SurCo: Learning Linear SURrogates for COmbinatorial Nonlinear Optimization Problems

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Abstract

Optimization problems with expensive nonlinear cost functions and combinatorial constraints appear in many real-world applications, but remain challenging to solve efficiently. Existing combinatorial solvers like Mixed Integer Linear Programming can be fast in practice but cannot readily optimize nonlinear cost functions, while general nonlinear optimizers like gradient descent often do not handle complex combinatorial structures, may require many queries of the cost function, and are prone to local optima. To bridge this gap, we propose ***SurCo*** that learns linear Surrogate costs which can be used by existing Combinatorial solvers to output good solutions to the original nonlinear combinatorial optimization problem, combining the flexibility of gradient-based methods with the structure of linear combinatorial optimization. Two variants of `SurCo` are proposed: `impromptu` which aims to solve a single nonlinear problem and `pre-trained` that tackles a distribution of problems by training a model that predicts linear surrogate costs for unseen problem instances. `pre-trained` gains domain knowledge and quickly produces high-quality solutions on held-out instances drawn from the same problem family. We analyze our method theoretically and empirically, showing smooth convergence and improved performance. Experiments show that compared to state of the art approaches and expert-designed heuristics, `SurCo` leads to lower cost solutions with comparable or faster solve time for two real-world industry-level applications: embedding table sharding and inverse photonic design.

1 Introduction

Combinatorial optimization problems with linear objective functions, like linear programming (LP) (Chvatal et al., 1983) and mixed integer linear programming (MILP) (Wolsey, 2007), have been extensively studied in operations research (OR). The resulting high-performance solvers like Gurobi (Gurobi Optimization, LLC, 2022) can solve industrial-scale optimization problems with ten of thousands of variables in a few minutes.

However, even with perfect solvers, one issue remains: the cost functions $f(\boldsymbol{x})$ in many practical problems are *nonlinear*, and the highly-optimized solvers mainly handle linear or convex formulations while real-world problems have less constrained objectives. For example, in embedding table sharding (Zha et al., 2022a) one needs to distribute embedding tables to multiple GPUs for the deployment of recommendation systems. Due to the batching behaviors within a single GPU and communication cost among different GPUs, the overall latency (cost function) in this application depends on interactions of multiple tables and thus can be highly nonlinear (Zha et al., 2022a). In this and similar cases, the optimal solution to a heuristic linear formulation may not be useful.

To obtain useful solutions to the real-world problems, one may choose to directly optimize the nonlinear cost, which is either a black-box output of a simulator (Gosavi et al., 2015; Ye et al., 2019), or a cost estimator learned by machine learning techniques (e.g., deep models) from offline data (Steiner et al., 2021; Koziel et al., 2021; Wang et al., 2021b; Cozad et al., 2014). However, many of these direct optimization approaches either rely on human-defined heuristics (e.g., greedy (Korte & Hausmann, 1978; Reingold & Tarjan, 1981; Wolsey, 1982), local improvement (Voß et al., 2012; Li et al., 2021)), or resort to general nonlinear optimization techniques (e.g., gradient descent (Ruder, 2016), reinforcement learning (Mazyavkina et al., 2021), evolutionary algorithms (Simon, 2013)). While these approaches can work in practice, they may lead to a slow optimization process, in

particular when the cost function is expensive to evaluate, and they ignore the combinatorial nature of most real-world applications (encoded in the feasible set $\boldsymbol{x} \in \Omega$).

In this work, we propose a systematic framework ***SurCo*** that leverages existing efficient combinatorial solvers to find solutions to nonlinear combinatorial optimization problems arising in real-world scenarios. There are two versions of SurCo, the impromptu version and the pre-trained version. In impromptu, given a nonlinear *differentiable* cost $f(\boldsymbol{x})$ to be minimized, we optimize a *linear surrogate* cost $\hat{\boldsymbol{c}}$ so that the *surrogate optimizer* (SO) $\min_{\boldsymbol{x} \in \Omega} \hat{\boldsymbol{c}}^\top \boldsymbol{x}$ outputs a solution that is expected to be optimal w.r.t. the *original* nonlinear cost $f(\boldsymbol{x})$. Due to its linear nature, SO can be solved efficiently with existing solvers, and the surrogate cost $\hat{\boldsymbol{c}}$ can be optimized in an end-to-end manner by back-propagating *through* the solver (Pogančić et al., 2019; Niepert et al., 2021; Berthet et al., 2020). In pre-trained, we consider a family of nonlinear differentiable functions $f(\boldsymbol{x}; \boldsymbol{y})$, where $\boldsymbol{y}$ parameterizes problem descriptions. We train the linear surrogate $\hat{\boldsymbol{c}}(\boldsymbol{y})$ on a set of optimization problems (called the training set $\{\boldsymbol{y}_i\}$), and evaluate on a held-out problem $\boldsymbol{y}'$, by directly optimizing SO: $\boldsymbol{x}^*(\boldsymbol{y}') := \arg\min_{\boldsymbol{x} \in \Omega(\boldsymbol{y})} \hat{\boldsymbol{c}}^\top(\boldsymbol{y}')\boldsymbol{x}$, which avoids optimizing the cost $f(\boldsymbol{x}; \boldsymbol{y}')$ from scratch.

Both versions of SurCo are evaluated in two real-world nonlinear optimization problems: embedding table sharding (Zha et al., 2022a), and photonic inverse design (Schubert et al., 2022). In both cases, we show that in impromptu, SurCo achieves higher quality solutions in comparable or less runtime, faster optimization in wall-clock time with lower solution cost, thanks to the help of an efficient combinatorial solver; in pre-trained, our method obtains better solutions in held-out problems compared to other methods that require training (e.g., reinforcement learning). More specifically, in table sharding SurCo-imp obtains between 14% to 85% improvement in solution quality with between 2% and 23% increase in runtime overhead compared to the greedy baseline, additionally SurCo-tr obtains between 47% and 71% runtime improvement against the state of the art RL-based table sharding algorithm Zha et al. (2022b). In photonic inverse design, SurCo-imp finds 21% more viable solutions for the beam splitter and twice as many solutions for the wavelength demultiplexers with all problems solving successfully for the mode converter and bend problems, taking between 10% to 64% less time than the pass-through approach from previous work (Schubert et al., 2022). While the offline trained SurCo-tr misses some optimal solutions in the different settings, it frequently obtains solutions in 0.5% to 2% of the runtime due to not needing to evaluate the objective and perform gradient steps.

We additionally present theoretical results that help motivate why training a model to predict surrogate linear coefficients exhibits better sample complexity than directly predicting the optimal solution (Li et al., 2018; Ban & Rudin, 2019).

2 PROBLEM SPECIFICATION

Our goal is to solve the following nonlinear optimization problem describe by $\boldsymbol{y}$:

$$\min_{\boldsymbol{x}} f(\boldsymbol{x}; \boldsymbol{y}) \qquad \text{s.t.} \quad \boldsymbol{x} \in \Omega(\boldsymbol{y}) \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ are the variables to be optimized, $f(\boldsymbol{x}; \boldsymbol{y})$ is the nonlinear differentiable cost function to be minimized, $\Omega(\boldsymbol{y})$ is the feasible region, typically specified by linear (in)equalities and integer constraints, and $\boldsymbol{y} \in Y$ are the problem instance parameters drawn from a distribution $\mathcal{D}$ over Y. For example, in the traveling salesman problem, $\boldsymbol{y}$ can be the distance matrix among cities. We often consider solving a family of optimization problems, described as $\boldsymbol{y} \in Y$.

Differentiable cost function. The nonlinear cost function $f(\boldsymbol{x}; \boldsymbol{y})$ can either be the result of a differentiable simulator (e.g., JAX (Bradbury et al., 2018)), or a cost model that is learned from an offline dataset, often generated via sampling multiple feasible solutions within $\Omega(\boldsymbol{y})$, and recording their costs. The cost model often takes the form of a deep neural network. In this work, we assume the following property of $f(\boldsymbol{x}; \boldsymbol{y})$:

Assumption 2.1 (Cost function)**.** *During optimization, the cost function $f(\boldsymbol{x}; \boldsymbol{y})$ and its partial derivative $\partial f / \partial \boldsymbol{x}$ are accessible.*

Learning a good nonlinear cost model f is highly non-trivial for practical applications (e.g., AlphaFold (Jumper et al., 2021), Density Functional Theory (Nagai et al., 2020), cost model for embedding tables (Zha et al., 2022a)) and is beyond the scope of this work.

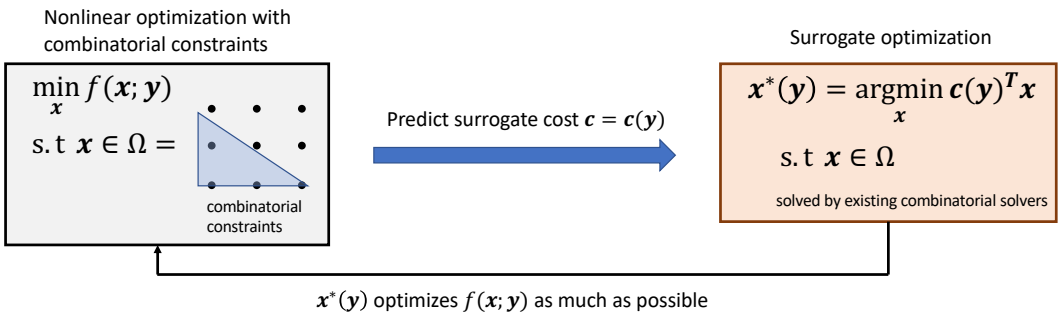


Figure 1: Overview of our proposed algorithm SurCo.

Evaluation Metric. In real-world applications, querying f can be slow and expensive, and thus a lower number of queries while getting better quality solution is the goal. We mainly focus on two aspects: how good the solution $\hat{\boldsymbol{x}}$ is, by checking the value of $f(\hat{\boldsymbol{x}};\boldsymbol{y})$, and how many queries of the nonlinear function f are needed during optimization in order to achieve the solution $\hat{\boldsymbol{x}}$.

Linear/nonlinear cost function. When $f(\boldsymbol{x};\boldsymbol{y})$ is linear w.r.t $\boldsymbol{x}$, and the feasible region can be encoded using mixed integer programs or other mathematical programs, the problem can be solved efficiently using existing scalable optimization solvers. When $f(\boldsymbol{x};\boldsymbol{y})$ is nonlinear, we propose SurCo that learns a surrogate linear objective function, which allow us to leverage these existing scalable optimization solvers, and which results in a solution that has high quality with respect to the original hard-to-encode objective function $f(\boldsymbol{x};\boldsymbol{y})$. We will elaborate in the following sections.

3 SurCo: Learning Linear surrogate

3.1 The impromptu case

We start from the simplest case in which we focus on a single instance with $f(\boldsymbol{x}) = f(\boldsymbol{x};\boldsymbol{y})$ and $\Omega = \Omega(\boldsymbol{y})$. SurCo aims to optimize the following objective:

$$(\texttt{SurCo-imp}): \quad \min_{\boldsymbol{c}} \mathcal{L}_{\texttt{imp}}(\boldsymbol{c}) := f(\boldsymbol{g}_\Omega(\boldsymbol{c})) \tag{2}$$

where the surrogate optimizer $\boldsymbol{g}_\Omega : \mathbb{R}^n \mapsto \mathbb{R}^n$ is the output of certain combinatorial solvers with linear cost weight $\boldsymbol{c} \in \mathbb{R}^n$ and feasible region $\Omega \subseteq \mathbb{R}^n$. For example, $\boldsymbol{g}_\Omega$ can be the following (n is the number of variables to be optimized):

$$\boldsymbol{g}_\Omega(\boldsymbol{c}) := \arg\min_{\boldsymbol{x}} \boldsymbol{c}^\top \boldsymbol{x} \quad \text{s.t.} \;\; \boldsymbol{x} \in \Omega := \{A\boldsymbol{x} \leq \boldsymbol{b}, \boldsymbol{x} \in \mathbb{Z}^n\} \tag{3}$$

which is the output of a MILP solver. Thanks to previous works (Ferber et al., 2020; Pogančić et al., 2019), we can efficiently compute the partial derivative $\partial \boldsymbol{g}_\Omega(\boldsymbol{c})/\partial \boldsymbol{c}$. Intuitively, this means that $\boldsymbol{g}_\Omega(\boldsymbol{c})$ can be *backpropagated* through.

Since f is also differentiable with respect to the solution it is evaluating, we thus can optimize Eqn. 2 in an end-to-end manner using any gradient-based optimizer. That is, $\boldsymbol{c}(t+1) = \boldsymbol{c}(t) - \alpha \frac{\partial \boldsymbol{g}_\Omega}{\partial \boldsymbol{c}} \frac{\partial f}{\partial \boldsymbol{x}}$, where α is the learning rate. The procedure starts from a randomly initialized $\boldsymbol{c}(0)$ and converges at a local optimal solution of $\boldsymbol{c}$.

While Eqn. 2 is still nonlinear optimization and there is no guarantee about the quality of the final solution $\boldsymbol{c}$, we argue that optimizing Eqn. 2 is better than optimizing the original nonlinear cost $\min_{\boldsymbol{x}\in\Omega} f(\boldsymbol{x})$. Furthermore, while we cannot guarantee optimality, we are able to guarantee feasibility by leveraging a linear combinatorial solver. Intuitively, instead of optimizing directly over the solution space $\boldsymbol{x}$, we optimize over the space of surrogate costs $\boldsymbol{c}$, and delegate the combinatorial feasibility requirements of the nonlinear problem to SoTA combinatorial solvers. Compared to naive approaches that directly optimize $f(\boldsymbol{x})$ via general optimization techniques, our method avoids handling complex constraints of the feasible regions, and thus makes the optimization procedure easier. Furthermore, it also helps escape from local minima, thanks to the embedded search component of existing combinatorial solvers (e.g., branch-and-bound (Land & Doig, 2010) in MILP solvers). As we see in the experiments, this is particularly important when the problem becomes large-scale with more local optima.

3.2 THE PRE-TRAINED CASE

We now discuss more general cases, where the nonlinear loss function $f(\boldsymbol{x}; \boldsymbol{y})$ represents *a family* of cost function to be optimized. Here the description of each *problem instance* $\boldsymbol{y}$ is drawn from a fixed problem distribution $\mathcal{D}$. We then ask the following question: how can we find solutions to a batch of training instances $\mathcal{D}_{\text{train}} := \{\boldsymbol{y}_i\}_{i=1}^N$, gain useful knowledge of the cost functions, and leverage such knowledge in held-out evaluation problem instances $\mathcal{D}_{\text{eval}}$ to accelerate the optimization procedure?

Following standard machine learning practice, let us first consider a naive two-stage approach. In the data collection stage, we simply apply `SurCo-imp`(Eqn. 2) to every $\boldsymbol{y}_i$ separately to get N surrogate cost vectors $\boldsymbol{c}_i$. Then in the training stage, we train a regressor $\hat{\boldsymbol{c}} = \hat{\boldsymbol{c}}(\boldsymbol{y}; \boldsymbol{\theta})$ on the dataset $\{(\boldsymbol{y}_i, \boldsymbol{c}_i)\}$ to learn to predict the surrogate costs from the problem features. Here $\hat{\boldsymbol{c}}$ is a parameterized model (e.g., a deep network) with the parameters $\boldsymbol{\theta}$ to be learned. This learned regressor $\hat{\boldsymbol{c}}(\boldsymbol{y}; \boldsymbol{\theta})$ can thus be used for a held-out problem instance $\boldsymbol{y}'$ to directly predict $\boldsymbol{c}' = \hat{\boldsymbol{c}}(\boldsymbol{y}'; \boldsymbol{\theta})$ and get the solution $\boldsymbol{x}' = \boldsymbol{g}_{\Omega(\boldsymbol{y}')}(\boldsymbol{c}')$ via surrogate optimizer (SO).

While this approach is simple, the N optimization procedures in the data collection stage are independent of each other, and can lead to excessive number of calls to f that are not helpful. E.g., if an optimization procedure converges to a bad local solution, then even if it achieves perfect convergence, which requires a lot of function calls, the resulting data point is still of low quality.

This motivates us to add a *regularizer* for the optimization:

$$(\texttt{SurCo-tr-}\lambda): \quad \min_{\boldsymbol{\theta}, \{\boldsymbol{c}_i\}} \mathcal{L}_{\texttt{tr}}(\boldsymbol{\theta}, \{\boldsymbol{c}_i\}; \lambda) := \sum_{i=1}^{N} f(\boldsymbol{g}_{\Omega(\boldsymbol{y}_i)}(\boldsymbol{c}_i); \boldsymbol{y}_i) + \lambda \|\boldsymbol{c}_i - \hat{\boldsymbol{c}}(\boldsymbol{y}_i; \boldsymbol{\theta}))\|_2 \tag{4}$$

Note that when $\lambda = 0$, it reduces to N independent optimizations, while when $\lambda > 0$, the surrogate costs $\{\boldsymbol{c}_i\}$ interact with each other. The intuition is that, the regressor $\hat{\boldsymbol{c}}(\boldsymbol{y}; \boldsymbol{\theta})$, even if not trained fully, can be very useful to guide $\boldsymbol{c}_i$ rather than just using its randomly initialized version. Furthermore, if $\hat{\boldsymbol{c}}$ is a mapping to global optimal solution of $\boldsymbol{c}$, then it will pull the solutions out of local optima to re-target towards global ones, even when starting from poor initialization, yielding fast convergence and better final solutions for individual optimization instances.

A special case is when $\lambda \to +\infty$, we arrive at a novel objective that jointly learns the surrogate cost function, given the training set $\mathcal{D}_{\text{train}}$:

$$(\texttt{SurCo-tr}): \quad \min_{\boldsymbol{\theta}} \mathcal{L}_{\texttt{tr}}(\boldsymbol{\theta}) := \sum_{i=1}^{N} f(\boldsymbol{g}_{\Omega}(\hat{\boldsymbol{c}}(\boldsymbol{y}_i; \boldsymbol{\theta})); \boldsymbol{y}_i) \tag{5}$$

3.3 COST REGRESSION VERSUS SOLUTION REGRESSION: A THEORETICAL ANALYSIS

We also want to compare `SurCo` with the previous works on ML optimizers (Ban & Rudin, 2019) that try to directly learn the mapping from problem description $\boldsymbol{y}$ to the solution, i.e. solution regression. Given a set of training instances $\mathcal{D}_{\text{train}}$ from distribution $\mathcal{D}$, these approaches first collect a set of training samples $\mathcal{D}_{\text{direct}} := \{\boldsymbol{y}, \boldsymbol{x}^*(\boldsymbol{y}) : \boldsymbol{y} \in \mathcal{D}_{\text{train}}\}$, and then learn a function $\tilde{\boldsymbol{x}}^*(\boldsymbol{y})$ to fit the training samples.

While this is conceptually simple, there exist fundamental difficulties to learn such a direct mapping. First, as mentioned above, it can be quite expensive to obtain the optimal solution $\boldsymbol{x}^*(\boldsymbol{y})$ due to the nature of nonlinear optimization and the query cost. Second, even if a perfect dataset $\mathcal{D}_{\text{direct}}$ is accessible, the number of samples needed to learn a mapping to directly predict $\boldsymbol{x}^*(\boldsymbol{y})$ is related to the *Lipschitz constant* L of the mapping, and for a direct mapping, L can be very large.

3.3.1 LIPSCHITZ CONSTANT AND SAMPLE COMPLEXITY

Let us first consider the sample complexity of solution regression methods as described above.

Formally, consider fitting any function $\boldsymbol{\phi} : \mathbb{R}^d \supseteq Y \mapsto \mathbb{R}^m$ with a dataset $\{\boldsymbol{y}_i, \boldsymbol{\phi}_i\}$. Here Y is a compact region with finite volume $\text{vol}(Y)$. The Lipschitz constant L is the smallest number so that $\|\boldsymbol{\phi}(\boldsymbol{y}_1) - \boldsymbol{\phi}(\boldsymbol{y}_2)\|_2 \leq L\|\boldsymbol{y}_1 - \boldsymbol{y}_2\|_2$ holds for any $\boldsymbol{y}_1, \boldsymbol{y}_2 \in Y$. The following theorem shows that if the dataset covers the space Y, we could achieve high accuracy prediction: $\|\boldsymbol{\phi}(\boldsymbol{y}) - \hat{\boldsymbol{\phi}}(\boldsymbol{y})\|_2 \leq \epsilon$ for any $\boldsymbol{y} \in Y$.

Definition 3.1 (δ-cover)**.** *A dataset $\mathcal{D}_{\text{direct}} := \{(\boldsymbol{y}_i, \boldsymbol{\phi}_i)\}_{i=1}^N$ δ-covers the space Y, if for any $\boldsymbol{y} \in Y$, there exists at least one $\boldsymbol{y}_i$ so that $\|\boldsymbol{y} - \boldsymbol{y}_i\|_2 \leq \delta$.*

Lemma 3.1 (Sufficient condition of prediction with ϵ-accuracy)**.** *If the dataset $\mathcal{D}_{\text{direct}}$ (ϵ/L)-cover Y, then for any $\boldsymbol{y} \in Y$, a 1-nearest-neighbor regressor $\hat{\boldsymbol{\phi}}$ leads to $\|\hat{\boldsymbol{\phi}}(\boldsymbol{y}) - \boldsymbol{\phi}(\boldsymbol{y})\|_2 \leq \epsilon$.*

Lemma 3.2 (Lower bound of sample complexity for ϵ/L-cover)**.** *To achieve ϵ/L-cover of Y, the size of the training set $N \geq N_0(\epsilon) := \frac{\text{vol}(Y)}{\text{vol}_0} \left(\frac{L}{\epsilon}\right)^d$, where vol_0 is the volume of unit ball in d-dimension.*

Please find all proofs in the Appendix. While we do not rule out a more advanced regressor than 1-nearest-neighbor that leads to better sample complexity, the lemmas demonstrate that the Lipschitz constant L plays an important role in sample complexity.

3.3.2 Difference between Cost and Solution Regression

In the following we will show that in certain cases, the direct prediction $\boldsymbol{y} \mapsto \boldsymbol{x}^*(\boldsymbol{y})$ could have an infinitely large Lipschitz constant L.

To show this, let us consider a general mapping $\boldsymbol{\phi} : \mathbb{R}^d \supseteq Y \mapsto \mathbb{R}^m$. Let $\boldsymbol{\phi}(Y)$ be the image of Y under mapping $\boldsymbol{\phi}$ and $\kappa(Y)$ be the number of connected components for region Y.

Theorem 3.1 (A case of infinite Lipschitz constant)**.** *If the minimal distance $d_{\min}$ for different connected components of $\boldsymbol{\phi}(Y)$ is strictly positive, and $\kappa(\boldsymbol{\phi}(Y)) > \kappa(Y)$, then the Lipschitz constant of the mapping $\boldsymbol{\phi}$ is infinite.*

Note that this theorem applies to a wide variety of combinatorial optimization problems. For example, when Y is a connected region and the optimization problem can be formulated as an integer program, the optimal solution set $\boldsymbol{x}^*(Y) := \{\boldsymbol{x}^*(\boldsymbol{y}) : \boldsymbol{y} \in Y\}$ is a discrete set of integral vertices, so the theorem applies. Combined with analysis in Sec. 3.3.1, we know the mapping $\boldsymbol{y} \mapsto \boldsymbol{x}^*(\boldsymbol{y})$ is hard to learn even with a lot of samples.

On the other hand, the mapping $\boldsymbol{y} \mapsto \boldsymbol{c}(\boldsymbol{y})$ can avoid too many connected components in its image $\boldsymbol{c}(Y)$, by connecting disjoint components of $\boldsymbol{x}^*(Y)$ together.

4 Settings

We evaluate the two variants of `SurCo` on two real-world settings, embedding table sharding and inverse photonic design. Both have industrial application. Each setting consists of a family of problem instances with varying feasible region and nonlinear objective function.

4.1 Embedding Table Sharding

The task of sharding embedding tables arises in the deployment of large scale neural network models which operate over both sparse and dense inputs (e.g., in recommendation systems (Zha et al., 2022a;b; Sethi et al., 2022)). Given T embedding tables and D homogeneous devices, the goal is to distribute the tables among the devices such that no device's memory limit is exceeded, while the tables are processed efficiently. Formally, let $x_{t,d}$ be the binary variable indicating whether table t is assigned to device d, and $\boldsymbol{x} := \{x_{t,d}\} \in \{0,1\}^{TD}$ be the collection of the variables. The optimization problem is:

$$\min_{\boldsymbol{x}} f(\boldsymbol{x}; \boldsymbol{y}) \quad \text{s.t.} \quad \boldsymbol{x} \in \Omega(\boldsymbol{y}) := \left\{ \boldsymbol{x} : \quad \forall t, \sum_t x_{t,d} = 1, \quad \forall d, \sum_t m_t x_{t,d} \leq M \right\} \tag{6}$$

Here the problem description $\boldsymbol{y}$ includes table memory usage $\{m_t\}$, and capacity M of each device. $\sum_d x_{t,d} = 1$ means each table t should be assigned to exactly one device, and $\sum_t m_t x_{t,d} \leq M$ means the memory consumption at each device d should not exceed its capacity. The nonlinear cost function $f(\boldsymbol{x}; \boldsymbol{y})$ is the runtime of the longest-running device. Due to shared computation (e.g., batching) among the group of assigned tables, and communication costs across devices, the objective is highly nonlinear. $f(\boldsymbol{x}; \boldsymbol{y})$ is well-approximated by a sharding plan runtime estimator proposed by Dreamshard (Zha et al., 2022b).

SurCo learns to predict $T \times D$ surrogate cost $\hat{c}_{t,d}$, one for each potential table-device assignment. During training, the gradients through combinatorial solver $\partial \boldsymbol{g}/\partial \boldsymbol{c}$ are computed via CVXPYLayers (Agrawal et al., 2019a) and the integrality constraints are relaxed. At test time we solve for the optimal integer solution using SCIP (Achterberg, 2009).

[This should be put to appendix]: The table features are the same used in Zha et al. (2022b), and sinusoidal positional encoding Vaswani et al. (2017) is used as device features so that the learning model is able to break symmetries between the different tables and effectively group them onto homogeneous devices. The table and device features are concatenated and then fed into Dreamshard's initial fully-connected table encoding module to obtain scalar predictions $\hat{c}_{t,d}$ for each desired objective coefficient.

We evaluate SurCo on the publicly available Deep Learning Recommendation Model (DLRM) dataset (Naumov et al., 2019). We consider 6 settings: 10, 20, 30, 40, 50, and 60 tables are placed to 4 devices with each GPU device having a 5GB memory limit. Each setting has 100 problem instances (50 training and 50 test). We evaluate against Dreamshard, the state of the art embedding table sharding algorithm, and domain-expert algorithm of greedily allocating tables to balance the aggregate dimension (Zha et al., 2022b).

To ensure our sharding plan can be deployed, we also calculate the true longest running time evaluated in the actual GPU hardware and report the performance in Tbl. **??**. We can see the predicted time spent is very close to the actual one.

4.2 Inverse Photonic Design

Photonic devices play an important role in high-speed communication (Marpaung et al., 2019), quantum computing (Arrazola et al., 2021), and machine learning hardware acceleration (Wetzstein et al., 2020). The photonic components can be formulated as a 2D device with grid cells either being filled or void

It remains challenging to design that are both manufacturable and satisfy certain design specifications (e.g., beam bending, wavelength-sensitive splitting). Given the design, existing work (Hughes et al., 2019) provides a differentiable cost function to compute design misspecification, evaluating how far off the transmission of the wavelengths of interest is from the desired locations.

Researchers also develop a standard benchmark of inverse photonic design problems (Schubert et al., 2022).

The inverse photonic design (Schubert et al., 2022) is to we need to design a 2D device with . Ultimately, we want light to be fed through specific input waveguides and then processed so that specific wavelengths are transmitted at desired levels in output locations. Additionally, we consider that we need to construct this design using a physical brush instrument which is a 3 by 3 cross shape. We visualize an example solution and the resulting wave intensities in Figures 4b and 4c respectively, where we are asked to split a beam that is fed in through the top into two wavelengths, each routed individually to the bottom left and right respectively. Overall this solution is physically viable since a 3 by 3 cross can fit in all filled space, and symmetrically, a 3 by 3 cross can fit in all void space.

The input design specification is fed in as a 2D image and is passed through a 3 layer convolutional neural network with ReLU activations and a final layer composed of filtering with the brush and then using a tanh activation to obtain surrogate coefficients, since previous work (Schubert et al., 2022) proposed using this filter and tanh operation to transform the latent parameters into a continuous solution that is projected onto the space of physically feasible solutions. Previous work formulated the projection as finding a discrete solution that minimized the dot product of the input continuous solution and proposed discrete solution. The authors then updated the continuous solution by computing gradients of the loss with respect to the discrete solution and using pass-through gradients to update the continuous solution. By comparison, our approach treats the projection as an optimization problem and updates the objective coefficients so that the resulting projected solution moves in the direction of the desired gradient. To enable differentiation of this blackbox projection solver, we leverage the approach suggested by Pogančić et al. (2019) which calls the solver twice, once with the original coefficients, and again with coefficients that are perturbed in the direction of the incoming solution gradient as being an "improved solution". The gradient with respect to the input

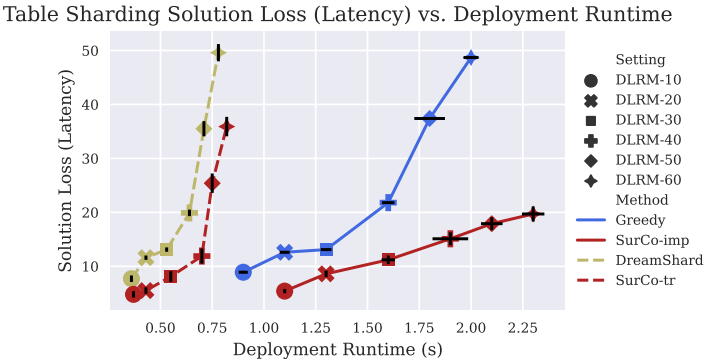
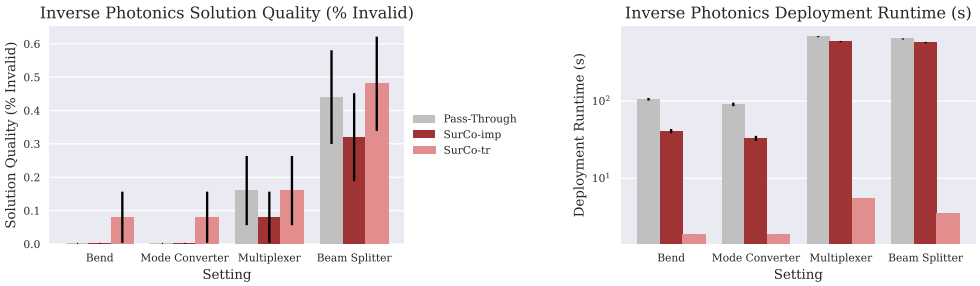


Figure 2: Table placement solution quality as measured by sharding plan latency and test time solver runtime. We evaluate our approaches against Dreamshard (Zha et al., 2022b) a state of the art embedding table sharding tool that requires offline training of an RL policy and a domain-expert greedy heuristic of iteratively adding tables based on their dimension. Dotted lines represent approaches that require pre-training.

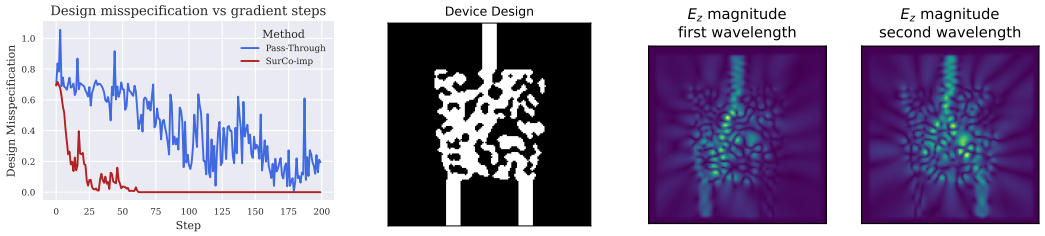


(a) (b)

Figure 3: Inverse photonics solution quality as measured by the percent of instances in which we fail to achieve 0 specification loss (a) and test time solver runtime in log scale (b), lower is better for both. We compare against the pass-through gradient approach proposed in Schubert et al. (2022). We observe that `SurCo-tr` is able to get similar success rates to the previous approach `Pass-through` with a substantially improved runtime due to not needing to run gradient-based optimization steps. Additionally, `SurCo-imp` outperforms or matches the runtime of the previous impromptu approach, while finding more solutions with 0 loss than `Pass-through`. Shaded bars represent approaches that pre-train a model.

coefficients are then the difference between the "improved solution" and the solution for the current objective coefficients.

We train and evaluate our approaches and compare against the pass-through method described above (Schubert et al., 2022) on randomly generated instances from the four types of problems implemented in Schubert et al. (2022), Waveguide Bend, Mode Converter, Wavelengths Division Multiplexer, and Beam Splitter. In each of these settings, we generate random instances by varying the locations of the input and output waveguides. Specifically, for mode converter we randomly select the right and left waveguide width, for the bend setting we randomize the waveguide width and length, for the beam splitter we randomize the waveguide separation as well as the waveguide width and length, and for the wavelength division multiplexer we randomize the input and output waveguide locations. In each setting we consider that we have two wavelengths of 1270nm and 1290nm, and are optimizing at a resolution of 40nm. We generate 50 instances from each of the 4 settings, 25 for training and 25 for testing. We visualize the results on test instances in Fig. 3.



(a) Loss Convergence (b) Device (c) Wave Mangitude

Figure 4: Inverse photonic design convergence for a ceviche challenge instance (Schubert et al., 2022). `SurCo-imp` smoothly lowers the loss while the pass-through baseline yields noisier convergence. We also visualize the resulting solution and magnitude of the two wavelengths of interest which we successfully route from the input at the top to the two different waveguides at the bottom.

5 Experimental Results

We compare `SurCo`'s performance against the relevant state of the art and human-expert baselines in Figures 2 and 3. For table placement, Fig. 2, `SurCo-imp` finds lower latency sharding plans than the `impromptu` Greedy baseline, even though it takes slightly longer due to using gradient-based optimization steps rather than simply greedily allocating tables to devices while observing memory limits. Comparing pretraining methods, Dreamshard and `SurCo-tr`, we find that `SurCo-tr` obtains lower latency solutions in about the same time with a slight increase in overhead due to using SCIP (Achterberg, 2009), a branch and bound MILP solver. In smaller problem instances where only 10 to 40 tables are placed on the 4 devices, `SurCo-tr` obtains better quality solutions than its impromptu counterpart, `SurCo-imp`, likely due to training on a variety of examples and being able to better escape local optima in any given problem instance as might be the case with the impromptu solver. However, as the problem size increases and more tables are available for placement, `SurCo-imp` gives better performance by optimizing for the test instances in question as opposed to `SurCo-tr` which only uses training data to obtain surrogate costs.

For inverse photonics, Fig. 3, `SurCo-imp` consistently finds as many or more valid photonic devices compared to the `Pass-through` baseline (Schubert et al., 2022). Additionally, since we terminate the two impromptu solvers when they either find a valid solution, or reach a maximum of 200 steps, the runtime of the impromptu solver `SurCo-imp` is slightly lower than the impromptu `Pass-through` baseline. Finally, `SurCo-tr` obtains similar success rates as the `Pass-through` baseline while taking two orders of magnitude less time as it does not require expensive impromptu optimization or multiple evaluations of the nonlinear objective, making `SurCotr` a promising approach for large-scale settings or when many slightly-varied instances.

We visualize the convergence of impromptu solvers in Fig. 4a where `SurCo-imp` has smoother and faster convergence compared to the `Pass-through` approach. Here, aligned gradients from a differentiable solver rather than pass-through gradients enables `SurCo-imp` to modify the surrogate coefficients to improve the design loss.

6 Related Work

Differentiable Optimization Previous work differentiated through several optimization problems, calculating how changes in input parameters impact the optimal solution. Initially, a differentiable convex quadratic programming solver called OptNet (Amos & Kolter, 2017) proposed to implicitly differentiate the optimal solution with respect to input parameters through the KKT optimality conditions, a set of linear equations that determined the optimal solution. Following this, researchers differentiated through linear programs (Wilder et al., 2019a), submodular optimization problems (Djolonga & Krause, 2017; Wilder et al., 2019a), cone programs (Agrawal et al., 2019a;b), MaxSAT (Wang et al., 2019), Mixed Integer Linear Programming (Ferber et al., 2020; Mandi et al., 2020), Integer Linear Programming (Mandi et al., 2020), dynamic programming solvers Demirovic et al. (2020), blackbox discrete linear optimizers (Pogančić et al., 2019; Rolínek et al., 2020a;b), maxi-

mum likelihood estimation (Niepert et al., 2021), kmeans clustering (Wilder et al., 2019b), knapsack (Guler et al., 2022; Demirović et al., 2019), the cross-entropy method (Amos & Yarats, 2020), and SVM training (Lee et al., 2019). Additionally, Wang et al. (2020a) learned to linearly combine LP variables. `SurCo` can use these differentiable surrogates based on the problem domain.

Learning Heuristics for Optimization Researchers have investigated task-based learning, where we are given a distribution of linear or quadratic optimization problems and asked to train an optimizer with the true objective hidden at test time but available for training (Elmachtoub & Grigas, 2022; Donti et al., 2017). There are several theoretically motivated approaches (El Balghiti et al., 2019; Liu & Grigas, 2021; Hu et al., 2022); however, they assume access to ground truth linear or quadratic coefficients at train time but not at test time, unlike our considered nonlinear nonconvex combinatorial settings. Donti et al. (2021) predicts solutions for continuous optimization problems with nonlinear constraints. The approach relies on the continuous feasible region to predict and correct initial solutions in a differentiable manner, something not trivially applicable to the combinatorial setting. Machine learning is also used to guide combinatorial algorithms. Several approaches produce solutions to combinatorial problems (Zhang & Dietterich, 1995; Khalil et al., 2017; Kool et al., 2018; Nazari et al., 2018; Zha et al., 2022a;b). Here, the approaches are limited to searching simple feasible regions by iteratively building solutions for problems like routing, assignment, or covering. Other approaches set parameters that improve the runtime of exact solvers (Khalil et al., 2016; Bengio et al., 2021). However, these approaches are unable to handle more complex combinatorial constraints that arise in practice such as those in inverse photonic design.

Learning Latent Space for Optimization As we learn latent linear objectives to optimize nonlinear functions, other approaches learn latent embeddings for faster solving. Faloutsos & Lin (1995) proposed FastMap, which learns latent object embeddings for efficient search. Variants of FastMap are used in graph optimization and shortest path (Cohen et al., 2018; Hu et al., 2022; Li et al., 2019). Wang et al. (2020b; 2021a); Yang et al. (2021); Zhao et al. (2022) use monte carlo tree search to perform single and multi-objective blackbox optimization by learning to split the search space.

Mixed Integer Nonlinear Programming (MINLP) `SurCo-imp` falls into the broad family of MINLP solvers, and specifically we are addressing problems with nonlinear and nonconvex objectives and discrete feasible regions with linear constraints. Specialized solvers handle many problem variants in the MINLP space (Burer & Letchford, 2012; Belotti et al., 2013); however, scalabliliy in the nonconvex setting is usually obtained by optimization experts who rely on problem-specific solving techniques such as making piecewise linear approximations, convexifying the objective, or exploiting special structure.

7 CONCLUSION

We introduced `SurCo`, a method for learning linear surrogates for combinatorial nonlinear optimization problems. `SurCo` learns linear objective coefficients for a surrogate solver which results in solutions that minimize the nonlinear loss via gradient descent. At its core, `SurCo` differentiates the surrogate solver which maps the predicted coefficients to a combinatorially feasible solution, combining the flexibility of gradient-based optimization with the structure of combinatorial solvers. We presented two variants of `SurCo`, `impromptu` which optimizes individual instances, and `pre-trained` which trains a coefficient prediction model offline. We evaluated these approaches on two domains against the state of the art approaches used previously, demonstrating improved performance in metrics of interest, obtaining better solution quality for similar or better runtime in the embedding table sharding domain, and quickly identifying viable photonic devices.

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A PROOFS

Lemma 3.1 (Sufficient condition of prediction with ϵ-accuracy)**.** *If the dataset $\mathcal{D}_{\text{direct}}$ (ϵ/L)-cover Y, then for any $\boldsymbol{y} \in Y$, a 1-nearest-neighbor regressor $\hat{\boldsymbol{\phi}}$ leads to $\|\hat{\boldsymbol{\phi}}(\boldsymbol{y}) - \boldsymbol{\phi}(\boldsymbol{y})\|_2 \leq \epsilon$.*

Proof. Since the dataset is a ϵ/L-cover, for any $\boldsymbol{y} \in Y$, there exists at least one $\boldsymbol{y}_i$ so that $\|\boldsymbol{y} - \boldsymbol{y}_i\|_2 \leq \epsilon/L$. Let $\boldsymbol{y}_{\text{nn}}$ be the nearest neighbor of $\boldsymbol{y}$, and we have:

$$\|\boldsymbol{y} - \boldsymbol{y}_{\text{nn}}\|_2 \leq \|\boldsymbol{y} - \boldsymbol{y}_i\|_2 \leq \epsilon/L \tag{7}$$

From the Lipschitz condition and the definition of 1-nearest-neighbor classifier ($\hat{\boldsymbol{\phi}}(\boldsymbol{y}) = \boldsymbol{\phi}(\boldsymbol{y}_{\text{nn}})$), we know that

$$\|\boldsymbol{\phi}(\boldsymbol{y}) - \hat{\boldsymbol{\phi}}(\boldsymbol{y})\|_2 = \|\boldsymbol{\phi}(\boldsymbol{y}) - \boldsymbol{\phi}(\boldsymbol{y}_{\text{nn}})\|_2 \leq L\|\boldsymbol{y} - \boldsymbol{y}_{\text{nn}}\|_2 \leq \epsilon \tag{8}$$

□

Lemma 3.2 (Lower bound of sample complexity for ϵ/L-cover)**.** *To achieve ϵ/L-cover of Y, the size of the training set $N \geq N_0(\epsilon) := \frac{\text{vol}(Y)}{\text{vol}_0}\left(\frac{L}{\epsilon}\right)^d$, where vol_0 is the volume of unit ball in d-dimension.*

Proof. We prove by contradiction. If $N < N_0(\epsilon)$, then for each training sample $(\boldsymbol{y}_i, \boldsymbol{\phi}_i)$, we create a ball $B_i := B(\boldsymbol{y}_i, \epsilon/L)$. Since

$$\mathrm{vol}\left(\bigcup_{i=1}^{N} B_i \cap Y\right) \leq \mathrm{vol}\left(\bigcup_{i=1}^{N} B_i\right) \leq \sum_{i=1}^{N} \mathrm{vol}(B_i) = N\mathrm{vol}_0\left(\frac{\epsilon}{L}\right)^d < \mathrm{vol}(Y) \tag{9}$$

Therefore, there exists at least one $\boldsymbol{y} \in Y$ so that $\boldsymbol{y} \notin B_i$ for any $1 \leq i \leq N$. This means that $\boldsymbol{y}$ is not ϵ/L-covered. □

Theorem 3.1 (A case of infinite Lipschitz constant)**.** *If the minimal distance $d_{\min}$ for different connected components of $\boldsymbol{\phi}(Y)$ is strictly positive, and $\kappa(\boldsymbol{\phi}(Y)) > \kappa(Y)$, then the Lipschitz constant of the mapping $\boldsymbol{\phi}$ is infinite.*

Proof. Let $R_1, R_2, \ldots, R_K$ be the $K = \kappa(\boldsymbol{\phi}(Y))$ connected components of $\boldsymbol{\phi}(Y)$, and $Y_1, Y_2, \ldots, Y_J$ be the $J = \kappa(Y)$ connected components of Y. From the condition, we know that $\min_{k \neq k'} \mathrm{dist}(R_k, R_{k'}) = d_{\min} > 0$.

We have $R_k \cap R_{k'} = \emptyset$ for $k \neq k'$. Each R_k has a pre-image $S_k := \boldsymbol{\phi}^{-1}(R_k) \subseteq Y$. These pre-images $\{S_k\}_{k=1}^K$ form a partition of Y since

- $S_k \cap S_{k'} = \emptyset$ for $k \neq k'$ since any $\boldsymbol{y} \in Y$ cannot be mapped to more than one connected components;
- $\bigcup_{k=1}^K S_k = \bigcup_{k=1}^K \boldsymbol{\phi}^{-1}(R_k) = \boldsymbol{\phi}^{-1}\left(\bigcup_{k=1}^K R_k\right) = \boldsymbol{\phi}^{-1}(\boldsymbol{\phi}(S)) = S.$

Since $K = \kappa(\boldsymbol{\phi}(Y)) > \kappa(Y)$, by pigeonhole principle, there exists one Y_j that contains at least part of the two pre-images S_k and $S_{k'}$ with $k \neq k'$. This means that

$$S_k \cap Y_j \neq \emptyset, \quad S_{k'} \cap Y_j \neq \emptyset \tag{10}$$

Then we pick $\boldsymbol{y} \in S_k \cap Y_j$ and $\boldsymbol{y}' \in S_{k'} \cap Y_j$. Since $\boldsymbol{y}, \boldsymbol{y}' \in Y_j$ and Y_j is a connected component, there exists a continuous path $\gamma : [0,1] \mapsto Y_j$ so that $\gamma(0) = \boldsymbol{y}$ and $\gamma(1) = \boldsymbol{y}'$. Therefore, we have $\boldsymbol{\phi}(\gamma(0)) \in R_k$ and $\boldsymbol{\phi}(\gamma(1)) \in R_{k'}$. Let $t_0 := \sup\{t : t \in [0,1], \boldsymbol{\phi}(\gamma(t)) \in R_k\}$, then $0 \leq t_0 < 1$. For any sufficiently small $\epsilon > 0$, we have:

- By the definition of sup, we know there exists $t_0 - \epsilon \leq t' \leq t_0$ so that $\boldsymbol{\phi}(\gamma(t')) \in R_k$.
- Picking $t'' = t_0 + \epsilon < 1$, then $\boldsymbol{\phi}(\gamma(t'')) \in R_{k''}$ with some $k'' \neq k$.

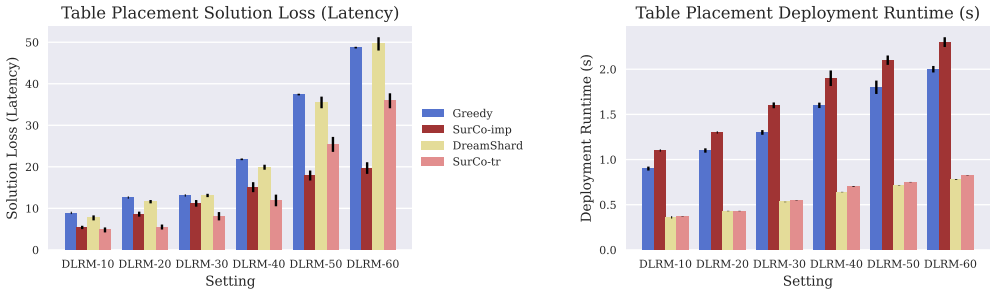
On the other hand, by continuity of the curve γ, there exists a constant $C(t_0)$ so that $\|\gamma(t') - \gamma(t'')\|_2 \leq C(t_0)\|t' - t''\|_2 \leq 2C(t_0)\epsilon$. Then we have

$$L = \max_{\boldsymbol{y}, \boldsymbol{y}' \in Y} \frac{\|\boldsymbol{\phi}(\boldsymbol{y}) - \boldsymbol{\phi}(\boldsymbol{y}')\|_2}{\|\boldsymbol{y} - \boldsymbol{y}'\|_2} \geq \frac{\|\boldsymbol{\phi}(\gamma(t')) - \boldsymbol{\phi}(\gamma(t''))\|_2}{\|\gamma(t') - \gamma(t'')\|_2} \geq \frac{d_{\min}}{2C(t_0)\epsilon} \to +\infty \tag{11}$$

□

B Experiment Details

Experiments are performed on a cluster of identical machines, each with 4 Nvidia A100 GPUs and 32 CPU cores, with 1T of RAM and 40GB of GPU memory. Additionally, we perform all operations in Python (Van Rossum & Drake, 2009) using Pytorch (Paszke et al., 2019). For embedding table placement, the nonlinear cost estimator is trained for 200 iterations and the offline-trained models of Dreamshard and `SurCo-tr` are trained against the pretrained cost estimator for 200 iterations. The DLRM Dataset Naumov et al. (2019) is available at `https://github.com/facebookresearch/dlrm_datasets`, and the dreamshard (Zha et al., 2022b) code is available at `https://github.com/daochenzha/dreamshard`. Additional details on dreamshard's model architecture and features can be obtained in the paper and codebase.



(a) (b)

Figure 5: Table placement solution quality as measured by sharding plan latency and test time solver runtime. We evaluate our approaches against Dreamshard (Zha et al., 2022b) a state of the art embedding table sharding tool that requires offline training of an RL policy and a domain-expert greedy heuristic of iteratively adding tables based on their dimension.