TTERGM: Social Theory-Driven network simulation

Abstract

Temporal exponential random graph models (TERGM) are powerful statistical
models that can be used to infer the temporal pattern of edge formation and
elimination in complex networks (e.g., social networks). TERGMs can also be used
in a generative capacity to predict longitudinal time series data in these evolving
graphs. However, parameter estimation within this framework fails to capture
many real-world properties of social networks, including: triadic relationships,
small world characteristics, and social learning theories which could be used to
constrain the probabilistic estimation of dyadic covariates. Here, we propose triadic
temporal exponential random graph models (TTERGM) to fill this void, which
includes these hierarchical network relationships within the graph model. We
represent social network learning theory as an additional probability distribution
that optimizes Gibbs entropy in the graph vector space. The new parameters are
then approximated via Markov chain Monte Carlo maximum likelihood estimation.
We show that our TTERGM model achieves improved fidelity and more accurate
predictions compared to several benchmark methods on GitHub network data.

1 Introduction

Social networks have been studied for over a century, and graph theory techniques have been used
for decades to model relationship patterns amongst individuals and social entities **?**. Graph network
statistics have provided fundamental and theoretical insight into social phenomenal across political,
economic and behavioral data. However, historically, these techniques have been applied to study
static 'snapshots' of a network, limiting the ability to make large-scale inferences about social network
dynamics.

In recent years, online social networks have provided a wealth of open source empirical data, providing
new opportunities to test and adjudicate amongst competing theories and quantitative models Borgatti
et al. [2009]. Online social networks are formed by nodes (e.g., individuals) and edges, embedding
social relationships (e.g., friends, relatives, colleagues) within a complex graph network. Both
network structure and nodal connectivity constraint the flow of information or resources through
the network Kane et al. [2014]. However, online social networks are ever expanding, and their
connectivity evolves rapidly over time. Thus, statistically modeling these data for generative and
predictive purposes is challenging.

In recent years, temporal exponential random graph models (TERGM) have become a core tool
for modeling dynamic social networks. Although a variety of approaches including variations of
TERGM have been applied to education Mamas et al. [2020], finance, Park et al. [2018] and political
data Abrams [2019], few TERGM variations have attempted to quantify the effect of "influencers"
alongside triadic relationships within a dynamic network. Here, we expand the classic TERGM
model to support triadic relationships to make predictions on dynamic graphs. We apply this new
triadic temporal exponential random graph model (TTERGM) to data derived from Github. Github
has been studied extensively within the context of graphical network modeling due to its popularity,

open access, and transparency. Thus, these data provide a benchmark for model comparison. GitHub also supports social network coding features Borges et al. [2016], providing the opportunity to study "influencers", their affect on network structure, and triadic relationships within a dynamic network.

2 Related Work

Social network models have been applied for many purposes to include: modeling an individual's behavioral patterns to predict future nodal attributes (e.g., connections) over time McConnell et al. [2018] McAvoy et al. [2020], modeling interactions and cluster formation within online communities Fortunato and Hric [2016] Xu et al. [2020] Liu et al. [2018], and modeling how network characteristics (e.g., centrality) influences its users Qiu et al. [2017]. Overall, these models attempt to characterize the relationship amongst network structure and information diffusion, decision making, and individual behavior. Jackson et al. [2017].

General classes of network formation methods include: 1) exponential random graph models (ERGMs) Lusher et al. [2013]Pattison and Wasserman [1999], meta-networks, and meta-matrices Carley and Hill [2001]Krackhardt and Carley [1998] for multilayer social networks. 2) block modeling Guimerà and Sales-Pardo [2009] 3) geographic or characteristic based approaches, Boucher and Mourifié [2012]Leung [2014]; 4) link formation techniques Christakis et al. [2010]Bramoullé et al. [2012], and 5) subgraph model-based approaches (SUGMs) Chandrasekhar and Jackson [2016]. Numerous studies have examined the formation of cascades of network activity, characterizing and predicting network growth Bakshy et al. [2012]Gruhl et al. [2004]Yang and Counts [2010]. Typically, spikes of activity occur within a few days of content's introduction into the network. This property forms the backdrop to a line of temporal analyses that focus on the basic rising-and-falling pattern that characterizes the initial occurrence of information burst.

Influence on the Github platform can be quantified by the number of followers, stars, mentions, quotes, and up-votes received from other users. Social network metrics such as centrality indicate how broadly influence extends (e.g. geographic interest) Weber and Luo [2014]. Other features include project volume, documentation volume, presence of supporting files, code volume and standard library usage. The popularity velocity can be measured by (Total_Stars / project_life). Few studies have examined influence of user-popularity, repo-popularity, and triadic relationships in dynamic graphs.

3 Methodology

3.1 Preliminaries

Exponential random graph models (ERGM) are static. Temporal exponential random graph models (TERGM) are an extension of ERGMs to handle dynamic information in real-world online social networks Hanneke et al. [2010]. ERGM can be written as in (1). TERGM with Markov assumption can be written as in (2).

$$P(N) = \frac{1}{Z}exp(\sum \gamma(N)) \tag{1}$$

$$P(N^t|N^{t-1}) = \frac{1}{Z_{t-1}}exp(\sum_{ij} \sigma(N_{ij}^t, N_{ij}^{t-1})) \tag{2}$$

$P(N)$ represents the probability of the network N; Z represents the normalization constant that is usually difficult to computer; γ is the vector of network characteristics such as number of edges, triangles, 2-stars, etc. t represents the sequence of network observations; σ is the vector of social-theory driven temporal network characteristics such as homophily, transitivity, reciprocity, etc. t represents the sequence of network observations; Compared to ERGMs, TERGMs are able to model the distribution on time series data (either embedded in the network or in separate timesteps), hence certain temporal patterns can be captured and reflected in the parameter values for phenomenon interpretation. Examples of these dynamic patterns of triadic effects in influencer networks are shown in Figure 1. TERGM provides significant advantages over ERGMs, since certain static

Figure 1: Examples of Triadic relationship in Influencer Networks

Figure 2: Design of the Triadic Temporal Exponential Random Graph Model

patterns can be enriched in higher dimensions when the sequence order is considered. TERGMs are also capable of modeling observed friendship networks with bootstrap methods estimated by maximum pseudolikelihood Leifeld et al. [2018], or networks of infectious disease transmission using statistical methods in network analysis Jenness et al. [2018]. The flexibility of TERGMs make it possible to adapt to a variety of input data types, such as cross-sectional or longitude data Henry et al. [2016]Block et al. [2018].

3.2   Triadic Temporal Exponential Random Graph Model

TERGM models estimated within the markov chain assumption are typically incapable of generating and reproducing realistic dynamics observed in real-world online social networks. We hypothesized that increasing the model's capacity to describe triadic network properties would reduce the error between the model and empirical observations. We propose TTERGM here to sequentially predict network probabilities by integrating the dynamics between influencers and followers (Code is available at https://github.com/alvin68633466/TTERGM-Social-Theory-Driven-network-simulation). TTERGM was run on a computer with 12900K CPU, 1080TI and 128GB RAM. Figure 2 shows the framework of TTERGM that has five major components - data collection module, network processing module, feature extraction module, pattern analysis module, and a generative network module.

The data collection module is discussed in Section 3.3. Influencer-follower networks N_t were identified and constructed by connecting users by events in Table 2. In the subsequent module, temporal features and triadic features listed in Table 1 were extracted from the influencer-follower networks. Network characteristics were then estimated using the Markov chain Monte Carlo (MCMC) method. In the pattern analysis module, the TTERGM model was applied to model the data. The general form of TTERGM can be written as in Equation (3). $P(N)$ represents the probability of a given network architechture, N; Z represents the normalization constant as done in classic TERGM models; γ is a vector of network characteristics (e.g, number of edges, triangles, 2-stars), t represents the temporal sequence of network observations, and σ is the vector of social-theory driven temporal network characteristics such as homophily, transitivity, reciprocity, etc. To fit the TTERGM model to the data, algorithm 1 is used to initialize and traverse each node in the network to construct the generative influencer-follower networks. Finally, we used these observed features to simulate real-world influencer-follower networks and compared the predictive of TTERGM performance with the classic TERGM model and the Block Model using left-out validation data.

$$P(N^t|N^{t-1}) = \frac{1}{Z_{t-1}}exp(\sum_{ij} \sigma(N_{ij}^t, N_{ij}^{t-1})) + ... + \frac{1}{Z_1}exp(\sum_{ij} \sigma(N_{ij}^2, N_{ij}^1)) \qquad (3)$$

Table 1: Features Extracted from Influencer Network

Measurement Type	Measurement Name	Description
Network connection	# direct links	Number of direct links influencers have with their followers
	# indirect length-2 links	Number of indirect links of length 2 from influencers to their followers
	# indirect length-3 links	Number of indirect links of length 3 from influencers to their followers
	# triangles	Influencer's activity on a repository triggered his/her follower's activity on the same repository, then a triangle forms
Network topology	Average shortest path	Average shortest path length of pairs of nodes in the network
	Assortativity	Pearson correlation coefficient of degree between pairs of linked nodes
	# of connected components	Number of connected components in the network
	Average Clustering Coefficient	Ratio of number of triangles over maximum possible number of triangles
	# nodes	Number of nodes in the network
	# edges	Number of edges in the network

Algorithm 1: Triadic Temporal Exponential Random Graph for Network Generation

Input: Online social network N

1   Initialization $G = G_{t_0}$ **repeat**

2   **while** $t \neq t_n$ **do**

3   read current node n_i;

4   **if** n_i *is not traversed* **then**

5   **for** *each link l_{ij} in graph G_{t_0}* **do**

6   calculate the network statistics of each link and

7   fill the values to the feature vector N_{ij}

8       $Prob(l_{ij}) = \sum_{p=0,q=0}^{pq} Prob(l_{ij}|l_{pq})$

9   **else**

10   read the next node n_{i+1};

11     $t = t + 1$;

12   Calculate $P(N^t) = \frac{1}{Z} exp(\sum_{ij} \sigma(N_{ij}^t, N_{ij}^{t-1}, ..., N_{ij}^{t_0})$ where

13    $Z = MCMC(exp(\sum_{ij} u(N_{ij}^t, N_{ij}^{t-1}, ..., N_{ij}^{t_0})))$ where

14   Z is the normalization constant, u is the network statistics, $MCMC$ is the Markov Chain Monte Carlo method

15 **until** *k iterations*;

3.3 Data Collection

The dataset used in this paper spans from 2015-01-01 to 2017-08-31 with 13.5 million software repositories, 2.6 million users and 245,501 influencer followers. The entire graph of our data has 52,260,372 nodes and 870,532,947 edges. We selected the top 100 repositories because we aimed to characterize highly popular GitHub repositories, and the impact of influencers on the popularity of repositories using the TTERGM model. The number of repositories is a hyperparameter which can

be adjusted, depending on the goal of the model. We selected the top 10 users, in terms of number of followers, as the influencers in this study. This threshold was chosen because the number of followers drops off sharply after that point, but can be chosen arbitrarily for different datasets. All data was obtained using the GitHub API Gousios and Spinellis [2012], which provides services to search public repositories and to retrieve specific data about them (e.g., number of follows, number of code updates, posted issues, and project participations). To retrieve the number of "Watch" events on different dates for each repository, we used service from the API Gousios and Spinellis [2012] to get all the events of our target repository. The data contains 14 types of events which are listed in Table 2.

Table 2: Event Categories and Descriptions

Event Category	Event Type	Description
Receptive Events	WatchEvent	When someone stars a repository
	PullRequestReviewCommentEvent	When comment on a pull request's unified changes
	IssueCommentEvent	When an issue comment is created or edited
	MemberEvent	When a user is invited or removed as a collaborator to a repository
	IssuesEvent	When an issue is created or edited
	GollumEvent	When a wiki page is created or edited
Contributive Events	WatchEvent	When someone stars a repository
	ForkEvent	When an user forks a repository
	ReleaseEvent	When a release is published or edited
	PublicEvent	When a private repository is made public
	PullRequestEvent	When a pull request is created or edited
	PushEvent	When a push is happened to a repository branch
	DeleteEvent	When a branch or tag is deleted
	CommitCommentEvent	When is commit comment is created
	CreateEvent	When a branch or tag is created

4 Experiment Results

We implemented our TTERGM technique to model the dynamics between repository popularity and networks. We examine how the heavy-tailed degree distribution, user interactions, small-world property and influencers' roles effected the popularity of repositories. For example, the most highly followed influencer had 52,722 followers. Subsequently ranked influencers had 30,161 and 25,827 followers respectively. All of the top ten influencers had at least 14,000 followers. Figures 34 shows the respective contributive and receptive popularity of repositories that the influencers have participated in, spanning from 01-01-2015 to 01-31-2018. For contributive events, users generally are developers and can modify and update the repository; while for receptive events, users typically receive update information and notifications passively. The x-axis represents the order of the months from 2015-01-01 to 2018-01-31 and the y-axis shows the monthly contributive event counts and receptive event counts, respectively. From our empirical observations, influencers that have more followers tend to have more receptive events in repositories that they participate in. The same holds true for contributive events, although the overall number of events correlates to the influencer's popularity. The number of contributive events peaked at the 1st month with over 5,000 events, and the number of receptive events peaked at the 6th month with over 25,000 events. The #1 influencer has much higher influence in receptive events than in contributive events in the early stages while others have normal fluctuations over the 3 years. Figure 5 (left) illustrates the influencer's triadic influence on followers. When an influencer commits an activity in a repository (e.g., WatchEvent, ForkEvent, PushEvent or CreateEvent), it triggers his/her followers to commit activities in the same repository

Figure 3: Monthly contributive events for five identified "rockstar" users on Github

(e.g., WatchEvent, ForkEvent, IssueEvent, IssueCommentEvent, PushEvent, PullRequestEvent or CommitCommentEvent).

From Figure 3 we can see that influencers with more followers have stronger triadic influence on the repositories that he/she participates in (with more overall monthly events). The difference in the number of events between the top influencer and other influencers is much higher in receptive events than in contributive events, indicating higher triadic influence from top influencers in those events for technology development in distributed management systems Bandura and Walters [1977]. Apart from the triadic influence, there are more network dynamics worth studying for social network links informing probability predictions, such as two-star sub-graphs (e.g., number of influencers with FollowEvents from two users), three-star sub-graphs (e.g., number of repositories with WatchEvents from three users), triangle sub-graphs (e.g., three users forming a following chain), and cliques (e.g., four users that have events in every pairs of them) Chandrasekhar and Jackson [2016]. Oftentimes, only the final network is observed but the formation process that reveals the online social dynamics is neglected, hence the temporal aspects of social network link formation are an important aspect for this study.

Table 3: Number of Followers of the Top 10 Influencers

Rank	Anonymized Influencer Id	Number of Followers
1	lBMOoXAjxIN_Dc3alQNLZQ	52722
2	BhQS5KA8AvmQJXbsVeusdw	30161
3	s0jAeLRt2onrivaUCqdJrg	25827
4	QFB1aZ8GXkNYHyfWe7aEeA	24604
5	jAGnWUFUmnBc9ydeQbIfDQ	24510
6	hXalEIoEWnEbCSfiQI1LNA	23076
7	eUnkVgArKJiNOBhb0w53_Q	18522
8	VRyyOPSJUCS5jRlDtwjefA	15755
9	wNDkYd6NACSuvLCnxog23w	15396
10	wHfAzUFXU8D186qTl9c54w	14928

Figure 4: Monthly participative events for five identified "rockstar" users on Github

Figure 5: How influencers' activities can affect their followers' activities by the triadic effect

Table 4: Block, TERGM, and TTERGM models were used to generate predicted distributions for network characteristics on out-of-sample temporal observations

Network Model	2017-07 in-deg	2017-07 out-deg	2017-08 in-deg	2017-08 out-deg
Block Model	4.39	5.08	5.32	4.56
TERGM	4.65	4.78	5.13	4.35
TTERGM	3.42*	4.15*	4.25*	3.25*

Note: ∗ indicates p-value < 0.05 comparing to Block Model and TERGM respectively

To evaluate the performance of the TTERGM, we compared the simulation with two benchmark models - TERGM and Block model. Each model is evaluated using a set of metrics (average degree of incoming edges and average degree of outgoing edges) to see how well the predicted network

distribution matched the real-world network characteristics. The evaluation result (average of 30 runs) was shown in Table 4. The Block model and TERGM perform similarly in month 2017-07 and 2017-08. Comparing to TERGM, TTERGM has 26.45% less errors in 2017-07 for incoming degree of errors, 13.17% less errors in 2017-07 for outgoing degree of errors, 17.15% less errors in 2017-08 incoming degree of errors, and 25.28% less errors in 2017-08 for outgoing degree of errors. We believe the consistent improvement stems from the extra computation from TTERGM in the markov chain.

5   Conclusion

We presented a network-based approach for bipartite social network modeling to examine the effect of including influencers and triadic relatinships on predicting future network dynamics. GitHub provides features for developers to collaborate in distributed teams and interact with community members. As popular repositories are formed or influential users act, the structure and the dynamics of the social network evolves, altering network metrics. In this work, we constructed daily bipartite social networks over two years (2015 to 2017) across 13.5 million software repositories and 2.6 million GitHub users, and observed the evolution of behavior across 245,501 influencer-followers. The TTERGM technique build upon previous statistical models by incorporating information flow across hierarchical configuration features. We represented social network learning theory as an additional probability distribution that optimizes Gibbs entropy in the graph vector space. The new parameters are then approximated via Markov chain Monte Carlo maximum likelihood estimation. The TTERGM model is capable of reproducing the dynamics observed empirically in large-scale social network data, and produced more accurate predictions on left-out data compared to classic TERGM and block models. However, the TTERGM model imposes additional computational burden during parameter estimation, which may hinder its ability to scale to larger data sets. Future work may include expanding this approach to model the influence of more "distant" users in the network, or those that do not directly follower an influencer (Figure 5 right), and how two-star, three-star, triangle and clique subgraphs effect social network dynamics.

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