STATISTICAL PROPERTY TESTING FOR GENERATIVE MODELS

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ABSTRACT

Generative models that produce image, text, or various sorts of other data are recently gaining more and more capabilities. But in some use cases of the generated data (e.g. using it for model training) we need to ensure that the synthetic datapoints will satisfy some properties, that make them suitable for the intended use, and can justify their usage. Towards this goal, in this work we present a simple framework for statistically checking if the data produced by a generative model satisfy some property with a given confidence level. We apply our methodology on standard image, as well as on image/text generative models.

1 INTRODUCTION & RELATED WORK

Generative models have achieved a remarkable progress in the last 4-5 years. First, models such as Generative Adversarial Networks (GANs) Goodfellow et al. (2020) or Variational Autoencoders (VAE) Kingma et al. (2019) have managed to create artificial images that are very natural - looking. Moreover, there have been extensions of GANs and VAEs for more specifi use cases, such as for creating synthietic data of given sorts, for generating images with given styles etc. More recently, Diffusion Models Ho et al. (2020) have matched or surpassed GANs in performance.

Beyond images, very recently, multi-modal generative models have been developed, for example for NLP applications Brown et al. (2020), or for creating artificial images based on text prompts Ramesh et al. (2021); Rombach et al. (2021).

However, depending on the use-case of the generated data, it might be necessary to make sure that the generated data satisfies some given properties of interest. For example, in the case where we want to use synthetic data for training other Machine Learning (ML) models on them, we need to ensure e.g. that the generated data is realistic, or that for example, synthetic images of cars in Autonomous Driving indeed contain cars, or that the depicted roads are correct etc.

In this work, we present a simple framework in order to test this, e.g. that the data produced by a generative model satisfy some given property at a specified confidence level. We showcase our method for images generated by GANs, as well as for text to image generative models.

In terms of related work, there is a growing body of work concerned about the safety properties of Deep Neural Networks (DNNs), for example from the perspective of formal methods Huang et al. (2017); Cheng et al. (2020), or statistics Angelopoulos & Bates (2021). However, most of these works focus on deriving safety claims of discrimitative models, and for that assumptions on the underlying data need to be made, .e.g that they are sampled i.i.d. (independently and identically distributed), that they cover the whole operational domain, etc. These assumptions are difficult to verify, as the real-world data distribution is unknown in most cases. But, in the case of generative models, the data distribution is well defined (e.g. by a DNN), and therefore sound statistical claims can be derived.

2 METHODOLOGY

Let G be a generative model that outputs data-points $\mathbf{x} \in \mathbb{R}^d$ based on a distribution $p_G(.)$; we slightly abuse notation and write $\mathbf{x} \sim G$. For example, $\mathbf{x}$ could represent images, text tokens, or other kinds of data. For some generative models (for example text to image), the generated

Table 1: Results: In experiment 1, we test property ϕ_1 on the GTSRB GAN. We generate $n = 500$ stop signs, out of which 351 are correctly recognized. In experiment 2, we test property ϕ_2 on SD. (a) In the 1st case, the results for the easier prompts are shown; we generated $n_1 = 300$ prompts, and SD failed to generate a car in 12 cases. (b) In the 2nd experiments we have the results for the harder prompts; we generated $n_1 = 200$ prompts, and SD failed to generate a car in 52 cases.

Experiment:	Hypothesis H_0	$p_0 = 50\%$	$p_0 = 60\%$	$p_0 = 70\%$	$p_0 = 80\%$	$p_0 = 90\%$
Experiment 1	confidence $1 - \delta$	1.00	0.99	0.00	-	-
Experiment 2 (a)	confidence $1 - \delta$	1.00	1.00	1.00	0.99	0.88
Experiment 2 (b)	confidence $1 - \delta$	1.00	0.99	0.42	-	-

samples are conditional to a given prompt, $\mathbf{c} \in \mathbb{R}^k$, and in that case we write $\mathbf{x} \sim G(\mathbf{c})$, with the understanding that the data $\mathbf{x}$ follow a distribution $p_G(.|\mathbf{c})$ conditional to $\mathbf{c}$.

Further, let $\phi : \mathbb{R}^d \to \{0, 1\}$ be a property that we want to verify. We imagine ϕ as a function, that, upon input of a given $\mathbf{x}$, returns 1 (true) or 0 (false), depending whether the property is satisfied or not. In practice, ϕ can be implemented by an oracle, e.g. human inspection, or by a discriminative model that classifies whether its input $\mathbf{x}$ satisfies the property or not.

We want to test the hypothesis that G outputs data that satisfy the property with sufficiently high probability, e.g. we formulate the (null) hypothesis $H_0 = \{p \leq p_0\}$, where $p = P_{mathbfx \sim G}[\phi(\mathbf{x}) = 1]$ is the probability that G's outputs satisfy the property, and p_0 is a required lower bound that we want to achieve. The goal is then to verify that H_0 holds with sufficiently high confidence. From Hoeffding's inequality, we can easily derive the following:

Proposition 1: Let $\mathbf{x}_1, ..., \mathbf{x}_n$ be n i.i.d. samples from G, and put $y_i = \phi(\mathbf{x}_i), i = 1, ..., n$. Further, consider the proportion $\bar{S}_n = (y_1 + ... + y_n)/n$. If $\bar{S}_n \geq p_0$, then the null hypothesis $H_0 = \{p \geq p_0\}$ (as defined above) holds with confidence of at least $1 - \delta$, where $\delta = \exp(-2n(\bar{S}_n - p_0)^2)$.

In the case of conditional generative models, we want to test whether the model's outputs satisfy the given property ϕ (with sufficiently high confidence), when the prompts satisfy the property as well, or comply with it. For example, we want to test whether the generated images contain cars, provided that the prompts are descriptions of cars. Thus, we want to test the conditional probability $p_G(.|\mathbf{c})$ where prompts $\mathbf{c}$ comply with ϕ. For that, Proposition 1 can be used as well.

3 EXPERIMENTS

In our first experiment, we generate artificial traffic sign images using the GAN of Spata et al. (2019), that intend to look similar to the GTSRB dataset Stallkamp et al. (2011). We generate synthetic stop signs, and want to test whether the synthetic images are indeed stop signs. For that, as our property ϕ_1 we use a custom Convolutional Neural Net (CNN) that achieves $\geq p5\%$ accuracy on the GTSRB test set. For various confidence levels we find the following results, shown in Table 1

For our 2nd experiment, we work with the Stable Diffusion (SD) Rombach et al. (2021) text to image generative model. We create $n_1 = 300$ short prompts describing a car (e.g. "A self-driving car navigating a busy intersection."), and test the property ϕ_2 on whether the generated image indeed contains a car. To test ϕ_2, we use an object detection model (YOLOv8 Jocher et al. (2023)) and return true if it detects a car. Further, we do a second round of the experiment, where we create $n_2 = 200$ longer and more complicated prompts. The results are shown in Table 1.

4 CONCLUSION

In this work, we discuss how to statistically test properties of data created by generative models, and obtain sound confidence levels. Beyond the presented approach, many directions for future work are possible. For example, it is not very clear exactly which properties synthetic data should satisfy, if it is to be used for subsequent model training. Another interesting avenue is how one can train generative models in order satisfy a given set of properties with high confidence.

URM STATEMENT

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A DEFERRED PROOFS

Proof (Proposition 1): From Hoeffding's inequality Shalev-Shwartz & Ben-David (2014), if $Y_1, ..., Y_n$ are independent random variables with $a_i \leq Y_i \leq b_i$ almost surely, then, setting $S_n = Y_1 + ... + Y_n$ we have, for all $t > 0$:

$$\mathbb{P}[S_n - \mathbb{E}[S_n] \geq t] \leq \exp\left[-\frac{2t^2}{\sum_{i=1}^{n}(b_i - a_i)^2}\right] \tag{1}$$

In our case, the random variables $Y_i = \phi(X_i), X_i \sim G$ are Bernoulli and bounded in $[0, 1]$, hence $a_i = 0, b_i = 1$ and $\sum_{i=1}^{n}(b_i - a_i)^2 = n$, so eq. 1 becomes:

$$\mathbb{P}[S_n - \mathbb{E}[S_n] \geq t] \leq \exp\left[-\frac{2t^2}{n}\right] \tag{2}$$

Now, we introduce the ratio $\bar{S_n} = S_n/n$, and $\mathbb{E}[\bar{S_n}] = \mathbb{E}[S_n]/n$ respectively. With these, the probability statement on the left-hand side of eq. 2 can be written equivalently as:

$$S_n - \mathbb{E}[S_n] \geq t \Longleftrightarrow \bar{S_n} - \mathbb{E}[\bar{S_n}] \geq t/n \tag{3}$$

Plugging 3 in 2, Setting $t/n = \epsilon$ and solving for ϵ, we get:

$$\mathbb{P}[\bar{S_n} - \mathbb{E}[\bar{S_n}] \geq \epsilon] \leq \exp[-2n\epsilon^2] \tag{4}$$

Next, since Y_i are Bernoullis with (unknown) probability p, we have that $\mathbb{E}[\bar{S_n}] = p$, which gives us:

$$\mathbb{P}[\bar{S_n} - p \geq \epsilon] \leq \exp[-2n\epsilon^2] \tag{5}$$

Finally, write the statement inside the probability equivalently as $p \leq \bar{S_n} - \epsilon$, and set $\bar{S_n} - \epsilon = p_0$, which gives:

$$\mathbb{P}[p \leq p_0] \leq \exp[-2n\epsilon^2] \tag{6}$$

Finally, eliminate ϵ on the right-hand side by replacing it with $\epsilon = \bar{S_n} - p_0$, and call the right-hand side δ. This gives:

$$\mathbb{P}[p \leq p_0] \leq \exp[-2n(\bar{S_n} - p_0)^2] := \delta \tag{7}$$

Therefore, for the opposite statement, we get $\mathbb{P}[p \geq p_0] \geq 1 - \delta$, as required. □

B EXPERIMENTAL DETAILS

In this section we present some further experimental details, omitted in the main text.

For experiment 1, we train a CNN with the following architecture to recognize traffic signs:

$RELU(BN(CONV(40))) \rightarrow MaxPool \rightarrow RELU(BN(CONV(20))) \rightarrow MaxPool \rightarrow RELU(fc(240)) \rightarrow RELU(fc(84)) \rightarrow fc(43)$

Convolutional layers ($CONV$) have (5,5) kernel and stride 1, while Max Pooling ($MaxPool$) layers are 2x2. fc denotes fully-connected layers, BN batch normalization, and $RELU$ is the ReLU activation function. We train this CNN with the Adam optimizer for 10 epochs and learning rate 10^{-3}, until it achieves an accuracy 96% on the GTSRB test-set.

For experiment 2, we use SD version 1.4 from the official repository. For the prompts, we generate them with chatGPT Brown et al. (2020) and then inspect them manually, to check that they are all correct, e.g. contain descriptions of cars.

In experiment 2a, we generate short prompts of a few words, such as the examples below:

- "A red sports car speeding down a coastal highway."
- "A silver Porsche driving through a snowy landscape."
- "A red Audi sports car driving through the countryside."
- "A black Ford F-150 parked in front of a construction site."

For experiments 2b, we generate longer prompts, that intend to be more complicated and contain several objects (besides cars). Some examples are the following:

- "Two cars driving on a narrow mountain road with sharp turns, with a mountain biker in the middle of the road."
- "A car driving through a city during a carnival, with elaborate costumes and floats parading down the street."
- "A car driving through a city during a technology conference, with keynote speakers and tech demos."
- "A car driving through a small town during a peach festival, with peach pie and other peach-themed treats."

Below we show the generated images for the above randomly selected prompts.

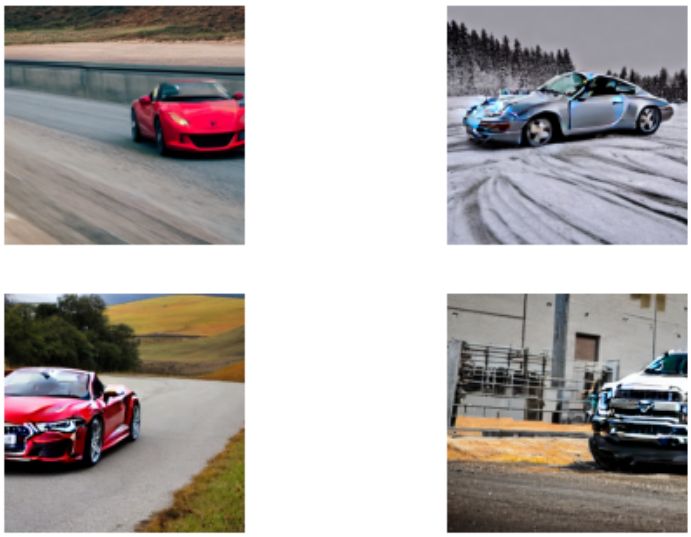

Figure 1: Generated images for the above random prompts from experiment 2a.

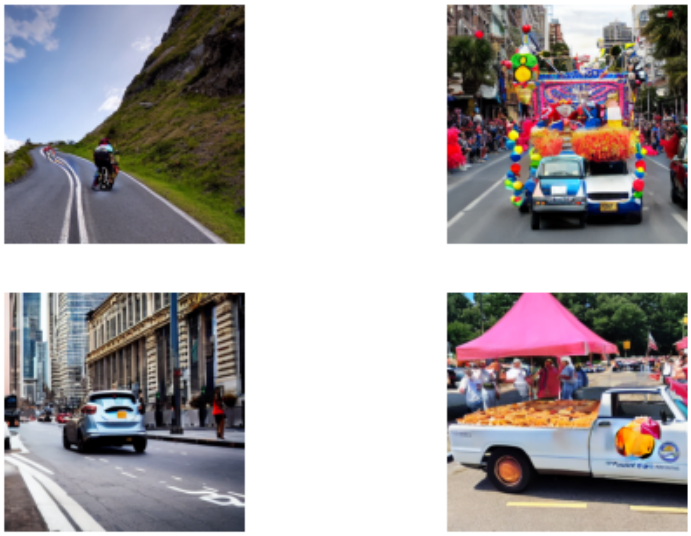

Figure 2: Generated images for the above random prompts from experiment 2b.