EGSDE: Unpaired Image-to-Image Translation via Energy-Guided Stochastic Differential Equations

Abstract

Score-based diffusion generative models (SDGMs) have achieved the SOTA FID results in unpaired image-to-image translation (I2I). However, we notice that existing methods totally ignore the training data in the source domain, leading to sub-optimal solutions for unpaired I2I. To this end, we propose energy-guided stochastic differential equations (EGSDE) that employs an energy function pretrained on both the source and target domains to guide the inference process of a pretrained SDE for realistic and faithful unpaired I2I. Building upon two feature extractors, we carefully design the energy function such that it encourages the transferred image to preserve the domain-independent features and discard domain-specific ones. Further, we provide an alternative explanation of the EGSDE as a product of experts, where each of the three experts (corresponding to the SDE and two feature extractors) solely contributes to faithfulness or realism. Empirically, we compare EGSDE to a large family of baselines on three widely-adopted unpaired I2I tasks under four metrics. EGSDE not only consistently outperforms existing SDGMs-based methods in almost all settings but also achieves the SOTA realism results (e.g., FID of 65.82 in Cat $\rightarrow$ Dog and FID of 59.75 in Wild $\rightarrow$ Dog on AFHQ) without harming the faithful performance.

1 Introduction

Unpaired image-to-image translation (I2I) aims to transfer an image from a source domain to a related target domain, which involves a wide range of computer vision tasks such as style transfer, super-resolution and pose estimation [32]. In I2I, the translated image should be *realistic* to fit the style of the target domain by changing the domain-specific features accordingly, and *faithful* to preserve the domain-independent features of the source image. Over the past few years, generative adversarial networks [12] (GANs)-based methods [10, 51, 47, 33, 3, 49, 39, 18, 17, 25, 10] dominated this field due to their ability to generate high-quality samples.

In contrast to GANs, score-based diffusion generative models (SDGMs) [43, 16, 31, 44, 2] perturb data to a Gaussian noise by a diffusion process and learn the reverse process to transform the noise back to the data distribution. Recently, SDGMs achieved competitive or even superior image generation performance to GANs [9] and thus were naturally applied to unpaired I2I [7, 29], which have achieved the state-of-the-art FID [13] and KID [4] results empirically. However, we notice that these methods *did not leverage the training data in the source domain at all*. Indeed, they trained a diffusion model solely on the target domain and exploited the test source image during inference (see details in Sec. 2.2). Therefore, we argue that if the training data in the source domain can be exploited together with those in the target domain, one can learn domain-specific and domain-independent features to improve both the realism and faithfulness of the SDGMs in unpaired I2I.

Submitted to 36th Conference on Neural Information Processing Systems (NeurIPS 2022). Do not distribute.

Figure 1: (a) Apart from the SDE, the EGSDE incorporates a realism expert and a faithful expert to preserve the domain-independent features and discard domain-specific ones. (b) Representative translation results on three unpaired I2I tasks.

To this end, we propose *energy-guided stochastic differential equations* (EGSDE) that employs an energy function pretrained across the two domains to guide the inference process of a pretrained SDE for realistic and faithful unpaired I2I. Formally, EGSDE defines a valid conditional distribution via a reverse time SDE that composites the energy function and the pretrained SDE. Ideally, the energy function should encourage the transferred image to preserve the domain-independent features and discard domain-specific ones. To achieve this, we introduce two feature extractors that learn domain-independent features and domain-specific ones respectively, and define the energy function upon the similarities between the features extracted from the transferred image and the test source image. Further, we provide an alternative explanation of the discretization of EGSDE in the formulation of *product of experts* [15]. In particular, the pretrained SDE and the two feature extractors in the energy function correspond to three experts and each solely contributes to faithfulness or realism.

Empirically, we validate our method on the widely-adopted AFHQ [8] and CeleA-HQ [19] datasets including Cat → Dog, Wild → Dog and Male → Female tasks. We compare to a large family of baselines, including the GANs-based ones [33, 51, 17, 25, 3, 10, 49, 50] and SDGMs-based ones [7, 29] under four metrics (e.g., FID). EGSDE not only consistently outperforms the direct competitor [7, 29] in almost all settings but also achieves the SOTA realism results (FID of 65.82 in Cat → Dog and FID of 59.75 in Wild → Dog on AFHQ) without harming the faithful performance. Furthermore, our method can flexibly trade off realism and faithfulness by tuning weighting hyper-parameters in the energy function and be extended to multi-domain translation easily.

2 Background

2.1 Score-based Diffusion Generative Models

Score-based diffusion generative models (SDGMs) gradually perturb data by a forward diffusion process, and then reverse it to recover the data [44, 2, 42, 16, 9]. Let $q(\boldsymbol{y}_0)$ be the unknown data distribution on $\mathbb{R}^D$. The forward diffusion process $\{\boldsymbol{y}_t\}_{t \in [0,T]}$, indexed by time t, can be represented by the following forward SDE:

$$d\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{y}, t)dt + g(t)d\boldsymbol{w}, \tag{1}$$

where $\boldsymbol{w} \in \mathbb{R}^D$ is a standard Wiener process, $\boldsymbol{f}(\cdot, t) : \mathbb{R}^D \to \mathbb{R}^D$ is the drift coefficient and $g(t) \in \mathbb{R}$ is the diffusion coefficient. The $f(\boldsymbol{y}, t), g(t)$ is related into the noise size and determines

the perturbation kernel $q_{t|0}(\boldsymbol{y}_t|\boldsymbol{y}_0)$ from time 0 to t. In practice, the $f(\boldsymbol{y}, t)$ is usually affine so that the the perturbation kernel is a linear Gaussian distribution and can be sampled in one step.

Let $q_t(\boldsymbol{y})$ be the marginal distribution of the SDE at time t in Eq. (1). Its time reversal can be described by another SDE [44]:

$$\mathrm{d}\boldsymbol{y} = [\boldsymbol{f}(\boldsymbol{y}, t) - g(t)^2 \nabla_{\boldsymbol{y}} \log q_t(\boldsymbol{y})]\mathrm{d}t + g(t)\mathrm{d}\overline{\boldsymbol{w}}, \qquad (2)$$

where $\overline{\boldsymbol{w}}$ is a reverse-time standard Wiener process, and $\mathrm{d}t$ is an infinitesimal negative timestep. We wish to solve the reverse SDE in Eq. (2) to generate data, where we need to learn the score function $\nabla_{\boldsymbol{y}} \log q_t(\boldsymbol{y})$. [44] adopts a score-based model $\boldsymbol{s}(\boldsymbol{y}, t)$ to approximate the unknown $\nabla_{\boldsymbol{y}} \log q_t(\boldsymbol{y})$ by score matching, thus inducing a score-based diffusion generative model (SDGM), which is defined by a SDE:

$$\mathrm{d}\boldsymbol{y} = [\boldsymbol{f}(\boldsymbol{y}, t) - g(t)^2 \boldsymbol{s}(\boldsymbol{y}, t)]\mathrm{d}t + g(t)\mathrm{d}\overline{\boldsymbol{w}}. \qquad (3)$$

There are numerous SDE solver to solve the Eq. (3) to generate images. [44] discretizes it using the Euler-Maruyama solver. Formally, adopting a step size of h, the iteration rule from s to $t = s - h$ is:

$$\boldsymbol{y}_t = \boldsymbol{y}_s - [\boldsymbol{f}(\boldsymbol{y}_s, s) - g(s)^2 \boldsymbol{s}(\boldsymbol{y}_s, s)]h + g(s)\sqrt{h}\boldsymbol{z}, \quad \boldsymbol{z} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{I}). \qquad (4)$$

2.2 SDGMs in Unpaired Image to Image Translation

Given unpaired images from the source domain $\mathcal{X} \subset \mathbb{R}^D$ and the target domain $\mathcal{Y} \subset \mathbb{R}^D$ as the training data, the goal of unpaired I2I is to transfer an image from the source domain to the target domain. Such a process can be formulated as designing a distribution $p(\boldsymbol{y}_0|\boldsymbol{x}_0)$ on the target domain $\mathcal{Y}$ conditioned on an image $\boldsymbol{x}_0 \in \mathcal{X}$ to transfer. The translated image should be *realistic* for the target domain by changing the domain-specific features and *faithful* for the source image by preserving the domain-independent features.

ILVR [7] uses a diffusion model on the target domain for *realism*. Formally, ILVR starts from $\boldsymbol{y}_T \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{I})$ and samples from the diffusion model according to Eq. (4) to obtain $\boldsymbol{y}_t$. For *faithfulness*, it further refines $\boldsymbol{y}_t$ by adding the residual between the sample $\boldsymbol{y}_t$ and the perturbed source image $\boldsymbol{x}_t$ through a non-trainable low-pass filter

$$\boldsymbol{y}_t \leftarrow \boldsymbol{y}_t + \Phi(\boldsymbol{x}_t) - \Phi(\boldsymbol{y}_t), \quad \boldsymbol{x}_t \sim q_{t|0}(\boldsymbol{x}_t|\boldsymbol{x}_0), \qquad (5)$$

where $\Phi(\cdot)$ is a low-pass filter and $q_{t|0}(\cdot|\cdot)$ is the perturbation kernel determined by the forward SDE in Eq. (1).

Similarly, SDEdit [29] also adopts a SDGM on the target domain for *realism*, i.e., sampling from the SDGM according to Eq. (4). For *faithfulness*, SDEdit starts the generation process from the noisy source image $\boldsymbol{y}_M \sim q_{M|0}(\boldsymbol{y}_M|\boldsymbol{x}_0)$, where M is a middle time between 0 and T, and is chosen to preserve the original overall structure and discard local details. We use $p_{r1}(\boldsymbol{y}_0|\boldsymbol{x}_0)$ to denote the marginal distribution defined by such SDE conditioned on $\boldsymbol{x}_0$.

Notably, these methods did not leverage the training data in the source domain at all and thus can be sub-optimal in terms of both the realism and faithfulness in unpaired I2I.

3 Method

To overcome the limitations of existing methods [7, 29] as highlighted in Sec. 2.2, we propose *energy-guided stochastic differential equations* (EGSDE) that employs an energy function pre-trained across the two domains to guide the inference process of a pretrained SDE for realistic and faithful unpaired I2I (see Fig. 2). EGSDE defines a valid conditional distribution $p(\boldsymbol{y}_0|\boldsymbol{x}_0)$ by compositing a pretrained SDE and a pretrained energy function under mild regularity conditions[1] as follows:

$$\mathrm{d}\boldsymbol{y} = [\boldsymbol{f}(\boldsymbol{y}, t) - g(t)^2 (\boldsymbol{s}(\boldsymbol{y}, t) - \nabla_{\boldsymbol{y}} \mathcal{E}(\boldsymbol{y}, \boldsymbol{x}_0, t))]\mathrm{d}t + g(t)\mathrm{d}\overline{\boldsymbol{w}}, \qquad (6)$$

where $\overline{\boldsymbol{w}}$ is a reverse-time standard Wiener process, $\mathrm{d}t$ is an infinitesimal negative timestep, $\boldsymbol{s}(\cdot, \cdot) : \mathbb{R}^D \times \mathbb{R} \to \mathbb{R}^D$ is the score-based model in the pretrained SDE and $\mathcal{E}(\cdot, \cdot, \cdot) : \mathbb{R}^D \times \mathbb{R}^D \times \mathbb{R} \to \mathbb{R}$ is the

---

[1] The assumptions are very similar to those in prior work [44]. We list them for completeness in Appendix A.1

Figure 2: The overview of our EGSDE. Starting from the noisy source image, we can run the EGSDE for unpaired I2I, which employs an energy function $\mathcal{E}(\boldsymbol{y}, \boldsymbol{x}, t)$ pretrained on both the source and target domains to guide the inference process of a pretrained SDE ($s(\boldsymbol{y}, t)$, realism expert 1). The energy function is decomposed into two terms further, where the realistic expert 2 $\mathcal{E}_s(\boldsymbol{y}, \boldsymbol{x}, t)$ encourages the transferred image to discard domain-specific features and the faithful expert $\mathcal{E}_i(\boldsymbol{y}, \boldsymbol{x}, t)$ aims to preserve the domain-independent ones.

energy function. The start point $\boldsymbol{y}_M$ is sampled from the perturbation distribution $q_{M|0}(\boldsymbol{y}_M|\boldsymbol{x}_0)$ [29], where $M = 0.5T$ typically. We obtain the transferred images by taking the samples at endpoint $t = 0$ following the SDE in Eq. (6).

Similar to the prior work [7, 29], EGSDE employs an SDE trained solely in the target domain as in Eq. (2), which defines a marginal distribution of the target images and mainly contributes to the realism of the transferred samples. In contrast, the energy function involves the training data across both the source and target domain, making EGSDE distinct from the prior work [7, 29]. Notably, although many other possibilities exist, we carefully design the energy function such that it (approximately) encourages the sample to retain the domain-independent features and discard the domain-specific ones to improve both the faithfulness and realism of the transferred sample. Below, we formally formulate the energy function.

3.1 Choice of Energy

In this section, we show how to design the energy function. Intuitively, during the translation, the domain-independent features (pose, color, *etc.* on Cat $\rightarrow$ Dog) should be preserved while the domain-specific features (beard, nose, *etc.* on Cat $\rightarrow$ Dog) should be changed accordingly. Motivated by this, we decompose the energy function $\mathcal{E}(\boldsymbol{y}, \boldsymbol{x}, t)$ as the sum of two log potential functions [5]:

$$\mathcal{E}(\boldsymbol{y}, \boldsymbol{x}, t) = \lambda_s \mathcal{E}_s(\boldsymbol{y}, \boldsymbol{x}, t) + \lambda_i \mathcal{E}_i(\boldsymbol{y}, \boldsymbol{x}, t) \tag{7}$$

$$= \lambda_s \mathbb{E}_{q_{t|0}(\boldsymbol{x}_t|\boldsymbol{x})} \mathcal{S}_s(\boldsymbol{y}, \boldsymbol{x}_t, t) - \lambda_i \mathbb{E}_{q_{t|0}(\boldsymbol{x}_t|\boldsymbol{x})} \mathcal{S}_i(\boldsymbol{y}, \boldsymbol{x}_t, t), \tag{8}$$

where $\mathcal{E}_i(\cdot, \cdot, \cdot) : \mathbb{R}^D \times \mathbb{R}^D \times \mathbb{R} \to \mathbb{R}$ and $\mathcal{E}_s(\cdot, \cdot, \cdot) : \mathbb{R}^D \times \mathbb{R}^D \times \mathbb{R} \to \mathbb{R}$ are the log potential functions, $\boldsymbol{x}_t$ is the perturbed source image in the forward SDE, $q_{t|0}(\cdot|\cdot)$ is the perturbation kernel from time 0 to time t in the forward SDE, $\mathcal{S}_s(\cdot, \cdot, \cdot) : \mathbb{R}^D \times \mathbb{R}^D \times \mathbb{R} \to \mathbb{R}$ and $\mathcal{S}_i(\cdot, \cdot, \cdot) : \mathbb{R}^D \times \mathbb{R}^D \times \mathbb{R} \to \mathbb{R}$ are two functions measuring the similarity between the sample and perturbed source image, and $\lambda_s \in \mathbb{R}_{>0}, \lambda_i \in \mathbb{R}_{>0}$ are two weighting hyper-parameters. Note that the expectation w.r.t. $q_{t|0}(\boldsymbol{x}_t|\boldsymbol{x})$ in Eq. (7) guarantees that the energy function changes slowly over the trajectory to satisfy the regularity conditions in Appendix A.1.

To specify $\mathcal{S}_s(\cdot, \cdot, \cdot)$, we introduce a time-dependent domain-specific feature extractor $E_s(\cdot, \cdot) : \mathbb{R}^D \times \mathbb{R} \to \mathbb{R}^{C \times H \times W}$, where C is the channel-wise dimension, H and W are the dimension of height and width. In particular, $E_s(\cdot, \cdot)$ is the all but the last layer of a classifier that is trained on both domains to predict whether an image is from the source domain or the target domain. Intuitively, $E_s(\cdot, \cdot)$ will preserve the domain-specific features and discard the domain-independent features for accurate predictions. Building upon it, $\mathcal{S}_s(\cdot, \cdot, \cdot)$ is defined as the cosine similarity between the

features extracted from the generated sample and the source image as follows:

$$\mathcal{S}_s(\boldsymbol{y}, \boldsymbol{x}_t, t) = \frac{1}{HW} \sum_{h,w} \frac{E_s^{hw}(\boldsymbol{x}_t, t)^\top E_s^{hw}(\boldsymbol{y}, t)}{||E_s^{hw}(\boldsymbol{x}_t, t)||_2 \, ||E_s^{hw}(\boldsymbol{y}, t)||_2}, \tag{9}$$

where $E_s^{hw}(\cdot, \cdot) \in \mathbb{R}^C$ denote the channel-wise feature at spatial position (h, w). Here we employ the cosine similarity since it preserves the spatial information and helps to improve the FID score empirically (see Appendix C.1 for the ablation study). Intuitively, reducing the energy value in Eq. (7) encourages the transferred sample to discard the domain-specific features to improve realism.

To specify $\mathcal{S}_i(\cdot, \cdot, \cdot)$, we introduce a domain-independent feature extractor $E_i(\cdot, \cdot) : \mathbb{R}^D \times \mathbb{R} \to \mathbb{R}^D$, which is a low-pass filter. Intuitively, $E_i(\cdot, \cdot)$ will preserve the overall structures (i.e., domain-independent features) and discard local information like textures (i.e., domain-specific features). Building upon it, $\mathcal{S}_i(\cdot, \cdot, \cdot)$ is defined as the negative squared L_2 distance between the features extracted from the generated sample and source image as follows:

$$\mathcal{S}_i(\boldsymbol{y}, \boldsymbol{x}_t, t) = -||E_i(\boldsymbol{y}, t) - E_i(\boldsymbol{x}_t, t)||_2^2. \tag{10}$$

Here, we choose negative squared L_2 distance as the similarity metric because it helps to preserve more domain-independent features empirically (see Appendix C.1 for the ablation study). Intuitively, reducing the energy value in Eq. (7) encourages the transferred sample to preserve the domain-independent features to improve faithfulness. In this paper we employ a low-pass filter for its simpleness and effectiveness while we can train more sophisticated E_i, e.g., based on disentangled representation learning methods [38, 6, 14, 22, 27], on the data in the two domains.

In our preliminary experiment, alternative to Eq. (7), we consider a simpler energy function that only involves the original source image $\boldsymbol{x}$ as follows:

$$\mathcal{E}(\boldsymbol{y}, \boldsymbol{x}, t) = \lambda_s \mathcal{S}_s(\boldsymbol{y}, \boldsymbol{x}, t) - \lambda_i \mathcal{S}_i(\boldsymbol{y}, \boldsymbol{x}, t), \tag{11}$$

which does not require to take the expectation w.r.t. $\boldsymbol{x}_t$. We found that it did not perform well because it is not reasonable to measure the similarity between the noise-free source image and the transferred sample in a gradual denoising process. See Appendix C.2 for empirical results.

3.2 Solving the Energy-guided Reverse-time SDE

Based on the pretrained score-based model $\boldsymbol{s}(\boldsymbol{y}, t)$ and energy function $\mathcal{E}(\boldsymbol{y}, \boldsymbol{x}, t)$, we can solve the proposed energy-guided SDE to generate samples from conditional distribution $p(\boldsymbol{y}_0|\boldsymbol{x}_0)$. There are numerical solvers to approximate trajectories from SDEs. In this paper, we take the Euler-Maruyama solver following [29] for a fair comparison. Given the EGSDE as in Eq. (6) and adopting a step size h, the iteration rule from s to $t = s - h$ is:

$$\boldsymbol{y}_t = \boldsymbol{y}_s - [\boldsymbol{f}(\boldsymbol{y}, s) - g(s)^2(\boldsymbol{s}(\boldsymbol{y}_s, s) - \nabla_{\boldsymbol{y}} \mathcal{E}(\boldsymbol{y}_s, \boldsymbol{x}_0, s))]h + g(s)\sqrt{h}\boldsymbol{z}, \quad \boldsymbol{z} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{I}). \tag{12}$$

The expectation in $\mathcal{E}(\boldsymbol{y}_s, \boldsymbol{x}_0, s)$ is estimated by the Monte Carlo method of a single sample for efficiency. For brevity, we present the general sampling procedure of our method in Algorithm 1. The sampling procedure for the specific variance preserve energy-guided SDE (VP-EGSDE) [44, 16] is explained in Appendix A.3 and is conducted in our experiments. Following SDEdit [29], we further extend this by repeating the Algorithm 1 K times (see details in Algorithm 2 in Appendix A.2).

3.3 EGSDE as Product of Experts

Inspired by the posterior inference process in diffusion models [41], we present a *product of experts* [15] explanation for the discretized sampling process of EGSDE, which formalizes our motivation in an alternative perspective and provides insights on the role of each component in EGSDE.

We first define a conditional distribution $\tilde{p}(\boldsymbol{y}_t|\boldsymbol{x}_0)$ at time t as a product of experts:

$$\tilde{p}(\boldsymbol{y}_t|\boldsymbol{x}_0) = \frac{p_{r1}(\boldsymbol{y}_t|\boldsymbol{x}_0)p_e(\boldsymbol{y}_t|\boldsymbol{x}_0)}{Z_t}, \tag{13}$$

where Z_t is the partition function, $p_e(\boldsymbol{y}_t|\boldsymbol{x}_0) \propto \exp(-\mathcal{E}(\boldsymbol{y}_t, \boldsymbol{x}_0, t))$ and $p_{r1}(\boldsymbol{y}_t|\boldsymbol{x}_0)$ is the marginal distribution at time t defined by SDEdit based on a pretrained SDE on the target domain.

Algorithm 1 EGSDE for unpaired image-to-image translation

Require: the source image $\boldsymbol{x}_0$, the initial time M, denoising steps N, weighting hyper-parameters λ_s, λ_i, the similarity function $\mathcal{S}_s(\cdot, \cdot, \cdot), \mathcal{S}_i(\cdot, \cdot, \cdot)$, the score function $\boldsymbol{s}(\cdot, \cdot)$

$\boldsymbol{y} \sim q_{M|0}(\boldsymbol{y}|\boldsymbol{x}_0)$ # the start point

$h = \frac{M}{N}$

for $i = N$ to 1 **do**

    $s \leftarrow ih$

    $\boldsymbol{x} \sim q_{s|0}(\boldsymbol{x}|\boldsymbol{x}_0)$ # sample perturbed source image from the perturbation kernel

    $\mathcal{E}(\boldsymbol{y}, \boldsymbol{x}, s) \leftarrow \lambda_s \mathcal{S}_s(\boldsymbol{y}, \boldsymbol{x}, s) - \lambda_i \mathcal{S}_i(\boldsymbol{y}, \boldsymbol{x}, s)$ # compute energy with one Monte Carlo

    $\boldsymbol{y} \leftarrow \boldsymbol{y} - [\boldsymbol{f}(\boldsymbol{y}, s) - g(s)^2(\boldsymbol{s}(\boldsymbol{y}, s) - \nabla_{\boldsymbol{y}}\mathcal{E}(\boldsymbol{y}, \boldsymbol{x}, s))]h$ # the update rule in Eq. (12)

    $\boldsymbol{z} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{I})$ if $i > 1$, else $\boldsymbol{z} = \boldsymbol{0}$

    $\boldsymbol{y} \leftarrow \boldsymbol{y} + g(s)\sqrt{h}\boldsymbol{z}$

end for

$\boldsymbol{y}_0 \leftarrow \boldsymbol{y}$

return $\boldsymbol{y}_0$

To sample from $\tilde{p}(\boldsymbol{y}_t|\boldsymbol{x}_0)$, we need to construct a transition kernel $\tilde{p}(\boldsymbol{y}_t|\boldsymbol{y}_s)$, where $t = s - h$ and h is small. Following [41], using the desirable equilibrium $\tilde{p}(\boldsymbol{y}_t|\boldsymbol{x}_0) = \int \tilde{p}(\boldsymbol{y}_t|\boldsymbol{y}_s)\tilde{p}(\boldsymbol{y}_s|\boldsymbol{x}_0)d\boldsymbol{y}_s$, we construct the $\tilde{p}(\boldsymbol{y}_t|\boldsymbol{y}_s)$ as follows:

$$\tilde{p}(\boldsymbol{y}_t|\boldsymbol{y}_s) = \frac{p(\boldsymbol{y}_t|\boldsymbol{y}_s)p_e(\boldsymbol{y}_t|\boldsymbol{x}_0)}{\tilde{Z}_t(\boldsymbol{y}_s)}, \tag{14}$$

where $\tilde{Z}_t(\boldsymbol{y}_s)$ is the partition function and $p(\boldsymbol{y}_t|\boldsymbol{y}_s) = \mathcal{N}(\boldsymbol{\mu}(\boldsymbol{y}_s, h), \Sigma(s, h)\boldsymbol{I})$ is the transition kernel of the pretrained SDE in Eq. (4), i.e., $\boldsymbol{\mu}(\boldsymbol{y}_s, h) = \boldsymbol{y}_s - [\boldsymbol{f}(\boldsymbol{y}_s, s) - g(s)^2\boldsymbol{s}(\boldsymbol{y}_s, s)]h$ and $\Sigma(s, h) = g(s)^2 h$. Assuming that $\mathcal{E}(\boldsymbol{y}_t, \boldsymbol{x}_0, t)$ has low curvature relative to $\Sigma(s, h)^{-1}$, it can be approximated using Taylor expansion around $\boldsymbol{\mu}(\boldsymbol{y}_s, h)$ and further we can obtain

$$\tilde{p}(\boldsymbol{y}_t|\boldsymbol{y}_s) \approx \mathcal{N}(\boldsymbol{\mu}(\boldsymbol{y}_s, h) - \Sigma(s, h)\nabla_{\boldsymbol{y}'}\mathcal{E}(\boldsymbol{y}', \boldsymbol{x}_0, t)|_{\boldsymbol{y}'=\boldsymbol{\mu}(\boldsymbol{y}_s, h)}, \Sigma(s, h)\boldsymbol{I}). \tag{15}$$

More details about derivation are available in Appendix A.4. We can observe the transition kernel $\tilde{p}(\boldsymbol{y}_t|\boldsymbol{y}_s)$ in (15) is equal to the discretization of our EGSDE in Eq. (12). Therefore, solving the energy-guided SDE in a discretization manner is approximately equivalent to drawing samples from a product of experts in Eq. (13). Note that $\mathcal{E}(\boldsymbol{y}_t, \boldsymbol{x}_0, t) = \lambda_s \mathcal{E}_s(\boldsymbol{y}_t, \boldsymbol{x}_0, t) + \lambda_i \mathcal{E}_i(\boldsymbol{y}_t, \boldsymbol{x}_0, t)$, the $\tilde{p}(\boldsymbol{y}_t|\boldsymbol{x}_0)$ can be rewritten as:

$$\tilde{p}(\boldsymbol{y}_t|\boldsymbol{x}_0) = \frac{p_{r1}(\boldsymbol{y}_t|\boldsymbol{x}_0)p_{r2}(\boldsymbol{y}_t|\boldsymbol{x}_0)p_f(\boldsymbol{y}_t|\boldsymbol{x}_0)}{Z_t}, \tag{16}$$

where $p_{r2}(\boldsymbol{y}_t|\boldsymbol{x}_0) \propto \exp(-\lambda_s \mathcal{E}_s(\boldsymbol{y}_t, \boldsymbol{x}_0, t))$, $p_f(\boldsymbol{y}_t|\boldsymbol{x}_0) \propto \exp(-\lambda_i \mathcal{E}_i(\boldsymbol{y}_t, \boldsymbol{x}_0, t))$.

In Eq. (16), by setting $t = 0$, we can explain that the transferred samples approximately follow the distribution defined by the product of three experts, where $p_{r1}(\boldsymbol{y}_t|\boldsymbol{x}_0)$ and $p_{r2}(\boldsymbol{y}_t|\boldsymbol{x}_0)$ are the *realism experts* and $p_f(\boldsymbol{y}_t|\boldsymbol{x}_0)$ is the *faithful expert*, corresponding to the score function $s(\boldsymbol{y}, t)$ and the log potential functions $\mathcal{E}_s(\boldsymbol{y}, \boldsymbol{x}, t)$ and $\mathcal{E}_i(\boldsymbol{y}, \boldsymbol{x}, t)$ respectively. Such a formulation clearly explains the role of each expert in EGSDE and supports our empirical results.

4 Related work

Apart from the prior work mentioned before, we discuss other related work including GANs-based methods for unpaired I2I and SDGMs-based methods for image translation.

GANs-based methods for Unpaired I2I. The methods in two-domain unpaired I2I are mainly divided into two classes: two-side and one-side mapping [32, 49]. In the two-side framework [51, 47, 24, 28, 26, 23, 11, 1, 46, 48, 20], the cycle-consistency constraint is the most widely-used strategy such as in CycleGAN [51], DualGAN [47] and DiscoGAN [24]. The key idea is that the translated image should be able to be reconstructed by an inverse mapping. More recently, there are numerical studies to improve this such as SCAN [26] and U-GAT-IT [23]. Since such bijective projection is too restrictive, several studies are devoted to one-side mapping [33, 3, 10, 51, 49, 34, 18]. One representative

Figure 3: The qualitative comparison on Cat → Dog, Wild → Dog and Male → Female. Our method achieved better visual quality for both *realism* and *faithfulness*.

approach is to design some kind of geometry distance to preserve content [32]. For example, DistanceGAN [3] keeps the distances between images within domains. GCGAN [10] maintains geometry-consistency between input and output. CUT [33] maximizes the mutual information between the input and output using contrastive learning. LSeSim [49] learns spatially-correlative representation to preserve scene structure consistency via self-similarities.

SDGMs-based methods for Image Translation. Several studies leveraged SDGMs for image translation due to their powerful generative ability and achieved good results. For example, Diffusion-CLIP [21] fine-tune the score network with CLIP [35] loss and has achieved great performance in a wide range of applications including text-driven image manipulation, zero-shot image manipulation between unseen domains and multi-attribute transfer. GLIDE [30] focus on text-to-image translation. As for I2I, SR3 [37] and Palette [36] learn a conditional SDGM and outperform state-of-art GANs-based methods on super-resolution, colorization and so on tasks, which needs paired data. For unpaired I2I, ILVR [7] and SDEdit [29] utilize a SDGM on the target domain and exploited the test source image to refine inference, which ignored the training data in the source domain. Compared with these methods, our method employs an energy function pretrained across both the source and target domains to improve the realism and faithful of translated images.

5   Experiment

Datasets. We validated the EGSDE on following datasets, where all images are resized to 256×256: (1) CelebaA-HQ [19], where we perform *Male→Female*. (2) AFHQ [8], where we perform *Cat→Dog* and *Wild→Dog*. More details is available in Appendix B.

Implementation. The time-dependent domain-specific extractor $E_s(\boldsymbol{x}, t)$ is trained based on the backbone in [9]. The resize function including downsampling and upsampling operation is used as low-pass filter and is implemented by [40]. For generation process, the initial time M is set 0.5. Denoising steps N is set 500. By default, the weight parameter λ_s and λ_i are set 500 and 2 respectively. More details about implementation are available in Appendix B.

Table 1: Quantitative comparison. ILVR [7], SDEdit [29], StarGAN v2 [8] and CUT [33] are reproduced using public code. The methods marked by * are public results from CUT[33] and ITTR [50]. All SDGMs-based methods and StarGAN v2 are repeated 5 times to eliminate randomness. CUT is conducted once since it learns a deterministic mapping. AMT show the preference rate of EGSDE against baselines. The EGSDE use the default-parameters ($\lambda_s = 500, \lambda_i = 2, M = 0.5T$) and EGSDE[†] use the parameters ($\lambda_s = 700, \lambda_i = 10, M = 0.6T$).

Model	FID ↓	L2 ↓	PSNR ↑	SSIM ↑	AMT ↑
		Cat → Dog			
CycleGAN* [51]	85.9	-	-	-	-
MUNIT* [17]	104.4	-	-	-	-
DRIT* [25]	123.4	-	-	-	-
Distance* [3]	155.3	-	-	-	-
SelfDistance* [3]	144.4	-	-	-	-
GCGAN* [10]	96.6	-	-	-	-
LSeSim* [49]	72.8	-	-	-	-
ITTR (CUT)* [50]	68.6	-	-	-	-
StarGAN v2 [8]	54.88 ± 1.01	133.65 ± 1.54	10.63 ± 0.10	0.27 ± 0.003	-
CUT* [33]	76.21	59.78	17.48	**0.601**	79.6%
ILVR [7]	74.37 ± 1.55	56.95 ± 0.14	17.77 ± 0.02	0.363 ± 0.001	75.4%
SDEdit [29]	74.17 ± 1.01	47.88 ± 0.06	19.19 ± 0.01	**0.423 ± 0.001**	65.2%
EGSDE	**65.82 ± 0.77**	**47.22 ± 0.08**	**19.31 ± 0.02**	0.415 ± 0.001	-
EGSDE[†]	**51.04 ± 0.37**	62.06 ± 0.10	17.17 ± 0.02	0.361 ± 0.001	-
		Wild → Dog			
CUT [33]	92.94	62.21	17.2	**0.592**	82.4%
ILVR [7]	75.33 ± 1.22	63.40 ± 0.15	16.85 ± 0.02	0.287 ± 0.001	73.4%
SDEdit [29]	68.51 ± 0.65	55.36 ± 0.05	17.98 ± 0.01	**0.343 ± 0.001**	57.2%
EGSDE	**59.75 ± 0.62**	**54.34 ± 0.08**	**18.14 ± 0.01**	**0.343 ± 0.001**	-
EGSDE[†]	**50.43± 0.52**	66.52± 0.09	16.40± 0.01	0.300± 0.001	-
		Male → Female			
CUT [33]	**36.99**	47.71	19.72	**0.74**	58.6%
ILVR [7]	46.12 ± 0.33	52.17 ± 0.10	18.59 ± 0.02	0.510 ± 0.001	88.2%
SDEdit [29]	49.43 ± 0.47	43.70 ± 0.03	20.03 ± 0.01	0.572 ± 0.000	74.4%
EGSDE	**41.93 ± 0.11**	**42.04 ± 0.03**	**20.35 ± 0.01**	**0.574 ± 0.000**	-
EGSDE[†]	**30.61 ± 0.19**	53.44 ± 0.09	18.32 ± 0.02	0.510 ± 0.001	-

Evaluation Metrics. For realism, we report the widely-used Frechet Inception Score (FID) [13] between translated images and the target dataset. To quantify faithfulness, we report the L_2 distance, PSNR and SSIM [45] between each input-output pair. To quantify both faithfulness and realism, we leverage Amazon Mechanical Turk(AMT) human evaluation to perform pairwise comparisons between the baselines and EGSDE.

5.1 Two-Domain Unpaired Image Translation

In this section, we compare EGSDE with the following state-of-the-art I2I methods in three tasks: SDGMs-based methods including ILVR [7] and SDEdit [29], and GANs-based methods including CUT [33], which are reproduced using public code. We also report the performance of other state-of-the-art GANs-methods on Cat → Dog, which are public results from CUT[33] and ITTR [50]. We provide more details about reproductions and evaluation in Appendix C.

The quantitative comparisons and qualitative results are shown in Table 1 and Figure 3. We can derive several observations. *First*, our method outperforms the SDGMs-based methods significantly in almost all *realism* and *faithfulness* metrics, suggesting the effectiveness of employing energy

| Source | K=1 | K=2 | K=3 | K=1 | K=2 | K=3 |

Figure 4: The results of repeating the Algorithm 1 K times. With the increase of K, SDEdit [29] tend to discard the domain-independent information of the source image (e.g., color and background) while our method still preserve them without harming realism.

function pretrained on both domains to guide the generation process. Especially, compared with the most direct competitor, i.e., SDEdit, with a lower L_2 distance at the same time, EGSDE improves the FID score by 8.35, 8.76 and 7.5 on Cat $\rightarrow$ Dog, Wild $\rightarrow$ Dog and Male $\rightarrow$ Female respectively. *Second*, EGSDE outperforms the current state-of-art GANs-based methods by a large margin on the challenging AFHQ dataset including Cat $\rightarrow$ Dog and Wild $\rightarrow$ Dog tasks. In addition, the human evaluation shows that EGSDE are preferred compared to all baselines ($> 50\%$). The qualitative results in Figure 3 agree with quantitative comparisons in Table 1, where our method achieved the results with the best visual quality for both *realism* and *faithfulness*. We show more results in Appendix C.

5.2 Ablation Studies

Repeating K Times. Following SDEdit [29], we show the results of repeating the Algorithm 1 K times. The quantitative and qualitative results are depicted in Table 2 and Figure 4. The experimental results show the EGSDE outperforms SDEdit in each K step in all metrics. With the increase of K, the SDEdit generates more realism images but the faithful metrics decrease sharply, because it only utilizes the source image at the initial time M.

The function of each expert. We validate the function of *realistic expert* $\mathcal{E}_s(\boldsymbol{y}, \boldsymbol{x}, t)$ and *faithful expert* $\mathcal{E}_i(\boldsymbol{y}, \boldsymbol{x}, t)$ by changing the weighting hyper-parameter λ_s and λ_i. As shown in Table 3 and Figure 1, larger λ_s results in more realistic images and larger λ_i results in more faithful images.

Table 2: Comparison with SDEdit [29] under different K times on Male $\rightarrow$ Female. The results on Cat $\rightarrow$ Dog and Wild $\rightarrow$ Dog are reported in Appendix C.4.

Methods	K	FID↓	L2↓	PSNR↑	SSIM↑
SDEdit [29]	1	49.95	43.71	20.03	0.572
EGSDE		**42.17**	**42.07**	**20.35**	**0.573**
SDEdit [29]	2	46.26	50.70	18.77	0.542
EGSDE		**38.68**	**47.10**	**19.40**	**0.548**
SDEdit [29]	3	45.19	55.03	18.08	0.527
EGSDE		**37.55**	**49.63**	**18.96**	**0.536**

Table 3: The results of different λ_s and λ_i on Wild $\rightarrow$ Dog. See results in other tasks in Appendix C.5. $\lambda_s = \lambda_i = 0$ corresponds to SDEdit [29].

λ_s, λ_i	FID↓	L2↓	PSNR↑	SSIM↑
$\lambda_s = 0, \lambda_i = 0$	67.87	55.39	17.97	0.344
$\lambda_s = 100, \lambda_i = 0$	60.80	56.19	17.85	0.341
$\lambda_s = 500, \lambda_i = 0$	53.72	58.65	17.47	0.335
$\lambda_s = 800, \lambda_i = 0$	53.01	60.02	17.27	0.331
$\lambda_s = 0, \lambda_i = 0.5$	68.31	53.23	18.32	0.347
$\lambda_s = 0, \lambda_i = 2$	71.10	51.99	18.52	0.349
$\lambda_s = 0, \lambda_i = 5$	72.70	51.44	18.61	0.351

6 Conclusions and Discussions

In this paper, we propose energy-guided stochastic differential equations (EGSDE) for realistic and faithful unpaired I2I, which employs an energy function pretrained on both domains to guide the generation process of a pretrained SDE. Building upon two feature extractors, we carefully design the energy function to preserve the domain-independent features and discard domain-specific ones of the source image. We demonstrate the EGSDE by outperforming state-of-art I2I methods on three widely-adopted unpaired I2I tasks.

One limitation of this paper is we employ a low-pass filter as the domain-independent feature extractor for its simpleness and effectiveness while we can train more sophisticated extractor, e.g. based on disentangled representation learning methods [38, 6, 14, 22, 27], on the data in the two domains. We leave this issue in future work. In addition, we must take care to exploit the method to avoid the potential negative social impact (i.e., generating fake images to mislead people).

References

[1] Matthew Amodio and Smita Krishnaswamy. Travelgan: Image-to-image translation by transformation vector learning. In *Proceedings of the ieee/cvf conference on computer vision and pattern recognition*, pages 8983–8992, 2019.

[2] Fan Bao, Chongxuan Li, Jun Zhu, and Bo Zhang. Analytic-dpm: an analytic estimate of the optimal reverse variance in diffusion probabilistic models. *arXiv preprint arXiv:2201.06503*, 2022.

[3] Sagie Benaim and Lior Wolf. One-sided unsupervised domain mapping. *Advances in neural information processing systems*, 30, 2017.

[4] Mikołaj Bińkowski, Danica J Sutherland, Michael Arbel, and Arthur Gretton. Demystifying mmd gans. *arXiv preprint arXiv:1801.01401*, 2018.

[5] Christopher M. Bishop. *Pattern Recognition and Machine Learning*. Springer, 2006.

[6] Xi Chen, Yan Duan, Rein Houthooft, John Schulman, Ilya Sutskever, and Pieter Abbeel. Infogan: Interpretable representation learning by information maximizing generative adversarial nets. *Advances in neural information processing systems*, 29, 2016.

[7] Jooyoung Choi, Sungwon Kim, Yonghyun Jeong, Youngjune Gwon, and Sungroh Yoon. Ilvr: Conditioning method for denoising diffusion probabilistic models. In *Proceedings of the IEEE/CVF International Conference on Computer Vision*, pages 14367–14376, 2021.

[8] Yunjey Choi, Youngjung Uh, Jaejun Yoo, and Jung-Woo Ha. Stargan v2: Diverse image synthesis for multiple domains. In *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*, pages 8188–8197, 2020.

[9] Prafulla Dhariwal and Alexander Nichol. Diffusion models beat gans on image synthesis. *Advances in Neural Information Processing Systems*, 34, 2021.

[10] Huan Fu, Mingming Gong, Chaohui Wang, Kayhan Batmanghelich, Kun Zhang, and Dacheng Tao. Geometry-consistent generative adversarial networks for one-sided unsupervised domain mapping. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pages 2427–2436, 2019.

[11] Aaron Gokaslan, Vivek Ramanujan, Daniel Ritchie, Kwang In Kim, and James Tompkin. Improving shape deformation in unsupervised image-to-image translation. In *Proceedings of the European Conference on Computer Vision (ECCV)*, pages 649–665, 2018.

[12] Ian Goodfellow, Jean Pouget-Abadie, Mehdi Mirza, Bing Xu, David Warde-Farley, Sherjil Ozair, Aaron Courville, and Yoshua Bengio. Generative adversarial nets. *Advances in neural information processing systems*, 27, 2014.

[13] Martin Heusel, Hubert Ramsauer, Thomas Unterthiner, Bernhard Nessler, and Sepp Hochreiter. Gans trained by a two time-scale update rule converge to a local nash equilibrium. *Advances in neural information processing systems*, 30, 2017.

[14] Irina Higgins, Loic Matthey, Arka Pal, Christopher Burgess, Xavier Glorot, Matthew Botvinick, Shakir Mohamed, and Alexander Lerchner. beta-vae: Learning basic visual concepts with a constrained variational framework. 2016.

[15] Geoffrey E Hinton. Training products of experts by minimizing contrastive divergence. *Neural computation*, 14(8):1771–1800, 2002.

[16] Jonathan Ho, Ajay Jain, and Pieter Abbeel. Denoising diffusion probabilistic models. *Advances in Neural Information Processing Systems*, 33:6840–6851, 2020.

[17] Xun Huang, Ming-Yu Liu, Serge Belongie, and Jan Kautz. Multimodal unsupervised image-to-image translation. In *Proceedings of the European conference on computer vision (ECCV)*, pages 172–189, 2018.

[18] Liming Jiang, Changxu Zhang, Mingyang Huang, Chunxiao Liu, Jianping Shi, and Chen Change Loy. Tsit: A simple and versatile framework for image-to-image translation. In *European Conference on Computer Vision*, pages 206–222. Springer, 2020.

[19] Tero Karras, Timo Aila, Samuli Laine, and Jaakko Lehtinen. Progressive growing of gans for improved quality, stability, and variation. *arXiv preprint arXiv:1710.10196*, 2017.

[20] Oren Katzir, Dani Lischinski, and Daniel Cohen-Or. Cross-domain cascaded deep translation. In *European Conference on Computer Vision*, pages 673–689. Springer, 2020.

[21] Gwanghyun Kim and Jong Chul Ye. Diffusionclip: Text-guided image manipulation using diffusion models. *arXiv preprint arXiv:2110.02711*, 2021.

[22] Hyunjik Kim and Andriy Mnih. Disentangling by factorising. In *International Conference on Machine Learning*, pages 2649–2658. PMLR, 2018.

[23] Junho Kim, Minjae Kim, Hyeonwoo Kang, and Kwanghee Lee. U-gat-it: Unsupervised generative attentional networks with adaptive layer-instance normalization for image-to-image translation. *arXiv preprint arXiv:1907.10830*, 2019.

[24] Taeksoo Kim, Moonsu Cha, Hyunsoo Kim, Jung Kwon Lee, and Jiwon Kim. Learning to discover cross-domain relations with generative adversarial networks. In *International conference on machine learning*, pages 1857–1865. PMLR, 2017.

[25] Hsin-Ying Lee, Hung-Yu Tseng, Jia-Bin Huang, Maneesh Singh, and Ming-Hsuan Yang. Diverse image-to-image translation via disentangled representations. In *Proceedings of the European conference on computer vision (ECCV)*, pages 35–51, 2018.

[26] Minjun Li, Haozhi Huang, Lin Ma, Wei Liu, Tong Zhang, and Yugang Jiang. Unsupervised image-to-image translation with stacked cycle-consistent adversarial networks. In *Proceedings of the European conference on computer vision (ECCV)*, pages 184–199, 2018.

[27] Alexander H Liu, Yen-Cheng Liu, Yu-Ying Yeh, and Yu-Chiang Frank Wang. A unified feature disentangler for multi-domain image translation and manipulation. *Advances in neural information processing systems*, 31, 2018.

[28] Ming-Yu Liu, Thomas Breuel, and Jan Kautz. Unsupervised image-to-image translation networks. *Advances in neural information processing systems*, 30, 2017.

[29] Chenlin Meng, Yutong He, Yang Song, Jiaming Song, Jiajun Wu, Jun-Yan Zhu, and Stefano Ermon. Sdedit: Guided image synthesis and editing with stochastic differential equations. In *International Conference on Learning Representations*, 2021.

[30] Alex Nichol, Prafulla Dhariwal, Aditya Ramesh, Pranav Shyam, Pamela Mishkin, Bob McGrew, Ilya Sutskever, and Mark Chen. Glide: Towards photorealistic image generation and editing with text-guided diffusion models. *arXiv preprint arXiv:2112.10741*, 2021.

[31] Alexander Quinn Nichol and Prafulla Dhariwal. Improved denoising diffusion probabilistic models. In *International Conference on Machine Learning*, pages 8162–8171. PMLR, 2021.

[32] Yingxue Pang, Jianxin Lin, Tao Qin, and Zhibo Chen. Image-to-image translation: Methods and applications. *IEEE Transactions on Multimedia*, 2021.

[33] Taesung Park, Alexei A Efros, Richard Zhang, and Jun-Yan Zhu. Contrastive learning for unpaired image-to-image translation. In *European Conference on Computer Vision*, pages 319–345. Springer, 2020.

[34] Taesung Park, Jun-Yan Zhu, Oliver Wang, Jingwan Lu, Eli Shechtman, Alexei Efros, and Richard Zhang. Swapping autoencoder for deep image manipulation. *Advances in Neural Information Processing Systems*, 33:7198–7211, 2020.

[35] Alec Radford, Jong Wook Kim, Chris Hallacy, Aditya Ramesh, Gabriel Goh, Sandhini Agarwal, Girish Sastry, Amanda Askell, Pamela Mishkin, Jack Clark, et al. Learning transferable visual models from natural language supervision. In *International Conference on Machine Learning*, pages 8748–8763. PMLR, 2021.

[36] Chitwan Saharia, William Chan, Huiwen Chang, Chris A Lee, Jonathan Ho, Tim Salimans, David J Fleet, and Mohammad Norouzi. Palette: Image-to-image diffusion models. In *NeurIPS 2021 Workshop on Deep Generative Models and Downstream Applications*, 2021.

[37] Chitwan Saharia, Jonathan Ho, William Chan, Tim Salimans, David J Fleet, and Mohammad Norouzi. Image super-resolution via iterative refinement. *arXiv preprint arXiv:2104.07636*, 2021.

[38] Eduardo Hugo Sanchez, Mathieu Serrurier, and Mathias Ortner. Learning disentangled representations via mutual information estimation. In *European Conference on Computer Vision*, pages 205–221. Springer, 2020.

[39] Zhiqiang Shen, Mingyang Huang, Jianping Shi, Xiangyang Xue, and Thomas S Huang. Towards instance-level image-to-image translation. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pages 3683–3692, 2019.

[40] Assaf Shocher. Resizeright. https://github.com/assafshocher/ResizeRight, 2018.

[41] Jascha Sohl-Dickstein, Eric Weiss, Niru Maheswaranathan, and Surya Ganguli. Deep unsupervised learning using nonequilibrium thermodynamics. In *International Conference on Machine Learning*, pages 2256–2265. PMLR, 2015.

[42] Yang Song, Conor Durkan, Iain Murray, and Stefano Ermon. Maximum likelihood training of score-based diffusion models. *Advances in Neural Information Processing Systems*, 34, 2021.

[43] Yang Song and Stefano Ermon. Generative modeling by estimating gradients of the data distribution. *Advances in Neural Information Processing Systems*, 32, 2019.

[44] Yang Song, Jascha Sohl-Dickstein, Diederik P Kingma, Abhishek Kumar, Stefano Ermon, and Ben Poole. Score-based generative modeling through stochastic differential equations. In *International Conference on Learning Representations*, 2020.

[45] Zhou Wang, Alan C Bovik, Hamid R Sheikh, and Eero P Simoncelli. Image quality assessment: from error visibility to structural similarity. *IEEE transactions on image processing*, 13(4):600–612, 2004.

[46] Wayne Wu, Kaidi Cao, Cheng Li, Chen Qian, and Chen Change Loy. Transgaga: Geometry-aware unsupervised image-to-image translation. In *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*, pages 8012–8021, 2019.

[47] Zili Yi, Hao Zhang, Ping Tan, and Minglun Gong. Dualgan: Unsupervised dual learning for image-to-image translation. In *Proceedings of the IEEE international conference on computer vision*, pages 2849–2857, 2017.

[48] Yihao Zhao, Ruihai Wu, and Hao Dong. Unpaired image-to-image translation using adversarial consistency loss. In *European Conference on Computer Vision*, pages 800–815. Springer, 2020.

[49] Chuanxia Zheng, Tat-Jen Cham, and Jianfei Cai. The spatially-correlative loss for various image translation tasks. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pages 16407–16417, 2021.

[50] Wanfeng Zheng, Qiang Li, Guoxin Zhang, Pengfei Wan, and Zhongyuan Wang. Ittr: Unpaired image-to-image translation with transformers. *arXiv preprint arXiv:2203.16015*, 2022.

[51] Jun-Yan Zhu, Taesung Park, Phillip Isola, and Alexei A Efros. Unpaired image-to-image translation using cycle-consistent adversarial networks. In *Proceedings of the IEEE international conference on computer vision*, pages 2223–2232, 2017.