AutoSparse: Towards Automated Sparse Training

Abstract

Sparse training is emerging as a promising avenue for reducing the computational cost of training neural networks. Several recent studies have proposed pruning methods using learnable thresholds to efficiently explore the non-uniform distribution of sparsity inherent within the models. In this paper, we propose Gradient Annealing (GA), a gradient driven approach where gradients to pruned out weights are scaled down in a non-linear manner. GA eliminates the need for additional sparsity-inducing regularization by providing an elegant trade-off between sparsity and accuracy. We integrated GA with the latest learnable threshold based pruning methods to create an automated sparse training algorithm called AutoSparse. Our algorithm achieves state-of-the-art accuracy with 80% sparsity for ResNet50 and 75% sparsity for MobileNetV1 on Imagenet-1K. AutoSparse also results in $7\times$ reduction in inference FLOPS and $> 2\times$ reduction in training FLOPS for ResNet50 on ImageNet at 80% sparsity. Finally, GA generalizes well to fixed-budget (Top-K, 80%) sparse training methods, improving the accuracy of ResNet50 on Imagenet-1K, to outperform TopKAST+PP by 0.3%.

1 Introduction

Deep learning models have emerged as the preferred solution for many important problems in the domains of computer vision(15; 10), language modeling (7), recommender systems(28) and reinforcement learning(32). Models have grown larger and more complex over the years, as they are applied to increasingly difficult problems on ever-growing datasets. In addition, DNN models are designed to operate in overparameterized regime (3; 4; 2) to facilitate easier optimization using gradient descent based methods. As a consequence, computational costs of performing training and inference tasks on state-of-the-art models has been growing at an exponential rate(1). The excess model capacity also makes DNNs more resilient to noise during training – reduced precision training methods(24; 37; 34) have successfully exploited this aspect of DNNs to speed up training and inference tasks significantly. Today, state-of-the-art training hardware consists of significantly more reduced-precision FLOPs compared to traditional FP32 computations.

Sparsity is another avenue for improving the compute efficiency by exploiting excess model capacity, thereby reducing the number of FLOPs needed for each iteration. Several studies have shown that while overparameterized model helps to ease the training, only a fraction of that model capacity is needed to perform inference (20; 14; 20; 27; 33; 13). A wide array of studies have also proposed methods to prune dense networks to produce sparse models for inference (dense-to-sparse) (25; 41; 12; 30). More recently, there is a growing interest in sparse-to-sparse (12; 26; 5; 11; 19; 9; 17; 39; 31) methods training models with end-to-end sparsity to reduce the computational costs of training. This paper presents techniques to improve and generalize sparse training methods for easy integration into different training workflows and deliver real-world performance.

Sparse training methods can be broadly divided into two categories – a) **deterministic pruning** methods, that initialize the model with a desired fixed sparsity budget at each layer and enforce it throughout the training cycle, b) **learnable threshold pruning** methods attempt to discover the sparsity distribution within the model by learning layer-wise threshold parameters. While these latter methods can aim for a desired sparsity level by selecting appropriate combination of initialization and hyper-parameters, the final model sparsity may not be exactly what was desired – hence they are non-deterministic.

Deterministic Pruning

Deterministic pruning methods expect a prior knowledge of how much sparsity can be extracted out of any given model. This sparsity budget is often determined by trial-and-error or extrapolated from previously published studies. Once the sparsity budget is determined, a choice must be made between a uniform or a non-uniform distribution of sparsity across the layers. Majority of the methods in this category (12; 5; 11; 19; 17; 39; 31; 40) opt for uniform sparse distribution because it requires fewer hyperparameters – a subset of these methods (17; 39; 31) maintain first and last layers in dense while the sparsity budget is uniformly distributed across rest of the layers. Fewer methods in this category (26; 9) use non-uniform distribution across layers using dynamic weight reallocation heuristics. Non-uniform distribution allows more degrees of freedom to explore ways to maximize accuracy at any given sparsity budget. In practice however, these methods did not show any improvement in accuracy over uniform distribution methods. The best performing methods in this category are the ones that skip pruning the first and last layers.

1.1 LEARNABLE THRESHOLD PRUNING

Learnable threshold pruning methods offer a two-fold advantage over deterministic pruning methods, **1) computationally efficient**, as the expensive computation (e.g. choosing Top-K largest) used for deriving threshold values at each layer is eliminated, **2) learns the non-uniform sparsity distribution** inherent within the model automatically, eliminating the need for additional hyperparameters. Current state-of-the-art methods in this space (21; 18) rely on $L2$ regularization based approaches to guide threshold updates and penalize pruned weights. Dynamic Sparse Training (21) is sparse training method that starts with threshold values initialized to 'zero' and pushes the small thresholds up using an exponential loss function added to $L2$ regularization term. Soft Threshold Reparameterization (18) initializes threshold parameters with large negative values and controls the induction of sparsity using different weight decay (λ) values for achieving different sparsity levels using $L2$ regularization. Sparsity vs accuracy trade-off is a challenge and is discussed below.

Exploring Accuracy Vs. Sparsity Trade-Off:

Deterministic pruning methods, not withstanding their limitations, offer a consistent level of sparsity throughout the training which in turn can lead to predictable performance improvements. For learnable threshold methods, the performance can be measured in the form of reduction in training FLOPs measured across the entire duration of the training. In order to meet this goal, learnable threshold methods must have the ability to induce sparsity early in the training cycle and provide algorithmic means to explore the trade-off to maximize model sparsity while also minimizing accuracy loss. Empirical studies on aforementioned methods(21; 18) have indicated that $L2$ regularization based approach offers at best, a weak trade-off of better accuracy at the expense of lower average sparsity. We also found these methods to be susceptible to *runaway sparsity* (e.g. hit 100% model sparsity), if higher levels of sparsity were induced right from the start of the training. To mitigate this problem, DST (21) implements hard upper limit checks (e.g. 99%) on sparsity to trigger a reset of the offending threshold and all the associated weights to prevent loss of accuracy. Similarly, STR (18) methods uses a combination of small initial threshold values with an appropriate λ to delay the induction of sparsity until later in the training cycle (e.g. 30 epochs) to control unfettered growth of sparsity.

Motivated by the above challenges, we propose **Gradient Annealing (GA)** method, to address the aforementioned issues related to training sparse models. When compared existing methods, **GA** offers greater flexibility to explore the trade-off between model sparsity and accuracy, and provides greater stability by preventing divergence due to runaway sparsity. We also propose a unified training algorithm called **AutoSparse**, combining best of learnable threshold methods (18) with **GA** that attempts to pave the path towards full automation of sparse-to-sparse training. Additionally, we also demonstrated that when coupled with deterministic pruning methods (**TopKAST**), **Gradient Annealing** can extract better accuracy at a fixed (80%) sparsity budget.

The key contributions of this work are as follows:

• We present a novel *Gradient Annealing* (GA) method (+ hyper-parameter α) for training end-to-end sparse models with higher accuracy compared to existing methods and provides greater flexibility to explore sparsity vs accuracy trade-off.

• We propose *AutoSparse*, a unified algorithm that combines learnable threshold methods with *Gradient Annealing* to create an automated framework for end-to-end sparse training. Our algorithm outperformed state-of-the-art methods by achieving better accuracy while maintaining consistently high sparsity throughout the training for ResNet50 and MobileNetV1 on Imagenet-1K dataset.

• AutoSparse achieves best FLOPs efficiency for inference and training among methods with comparable sparsity and accuracy.

• We demonstrate the efficacy of *Gradient Annealing* as a general learning technique independent of *AutoSparse* by applying it to *TopKAST* method to improve the Top-1 accuracy of ResNet50 by 0.3% on ImageNet-1K dataset using same fixed sparsity budget of 80%.

2 GRADIENT ANNEALING (GA)

A typical pruning step of deep networks involves masking out weights that are below some threshold T. This sparse representation of weights benefits from sparse computation in forward pass and in computation of gradients of inputs. We propose the following pruning step, where w is a weight and T is a threshold that can be deterministic (e.g., TopK magnitude) or learnable:

$$\text{(sparse)} \quad \tilde{w} = \text{sign}(w) \cdot h_\alpha(|w| - T)$$

$$\text{Forward pass} \quad h_\alpha(x) = \begin{cases} x, & x > 0 \\ 0, & x \leq 0 \end{cases} \qquad \text{(Proxy) Gradient} \quad \frac{\partial h_\alpha(x)}{\partial x} = \begin{cases} 1, & x > 0 \\ \alpha, & x \leq 0 \end{cases} \qquad (1)$$

where $0 \leq \alpha \leq 1$. $\tilde{w}$ is 0 if $|w|$ is below threshold T. Magnitude-based pruning is a greedy, temporal view of parameter importance. However, some of the pruned-out weights (in early training epochs) might turn out to be important in later epochs when a more accurate sparse pattern emerges. For this, $h_\alpha(\cdot)$ in eqn (1) allows the loss gradient to flow to masked weights in order to avoid permanently pruning out some important weights. The proposed gradient approximation is inspired by the Straight Through Estimator (STE) (6) which replaces zero gradients of discrete sub-differentiable functions by proxy gradients in back-propagation. Furthermore, we decay this α as the training progresses. We call this technique the *Gradient Annealing*.

We anneal α at the beginning of every epoch and keep this value fixed for all the iterations in that epoch. We want to decay α slowly in early epochs and then decay it steadily. For this, we compare several choices for decaying α: fixed scale (no decay), linear decay, cosine decay (same as learning rate (2)), sigmoid decay (defined in (3)) and Sigmoid-Cosine decay (defined in (4)). For sigmoid decay in (3), $L_0 = -6$ and $L_1 = 6$. For total epochs T, scale for α in epoch i is

$$\begin{align} \text{Cosine-Decay}(i, T) \qquad c_i &= (1 + \text{cosine}(\pi \cdot i/T))/2 & (2) \\ \text{Sigmoid-Decay}(i, T) \qquad s_i &= 1 - \text{sigmoid}(L_0 + (L_1 - L_0) \cdot i/T) & (3) \\ \text{Sigmoid-Cosine-Decay}(i, T) \qquad &= \max\{s_i, c_i\} & (4) \end{align}$$

Figure 1 shows the effect of various linear and non-linear annealing of α on dynamic sparsity. Fixed scale with no decay does not give us a good control of dynamic sparsity. Linear decay is better than this but suffers from drop in sparsity towards the end of training. Non-linear decays in eqn (2, 3, 4) provide much superior trade-off between sparsity and accuracy. While eqn (2) and eqn (4) show very similar behavior, sharp drop of eqn (3) towards the end of training push up the sparsity a bit (incurring little more drop in accuracy). These plots are consistent with our analysis of convergence of GA in eqn (5). Annealing schedule of α closely follows learning rate decay schedule.

Analysis of Gradient Annealing

Here we analyze the effect of the transformation $h_\alpha(\cdot)$ on the convergence of the learning process using a simplified example as follows. Let $v = |w| - T$, $u = h_\alpha(v)$, optimal weights be w^* and optimal threshold be T^*, i.e., $v^* = |w^*| - T^*$. Let us consider the loss function as

$$\min_v \mathcal{L}(v) = 0.5 \cdot (h_\alpha(v) - v^*)^2$$

and let $\partial h_\alpha(v)$ denote the gradient $\frac{\partial h_\alpha(v)}{\partial v}$. We consider the following cases for loss gradient for v.

(a) Various Gradient Annealing Schedules

(b) Sparsity achieved for non-linear annealing

Figure 1: Various schedule for annealing α and their effect on dynamic sparsity for ResNet50.

$$\frac{\partial \mathcal{L}}{\partial v} = \partial h_\alpha(v)(h_\alpha(v) - v^*) = \begin{cases} \partial h_\alpha(v) \cdot 0 = 0 & \text{if} \quad h_\alpha(v) = v^* \\ \partial h_\alpha(v) \cdot (v - v^*) = 1 \cdot (v - v^*) & \text{if} \quad v > 0 \text{ and } v^* > 0 \\ \partial h_\alpha(v) \cdot (v + |v^*|) = 1 \cdot (v + |v^*|) & \text{if} \quad v > 0 \text{ and } v^* \leq 0 \\ \partial h_\alpha(v) \cdot (-v^*) = \alpha \cdot (-v^*) & \text{if} \quad v \leq 0 \text{ and } v^* > 0 \\ \partial h_\alpha(v) \cdot (|v^*|) = \alpha \cdot (|v^*|) & \text{if} \quad v \leq 0 \text{ and } v^* \leq 0 \end{cases} \quad (5)$$

Correct proxy gradients for $h_\alpha(\cdot)$ should move v towards v^* during pruning (e.g., opposite direction of gradient for gradient descent) and stabilize it at its optima (no more updates). Therefore, $\partial h_\alpha(v) > 0$ should be satisfied for better convergence of v to v^*. Our $h_\alpha(\cdot)$ satisfies this condition for $\alpha > 0$. Furthermore, for $v > 0$, v gets updated proportional to $v - v^*$, i.e., how far v is from v^*. As training progresses and v gets closer to v^*, v receives gradually smaller gradients to finally converge to v^*. However, for $v \leq 0$, v receives gradient proportional to magnitude of $\alpha \cdot v^*$, irrespective of how close v is to v^*. Also, note that we benefit from sparse compute when $v \leq 0$.

We set initial T high in order to achieve sparsity from early epochs. However, this likely leads to a large number of weights following condition 4 in eqn (5). Fixed, large α (close to 1) makes large correction to v and moves it to v^* quickly. Consequently, v moves from condition 4 to condition 2, losing out the benefit of sparse compute. A lower α 'delays' this transition and enjoys the benefits of sparsity. *This is why we choose $\alpha < 1$ rather than identity STE as proxy gradient* (unlike (35)). However, as training progresses, more and more weights move from condition 4 to condition 2 leading to a drop in sparsity. This behavior is undesirable to reach a target sparsity at the end of training. In order to overcome this, we propose to decay α with training epochs such that we enjoy the benefits of sparse compute while v being close to v^*. That is, GA provides a more controlled and stable trade-off between sparsity and accuracy throughout the training.

Note that, GA is applicable when we compute loss gradients for a superset of active (non-zero) weights that take part in forward sparse computation using gradient descend. For an iteration t, let the forward sparsity be S. If $\alpha = 0$, then we need to calculate gradient for only those non-zero weights as other weights would not receive gradients due to ReLU STE. In order to benefit from such computational reduction, we can set $\alpha = 0$ after several epochs of α annealing.

3 AUTOSPARSE : SPARSE TRAINING WITH GRADIENT ANNEALING

AutoSparse is the sparse training algorithm that combines the best of learnable threshold pruning techniques (18; 21) with Gradient Annealing (GA) method. AutoSparse meets the following requirements necessary for efficient training of sparse neural networks, as outlined in Section 1.1.

• **Learnable Pruning Thresholds** : Eliminate threshold computation, minimize sparsification overheads compared to deterministic pruning methods(17). Learn the non-uniform distribution of sparsity across the layers.

• **Sparse Model Discovery** : Discover the optimal trade-off between model accuracy vs. level of sparsity by applying Gradient Annealing method (as shown in Figure 1). Produce a sparse model at the end of the training with desired sparsity level guided by the hyper-parameter α.

• **Accelerate Training** : Reduce training FLOPs by training with sparse weights from scratch, maintain high levels of sparsity throughout the training epochs. Use sparsity in both forward and backward pass to accelerate the computation.

Previous proposals using leanable threshold methods such as (21) and (18) address the first criterion but do not effectively optimize accuracy vs sparsity trade-off as they are outperformed by our method. These methods also do not accelerate training since they are dense-to-sparse methods. Deterministic pruning methods such as (17) and (11) address the third criterion of accelerating training, but incur higher sparsfication overheads for computing threshold values and cannot automatically discover the non-linear distribution of sparsity.

Let $\mathcal{D} := \{(\mathbf{x_i} \in \mathbb{R}^{\mathbf{d}}, y_i \in \mathbb{R})\}$ be the observed data, $\mathcal{W}$ be the learnable network parameters, $\mathcal{L}$ be a loss function. For an L-layer DNN, $\mathcal{W}$ is divided into layer-wise trainable parameter tensors, $[\mathbf{W}_\ell]_{\ell=1}^L$. As various layers can have widely different number of parameters and also unequal sensitivity to parameter alteration, we use one trainable pruning parameter, s_ℓ for each layer ℓ, i.e., $\mathbf{s} = [s_1, ..., s_L]$ is the vector of trainable pruning parameter. Let $g : \mathbb{R} \to \mathbb{R}$ be applied element-wise. For layer ℓ, $T_\ell = g(s_\ell)$ is the pruning threshold for $\mathbf{W}_\ell$. We seek to optimize the following:

$$\min_{\mathcal{W}, \mathbf{s}} \mathcal{L}(\mathcal{S}_{h_\alpha, g}(\mathcal{W}, \mathbf{s}), \mathcal{D}) \tag{6}$$

where, function $\mathcal{S}_{h_\alpha, g}$, parameterized by $h_\alpha \in \mathbb{R} \to \mathbb{R}$ that is applied element-wise.

$$\hat{\mathbf{W}}_\ell = \mathcal{S}_{h_\alpha, g}(\mathbf{W}_\ell, s_\ell) = \text{sign}(\mathbf{W}_\ell) \cdot h_\alpha(|\mathbf{W}_\ell| - g(s_\ell)) \tag{7}$$

Gradient annealing is applied via $h_\alpha(\cdot)$ as discussed earlier.

Sparse Forward Pass: At iteration t, for layer ℓ, sparse weights are $\hat{\mathbf{W}}_\ell^{(t)} = \mathcal{S}_{h_\alpha, g}(\mathbf{W}_\ell^{(t)}, s_\ell^{(t)})$ as defined in eqn (7). Let the non-zero (active) set of weights $\mathcal{A}_\ell^{(t)} = \{i : \hat{\mathbf{W}}_{\ell, i}^{(t)} > 0\}$. For simplicity, let us drop the subscript notations. $\hat{\mathbf{W}}$ is used in the forward pass as $\mathbf{Y} = \mathbf{X} \otimes \hat{\mathbf{W}}$ where $\otimes$ is tensor MAC operation, e.g., convolution. Let A denote the fraction of non-zero elements of $\mathbf{W}$ belonging to $\mathcal{A}$, i.e, out forward sparsity is $1 - A$. $\hat{\mathbf{W}}$ is also used in computation of input gradient during backward pass. Every iteration, we update $\mathbf{W}$ and construct $\hat{\mathbf{W}}$ from updated $\mathbf{W}$ and learned threshold.

Sparse Compute of Input Gradient: For an iteration and a layer (we drop the subscripts t, ℓ for simplicity), output of sparse operations in forward pass need not be sparse, i.e., $\mathbf{Y} = \mathbf{X} \otimes \hat{\mathbf{W}}$ is typically dense. Consequently, gradient of output ∇_Y is also dense. We compute gradient of input ∇_X as $\hat{\mathbf{W}} \otimes \nabla_Y$. Computation of ∇_X is sparse due to sparsity in $\hat{\mathbf{W}}$.

Sparse Weight Gradient: Gradient of weights ∇_W is computed as $\mathbf{X} \otimes \nabla_Y$. Note that, for forward sparsity S, $\alpha = 0$ implies weight gradient sparsity S as no gradient flows to pruned weights. We can have a hyperparameter that specifies at which epoch we set $\alpha = 0$, to enjoy benefits of sparse weight gradients. However, we need to keep $\alpha \neq 0$ for several epochs to achieve high accuracy results, losing the benefits of sparse weight gradient. In order to overcome this, we propose the following for the epochs when $\alpha \neq 0$.

We can make sparse ∇_W if we compute loss gradient using a subset $\mathcal{B}$ of $\mathbf{W}$.

$$\mathcal{B} = \{i : \mathbf{W}_i \in \text{TopK}(\mathbf{W}, B)\}, \quad B \geq A \tag{8}$$

where $\mathcal{B}$ is a superset of $\mathcal{A}$ and TopK($\mathbf{W}$,k) picks indices of k largest magnitude elements from $\mathbf{W}$. This constitutes our weight gradient sparsity $1 - B$.

We apply gradient annealing on set $\mathcal{B} \setminus \mathcal{A}$, i.e., gradients of weights in $\mathcal{B} \setminus \mathcal{A}$ are decayed using α. Note that, $\alpha = 0$ implies $\mathcal{B} = \mathcal{A}$. Gradient computation for learnable parameters $\mathbf{W}$ and s are shown in Appendix.

4 RELATED WORK

Learnable Threshold Methods

STR (18) is a state-of-the-art method that learns pruning thresholds along with weights. STR prunes weights that are below the threshold using ReLU as a mask function. Setting $\alpha = 0$ in (7) reproduces

the formulation of STR (however, our method learns completely different distribution than that of STR). This $\alpha = 0$ implies identical forward and backward sparsity which are computationally beneficial. However, this prevents gradients from flowing to pruned out weights resulting in a sub-optimal trade-off between sparsity and accuracy. This forces STR to run fully dense training for many epochs (implemented by initializing threshold parameter s to a very large negative value, e.g., -3200 so that sigmoid$(s) \approx 0$, and weight decay on s takes many epochs to make sigmoid(s) large enough to induce sparsity. Note that, extracting sparsity from early epochs using STR results in run-away sparsity. Using gradient annealing technique, we are able to extract sparsity from early epochs to maintain high average sparsity throughout the training, yet achieve SOTA accuracy.

DST (21) is also a sparse training algorithm based on learned weights and thresholds, where a binary weight mask is created by suppressing the weights that are below the threshold (using step function). they impose exponential decay of thresholds as regularizer. A hyperparameter controls the amount of regularization, leading to a balance between sparsity and accuracy. In order to reduce the sensitivity of the hyperparameter, they manually reset the sparsity if it exceeds some predefined limit (thus occasionally falling into dense training regime). Further, they approximated the gradient of pruning step function by linearly scaling up the gradient of the elements that are close to the pruning threshold, and scaling down (fixed scale) the gradient of other elements. This helps some of the pruned out elements to receive loss gradient.

SCL (35) learns both network weights as well as mask parameters during training. These masks are binarized during forward pass to represent network sparsity. This mask information, along with a decaying connectivity hyperparameter are used as a sparsity-inducing regularizer in the objective function. During training, the mask is learned as dense parameter tensor. This increases the effective model size during training, which might create overhead moving parameters from memory.

Deterministic Pruning Methods

GMP (41) suggests to gradually increase the number of removed weights until desired sparsity is reached. RigL (11) proposes to prune out a fraction of weights, and activates/revives new ones iteratively using infrequent full gradient calculations. Such restriction of gradient flow causes more accuracy drop. TopKAST (17) always prunes Top-K weights, but updates a superset of active weights based on gradients so that the pruned out weights can be revived. Moreover, they penalize the non-active weights by a factor inversely proportional to density. This way, the active set of weights becomes stable with epochs as small fraction of pruned out weights become active again. ITOP (23) maintains a fixed amount of sparsity throughout training. They use gradient based weight regrowth used in RigL and SNFS (9). Applying ITOP in RigL grow stage, i.e., exploring new weights based on (full) gradient information in Rigl grow stage, RigL+ITOP achieves higher accuracy. PowerPropagation (31) is a technique to transform weight as $w = v|v|^{\alpha-1}$, s.t. it creates a heavy-tailed distribution of trained weights. As an effect, they observe that weights initialized close to 0 are likely to be pruned out, and weights are less likely to change sign. PP, when applied on TopKAST, i.e., TopKAST + PP, improves the accuracy of TopKAST. OptG (39) learns both weights and a pruning supermask in a gradient driven manner. They argued in favour of letting gradient flow to pruned weights so that it solves the 'independence paradox' problem that prevents from achieving high-accuracy sparse results. However, they achieve a given sparsity budget by increasing sparsity according to some sigmoid schedule (sparsity is extracted only after 40 epochs). This suffers from larger training FLOPs count.

Gradient Annealing of α, in our formulation, unifies all such observations of sparse training.

5 Experiments

5.1 Experimental Set up

Vision Models: ImageNet-1K ((8)) is a widely used largescale image classification dataset with 1K classes. We show sparse training results on ImageNet-1K for two popular CNN architectures: ResNet50 (15) and and MobileNetV1 (16), to demonstrate the generalizability of our method. For AutoSparse training, we use SGD as the optimizer, momentum 0.875, learning rate (max) 0.256 using a cosine annealing with warm-up of 5 epochs. We run all the experiments for 100 epochs using a batch size 256. We use weight decay $\lambda = 0.000030517578125$ (picked from STR (18)), label

smoothing 0.1, $s\_init= -5$. We presented our results only using Sigmoid-Cosine decay of α (defined in eqn (4)).

Language Models: We choose Transformer models (36) for language translation task on WMT14 English-German data. We have 6 encoder and 6 decoder layers with standard hyperparameter setting: optimizer is ADAM with betas $(0.9, 0.997)$, token size 10240, warm up 4000, , learning rate 0.000846 that follows inverse square root decay. We apply AutoSparse by introducing a learnable threshold for each linear layer, and we initialize them as $s = -7.0$. Also, initial $\alpha = 0.4$ is annealed according to exponential decay as follows. For epoch t and $\beta > 0$ (we use $\beta = 1$):

$$\text{Exponential-Decay } (t; \beta) = e^{(-\beta \cdot t)} \tag{9}$$

We keep the first and last layers of transformer dense and apply AutoSparse to train it for 44 epochs.

We repeat the experiments several times with different random seeds and report the average numbers.

Notation: We define the following notations that are used in the tables. 'Base': Dense baseline accuracy, 'Top1(S)': Top-1 accuracy for sparse models, 'Drop%': relative drop in accuracy for sparse models from Base, '$S\%$': percentage of model sparsity, 'Train F': fraction of training FLOPs comparing to baseline FLOPs, 'Test F': fraction of inference FLOPs comparing to bsaeline FLOPs, 'Back $S\%$': explicitly set sparsity in weight gradients (B in eqn (8)), 'BLEU(S)': BLEU score for sparse models. Smaller values of 'Train F' and 'Test F' suggests larger reduction in computation.

5.2 EFFICACY OF GRADIENT ANNEALING

Comparison with Other Learnable Threshold Methods:

We compare AutoSparse results with STR (18) and DST (21). Lack of gradient flow to pruned out elements prevents STR to achieve the optimal sparsity-accuracy trade-off. For example, they need dense training for many epochs in order to achieve high accuracy results, losing out the benefits of sparse training. Our gradient annealing overcomes such problems and achieves much superior sparsity-accuracy trade-off. In Table 1, we achieve better accuracy than STR for ResNet50 at various sparsity levels 80%, 90%, and 95%, while gaining in FLOPs reduction for both training and inference. Also, Table 3 shows our superior accuracy for MobileNetV1. Similarly, our method achieves higher accuracy than DST for both 80% and 90% sparsity budget for ResNet50 (Table 1). However, DST uses separate sparsity-inducing regularizer, whereas our gradient annealing itself does the trick. SCL (35) is another dynamic sparse method that learns both weights and sparsity mask parameters using explicit regularizer with the objective function. They report 0.23% drop in accuracy at 74% sparsity for ResNet50 on ImageNet. However, their much lower dense baseline, lower reported sparsity and lack of training FLOPs make it harder for a direct comparison (inference FLOPs $0.21\times$ of baseline).

Comparison with Deterministic Pruning Methods:

TopKAST always prunes Top-K weights but also updates a superset of active weights by letting the gradients flow to them (along with some regularization). This helps pruned out weights to be revived, if necessary. RigL (11) (more restrictive than TopKAST) activates/revives new weights iteratively using infrequent full gradient calculation. Such restriction of gradient flow causes more accuracy drop: 80% sparse model loses 2.86% top-1 accuracy from dense baseline (table 1). ITOP (23) maintains a fixed amount of sparsity throughout training. They use gradient based weight regrowth used in RigL and SNFS (9). Applying ITOP in RigL grow stage, i.e., exploring new weights based on (full) gradient information in Rigl grow stage, RigL+ITOP achieves accuracy 75.84 for sparsity budget $80\% \sim 1.25\%$ off from baseline. OptG (39) learns both weights and a pruning supermask in a gradient driven manner where pruned weights also receives loss gradient. AutoSparse is better than their accuracy at 90% sparsity budget, despite achieving significantly higher average sparsity. PowerPropagation (31) transforms weight as $w = v|v|^{\alpha-1}$, s.t. it creates a heavy-tailed distribution of trained weights that are more amenable to pruning. PP, when applied on TopKAST, i.e., TopKAST + PP, improves on TopKAST and achieves 0.73% drop in accuracy for 80% sparsity budget and 2% drop in accuracy for 90% sparsity budget. Gradient Annealing, when applied with TopKAST method (without additional regularization) improves these accuracies while maintaining similar training and inference FLOPs. Finally, SWAT-U sparsifies models in the forward pass and activation in the backpass to reduce the training FLOPs. For models with ReLU activation, we typically have 50% default sparsity. This would lead to up to $2\times$ further reduction of training FLOPs for AutoSparse. However, in our analysis we do not count such inherent sparsity of activations.

Method	Base	Top1(S)	Drop%	S%	Train F	Test F	comment
RigL(11)	76.8	74.6	2.86	80	0.33×	0.22×	sparse to sparse
SNFS+ERK(9)	77.01	75.2	2.35	80	–	–	
RigL+ERK(11)	76.8	75.1	2.21	80	–	–	
*SWAT-U(29)	76.8	75.2	2.08	80	0.24×	0.22×	TopK weight, act
*SWAT-ERK(29)	76.8	76	1.04	80	0.4×	0.41×	TopK weight, act
GMP(41)	77.01	75.6	1.83	80	–	0.2×	deterministic
DNW(38)	77.01	76	1.31	80	–	0.2×	deterministic
RigL+ITOP(23)	76.8	75.84	1.25	80	–	–	
GraNet(22)	76.8	75.8	1.3	80	–	–	
TopKAST*(17)	76.8	75.7	0.94	80	0.48×	0.22×	deterministic
TopKAST*+PP(31)	76.8	76.24	0.73	80	0.48×	0.22×	deterministic
TopKAST*+**GA**	76.8	76.47	0.43	80	0.48×	0.22×	deterministic
DST (21)	74.95	74.02	1.24	80.4	–	0.15×	dynamic
STR (18)	77.01	76.19	1.06	79.55	0.54×	0.18×	dense to sparse
AutoSparse	77.01	76.77	0.31	79.67	0.46×	0.14×	α=0 @ epoch 90
GMP(41)	77.01	73.91	4.03	90	–	0.1×	deterministic
DNW(38)	77.01	74	3.91	90	–	0.1×	deterministic
OptG (39)	77.01	74.28	3.55	90	–	–	
DST (21)	74.95	72.78	2.9	90.13	–	0.087×	dynamic
STR (18)	77.01	74.31	3.51	90.23	0.38×	0.083×	dense to sparse
AutoSparse	77.01	75.11	2.47	89.6	0.417×	0.078×	$\alpha \neq 0$
GMP (41)	77.01	70.59	8.34	95	–	0.05×	deterministic
DNW (38)	77.01	68.3	11.3	95	–	0.05×	deterministic
STR (18)	77.01	70.4	8.58	95.03	0.246×	0.039×	dense to sparse
AutoSparse	77.01	70.84	8.01	95.09	0.182×	0.036×	α=0 @ epoch 20

Table 1: ResNet50 on ImageNet: Comparing accuracy, sparsity and the FLOPS (dense $1\times$) for training and inference for selected sparse training methods. **TopKAST***: TopKAST with 0% backward sparsity. **TopKAST*+GA**: TopKAST* with Gradient Annealing boosts the accuracy despite having same training/inference FLOPS. For AutoSparse 79.67% sparsity, $\alpha = 0.75$ is decayed till epoch 90 and then set to 0 (implying $\sim 80\%$ forward and backward sparsity after epoch 90). Similarly, for AutoSparse 95.09% sparsity, $\alpha = 0.8$ is decayed till epoch 20 and then set to 0. Standard deviation of results for AutoSparse is less than 0.06.

Method	S%	Top1 (S)	Drop %	Train F	Test F	Back S%	comment
AutoSparse	85.72	74.81	2.86	0.298×	0.111×	50	same fwd & in grad sparsity, w grad 50% sparse
TopKAST	80	75	2.34	0.32×	0.22×	50	fwd & in grad 80%, w grad 50% sparse

Table 2: ResNet50 on ImageNet: AutoSparse with explicitly set sparsity for weight gradient. For AutoSparse 85.72% sparsity, $\alpha = 0.85$ is decayed till epoch 100 resulting in identical sparsity for forward and input gradients, along with 50% sparsity for weight gradient throughout the training (involves invoking TopK method).

Results with Backward Sparsity

In AutoSparse, the forward sparsity is determined by weights that are below the learned threshold (let this dynamic sparsity be S). We decay $\alpha = 0.85$ throughout sparse training and apply TopK to select indices of 50% largest magnitude weights (during forward pass) that are used in computation of loss gradients (S $\geq 50\%$). These weights are superset of non-zero weights used in forward computation. This way, we have sparsity S for forward and input gradient compute, and 50% sparsity for weight

Method	Base	Top1(S)	Drop%	S%	Train F	Test F	comment
GMP(41)	71.95	67.7	5.91	74.11	–	0.29×	
STR (18)	71.95	68.35	5	75.28	0.37×	0.18×	dense to sparse
AutoSparse	71.95	70.1	2.57	75.1	0.51×	0.22×	$\alpha \neq 0$
STR (18)	71.95	64.83	9.9	85.8	0.32×	0.1×	dense to sparse
AutoSparse	71.95	64.87	9.84	86.36	0.25×	0.1×	α=0 @ epoch 20
AutoSparse	71.95	64.18	10.8	87.72	0.22×	0.08×	α=0 @ epoch 20
STR (18)	71.95	62.1	13.7	89.01	–	0.07×	dense to sparse

Table 3: MobileNetV1 on ImageNet: Comparing accuracy, sparsity and FLOPS (dense 1×) for training and inference for selected sparse training methods. AutoSparse achieves significantly higher accuracy for comparable sparsity. For AutoSparse 75.1% sparsity, $\alpha = 0.4$ is decayed till epoch 100, resulting in identical sparsity for forward and input gradient computation. For AutoSparse (87.72%,86.36%) sparsity, ($\alpha = 0.6$,$\alpha = 0.8$) decayed till epoch 20 and then set to 0, which implies identical forward and backward sparsity after epoch 20.

Method	Dense	BLEU(S)	Drop %	S%
AutoSparse	27.83	27.57	0.93	60.99

Table 4: Transformer on WMT: For AutoSparse, $\alpha = 0.4$ is annealed exponentially till epoch 10 and then set to 0 for remaining epochs. This implies identical sparsity for forward and input gradient computation for 10 epochs, then identical sparsity for forward and backward pass.

gradient. Gradient annealing is applied on gradients of those weights that appear in these top 50% but were pruned in forward pass. We compare our results with TopKAST 80% forward sparsity and 50% backward sparsity in Table 2. We produce 85.7% sparse model that significantly reduces inference FLOPs (0.11× of baseline and 0.5× of TopKAST) while achieving similar accuracy using similar training FLOPs as TopKAST.

Various layers have different compute profile and parameter density. Distribution of layer-wise sparsity affects the overall compute despite having identical model sparsity (compression). AutoSparse appears to learn the most FLOPs-efficient sparsity distribution than other methods (Appendix fig 4)

5.3 AutoSparse for Language Models

We applied our AutoSparse on Transformer model using exponential decay for $\alpha = 0.4$. This annealing schedule resulted in a better trade-off between accuracy and sparsity. Also, we set $\alpha = 0$ after 10 epochs to take advantage of more sparse compute in backward pass.

AutoSparse achieves a BLEU score of 27.57 with overall sparsity 60.99% which incurs an accuracy drop of 0.93% from dense baseline BLEU score 27.83 (Table 4).

6 Conclusion

We propose Gradient Annealing, a novel technique that decays the gradients of pruned out weights in a non-linear manner. This potentially eliminates the need for explicit regularizer to induce sparsity in model training. This provides us with a tool to develop an automated sparse training method to explore the inherent model sparsity without the knowledge of an appropriate sparsity budget. AutoSparse helps us to achieve state-of-the-art sparsity/accuracy trade-off for two different type of CNN models: ResNet50 and MobiletNetV1 on ImageNet dataset. Further, we demonstrate the applicability of our method on Transformer models for language translation task. Interestingly, among methods producing comparable sparsity, AutoSparse learns a non-linear sparsity distribution that is the most FLOPs-efficient.

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REFERENCES

[1] D. Amodei and D. Hernandez. AI and Compute. `https://openai.com/blog/ai-and-compute/`.

[2] N. Ardalani, J. Hestness, and G. Diamos. Empirically Characterizing Overparameterization Impact on Convergence. In `https://openreview.net/forum?id=S1lPShAqFm`, 2019.

[3] S. Arora, N. Cohen, and E. Hazan. On the optimization of deep networks: Implicit acceleration by overparameterization. In Jennifer Dy and Andreas Krause, editors, *Proceedings of the 35th International Conference on Machine Learning*, volume 80 of *Proceedings of Machine Learning Research*, pages 244–253. PMLR, 10–15 Jul 2018.

[4] M. Belkin, D. Hsu, S. Ma, and S. Mandal. Reconciling modern machine-learning practice and the classical bias-variance trade-off. *Proceedings of the National Academy of Sciences*, 116(32):15849–15854, 2019.

[5] G. Bellec, D. Kappel, W. Maass, and R. Legenstein. Deep rewiring: Training very sparse deep networks. In *International Conference on Learning Representations*, 2018.

[6] Y. Bengio, N. Léonard, and A. Courville. Estimating or propagating gradients through stochastic neurons for conditional computation. In *arXiv preprint arXiv:1308.3432*, 2013.

[7] T. Brown, B. Mann, N. Ryder, M. Subbiah, J D Kaplan, P Dhariwal, A Neelakantan, P Shyam, G Sastry, A Askell, S Agarwal, A Herbert-Voss, G Krueger, T Henighan, R Child, A Ramesh, D Ziegler, J Wu, C Winter, C Hesse, M Chen, E Sigler, M Litwin, S Gray, B Chess, J Clark, C Berner, S McCandlish, A Radford, I Sutskever, and D Amodei. Language Models are Few-Shot Learners. In `https://arxiv.org/abs/2005.14165`, 2020.

[8] J. Deng, W. Dong, R. Socher, L.-J. Li, K. Li, and L. Fei-Fei. A large-scale hierarchical image database. In *IEEE conference on computer vision and pattern recognition*, 2009.

[9] T. Dettmers and L. Zettlemoyer. Sparse Networks from Scratch: Faster Training without Losing Performance. In `https://arxiv.org/pdf/1907.04840.pdf`, 2019.

[10] A. Dosovitskiy, L. Beyer, A. Kolesnikov, D. Weissenborn, X. Zhai, T. Unterthiner, M. Dehghani, M. Minderer, G. Heigold, S. Gelly, J. Uszkoreit, and N. Houlsby. An image is worth 16x16 words: Transformers for image recognition at scale. In *International Conference on Learning Representations*, 2021.

[11] U. Evci, T. Gale, J. Menick, P.S. Castro, and E. Elsen. Rigging the Lottery: Making All Tickets Winners. In `https://arxiv.org/pdf/1911.11134.pdf`, 2021.

[12] J. Frankle and M. Carbin. The lottery ticket hypothesis: Finding sparse, trainable neural networks. In *International Conference on Learning Representations*, 2019.

[13] T. Gale, E. Elsen, and S. Hooker. The State of Sparsity in Deep Neural Networks. In `https://arxiv.org/pdf/1902.09574.pdf`, 2019.

[14] S. Han, J. Pool, J. Tran, and W. Dally. Learning both weights and connections for efficient neural network. In *In Advances in Neural Information Processing Systems (NeurIPS)*, 2015.

[15] K. He, X. Zhang, S. ren, and J. Sun. Deep residual learning for image recognition. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, 2016.

[16] A.G. Howard, M. Zhu, B. Chen, D. Kalenichenko, W. Wang, T. Weyand, M. Andreetto, and H. Adam. Mobilenets: Efficient convolutional neural networks for mobile vision applications. In *arXiv preprint arXiv:1704.04861*, 2017.

[17] S.M. Jayakumar, R. Pascanu, J.W. Rae, S. Osindero, and E. Elsen. Top-KAST: Top-K Always Sparse Training. In , 2021.

[18] A. Kusupati, V. Ramanujan, R. Somani, M. Wortsman, P. Jain, S. Kakade, and A. Farhadi. Soft Threshold Weight Reparameterization for Learnable Sparsity. In *https://arxiv.org/abs/2002.03231*, 2020.

[19] Y. Lee. Differentiable Sparsification for Deep Neural Networks. In *https://openreview.net/forum?id=_x4A8IZ-rRv*, 2021.

[20] Z. Li, E. Wallace, S. Shen, K. Lin, K. Keutzer, D. Klein, and J. Gonzalez. Train Big, Then Compress: Rethinking Model Size for Efficient Training and Inference of Transformers. In *https://arxiv.org/pdf/2002.11794.pdf*, 2020.

[21] J. Liu, Z. Xu, R. Shi, R.C.C Cheung, and H.K.H So. Dynamic Sparse Training: Find Efficient Sparse Network from Scratch with Trainable Masked Layers. In *International Conference on Learning Representations (ICLR)*, 2020.

[22] S. Liu, T. Chen, X. Chen, Z. Atashgahi, L. Yin, H. Kou, L. Shen, M. Pechenizkiy, Z. Wang, and D.C. Mocanu. Sparse Training via Boosting Pruning Plasticity with Neuroregeneration. In *Conference on Neural Information Processing Systems (NeurIPS*, 2021.

[23] S. Liu, L Yin, D. C. Mocanu, and M. Pechenizkiy. Do We Actually Need Dense Over-Parameterization? In-Time Over-Parameterization in Sparse Training. In *https://arxiv.org/pdf/2102.02887.pdf*, 2021.

[24] P. Micikevicius, S. Narang, J. Alben, G. Diamos, E. Elsen, D. Garcia, B. Ginsburg, M. Houston, O. Kuchaiev, G. Venkatesh, et al. Mixed precision training. In *arXivpreprintarXiv:1710.03740*, 2017.

[25] D. Molchanov, A. Ashukha, and D. P. Vetrov. Variational Dropout Sparsifies Deep Neural Networks. In *http://proceedings.mlr.press/v70/molchanov17a.html*, 2017.

[26] H. Mostafa and X. Wang. Parameter efficient training of deep convolutional neural networks by dynamic sparse reparameterization. In *Proceedings of the 36th International Conference on Machine Learning*, pages 4646–4655. PMLR, 2019.

[27] S. Narang, G. F. Diamos, S. Sengupta, and E. Elsen. Exploring sparsity in recurrent neural networks. *CoRR*, abs/1704.05119, 2017.

[28] M. Naumov, D. Mudigere, H.-J. M. Shi, J. Huang, N.n Sundaraman, J. Park, X. Wang, U. Gupta, C-J Wu, A G. Azzolini, D. Dzhulgakov, A. Mallevich, I. Cherniavskii, Y. Lu, R. Krishnamoorthi, A. Yu, V. Kondratenko, S. Pereira, X. Chen, W. Chen, V. Rao, B. Jia, L. Xiong, and M. Smelyanskiy. Deep learning recommendation model for personalization and recommendation systems. *CoRR*, abs/1906.00091, 2019.

[29] Md. A. Raihan and T.M. Aamodt. Sparse Weight Activation Training. In *Conference on Neural Information Processing Systems (NeurIPS 2020)*, 2020.

[30] A. Renda, J. Frankle, and M. Carbin. Comparing rewinding and fine-tuning in neural network pruning. In *International Conference on Learning Representations*, 2020.

[31] J. Schwarz, S. M. Jayakumar, R. Pascanu, P.E. Latham, and Y. W. Teh. Powerpropagation: A sparsity inducing weight reparameterisation. In *https://arxiv.org/pdf/2110.00296.pdf*, 2021.

[32] D. Silver, J. Schrittwieser, K. Simonyan, I. Antonoglou, A. Huang, A. Guez, T. Hubert, L. Baker, M. Lai, A. Bolton, Y. Chen, T. Lillicrap, F. Hui, L. Sifre, G. van den Driessche, T. Graepel, and D. Hassabis. Mastering the game of go without human knowledge. *Nature*, 550:354–, October 2017.

[33] N. Ström. Sparse Connection And Pruning In Large Dynamic Artificial Neural Networks. In *http://www.nikkostrom.com/publications/euro97/euro97.pdf*, 1997.

[34] X. Sun, J. Choi, C-Y Chen, N. Wang, S. Venkataramani, V. Srinivasan, X. Cui, W. Zhang, and K. Gopalakrishnan. Hybrid 8-bit Floating Point (HFP8) Training and Inference for Deep Neural Networks. In *In The Conference and Workshop on Neural Information Processing Systems*, 2019.

[35] Z. Tang, L. Luo, B. Xie, Y. Zhu, R. Zhao, L. Bi, and C. Lu. Automatic Sparse Connectivity Learning for Neural Networks. In *https://arxiv.org/pdf/2201.05020.pdf*, 2022.

[36] A. Vaswani, N. Shazeer, N. Parmar, J. Uszkoreit, L. Jones, A.N. Gomez, Ł. Kaiser, and I. Polosukhin. Attention Is All You Need. In *Neural Information Processing Systems*, 2017.

[37] N. Wang, J. Choi, D. Brand, C-Y Chen, and K. Gopalakrishnan. Training Deep Neural Networks with 8-bit Floating Point Numbers. In *https://arxiv.org/pdf/1812.08011.pdf*, 2018.

[38] M. Wortsman, A. Farhadi, and M. Rastegari. Discovering neural wirings. In *Conference on Neural Information Processing Systems (NeurIPS*, 2019.

[39] Y. Zhang, M. Lin, M. Chen, Z. Xu, F. Chao, Y. Shen, K. Li, Y. Wu, and R. Ji. Optimizing Gradient-driven Criteria in Network Sparsity: Gradient is All You Need. In *https://arxiv.org/pdf/2201.12826.pdf*, 2022.

[40] A. Zhou, Y. Ma, J. Zhu, J. Liu, Z. Zhang, K. Yuan, W. Sun, and H. Li. Learning n:m fine-grained structured sparse neural networks from scratch. In *International Conference on Learning Representations*, 2021.

[41] M.H. Zhu and S. Gupta. To prune, or not to prune: exploring the efficacy of pruning for model compression. In *https://arxiv.org/pdf/1710.01878.pdf*, 2017.