DHRL: A Graph-Based Approach for Long-Horizon and Sparse Hierarchical Reinforcement Learning

Abstract

Hierarchical Reinforcement Learning (HRL) has made notable progress in complex control tasks by leveraging temporal abstraction. However, previous HRL algorithms often suffer from serious data inefficiency as environments get large. The extended components, $i.e.$, goal space and length of episodes, impose a burden on either one or both high-level and low-level policies since both levels share the total horizon of the episode. In this paper, we present a method of **D**ecoupling **H**orizons Using Graph in Hierarchical **R**einforcement **L**earning (DHRL) which can alleviate this problem by decoupling the horizons of high-level and low-level policies and bridging the gap between the length of both horizons using graph. DHRL provides a freely stretchable high-level action interval, which facilitates longer temporal abstraction and faster training in complex tasks. Our method outperforms state-of-the-art HRL algorithms in typical HRL environments. Moreover, DHRL achieves extremely long and complex locomotion and manipulation tasks.

1 Introduction

Reinforcement Learning (RL) has been successfully applied to a range of robot systems, such as locomotion tasks [23, 7], learning to control aerial robots [9, 12] and robot manipulation [13, 21]. Goal-conditioned RL, which augments state with the goal to train an agent for various goals [22, 18], further raised the applicability of RL in robot systems allowing the agent to achieve diverse tasks.

Hierarchical Reinforcement Learning (HRL), which trains multiple levels of goal-conditioned RL, has improved the performance of RL in complex and sparse tasks with long horizons using temporally extended policy [24, 25, 15]. On

Figure 1: **DHRL:** By decoupling the horizons of both levels of the hierarchical network, DHRL not only solves extremely long and sparse tasks but also significantly outperforms previous state-of-the-art algorithms.

the back of these strengths, HRL was adopted to solve various complex robotics tasks [19, 10, 17].

However, HRL often has difficulty in complex or large environments because of training inefficiency. Previous studies speculated that the cause of this problem is the large goal space, and restricted the high-level action space to alleviate this phenomenon [27, 11]. Nevertheless, this approach performs well only in limited length and complexity and still suffers from the same trouble in larger environments.

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We show that the fundamental reason is the inherent traits of HRL, which could not be solved by constraining high-level policy only. In previous HRL frameworks, the horizons of the low level and high level are related to each other structurally because they share the total length of the episode. This relation causes a tradeoff between the training burden of both levels; if the intervals between high-level actions increase (x-axis in Figure 2), the low-level policy has to cover a wider range, and in the opposite case, the high-level policy takes charge of the extended burden alone in large environments (y-axis in Figure 2). This is the reason why the previous HRL algorithms cannot cope with extended components of large environments.

Figure 2: Our method is scalable in large environments by breaking down the relations between the two levels and allowing both levels to operate at their suitable horizons.

We break down the fundamental traits of HRL: To break the relation between the horizons of both levels, we adopt a graph structure. In our method, the high-level policy can use a longer temporal abstraction while the lower one only takes charge of smaller coverage by decomposing the subgoal into several waypoints with a graph. In this way, the HRL algorithm obtains the capability to stretch the interval of high-level action freely and achieves extremely complex and large tasks, thanks to the enlarged strength of the HRL.

In summary, our main contributions are:

- We show that the previous HRL structures are not scalable in large environments and point out the fundamental reason.
- To break down the inherent traits of HRL, we propose DHRL which decouples the horizons of high-level and low-level policies and bridges the gap using a graph.
- Our algorithm outperforms state-of-the-art algorithms in typical HRL environments and achieves extremely complex and long tasks.

2   Preliminaries

We consider a finite-horizon Universal Markov Decision Process (UMDP) which can be represented as a tuple $(\mathcal{S}, \mathcal{G}, \mathcal{A}, \mathcal{T}, \mathcal{R}, \gamma)$ where $\mathcal{S}$, $\mathcal{A}$ and $\mathcal{G}$ are state space, goal space and action space respectively. The environment is defined by the transition distribution $\mathcal{T}(s_{t+1}|s_t, a_t)$ and reward function $\mathcal{R} : \mathcal{S} \times \mathcal{A} \times \mathcal{G} \to \mathbb{R}$. Also, total return of a trajectory $\tau = (s_0, a_0, ..., s_H, a_H)$ is $R(\tau, g) = \sum_{t=0}^{H-1} \gamma^t r(s_{t+1}, g)$ where $r(s_{t+1}, g)$ (or $r(s_t, a_t, g)$) is a goal conditioned reward function and γ is a discount factor. Subgoal sg, waypoint wp and goal g are defined in goal space $\mathcal{G}$ and we consider $\mathcal{G}$ that is a subspace of $\mathcal{S}$ with a mapping $\psi : \mathcal{S} \to \mathcal{G}$.

HRL framework typically has high-level policy π^{hi} and low-level policy π^{lo}, each maintaining a separated replay buffer $\mathcal{B}^{hi}$ and $\mathcal{B}^{lo}$. Our method also follows the general HRL framework and employs both buffers that store high-level $(s_t, g_t, sg_t, r_t, s_{t+c_h}) \in \mathcal{B}^{hi}$ and low-level transition data $(s_t, wp_t, a_t, r_t, s_{t+1}) \in \mathcal{B}^{lo}$ where c_h is the interval between high-level action.

One of the key problems in learning HRL is that the low-level policy π^{lo} is non-stationary and thus old data from past policies may contain different next-states s_{t+c} even though the high-level policy provides the same subgoal in the identical state. To bypass this off-policy discrepancy, HRL models use off-policy correction which relabels the subgoal of high-level policy to be $\tilde{sg}_t = \arg\min_{sg_t} (\sum_{n=t}^{t+k-1} \log \pi^{lo}(a_i|s_i, sg_i) - \log \pi_\beta^{lo}(a_i|s_i, sg_i))^2$, where π_β^{lo} is the previous policy used to collect data [16]. It can be interpreted as correcting the old subgoal to the most plausible subgoal that will result in the same transition with the current agent. In practical use of this off-policy correction, we choose $k = 1$ because of restrictions on access to the environment.

Graph-guided RL methods, which combine the strength of RL and planning by decomposing a long-horizon task into multi-step sub-problems, estimate the temporal distance between states and goals to construct a graph $\mathbf{G} = (\mathbf{V}, \mathbf{E})$ on goal space $\mathcal{G}$ without additional prior knowledge about

87 environments. Previous studies proposed various methods to recover distance from Q-value [3, 8, 26].
88 If the agent gets -1 reward at every step except when it is in a goal area where the agent gets 0 reward,
89 then, $Q_{lo}(s, a|g)$ can reveal the temporal distance between s to g as

$$Dist(s \rightarrow g) = \log_\gamma \left(1 - (1 - \gamma)Q_{lo}(s, \pi(s, g)|g)\right) \tag{1}$$

90 However, it is known that recovering temporal distance correctly from the vanilla Q-network in
91 this setting is challenging. For that reason, the previous methods use an additional Value function
92 approximator [26] or distributional Q-networks [3].

3 Related work

94 **Graph-guided RL.** Graph has recently been
95 used as a non-parametric model in reinforce-
96 ment learning (RL) to combine the advantages
97 of RL and planning [3, 8, 26, 2, 5]. By decom-
98 posing a long-horizon task into multi-step plan-
99 ning problems, these studies have shown bet-
100 ter performance and data efficiency. Search on
101 the Replay Buffer (SORB) [3] constructs a di-
102 rected graph based on the states randomly ex-
103 tracted from a replay buffer and Q-function-
104 based edge cost estimation. The follow-up stud-
105 ies further improved the performance of ear-
106 lier graph-guided RLs by combining additional
107 methods such as graph search on latent space or
108 model predictive control.

Figure 3: The differences between DHRL and the
previous graph-based HRL methods. Our algo-
rithm includes the graph structure between both
levels explicitly while the previous methods use
the graph only for training high-level policy or get-
ting waypoints.

109 However, previous papers sidestepped the exploration problems in complex tasks through 'random
110 starting condition' [3, 8, 26, 5], or work only on the dense reward settings [2]. We emphasize that the
111 'random starting condition' accesses privileged information about the environment during training by
112 generating the agent uniformly within the feasible area of the map. This greatly reduces the scope
113 of application of these algorithms. Unlike prior methods, ours can train from sparse reward settings
114 and a fixed initial state without knowledge of the agent's surroundings, which makes it practical for
115 physical settings.

116 **Constrained-subgoal HRL.** To mitigate the training inefficiency issue of HRL, several researchers
117 proposed methods that restrict the action of high-level agent to be placed in adjacent areas. Hier-
118 archical Reinforcement Learning with k-step Adjacency Constraint (HRAC) [27] limits the sub-
119 goal to be in the adjacency space of the current state. Hierarchical reinforcement learning Guided
120 by Landmarks (HIGL) [11] improved the data efficiency by adding novelty-based landmarks to
121 adjacency-constrained HRL. However, these improvements only work on the limited length and
122 complexity of the environment.

123 The previous work closest to our method is HIGL. However, there are three key differences between
124 the previous work and our approach. First, our model explicitly includes the whole graph structure
125 while HIGL needs to train high-level action to imitate the graph by using an additional loss term
126 corresponding to the Euclidean distance from landmark or pseudo-landmark. Second, we use a graph
127 to decouple the horizons of high level and low level, unlike the previous method which uses the
128 graph only for guidance. Most importantly, ours can achieve goals in extremely long and complex
129 environments. To the best of our knowledge, there is no prior HRL research to train a model that has
130 decoupled the horizons of the two levels.

4 Methods

132 We introduce Decoupling Horizons Using Graph in Hierarchical Reinforcement Learning (DHRL),
133 which can separate the time horizons of high-level and low-level policies and bridge the gap between
134 both horizons using graph. Our framework consists of high-level policy $\pi^{hi}(sg|s, g)$, low-level policy
135 $\pi^{lo}(a|s, wp)$, and graph **G**. Given a goal g_t in the environment, the high-level policy outputs a
136 subgoal sg_t (see Figure 4). Then, the shortest path from the current state s_t to sg_t is found on the

Figure 4: An overview of DHRL which includes the mid-level non-parametric policy between the high and low levels. The high-level (orange box) policy delivers subgoal $sg \in \mathcal{G}$ to the graph level (green box) and the graph instructs the low-level policy (blue box) to reach the waypoint $wp \in \mathcal{G}$. c_h represents how long each high-level action operates for. The low level is given $c_{l,i}$ steps to achieve the goal where $c_h \neq c_{l,i}$.

graph. To do so, s_t and sg_t are added to the existing graph structure, then a sequence of waypoints $(s_t, wp_{t,1}, wp_{t,2}, ..., sg_t)$ is returned using a graph search algorithm. Finally, the low-level policy tries to achieve $wp_{t,i}$ during $c_{l,i}$ steps.

The key point of our method is that the low-level horizon $c_{l,i}$ is unrelated to the high-level horizon, which is H/c_h in a H-steps task ($c_h > c_{l,i}$), because the graph decomposes it into several stages. In other words, unlike the previous HRL methods which have the relationship of

$$horizon^{high} \times horizon^{low} = H, \tag{2}$$

our algorithm does not have such relations, removing a big obstacle toward a scalable-RL algorithm.

In section 4.1, we explain how to construct a graph over states and find a path on the graph level. In section 4.2, we present a low-level policy which can recover temporal distance between states without overestimation. In section 4.3, We introduce a strategy to train our method through graph-agnostic off-policy learning. Finally, in section 4.4, we propose additional techniques for better data efficiency in large environments.

4.1 Graph level: planning over the graph

This section details the graph search part in DHRL planning. We emphasize that every distance in the DHRL model is based on temporal distance $Dist(\cdot \rightarrow \cdot)$, obtained through (1). Thus, our algorithm requires no further information about the environment (*e.g.* L2 norm distance between states) than general HRL settings.

Let $\psi : \mathcal{S} \rightarrow \mathcal{G}$ be the projection of states on the goal space. In the graph initialization phase, we samples n nodes (also called landmarks) using FPS algorithm (Algorithm 2 in Appendix A) from $\psi(s)$ where $s \in \mathcal{S}$ is state sampled from $\mathcal{B}^{lo}$. Then, we connect a directed edge $s_1 \rightarrow s_2$ if the temporal distance from s_1 to s_2 is less than the cutoff-threshold. In the planning phase, the graph $\mathbf{G}(\mathbf{V}, \mathbf{E})$ gets the subgoal sg_t from the high-level policy and adds the projection of current state $\psi(s_t)$ and sg_t to $\mathbf{V}$, so that the number of nodes in $\mathbf{G}$ becomes $n + 2$. Then, we connect the edges with costs less than the cutoff-threshold between the newly added nodes and existing nodes.

Next, we find the sequence of waypoints $\mathcal{W} : (wp_{t,0} = \psi(s_t), wp_{t,1}, wp_{t,2}, ..., wp_{t,k-1}, wp_{t,k} = sg_t)$ that connects from $\psi(s_t)$ to sg_t using a graph search algorithm. At this time, if there is no path from $\psi(s_t)$ to sg_t, we adopt a secondary path from $\psi(s_t)$ to $sg_t^{\dagger}$, where $sg_t^{\dagger}$ is the closest node to sg_t among the nodes connected from $\psi(s_t)$. After finding the waypoint sequence, the graph level provides $wp_{t,1}$ to the low-level policy and instructs it to reach $wp_{t,1}$. If it has been $Dist(wp_{t,i-1} \rightarrow wp_{t,i})$ steps since the agent started tracking $wp_{t,i}$ or it achieves $wp_{t,i}$, then we update the current tracking waypoint to the next waypoint $wp_{t,i+1}$. In order to reflect the newly discovered state, we update graph nodes at every N episodes.

However, it is challenging to train Q-network which recovers reliable temporal distance. We explain the reason and a solution in the next section.

4.2 Low level: Q-network for graph construction

Consider a situation where a low-level policy π^{lo} is given a waypoint $wp_{t,i}$. Until the low-level policy is learned enough to achieve $wp_{t,i}$ definitively, it may fail and the experience of failure remains in replay buffer. Because of this experience, Q-value decreases and raises serious overestimation of temporal distance reconstructed from (1). As a result, node selection algorithms (*e.g.* Algorithm 2 in Appendix A.) pick more and more graph nodes around the overestimated area (*e.g.*, where the agent is difficult to pass) since these algorithms select more nodes in spacious areas. This results in overcrowding of the graph nodes in those areas. The agent rarely can go through a 'difficult area' to achieve the goal beyond it because the distance estimator and graph search algorithm do not function properly around it.

To mitigate this limitation, we train Separate Q-networks for Graph and Critic (SQGC), composed of two identical Q-networks using different proportions of hindsight goal relabeling. In these networks, We use Hindsight Experience Replay (HER) [1] to relabel desired goal as achieved goal. The SQGC includes Q^{lo}_{critic} and Q^{lo}_{graph} where Q^{lo}_{critic} is for training π^{lo}, just like a typical application, and Q^{lo}_{graph} is for recovering temporal distance between nodes. We substitute wp_t with $\hat{wp}_t := ag_{t+t_{ftr}}$ in the sequential transition of a single episode $(s_t, wp_t, a_t, r(s_{t+1}, wp_t), s_{t+1})_{t=1:H-1}$ where ag means the achieved goal and t_{ftr} is a random integer drawn from the uniform distribution between 0 and $H - t$. To train Q^{lo}_{graph}, we relabel the whole wp_t in $(s_t, wp_t, a_t, r(s_{t+1}, wp_t), s_{t+1})_{t=1:H-1}$ as $\hat{wp}_t$, while we replace only a part of wp_t for Q^{lo}_{critic}.

Our method can prevent overestimation in the distance recovery by relabelling the goals in the whole transitions for training Q^{lo}_{graph} because the experiences of failure are replaced with the successful trajectory. Also, by maintaining the original Q^{critic}_{lo} to train π_{lo}, there is no degradation in the performance of the agent who might otherwise not be able to get negative feedback from failure since the failure will be relabeled as a desired goal.

However, it is still challenging to train HRL including a graph level using off-policy RL algorithms. We describe our approach to train DHRL using an off-policy algorithm in the next section.

4.3 High level: hindsight transitions for graph-agnostic off-policy learning

Thanks to decoupling the time horizons of both levels in HRL, the high-level policy in our method can look further without any additional burden on the low-level policy. In other words, We can stretch c_h, which represents how long each high-level action operates for. However, we experimentally found it challenging to train DHRL with an off-policy algorithm. The ability to train through an off-policy algorithm is important for data efficiency in that it can use the previous data from replay buffer. We propose two main reasons for this difficulty and a solution.

First, since the graph is gradually updated, it is challenging to train a model using an off-policy method from the data given by the previous graph. Second, as c_h gets longer, predicting the similarity of the trajectory of c_h steps with only the first step of $\pi^{low}(a_i|s_i, g_i)$ gets harder when we replace subgoal using off-policy correction. In short, the extended interval and non-stationary graph can cause a larger discrepancy between the data obtained from the previous policy to the current policy.

To facilitate the off-policy learning for longer c_h and changing graph in DHRL, We use hindsight action transitions in the high-level replay buffer data $(s_t, g_t, sg_t, r_t, s_{t+c_h}) \in \mathcal{B}^{hi}$. Alongside the original transitions data, we copy it and replace the subgoal $(sg_t = \pi^{high}_\beta(sg_t|s_t, g_t))$ with the achieved goal ag_{t+c_h} after c_h steps. Thus, we use both transitions $(s_t, g_t, sg_t, r_t, s_{t+c_h})$ and $(s_t, g_t, ag_{t+c_h}, r_t, s_{t+c_h})$ to train the high-level policy. This method is inspired by Hierarchical Actor-Critic (HAC) [14], which relabels subgoals and assumes that π^{lo}_β is the optimal low-level policy π^{lo*}.

By replacing the previous subgoal with the achieved goal, we can assume that this transition was obtained from a stationary graph and π^{lo*} with an error below a bound which is set to be a function of the density of the graph. We provide a theoretical analysis of the possibility of replacing the old off-policy graph with a virtual stationary graph in this section and Appendix B.

Definition 4.1. Given a compact state space $\mathcal{S}$, $\mathbf{G}(\mathbf{V}, \mathbf{E})$ is an $\epsilon - resolution$ graph $if\ ^\forall s \in \mathcal{S}, ^\exists v \in V\ s.t.\ \max(Dist(\psi(s) \to v), Dist(v \to \psi(s))) < \epsilon$, where $\mathbf{V} \subset \psi(\mathcal{S})$ and $E = \{(v_i, v_j) | v_i, v_j \in \mathbf{V}, Dist(v_i \to v_j) < c_l\}$.

Theorem 4.2. *Let $\mathbf{G}$ be an arbitrary $\epsilon - resolution$ graph ($\epsilon < c_l/2$). Also, let π_β^{lo} and $\mathbf{G}_\beta$ be the low-level policy and graph at the time of collecting the data. Off-policy error rate $\rho(\mathbf{G})$ is the normalized distance error with respect to the total traversal distance according to the change of π_β^{lo} and $\mathbf{G}_\beta$ to π^{lo*} and $\mathbf{G}$. If there is a path from s to g, the upper bound of off-policy error rate $\rho(\mathbf{G})$ using a path obtained from graph search over $\mathbf{G}$ is $2\epsilon/c_l$.*

Theorem 4.2 shows that if we replace the previous subgoal with the achieved goal, and assume that these transitions are obtained from a stationary $\epsilon - resolution$ graph $\mathbf{G}$, then the off-policy error ρ is less than $2\epsilon/c_l$. Thus, we can train DHRL through a graph-agnostic off-policy RL algorithm using substituted transition without additional off-policy correction.

4.4 Optional techniques: gradual penalty and frontier-based goal-shifting

In this section, we propose two additional techniques, gradual penalty and frontier-based goal-shifting (FGS). These optional techniques can boost the performance of DHRL on some extremely long tasks as shown in Figure 9(a) and (c). We note that these are not essential to train DHRL and ours outperforms previous HRL frameworks without these techniques.

Figure 5: Optional techniques for better data efficiency: gradual penalty encourages the high-level policy to output a subgoal that the low-level agent can achieve. Frontier-based goal-sampling replaces goals in the explored area with new goals positioned in the rim of the graph.

Gradual penalty. Similar to the subgoal testing in the previous method [14], we propose gradual penalty, which can impose the penalty more delicately when the action space of the high-level agent gets large. We evaluate sg_t of the original transition data $(s_t, g_t, sg_t, r_t, s_{t+c_h})$ by categorizing the following three cases; (a) close to the graph and low level actually achieved the subgoal $\in \mathbf{L}_1$, (b) close to the graph but the low-level policy could not achieve the subgoal $\in \mathbf{L}_2$ and (c) far from the graph $\in \mathbf{L}_3$. In this way, we can impose the more detailed penalties to respond to the expansion of the high-level action space as the high-level interval c_h stretches.

Frontier-based goal-shifting. This optional technique has been devised to accelerate learning in a complex environment. FGS moves the final goal g to the frontier area when g comes into the place where the graph is already laid out during training. To check whether the goal is in the graph area, we examine whether $\min_{v \in \mathbf{V}}(Dist(v \to g))$ is smaller than cut-off threshold. Alternative goals are the addition of random noise to nodes sampled from $v \in \mathbf{V}$ proportional to $-Q(start, \pi(start, v)|v)$. FGS is similar to the previous goal-directed exploration research [20] in that both use weighted samples, but ours does not maintain a generative model and samples the goals from graph level in DHRL. Note that we do not use FGS when we compare DHRL with the previous state-of-the-art, since this FGS is beyond the main contribution, which is about HRL structure.

5 Experiments

5.1 Environment description

We evaluate DHRL on robot environments based on the MuJoCo simulator including some extremely sparse and long-horizon tasks. Locomotion environments have fixed starting points and random goal points during training. In test episodes, they give the agent one of the most challenging goals (*i.e.*, the end of the maze). We also evaluate our algorithm in a robot arm environment in which the agent aims to make the end-effector touch the goal.

Figure 6: DHRL significantly outperforms prior state-of-the-art algorithms (success rate averaged over 4 random seeds and smoothed equally, and only the sparse settings for Reacher3D are tested as in the previous papers). Note that in AntMazeComplex, AntMazeBottleneck, and UR3Obstacle, the curves are not visible as they overlap at zero success rate.

- PointMaze / AntMazeSmall : The point / ant achieves the goal if it comes within 2.5 distance (success threshold) from the target point in 12×12 maze.
- AntMaze : 24×24 maze with success threshold 5
- AntMazeBottleneck : Bottleneck exists at the middle of the maze. The ant can barely pass through bottleneck.
- AntMazeComplex : 56×56 maze with success threshold 5
- Reacher3D : 7-dof robot arm aims to reach a goal.
- UR3Obstacle : 6-dof robot arm aims to reach a goal in an environment with several board-shaped obstacles.

We adopt TD3 algorithm [4] for high-level and low-level networks and use Dijkstra algorithm to find the shortest path in the graph level. To train the separate Q-networks for graph and critic in DHRL, We use 80% and 100% of HER transition for Q_{lo}^{critic} and Q_{lo}^{graph} respectively. Our algorithm automatically determines c_l based on the temporal distance to the nearest waypoint. Only a few hyperparameters have been changed across various locomotion experiments: the number of nodes, penalty, and c_h. The results of DHRL in this paper are obtained using only sparse reward.

5.2 Experiment result

Baselines. We compare our method with state-of-the-art algorithms with and without a graph respectively; HIGL [11] and HRAC [27]. For more comparison with shallow RL (SAC) [6] and vanilla HRL (HIRO) [16], see Table 1 in Appendix C.

Comparison to state-of-the-art algorithms. Results are shown in Figure 6. Thanks to decoupling the horizons of the levels, DHRL could stretch the high-level interval and shows high data efficiency and success rate in locomotion and goal-reaching tasks in simulation environments. Moreover, DHRL is the only algorithm that can succeed in extremely complex environments (AntMazeComplex and AntMazeBottleneck). When measuring the performance of the baselines, we selected c_h with the best performance among the values c_h=10, 20, ..., and 50 (80 in AntMazeComplex).

Figure 7: Robustness to c_h: HRAC vs Ours

As shown in Figure 6, previous HRL methods cannot solve long-horizon tasks. This is likely due to the coupling ($c_h = c_l$) of the high-level horizon ($horizon^{high} = H/c_h$) and the low-level horizon ($horizon^{low} = c_l$) resulting in an increased burden on either the high-level or the low-level policy. For long-horizon tasks (large H), if $c_h(= c_l)$ is fixed, then $horizon^{high}$ increases and the high-level performance plunges as shown in Figure 9(b). On the other hand, if $c_h(= c_l)$ is increased, the low level has to manage a wider area and the performance plunges as shown in Figure 7.

Figure 8: Locomotion and Reacher tasks in simulation: Additional to general tasks for HRL, we evaluated our method in extremely long and sparse tasks (see (a), (b), and (c) in the figure). The maps of the environments are not given to the agent.

Figure 9: **Ablation study.** (a): The separate Q-network (SQGC) effectively alleviates the overestimation problem of temporal distance and FGS improves data efficiency. (b): Extending the high-level horizon is crucial in long-horizon tasks and DHRL successfully separates the horizons. (c): Gradual penalty improves the data efficiency in extremely long tasks. (d): Why do we need π^{hi}?: Without an additional high-level policy above the graph level, the performance plunges without the random starting condition.

5.3 Ablation study

Decoupling horizons.   In this section, we examine how decoupling horizons affects the performance of long-horizon HRL and evaluate whether DHRL can stretch the interval of high-level policy. As shown in Figure 9(b), we tested various values of spacing of the high-level action, c_h.

The result shows that even in long intervals of the high-level policy, DHRL shows consistent or improved performance (Figure 9(b)) without major degradation unlike previous HRL methods (Figure 7). In particular, DHRL agent in long-horizon environments such as AntMazeComplex shows that extended high-level interval is crucial for exploration and performance (see the rightmost figure in Figure 9(b)). Considering that the baselines could not solve extremely long-horizon tasks even with various values of $c_h \in [10, 80]$, and the performance degrades with increasing c_h, we conclude that

our method successfully decoupled the time steps in both levels and stretched the interval between high-level actions. This trait allows the high-level policy to look further and take advantage of the extended temporal abstraction.

Separate Q-network for Graph and Critic (SQGC). In this section, we evaluate the effects of the Separate Q-network for Graph and Critic (SQGC) in our method. The red line in Figure 9(a), which is a variant of DHRL without a SQGC, clearly shows that the SQGC is critical to the performance. We empirically found that the overestimation of the temporal distance between states occurs especially near the obstacles (*e.g.* corners at the maze), and it is difficult for an agent to pass by without a separate Q network for graph construction. This is consistent with our analysis that the experiences of failure spoil the ability to recover the temporal distance from Q-network.

Ablate high-level policy. Graph-guided RL, which maintains graph level and low-level policy, is also a variant of HRL in that the graph level is a non-parametric version of the high-level policy (see Figure 3). From this point of view, the main difference between graph-guided RL and our method is the existence of the high-level policy. Then, why do we need additional high-level policy above the graph level?

Most graph-guided RL algorithms use 'random starting condition' to train the agent in complex environments and such assumption could be expensive, especially in the physical world where it is challenging to start from different positions each time. Figure 9(d) shows the result of the graph-guided RL algorithm, L3P [26], in a random and fixed starting environment. This indicates that without 'random starting condition', the performance and data efficiency of the previous graph-guided RL method drops seriously even in the smallest environment we experimented with. In contrast, DHRL can explore without 'random starting condition' even in extremely long environments thanks to the enlarged temporal abstraction and exploration performance. Considering that the high-level policy suggests a subgoal to graph level, we conclude that the high level facilitates better exploration of the graph-guided RL. Thus, DHRL that has exploration capability also can be seen as the improved version of graph-guided RL.

Optional techniques. The result in Figure 9(c) shows that the gradual penalty can improve data efficiency in extremely long tasks. By imposing a penalty on subgoals that are far from the current graph, the gradual penalty encourages the high-level agent to output a subgoal near the graph. Since the low-level agent can hardly achieve subgoals far from the explored area, the gradual penalty accelerates the training by providing more valid goals. FGS also shows better training efficiency in environments. The results in Figure 9(a) show the wider improvements as the training progresses. This means that as the more area the agent has explored in the environment, it becomes more difficult to get goals that help exploration and the data efficiency decreases. With FGS, DHRL learns efficiently using randomly generated goals by replacing easy goals with new goals located on the rim of the graph.

6 Conclusion

We proposed a Decoupling Horizons Using Graph in Hierarchical Reinforcement Learning (DHRL), which is a data-efficient HRL algorithm leveraging a graph to expand the range of problems that HRL can solve, by decoupling horizons and allowing both levels to operate at their suitable horizons. Our experimental results show that our method successfully separates the horizons of the levels and outperforms prior state-of-the-art methods. We believe that our method could present a more fundamental solution towards scalable RL rather than constraining high-level policy or stacking three or more levels of policies like the previous methods in that ours has a lower dependency on the horizon of the environment. In this paper, we adopted a vanilla graph construction algorithm and sparse reward settings. We expect that if a novelty-based node selection is added to the graph construction is applied, the performance will be further improved.

Limitation. While our algorithm shows successful results in complex and long-horizon tasks, it might be difficult to construct a graph in some tasks that require complex interactions with the environment because of the higher-dimensional state space. We expect that this limitation can be solved by extending our algorithms into latent state space, or by incorporating a node sparsification algorithm, which we leave for future works.

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 # A   Algorithms.

Algorithm 1 Training DHRL

1: sample $D_i^{lo} = (s_t, wp_t, a_t, r(s_{t+1}, wp_t), s_{t+1})_i \in \mathcal{B}^{lo}$
2: relabel $wp_t \leftarrow \hat{wp}_t = ag_{t+t_{ftr}}$ to make $\hat{D}_i^{lo}$
3: update Q_{critic,θ_1}^{lo} and $\pi_{\phi_1}^{lo}$ using $D_i^{lo} \cup \hat{D}_i^{lo}$
4: update Q_{graph,θ_2}^{lo} using $\hat{D}_i^{lo}$
5: **if** t mod d **then**
6:     sample $D_i^{hi} = (s_t, g_t, sg_t, r_t, s_{t+c_h})_i \in \mathcal{B}^{hi}$
7:     relabel $sg_t \leftarrow ag_{t+c_h}$ to make $\hat{D}_i^{hi}$
8:     **for** $(s_t, g_t, sg_t, r_t, s_{t+c_h})$ in D_i^{hi} **do**
9:         **if** $r(s_{t+c_h}, sg_t) = 0$ **then**
10:            $r_t \leftarrow r_t$
11:        **else**
12:            $r_t \leftarrow penalty$
13:        **end if**
14:    **end for**
15:    update $Q_{\theta_3}^{hi}$ and $\pi_{\phi_2}^{hi}$ using $D_i^{hi} \cup \hat{D}_i^{hi}$
16: **end if**

Algorithm 2 Farthest Point Sampling Algorithm

1: **Input:** set of states $\{s_1, s_2, ..s_K\}$, sampling number k, temporal distance function $Dist(\cdot \rightarrow \cdot)$
2: SelectedNode = []
3: DistList = [inf, inf, ... inf]
4: **for** $i = 1$ **to** k **do**
5:    FarthestNode $\leftarrow$ argmin(DistList)
6:    add FarthestNode to SelectedNode
7:    DistFromFarthest $\leftarrow [Dist(\text{FarthestNode} \rightarrow s_1), ..., Dist(\text{FarthestNode} \rightarrow s_K)]$
8:    DistList = ElementwiseMin(DistFromFarthest, DistList)
9: **end for**
10: **return** SelectedNode

Algorithm 3 Planning with DHRL

1: **while** not done **do**
2:    **if** t mod $graph\_construct\_freq$ **then**
3:        construct a graph $\mathbf{G}(\mathbf{V}, \mathbf{E})$: sample $\mathbf{V} = \psi(s)$ where $s \in D^{lo}$ through FPS algorithm and get edge cost $\mathbf{E}$ by (1)
4:    **end if**
5:    $sg_t = \pi^{hi}(s_t, g_t)$
6:    get $\mathcal{W} : (wp_{t,0} = \psi(s_t), wp_{t,1}, wp_{t,2}, ..., wp_{t,k-1}, wp_{t,k} = sg_t)$
7:    previous waypoint index $idp \leftarrow 0$; tracking waypoint index $idt \leftarrow 1$; tracking time $t_{tr} \leftarrow 0$
8:    **for** $\tau = 1$ **to** c_h **do**
9:        get low-level action a_τ from $\pi^{lo}(a_\tau | s_\tau, wp_{t,idt})$
10:       act a_τ in the environment and get $s_{\tau+1}$
11:       $t_{tr} += 1; t += 1$
12:       **if** agent achieve $wp_{t,idt}$ or $t^{tr} > Dist(wp_{t,idp} \rightarrow wp_{t,idt})$ **then**
13:           $idp += 1; idt += 1; t_{tr} \leftarrow 0$
14:       **end if**
15:    **end for**
16: **end while**

B Proofs of Theorems.

Definition B.1. $\mathcal{W}_{\mathbf{G}}(s_t, sg_t) = (wp_{t,0}, wp_{t,1}, ..., wp_{t,k})$ is a sequence of waypoint obtained by the graph search algorithm and $w(\mathcal{W}_{\mathbf{G}}, \tau) = wp_{t,i} \in \mathcal{W}_{\mathbf{G}}(s_t, sg_t)$ is the waypoint that is given to low-level policy at τ.

Given the transition distribution of the environment $\mathcal{T}(s_{\tau+1}|s_\tau, a_\tau)$, the transition data $(s_t, g_t, sg_t, r(s_{t+c_h}, g_t), s_{t+c_h})$ from the high-level policy's replay buffer has been obtained as

$$s_{t+c_h} = \prod_{\tau=t}^{t+c_h-1} \mathcal{T}(s_{\tau+1}|s_\tau, a_\tau) \cdot \pi_\beta^{lo}(a_\tau|s_\tau, w(\mathcal{W}_{\mathbf{G}_\beta}, \tau)), \tag{3}$$

where π_β^{lo} and $\mathbf{G}_\beta$ are the previous low-level policy and graph respectively. Also, by using a different graph $\mathbf{G}$ and an optimal policy π^{lo*}, we get a new transition data $(s_t, g_t, sg_t, r(s'_{t+c_h}, g_t), s'_{t+c_h})$, where

$$s'_{t+c_h} = \prod_{\tau=t}^{t+c_h-1} \mathcal{T}(s_{\tau+1}|s_\tau, a_\tau) \cdot \pi^{lo*}(a_\tau|s_\tau, w(\mathcal{W}_{\mathbf{G}}, \tau)). \tag{4}$$

For given s_t and s_{t+c_h}, we define the off-policy error rate, which is the normalized distance error with respect to the total traversal distance according to the change of π_β^{lo} and $\mathbf{G}_\beta$ to π^{lo*} and $\mathbf{G}$, as

$$\rho(\mathbf{G}) = \frac{Dist(\psi(s'_{t+c_h}) \to \psi(s_{t+c_h}))}{Dist(\psi(s_t) \to \psi(s_{t+c_h}))}. \tag{5}$$

Lemma B.2. *Suppose that $Dist(\cdot \to \cdot)$ in eqn (1) is Lipschitz continuous. Then, there exists a constant $L > 0$ such that $^\forall x$ and y, $\max(Dist(x \to y), Dist(y \to x)) \leq L||x - y||$, where $|| \cdot ||$ is the Euclidean norm, since $Dist(x \to x) = 0$. Then, any $\epsilon/L - resolution$ graph w.r.t the Euclidean norm, whose existence is trivial, is an $\epsilon - resolution$ graph w.r.t $Dist(\cdot \to \cdot)$.*

Proof of Theorem 4.2

Proof. Let $\mathcal{C}^{s \to g}$ be one of the shortest paths from s to g and T be the distance of $\mathcal{C}^{s \to g}$. Also let $p_1 \in \mathcal{C}^{s \to g}$ be a point that $Dist(\psi(s) \to p_1) = c_l - \epsilon$. Then, $^\exists wp_1 \in \mathbf{V}$ s.t. $\max(Dist(p_1 \to wp_1), Dist(wp_1 \to p_1)) < \epsilon$, because $\mathbf{G}$ is an $\epsilon - resolution$ graph. Since $Dist(\cdot \to \cdot)$ is a temporal distance, it satisfies the triangular inequality and then, $Dist(\psi(s) \to wp_1) \leq Dist(\psi(s) \to p_1) + Dist(p_1 \to wp_1) < (c_l - \epsilon) + \epsilon = c_l$ and $Dist(wp_1 \to g) \leq Dist(wp_1 \to p_1) + Dist(p_1 \to g) < \epsilon + (T - c_l + \epsilon) = T - c_l + 2\epsilon$.

Repeating the above procedure, let $p_{i+1} \in \mathcal{C}^{wp_i \to g}$ be a point that $Dist(wp_i \to p_{i+1}) = c_l - \epsilon$. Then, $^\exists wp_{i+1} \in \mathbf{V}$ s.t. $\max(Dist(p_{i+1} \to wp_{i+1}), Dist(wp_{i+1} \to p_{i+1})) < \epsilon$. Then, $Dist(wp_i \to wp_{i+1}) < c_l$ and $Dist(wp_{i+1} \to g) < T - (i+1)c_l + 2(i+1)\epsilon$. Consequently, the agent after T time-step will be closer than the $\lfloor T/c_l \rfloor^{th}$ waypoint from g. The remaining distance is less than

$$T - \lfloor T/c_l \rfloor c_l + 2\lfloor T/c_l \rfloor \epsilon. \tag{6}$$

Thus, if an agent follows the sequence of waypoints $\{s, wp_1, wp_2, ..., g\}$, which is generated from a graph search algorithm over $\mathbf{G}$ and π^{lo*}, the error rate over this path satisfies

$$\rho(\mathbf{G}) \leq \frac{T - \lfloor T/c_l \rfloor c_l + 2\lfloor T/c_l \rfloor \epsilon}{T} \leq \frac{T - (c_l - 2\epsilon)(T/c_l)}{T} = \frac{2\epsilon}{c_l}. \tag{7}$$

Thus the off-policy error rate ρ is equal or less than $2\epsilon/c_l$ during T. Since all path from s to g takes at least T time-steps, this upper-bound of error rate is also satisfied in all path from s to g. $\qquad\square$

 # C   Additional Results

Table 1: **Comparisons between our algorithm (DHRL) and baselines:** The numbers next to the environment names are the time-steps for training the models. The results are averaged over 4 random seeds and smoothed equally. '-D' and '-S' mean dense reward and sparse reward respectively. We use NVIDIA RTX A5000.

Success Rate		SAC	HIRO-D	HIRO-S	HRAC-D	HRAC-S	HIGL-D	HIGL-S	DHRL
PointMaze	0.25M	1.2%	73.8%	77.9%	6.1%	56.1%	25.5%	34.3%	**96.9%**
	0.5M	0.3%	93.6%	76.8%	33.5%	93.7%	88.3%	91.8%	**99.8%**
AntMazeSmall	0.5M	0.0%	0.0%	2.0%	9.2%	54.0%	36.3%	44.1%	**89.8%**
	1.0M	0.0%	24.1%	60.1%	83.4%	88.4%	83.7%	52.2%	**95.1%**
AntMaze	0.5M	0.0%	0.0%	0.0%	1.1%	0.0%	60.2%	32.7%	**71.5%**
	1.0M	0.0%	0.7%	0.8%	68.7%	48.9%	78.1%	60.3%	**91.1%**
Bottleneck	0.5M	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	**16.5%**
	1.0M	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	**38.7%**
Complex	2.5M	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	**20.4%**
	4.0M	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	**40.1%**
Reacher-3D	0.25M	49.2%	-	66.0%	-	26.5%	-	78.2%	**95.1%**
	0.5M	47.2%	-	67.1%	-	44.1%	-	47.1%	**90.6%**
UR3Obstacle		0.0%	2.5%	1.9%	0.5%	0.0%	11.1%	0.5%	**69.8%**

Figure 10: Comparison with shallow RL (SAC) and vanilla HRL (HIRO). The completely failed baselines are occluded by others.

Figure 11: Changes in the graph level over the training; DHRL can explore extremely long tasks with fixed starting conditions and limited knowledge about the environment.

D Hyperparameters

When evaluating the previous state-of-the-art algorithm, we used the same hyperparameters as used in their papers. We also tried various numbers of landmarks and c_h which may affect the performance in long-horizon tasks.

Table 2: Hyperparameters for HRL

	HIRO	HRAC	HIGL
high-level τ	0.005	0.005	0.005
π^{hi} lr	0.0001	0.0001	0.0001
Q^{hi} lr	0.001	0.001	0.001
high-level γ	0.99	0.99	0.99
high-level train freq	10	10	10
c_h		10-50	
low-level τ	0.005	0.005	0.005
π^{lo} lr	0.0001	0.0001	0.0001
Q^{lo} lr	0.001	0.001	0.001
low-level γ	0.95	0.95	0.95
hidden layer	(300,300)	(300,300)	(300,300)
number of coverage landmarks γ	-	-	20-250
number of novelty landmarks γ	-	-	20-250
batch size	128	128	128

Table 3: Hyperparameters for SAC

	SAC
hidden layer	(256, 256, 256)
actor lr	0.0003
critic lr	0.0003
entropy coef	0.2
τ	0.005
batch size	256
γ	0.99

Table 4: Hyperparameters for DHRL

	DHRL
hidden layer	(256, 256, 256)
start planning epoch	5
gradual penalty transition rate	0.2
high-level train freq	10
actor lr	0.0001
critic lr	0.001
τ	0.005
γ	0.99
number of landmarks	300-500
target update freq	10
actor update freq	2