M-k)!} = M * (M-1) * ... * (M-k+1)$$

Therefore, we need to solve with all k that:

$$\frac{|B_i| - k}{|B_j| - k} \geq \frac{|X_i| - k}{|X_j| - k} \tag{4}$$

Replace $B_i = \frac{|X_i|}{t}, B_j = \frac{|X_j|}{t}, |X_i| = m, |X_j| = n$, $m = xn$, $h = k - 1$, we achieve a new form need to solve:

$$\frac{\frac{xn}{t} - h}{\frac{n}{t} - h} \geq \frac{xn - h}{n - h} \tag{5}$$

Expanding the InEq. 5 we achieve:

$$nh(t-1)(x-1) \geq 0 \tag{6}$$

which is True. □

3.2 Adaptive temperature

In this section, we discuss the gradient of the contrastive learning model, which has played an important role in classical machine learning to mitigate the imbalance effects Tang et al. (2020); Ren et al. (2020). However, this result has not been explored in the contrastive setting. Therefore, based on the previous work by Wang & Liu (2021), we investigate its behavior in the context of the contrastive model, and introduce a technique called **adaptive temperature** based on temperature value, which can effectively reduce the magnitude difference of gradients between head and tail classes.

Gradient analysis Normalizing the gradient between head and tail classes has received much attention in cross-entropy learning (Tan et al., 2021). However, in contrastive learning, this problem has not been exploited much. As introduced in Khosla et al. (2020) with input x_i and embedding function $f()$, the gradient of embedding vector $z_i = f(x_i)$ is defined:

$$\frac{\partial L_i^{sup}}{\partial z_i} = \frac{1}{\tau} \left(\sum_{p \in P(i)} z_p \left(P_{ip} - \frac{1}{|P(i)|} \right) + \sum_{n \in N(i)} z_n P_{in} \right) \tag{7}$$

Here, L_i^{sup} is the L_{out}^{sup} in (Khosla et al., 2020), which is defined:

$$L_{out}^{sup} = -\frac{1}{|P(i)|}\sum_{p\in P(i)} \log \frac{\exp z_i \cdot z_p/\tau}{\sum_{a\in A(i)} \exp z_i \cdot z_a/\tau}$$

τ is a temperature parameter, $P(i)$ is a set of all positive samples of z_i, $N(i)$ is a set of all negative samples, and $P_{ip} = \exp{(z_i \cdot z_p/\tau)}/\sum_{a\in A(i)} \exp{(z_i \cdot z_a/\tau)}$ where $A(i)$ is a set of all samples in the batch excluding z_i.

In each batch, the gradient of a sample i will be equal to the gradient of a positive sample: $P(i)$ plus the gradient of negative samples $N(i)$ and then divide by temperature τ. For easier analysis, set:

$$\frac{\partial L_i^{sup}}{\partial z_i^+} = \sum_{p\in P(i)} z_p \left(P_{ip} - \frac{1}{|P(i)|}\right) \tag{8}$$

$$\frac{\partial L_i^{sup}}{\partial z_i^-} = \sum_{n\in N(i)} z_n P_{in} \tag{9}$$

Now we can rewrite the Eq. 7 as the combination of positive gradient and negative gradient as:

$$\frac{\partial L_i^{sup}}{\partial z_i} = \frac{1}{\tau}\left(\frac{\partial L_i^{sup}}{\partial z_i^+} + \frac{\partial L_i^{sup}}{\partial z_i^-}\right) \tag{10}$$

Applying the above formulas to GloCo, we will have $|P(i)| = k$, which means that the number of positive samples for each query x_i is fixed. The gradient of positive samples in Eq. 8 will be recalculated as follows:

$$\frac{\partial L_i^{sup}}{\partial z_i^+} = \sum_{p\in P(i,k)} z_p \left(\frac{\exp{(z_i \cdot z_p/\tau)}}{\sum_{a\in A(i)} \exp{(z_i \cdot z_a/\tau)}} - \frac{1}{k}\right)$$

where $P_{i,k}$ is the set of k positive samples across batches of query x_i. Thus, choosing only k positive samples during training will make the positive gradient of samples between different classes more balanced. This is another explanation for why KCL (Kang et al., 2020) works well on imbalanced data. However, as mentioned in Eq. 10, the gradient of embedding z_i is a combination of positive and negative gradients. Choosing k positive samples helps to balance the positive gradient but does not balance the negative gradient. Expanding Eq. 9 for the negative gradient we have:

$$\frac{\partial L_i^{sup}}{\partial z_i^-} = \sum_{n\in N(i)} z_n \left(\frac{\exp{(z_i \cdot z_n/\tau)}}{\sum_{a\in A(i)} \exp{(z_i \cdot z_a/\tau)}}\right) \tag{11}$$

From this formula, when the negative sample is similar to the query (hard negative), its gradient becomes larger. Therefore the negative gradient will be contributed mainly from the hard negative samples. Besides, we can divide hard negative samples into two types: false negative (samples have the same class as x_i but are considered negative) and true negative (similar samples of other classes). We have:

$$\frac{\partial L_i^{sup}}{\partial z_i^-} = \partial hard_{negative}^{false} + \partial hard_{negative}^{true} \tag{12}$$

In Eq. 12, while the true negative does not depend on the imbalance of the data, the false negative will be proportional to the imbalance factor, since classes with more samples (head class) will have more false negatives (because we just choose k positive samples for training, then the remained samples are considered as negative samples), resulting in the negative gradient of the head class being much larger than that of the tail classes, making training more difficult. Therefore, if we can remove the influence of false hard negatives, it will help the learning become more stable and balanced.

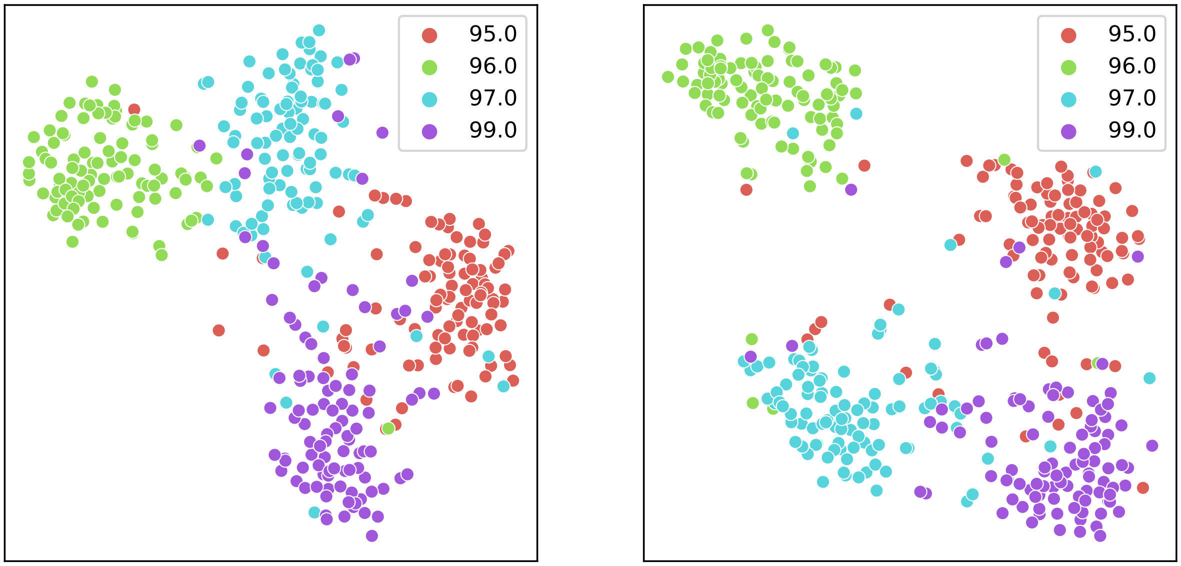

Figure 2: Feature space of four tail classes when using constant temperature (left) and adaptive temperature (right). Adaptive temperature helps the model learn more separable feature space.

Adaptive temperature formulation Work by Wang & Isola (2020); Wang & Liu (2021) has shown that when the value of τ is small, the model will focus only on points near x_i (hard negative), and as τ increases, the model will focus more on further points. Especially, when τ is large enough, the model considers all points (normal negative and hard negative) to contribute equally. Therefore in head classes, to reduce the influence of false-hard negative samples, we should increase the value of τ and vice versa, for tail classes, because the number of false-hard negatives is not large, the reduction of τ will not affect the negative gradient too much. Besides, τ can be viewed as a re-weighted parameter in Eq. 7, where tail classes with small τ will enlarge their gradient and vice versa. Based on these observations, instead of using temperature as a constant parameter. We can assign different temperature values to samples of different classes with the property that head classes will have large τ values and tail classes will have smaller τ values. To ensure the above property, we propose a formula where the temperature of each class will be proportional to the number of samples of that class:

$$\tau_i = \gamma + (1-\gamma) \cdot \frac{\mathrm{frequent(class_i)}}{\mathrm{frequent(class_{max})}} \tag{13}$$

where τ_i is the temperature of class i, γ is the minimum value of temperature, $frequent(class_i)$ is the number of samples in class i, and $frequent(class_{max})$ is the number of samples of largest class.

3.3 Prototype Learning via Coreset (ProCo)

Previous works (Li et al., 2020; Zhu et al., 2022) determine prototype as a mean representation of feature space for each class. During the training, these prototypes are viewed as positive samples in the contrastive loss. Therefore, prototype training is similar to clustering (Li et al., 2020), where it pulls samples in the same class closer to its prototype(the role of a prototype here is like a centroid of class) while pushing samples from

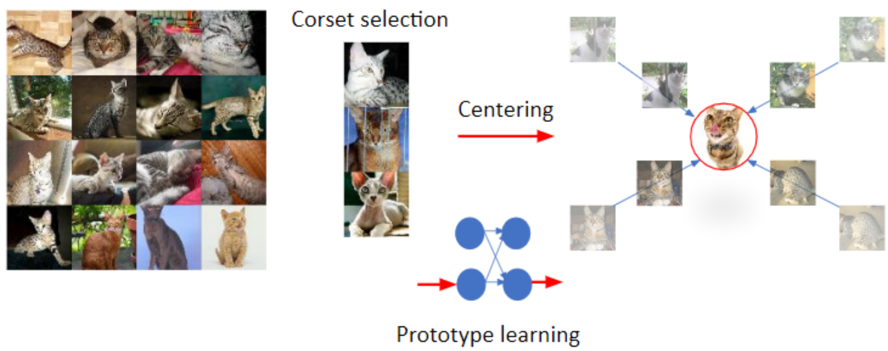

Figure 3: Prototype learning via coreset selection moves images closer to their centers. This will help the samples in the same class be closer together, thus increasing semantic discrimination feature space.

another class far away. Recently, a few applications using prototype learning have been proposed to mitigate imbalanced data problems. TSC (Li et al., 2022b) re-balance feature space by pre-define balanced targets, then move samples in class close to their assigned balanced prototypes based on the matching algorithm. BCL (Zhu et al., 2022) analyzes the behavior of supervised contrastive learning, then they propose two new concepts named: class-averaging, and class-complement based on prototypes to mitigate the imbalance effect in supervised contrastive learning. In this paper, we propose another approach of prototype learning. Firstly, we tackle the trade-off between global prototype representation and computation cost (the total running time for calculating prototype), our method achieves global mean representation for each class while saving a lot of computation cost due to the estimation mean being conducted on the coreset (the set contains important samples of the training data, where its size is significantly smaller than the training set). This makes our prototypes globally represent the feature in data without using all training samples. Secondly, we propose a new prototype loss function and empirically show our loss function can work well on imbalanced data and improve the contrastive learning model a lot.

Coreset selection for calculating prototype The main challenge in prototype learning is how can we efficiently calculate prototypes, both in computation time and representation ability. Previous works determine prototypes from the momentum queue feature in MoCo (Li et al., 2020; 2022b), which can yield consistent prototypes with low computation costs. However, this method can not adapt to another contrastive learning model which does not use a large queue size (Chen et al., 2020a). Besides, when the dataset size is extremely large (Krizhevsky et al., 2017), calculating the prototypes based on samples in the queue can yield local prototypes. To solve the above limitations, we distill the original dataset to extract its most important samples, then we will calculate prototypes according to this set. Our algorithm is as follows: we first use coreset generation algorithm (Killamsetty et al., 2021) as a data distillation method to extract the coreset. During the training, we update our prototypes based on the coreset at the beginning of each epoch. Calculating prototypes from the coreset help centers globally represent their class, while can easily adapt to any framework and dataset size.

Prototype loss function After having a coreset, we augment two views of it and calculate two prototype sets for every two views. Then we define a symmetric prototype loss function, based on contrastive learning loss. This loss function uses the calculated prototypes as the positive sample for a query having the same class label. Assume each query v_i augment two views $v^+_{i,1}, v^+_{i,2}$. We define our loss as the contrastive loss between each pair of the query sample and its prototypes:

$$-\frac{1}{4N}\sum_{i=1}^{N}\sum_{v_+\in\{v^+_{i,1},v^+_{i,2}\}}\sum_{q\in\{1,2\}}\log\frac{e^{\frac{v_+\times c_{i,q}}{\tau}}}{e^{\frac{v_+\times c_{i,q}}{\tau}}+\sum_{c\in C^-_q}e^{\frac{v_+\times c}{\tau}}},$$

where $c_{i,q}$ is the prototype (center) of query i in coreset q, v_i is calculated from coreset q, and C_q^- are the list prototypes from coreset q excluding $c_{i,q}$. Our proposed prototype formulation is different from others (Li et al., 2022b; Zhu et al., 2022) in its symmetric loss between different views of query and coreset, which can help the model learn more stability.

Join training GloCo and ProCo The final loss function will be the weighted combination between GloCo and Proco, which is defined as:

$$L_{final} = \alpha L_{GloCo} + (1-\alpha) L_{ProCo}$$

where α is the hyperparameter control of the contribution between GloCo and ProCo during training, and it needs to tune during the training.

4 Analysis

To further understand the benefit of our model, we extensively evaluate several aspects of the contrastive learning model based on two most important characteristics: Alignment and Uniformity. Besides, we discuss the limitations of the current formula for calculating Intra Alignment and Inter Uniformity, then introduce an improved version that can calculate more efficiently. Finally, we evaluate these characteristics in our models to further understand their behavior when learning on long-tailed data.

Intra Alignment This metric is the average distance between all samples in the same class on feature space (Li et al., 2022b). It is similar to the variance of class, meaning when the value of alignment is small, samples in class lie close together, and the sample extends larger when alignment is large. The alignment formula is defined as:

$$\text{Intra} = \frac{1}{C} \sum_{i=1}^{C} \frac{1}{\|F_i\|^2} \sum_{v_j, v_k \in F_i} \|v_j - v_k\|^2.$$

This formula is a good choice to estimate the spread out of samples in a class. However, it takes a lot of time to calculate. When a class has n samples, the time complexity of Intra Alignment is $O(n^2)$, so when the number of samples in a class is large, it is computationally expensive to calculate Intra Alignment. To overcome this issue, we can use class Variance here, it is similar to Intra-Alignment in the measurement but save computation cost from $O(n^2)$ to $O(n)$. We first need to construct centers for each class, then calculate the variance by:

$$\text{Variance} = \frac{1}{C} \sum_{i=1}^{C} variance_i$$

In this formula, $variance_i$ is the variance of class i and can be calculated by:

$$variance_i = \frac{1}{F_i} \sum_{v_j \in F_i} \|v_j - c_i\|^2$$

where c_i is the center of class i.

Inter Uniformity Different from Intra Alignment, Inter Uniformity is used to measure the distance between classes (Li et al., 2022b). It is the total distance between the centers of all classes. And when inter uniformity is large, it means feature space is more separable, and vice versa, the original formula of inter uniformity is defined as:

$$\text{Inter} = \frac{1}{C(C-1)} \sum_{i=1}^{C} \sum_{j=1, j \neq i}^{C} \|c_i - c_j\|^2.$$

Metric		CIFAR-10-LT			CIFAR-100-LT		
		Imbalance factor					
		100	50	10	100	50	10
Intra	KCL	0.43	0.42	0.42	0.55	0.57	0.45
	Ours	**0.41**	**0.41**	**0.39**	**0.44**	**0.43**	**0.41**
Variance	KCL	0.65	0.62	0.61	0.84	0.85	0.63
	Ours	**0.61**	**0.60**	**0.57**	**0.66**	**0.65**	**0.60**
Inter	KCL	0.70	0.75	0.83	0.90	0.95	0.93
	Ours	**0.73**	**0.79**	**0.85**	**0.98**	**1.00**	**0.98**
Improved Inter	KCL	1.10	1.25	2.03	1.91	2.00	2.31
	Ours	**1.20**	**1.29**	**2.20**	**2.23**	**2.36**	**2.41**

Table 1: We compare different metrics on Cifar-10-LT and Cifar-100-LT. The first row shows the baseline results (KCL) and the second row shows our model results. GloCo + ProCo improves alignment and uniformity characteristics.

Model	CIFAR-10-LT			CIFAR-100-LT		
	Imbalance factor					
	100	50	10	100	50	10
KCL	12.0	14.0	21.0	14.5	15.5	22.5
Global selection	28.5	34.0	50.5	34.0	37.5	57.5
Adaptive temperature	12.5	14.5	22.0	15.5	16.5	22.5
ProCo	22.0	25.0	35.0	26.0	27.5	36.0

Table 2: The running time (in seconds) of different settings for one training epoch. Global selection increases the time proportionally to the number of k (we choose k=3 in our model) positives, while ProCo adds extra time to update the class centers after each epoch and compute the prototype loss.

This formula still remains a problem: it does not use the alignment information, so sometimes the Inter Uniformity measure is not correct. The obvious case to point that Inter Uniformity is wrong is when we have two pairs of classes, which have the same center distance, but in the first pair each class has a large variance, and in the second pair each class has a low variance. The Inter Uniformity of the second pair should be larger because the boundary between classes is larger, but with the above formula, it will return the same result. Therefore, we modify the original uniformity formula by adding variance information to solve the above problem, the updated formula is:

$$\text{Inter}_{\text{improve}} = \frac{2}{C(C-1)} \sum_{i=1}^{C} \sum_{j=1, j\neq i}^{C} \frac{\|c_i - c_j\|^2}{|\text{variance}_i + \text{variance}_j|}.$$

The above formula estimates the uniformity of data better when combined with alignment information. Table 1 shows the effectiveness of this formula, while the original uniformity formula indicates that GloCo will have higher uniformity than GloCo + ProCo, its improved version yield a more consistent result that GloCo + ProCo gives a higher uniformity. The results from the improved uniformity formula are more reasonable because prototype learning helps the model learn more separable feature space, therefore increasing its uniformity. We will provide a detailed example of this problem in the supplemental section.

Table 3: The top-1% acc run on Cifar-10-LT and Cifar-100-LT with Resnet-32 backbone. In this table, GloCo + ProCo dominates the accuracy on all experiments, and gives a large margin with the previous state-of-the-art model, especially on Cifar-100, it increases 3% This impression result shows that our model can be viewed as a new baseline for contrastive learning in long-tailed recognition.

Method	CIFAR-10-LT			CIFAR-100-LT		
	Imbalance factor					
	100	50	10	100	50	10
CE	70.4	74.8	86.4	38.3	43.9	55.7
CB-CE (Cui et al., 2019)	72.4	78.1	86.8	38.6	44.6	57.1
Focal (Lin et al., 2017)	70.4	76.7	86.7	38.4	44.3	55.8
CE-DRW (Cao et al., 2019)	75.1	78.9	86.4	40.5	44.7	56.2
LDAM (Cao et al., 2019)	73.4	76.8	87.0	39.6	45.0	56.9
LDAM-DRW (Cao et al., 2019)	77.0	80.9	88.2	42.0	46.2	58.7
M2m-LDAM (Kim et al., 2020)	79.1	-	87.5	43.5	-	57.6
PCL (Cui et al., 2021)	-	-	-	52.0	56.0	64.2
BCL (Zhu et al., 2022)	84.3	87.2	91.1	51.9	56.6	64.9
KCL (Kang et al., 2020)	77.6	81.7	88.0	42.8	46.3	57.6
TSC (Li et al., 2022b)	79.7	82.9	88.7	43.8	47.4	59.0
GloCo	81.4±0.1	84.5±0.4	88.9±0.1	45.4±0.1	49.5±0.1	60.7±0.1
GloCo + ProCo	**81.6**±0.3	**85.5**±0.1	**89.4**±0.2	**46.0**±0.2	**50.4**±0.2	**60.8**±0.1

Table 4: The top-1% acc run on Imagenet-LT. Here KCL† means the baseline model that we have reproduced

Method	Many	Medium	Few	All
OLTR (Liu et al., 2019)	35.8	32.3	21.5	32.2
LWS (Kang et al., 2019)	57.1	45.2	29.3	47.7
PCL (Cui et al., 2021)	-	-	-	57.0
BCL (Zhu et al., 2022)	-	-	-	56.0
KCL (Kang et al., 2020)	61.8	49.4	30.9	51.5
TSC (Li et al., 2022b)	63.5	49.7	30.4	52.4
KCL†	59.5	50.3	38.5	49.5
Ours	59.0	49.5	40.9	49.8

5 Experiments

5.1 Dataset and implementation details

We conduct experiments on long-tailed recognition with different datasets, including Cifar-10-LT and Cifar-100-LT.[1] Similar to TSC (Li et al., 2022b), we use Mocov2 here with the same configurations in TSC: batch size 256, initial learning rate 0.1, SGD optimizer with momentum 0.9, and we train all the models for 1,000 epochs. The backbone is similar to other baseline models on Cifar data: ResNet32. After pre-training on our framework, we fine-tune the top-classifier layer of the pre-trained model with LDAM loss and Reweight for Cifar-10 and CE loss and Reweight for Cifar-100. Both of them use the same learning rate of 0.1 and batch size of 256. The result shows that our model can outperform the current state-of-the-art model on all datasets and imbalance factors by a large margin.

5.2 Result

[1]For each setting, we evaluate it with 3 seeds and report its accuracy and standard deviation. The results of other methods extracted from TSC (Li et al., 2022b) do not have a standard deviation.

Table 5: The accuracy of GloCo with different loss types. Unsupervised contrastive loss outperformance its supervised counterparts

Loss type	CIFAR-100-LT		
	Imbalance factor		
	100	50	10
Supervised	43.7 ±0.1	48.6 ±0.2	57.9 ±0.1
K-positive(KCL)	44.3 ±0.1	47.8 ±0.1	58.7 ±0.2
Unsupervised	**45.4** ±0.1	**50.4** ±0.2	**60.8** ±0.1

Cifar-10-LT and Cifar-100-LT Table 3 shows our model outperforms the current state-of-the-art model on both datasets with different imbalance factors. In particular, on Cifar-100, the model increased by approximately 3%, compared to the best model in the same group: TSC (Li et al., 2022b) (50.4% vs 47.4%). This improvement comes from the efficient extraction of positive samples from the k-global positive selection and unsupervised loss function, helping the model learn balance and semantic features. Besides, GloCo when training alone can still outperformance TSC (Li et al., 2022b) (k-positive + prototype learning). This points out that GloCo can become a stronger baseline in contrastive learning when solving long-tailed problems.

ImageNet-LT In table.4, We compare the performance of our model with other baselines on the ImageNet-LT dataset. Our model achieves a slight improvement of 0.3% over the baseline (KCL). This modest gain is due to the fact that we have not performed cross-validation to select the optimal hyperparameters for the ImageNet-LT dataset. Instead, we use the same hyperparameters as on the CIFAR-100-LT dataset. We expect that fine-tuning our model on ImageNet-LT will lead to a more significant improvement.

5.3 Ablation study

Running time of different setting Contrastive learning models can benefit from adding k-global selection or ProCo, but these modules also increase the running time. To better understand our architecture and apply it to different scenarios, we measure the running time of the baseline model when trained jointly with these modules. The detailed result is shown in Table.2

How each module contributes to the baseline We evaluate the contribution of each module by combining it with the baseline model: k-global selection, adaptive temperature, and prototype learning. Training with these modules individually improves learning accuracy. The table6. shows the detailed results.

How different loss functions change the model performance In the k-positive selection, sampling positive samples in the queue of MoCo leads to a trade-off between equalizing positive samples across classes, and semantic discrimination of the learned features. Reducing the value of k makes the model learn more balanced features but reduces the quality of the model, and vice versa. In GloCo, to avoid this trade-off, we construct two independent modules: while k-global positive selection has the role of learning semantic discrimination features, an unsupervised contrastive loss is responsible that a learned space balanced between classes. This learning strategy helps the model learn more efficiently. In table 5, we show the behavior of GloCo when using three different loss functions: unsupervised, supervised, and supervised contrastive with k-positive selection. While the model learns with unsupervised loss(the default loss function in GloCo) has the best accuracy, combining it with k-positive loss yields an accuracy higher than the supervised setting. Besides, GloCo + k-positive loss outperforms the original k-positive version with a large gap (1.3% - 1.5%). This indicates that GloCo can become a strong baseline for other methods in long-tailed learning.

6 Conclusion and Future work

Conclusion In this paper, we have overviewed the current works on long-tailed data with contrastive learning, both its achievements and limitations. After that, we introduced two new methods named GloCo

Table 6: The accuracy of Adaptive temperature and Proco alone on Cifar-100-LT dataset. Each of these modules independently improves the accuracy of the model. Therefore, it is possible to combine various model architectures with one of these techniques: k-global selection, adaptive temperature, or Proco to improve their performance.

Loss type	CIFAR-100-LT		
	Imbalance factor		
	100	50	10
Adaptive	43.7 ±0.1	48.6 ±0.2	57.9 ±0.1
Proco	44.3 ±0.1	47.8 ±0.1	58.7 ±0.2

and ProCo which can improve the training of contrastive learning models for long-tailed data in different ways. In each method, we create a stronger model based on utilizing the advantages of the previous method while improving its limitation. Then we conduct a variety of experiments to highlight the contribution of each method. Finally, we reviewed the previous evaluation metric in contrastive learning, discuss its remaining problems, and propose our improvement.

Future work Our model solely tackles data imbalance based on the general characteristics of contrastive learning. It is necessary to build a robust baseline model like GloCo. However, combining contrastive learning with other methods to mitigate data imbalance such as class sampling, class re-weighting, logit adjustment, etc. has not been explored enough. Therefore, it is necessary to include the above methods in GloCo to have both theoretical and experimental observations.

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