ROBUST FEDERATED LEARNING FOR NEURAL NETWORKS

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ABSTRACT

In federated learning, data is distributed among local clients which collaboratively train a prediction model using secure aggregation. To preserve the privacy of the clients, the federated learning paradigm requires each client to maintain a private local training data set, and only uploads its summarized model updates to the server. In this work, we show that this paradigm could lead to a degenerate model, which collapses in performance when the corrupted updates (under adversarial manipulations) are uploaded and aggregated at the server. To improve model robustness, we first decompose the aggregation error of the central server into bias and variance, and then, propose a robust federated learning framework (`Fed_BVA`) that performs on-device adversarial training using the bias-variance oriented adversarial examples supplied by the server via asymmetrical communications. The experiments are conducted on multiple benchmark data sets using a few prevalent neural network models, and the empirical results show that `Fed_BVA` is robust against white-box and black-box adversarial corruptions under both IID and non-IID settings.

1 INTRODUCTION

The explosive amount of decentralized user data collected from the ever-growing usage of smart devices, e.g., smartphones, wearable devices, home sensors, etc., has led to a surge of interest in the field of decentralized learning. To protect the privacy-sensitive data of the clients, federated learning (McMahan et al., 2017; Yang et al., 2019) has been proposed. Federated learning only allows a group of clients to train local models using their own data, and then collectively merge the model updates on a central server using secure aggregation (Acar et al., 2018). Due to its high privacy-preserving property, federated learning has attracted much attention in recent years with the prevalence of efficient light-weight deep models (Howard et al., 2017) and low-cost network communications (Wen et al., 2017; Konecný et al., 2016).

In federated learning, the central server only inspects the secure aggregation of the local models as a whole. Consequently, it is susceptible to clients' corrupted updates (e.g., system failures, adversarial manipulations, etc.). Recently, multiple robust federated learning models (Fang et al., 2019; Pillutla et al., 2019; Portnoy & Hendler, 2020; Mostafa, 2019) have been proposed. However, these works only focus on performing client-level robust training or designing server-level aggregation variants with hyper-parameter tuning, and largely ignore the underlying true cause of federated learning's vulnerability coming from the server's generalization error, as we will show in Section 4.1.

Our work bridges this gap by investigating the error incurred during secure aggregation of federated learning from the perspective of bias-variance decomposition (Pedro, 2000; Valentini & Dietterich, 2004). Specifically, we show that the generalization error of the aggregated model on central server can be decomposed as the combination of **bias** (triggered by the main prediction of these clients) and **variance** (triggered by the variations among clients' predictions). Next, we propose to perform the local robust training on clients by supplying them with a tiny amount of the bias-variance perturbed examples generated from the central server via asymmetrical communications. The experiments are conducted on neural networks with cross-entropy loss, however, other loss functions are also applicable as long as their gradients w.r.t. bias and variance are tractable to estimate. In this way, any gradient-based adversarial training strategies (Goodfellow et al., 2015; Madry et al., 2018) could be used. Compared with previous work, our major contributions include:

- We provide the exact solution of bias-variance analysis w.r.t. the generalization error which is perfectly suitable for neural network based federated learning. As a comparison, performing

adversarial attacks or training with conventional federated learning methods will only focus on the bias of the central model but ignore the variance.
- We demonstrate that the conventional federated learning framework is vulnerable to the strong attacking methods with increasing communication rounds even if the adversarial training using the locally generated adversarial examples is performed on each client.
- Without violating the clients' privacy, we show that providing a tiny amount of bias-variance perturbed data from the central server to the clients through asymmetrical communication could dramatically improve the robustness of the training model under various settings.

2 PRELIMINARIES

2.1 SETTINGS

In federated learning, there is a central server and K different clients, each with access to a private training set $\mathcal{D}_k = \{(x_i^k, t_i^k)\}_{i=1}^{n_k}$, where x_i^k, t_i^k, and n_k are the features, label, and number of training examples in the kth client $(k = 1, \cdots, K)$. Each data $\mathcal{D}_k$ is exclusively owned by client k and will not be shared with the central server or other clients. In addition, there is a small public training set $\mathcal{D}_s = \{(x_j^s, t_j^s)\}_{j=1}^{n_s}$ with n_s training examples from the server that is shared with clients, where $n_s \ll \sum_{k=1}^{K} n_k$. Note that this will not break the privacy constraints, for example, hospitals that contribute to a federated learned medical image diagnosis system could take a few publicly shared images as additional inputs. The goal of federated learning is to train a global classifier $f(\cdot)$ using knowledge from all the clients such that it generalizes well over test data $\mathcal{D}_{test}$. The notation used in this paper is summarized in the Appendix (see Table 4).

2.2 PROBLEM DEFINITION

Privacy-preserving federated learning trains the machine learning models without directly accessing to the clients' raw data. A group of clients could collaborate in achieving the learning objective under the coordination of a central server that aggregates the local clients' model updates asynchronously (Kairouz et al., 2019). In this paper, we study the adversarial robustness of neural networks[1] in federated learning setting, and we define robust decentralized learning as follows.

Definition 2.1. (Robust Federated Learning)
Input: (1). A set of private training data $\{\mathcal{D}_k\}_{k=1}^{K}$ on K different clients; (2). Tiny amount of training data $\mathcal{D}_s$ on the central server; (3). Learning algorithm $f(\cdot)$ and loss function $L(\cdot, \cdot)$.
Output: A trained model on the central server that is robust against adversarial perturbation.

We would like to point out that our problem definition has the following properties: **Asymmetrical communication:** The asymmetrical communication between each client and server cloud is allowed: the server provides both global model parameters and limited shared data to the clients; while each client only uploads its local model parameter back to the server. **Data distribution:** All training examples on the clients and the server are assumed to follow the same data distribution. However, the experiments show that our proposed algorithm also achieves outstanding performance under the non-IID setting, which could be common among personalized clients in real scenarios. **Shared learning algorithm:** All the clients are assumed to use the identical model $f(\cdot)$, including architectures as well as hyper-parameters (e.g., learning rate, local epochs, local batch size).

2.3 BIAS-VARIANCE TRADE-OFF

Following (Pedro, 2000; Valentini & Dietterich, 2004), we define the optimal prediction, main prediction as well as the bias, variance and noise for any real-valued loss function $L(\cdot, \cdot)$ as follows:

Definition 2.2. (**Optimal Prediction and Main Prediction**) Given a loss function $L(\cdot, \cdot)$ and a learning algorithm $f(\cdot)$, the optimal prediction y_* and main prediction y_m for an example are defined as:

$$y_*(x) = \arg\min_{y} \mathbb{E}_t[L(y, t)] \quad \text{and} \quad y_m(x) = \arg\min_{y'} \mathbb{E}_{\mathcal{D}}[L(f_{\mathcal{D}}(x), y')] \tag{1}$$

where t and $\mathcal{D}$ are viewed as the random variables to denote the class label and training set, and $f_{\mathcal{D}}$ denotes the model trained on $\mathcal{D}$. In short, the main prediction is the prediction whose average loss

[1] Our theoretical contribution mainly focuses on classification using neural networks with cross-entropy loss and mean squared loss. However, the proposed framework is generic to allow the use of other classification loss functions as well.

relative to all the predictions over data distributions is minimum, e.g., the main prediction for zero-one loss is the mode of predictions. In this work, we show that the main prediction is the average prediction of client models for mean squared (MSE) loss and cross-entropy (CE) loss in Section 4.1.

Definition 2.3. (**Bias, Variance and Noise**) Given a loss function $L(\cdot,\cdot)$ and a learning algorithm $f(\cdot)$, the expected loss $\mathbb{E}_{\mathcal{D},t}[L(f_{\mathcal{D}}(x),t)]$ for an example x can be decomposed[2] into bias, variance and noise as follows:

$$B(x) = L(y_m, y_*) \quad \text{and} \quad V(x) = \mathbb{E}_{\mathcal{D}}[L(f_{\mathcal{D}}(x), y_m)] \quad \text{and} \quad N(x) = \mathbb{E}_t[L(y_*, t)] \tag{2}$$

In short, bias is the loss incurred by the main prediction w.r.t. the optimal prediction, and variance is the average loss incurred by predictions w.r.t. the main prediction. Noise is conventionally assumed to be irreducible and independent to $f(\cdot)$.

Remark. *Our definitions on optimal prediction, main prediction, bias, variance and noise slightly differ from previous ones (Pedro, 2000; Valentini & Dietterich, 2004). For example, conventional optimal prediction was defined as $y_*(x) = \arg\min_y \mathbb{E}_t[L(t,y)]$, and it is equivalent to our definition when loss function is symmetric over its arguments, i.e., $L(y_1,y_2) = L(y_2,y_1)$.*

3 The Proposed Framework

A typical framework (Kairouz et al., 2019) of privacy-preserving federated learning can be summarized as follows: (1) *Client Update:* Each client updates local model parameters w_k by minimizing the empirical loss over its own training set; (2) *Forward Communication:* Each client uploads its model updates to the central server; (3) *Server Update:* It synchronously aggregates the received parameters; (4) *Backward Communication:* The global parameters are sent back to the clients. Our framework follows the same paradigm but with substantial modifications:

Server Update. Server has two components: The first component uses FedAvg (McMahan et al., 2017) algorithm to aggregate the local models' parameters, i.e., $w_G = \text{Aggregate}(w_1, \cdots, w_K) = \sum_{k=1}^{K} \frac{n_k}{n} w_k$ where $n = \sum_{k=1}^{K} n_k$ and w_k is the model parameters in the kth client. Meanwhile, another component is designed to produce adversarially perturbed examples which could be induced by a poisoning attack algorithm for the usage of robust adversarial training.

It has been well studied (Belkin et al., 2019; Pedro, 2000; Valentini & Dietterich, 2004) that in the classification setting, the generalization error of a learning algorithm on an example is determined by the bias, variance, and irreducible noise as defined in Eq. (2). Similar to the previous work, we also assume a noise free learning scenario where the class label t is a deterministic function of x (i.e., if x is sampled repeatedly, the same values of its class t will be observed). This motivates us to generate the adversarial examples by attacking the bias and variance induced by clients' models as:

$$\max_{\hat{x} \in \Omega(x)} B(\hat{x}; w_1, \cdots, w_K) + \lambda V(\hat{x}; w_1, \cdots, w_K) \quad \forall (x,t) \in \mathcal{D}_s \tag{3}$$

where $B(\hat{x}; w_1, \cdots, w_K)$ and $V(\hat{x}; w_1, \cdots, w_K)$ could be empirically estimated from a finite number of local training sets $\{\mathcal{D}_1, \mathcal{D}_2, \cdots, \mathcal{D}_K\}$. Here λ is a hyper-parameter to measure the trade-off of bias and variance, and $\Omega(x)$ is the perturbation constraint.

Note that $\mathcal{D}_s$ (on the server) is the candidate subset of all available training examples that would lead to their perturbed counterparts. This is a more feasible setting as compared to generating adversarial examples on clients' devices because the server usually has much powerful computational capacity in real scenarios that allows the usage of flexible poisoning attack algorithms. In this case, both poisoned examples and server model parameters would be sent back to each client (*Backward Communication*), while only clients' local parameters would be uploaded to the server (*Forward Communication*), i.e., the *asymmetrical communication* as discussed in Section 2.2.

Client Update. The robust training of one client's prediction model (i.e., w_k) can be formulated as the following minimization problem.

$$\min_{w_k} \left(\sum_{i=1}^{n_k} L(f_{\mathcal{D}_k}(x_i^k; w_k), t_i^k) + \sum_{j=1}^{n_s} L(f_{\mathcal{D}_k}(\hat{x}_j^s; w_k), t_j^s) \right) \tag{4}$$

where $\hat{x}_j^s \in \Omega(x_j^s)$ is the perturbed examples that is asymmetrically transmitted from the server.

[2] This decomposition is based on the weighted sum of bias, variance, and noise. In general, t is a non-deterministic function (Pedro, 2000) of x when the irreducible noise is considered. Namely, if x is sampled repeatedly, different values of t will be observed.

Remark. *Intuitively, the bias measures the systematic loss of a learning algorithm, and the variance measures the prediction consistency of the learner over different training sets. Therefore, our robust federated learning framework has the following advantages: (i) it encourages the clients to consistently produce the optimal prediction for perturbed examples, thereby leading to a better generalization performance; (ii) local adversarial training on perturbed examples allows to learn a robust local model, and thus a robust global model could be aggregated from clients.*

Theoretically, we could still have another possible robust federated training strategy:

$$\min_{w_k} \sum_{i=1}^{n_k} \max_{\hat{x}_i^k \in \Omega(x_i^k)} L(f(\hat{x}_i^k; w_k), t_i^k) \quad \forall k \in \{1, 2, \cdots, K\} \tag{5}$$

where the perturbed training examples for each client k is generated on local device from $\mathcal{D}_k$ instead of transmitted from the server. This min-max formula is similar to (Madry et al., 2018; Tramèr et al., 2018) where the inner maximization problem aims to synthesize the adversarial counterparts of clean examples, while the outer minimization problem finds the optimal model parameters over perturbed training examples. In this way, each local robust model is trained individually, nevertheless, poisoning attacks on device will largely increase the computational cost and memory usage. Meanwhile, it only considers the bias on server and is still vulnerable against the adversarial examples with increasing communication rounds. Both phenomena are observed in the experimental results.

4 Algorithm

4.1 Bias-Variance Attack

We first consider the maximization problem in Eq. (3) using bias-variance based adversarial attacks. It aims to find the adversarial example $\hat{x}$ (from the original example x) that would produce large bias and variance values w.r.t. clients' local models. Specifically, perturbation constraint $\hat{x} \in \Omega(x)$ forces the adversarial example $\hat{x}$ to be visually indistinguishable w.r.t. x. Here we consider the well-studied l_∞-bounded adversaries[3] (Goodfellow et al., 2015; Madry et al., 2018; Tramèr et al., 2018) such that $\Omega(x) := \{\hat{x} \big| ||\hat{x} - x||_\infty \leq \epsilon\}$ for a perturbation magnitude ϵ. Furthermore, we propose to consider the following two gradient-based algorithms to generate adversarial examples.

Bias-variance based Fast Gradient Sign Method (BV-FGSM): Following FGSM (Goodfellow et al., 2015), it linearizes the maximization problem in Eq. (3) with one-step attack as follows.

$$\hat{x}_{\text{BV-FGSM}} := x + \epsilon \cdot \text{sign}\left(\nabla_x \left(B(x; w_1, \cdots, w_K) + \lambda V(x; w_1, \cdots, w_K)\right)\right) \tag{6}$$

Bias-variance based Projected Gradient Descent (BV-PGD): PGD can be considered as a multi-step variant of FGSM (Kurakin et al., 2017) and might generate powerful adversarial examples. This motivated us to derive a BV-based PGD attack:

$$\hat{x}_{\text{BV-PGD}}^{l+1} := \text{Proj}_{\Omega(x)}\left(\hat{x}^l + \epsilon \cdot \text{sign}\left(\nabla_{\hat{x}^l}\left(B(\hat{x}^l; w_1, \cdots, w_K) + \lambda V(\hat{x}^l; w_1, \cdots, w_K)\right)\right)\right) \tag{7}$$

where $\hat{x}^l$ is the adversarial example at the lth step with the initialization $\hat{x}^0 = x$ and $\text{Proj}_{\Omega(x)}(\cdot)$ projects each step onto $\Omega(x)$.

Remark. *The proposed framework could be naturally generalized to any gradient-based adversarial attack algorithms where the gradients of bias $B(\cdot)$ and variance $V(\cdot)$ w.r.t. x are tractable when estimated from finite training sets. Compared with the existing attack methods (Carlini & Wagner 2017; Goodfellow et al., 2015; Kurakin et al., 2017; Moosavi-Dezfooli et al., 2016), our loss function the adversary aims to optimize is a linear combination of bias and variance, whereas existing work mainly focused on attacking the overall classification error that considers bias only.*

The following theorem states that bias $B(\cdot)$ and variance $V(\cdot)$ as well as their gradients over input x could be estimated using the clients' models.

Theorem 4.1. Assume that $L(\cdot, \cdot)$ is the cross-entropy loss function, then the empirical estimated main prediction y_m for an input example (x, t) has the following closed-form expression:

$$y_m(x; w_1, \cdots, w_K) = \frac{1}{K} \sum_{k=1}^{K} f_{\mathcal{D}_k}(x; w_k)$$

[3] l_∞ robustness is surely not the end-goal of robustness learning, we use this standard model to show the limitations of prior federated learning, and evaluate the improvements of our proposed framework.

Furthermore, the empirical bias and variance as well as their gradients over input x are estimated as:

$$B(x; w_1, \cdots, w_K) = \frac{1}{K}\sum_{k=1}^{K} L(f_{\mathcal{D}_k}(x; w_k), t); \qquad V(x; w_1, \cdots, w_K) = L(y_m, y_m) = H(y_m)$$

Here, $H(y_m) = -\sum_{j=1}^{C} y_m^{(j)} \log y_m^{(j)}$ is the entropy of the main prediction y_m and C is the number of classes. Easily, we can have their gradients in terms of the bias and variance as $\nabla_x B(x; w_1, \cdots, w_K) = \frac{1}{K}\sum_{k=1}^{K} \nabla_x L(f_{\mathcal{D}_k}(x; w_k), t)$ and $\nabla_x V(x; w_1, \cdots, w_K) = -\frac{1}{K}\sum_{k=1}^{K}\sum_{j=1}^{C}(\log y_m^{(j)} + 1)\nabla_x f_{\mathcal{D}_k}^{(j)}(x; w_k)$.

In addition, we also consider the case where $L(\cdot, \cdot)$ is the mean squared error (MSE) loss function. Although its main prediction y_m for an input example (x, t) has a closed-form expression which is exactly the same as the cross-entropy loss, its empirical bias and unbiased variance can only be estimated in the following formulas: $B(x; w_1, \cdots, w_K) = ||\frac{1}{K}\sum_{k=1}^{K} f_{\mathcal{D}_k}(x; w_k) - t||_2^2$ and $V(x; w_1, \cdots, w_K) = \frac{1}{K-1}\sum_{k=1}^{K} ||f_{\mathcal{D}_k}(x; w_k) - \frac{1}{K}\sum_{k=1}^{K} f_{\mathcal{D}_k}(x; w_k)||_2^2$. Note that the gradients of MSE's bias and variance are much more computational demanding comparing with the concise formulas that cross-entropy has. More details are illustrated in Appendix A.5.1.

4.2 FED_BVA

We present a novel robust federated learning algorithm with our proposed bias-variance attacks, named `Fed_BVA`. Following the framework defined in Eq. (3) and Eq. (4), key components of our algorithm are: bias-variance attacks for generating adversarial examples on the server, and adversarial training with poisoned server examples. and clean local examples on each client. Therefore, a robust federated learning model could be optimized by iteratively producing the adversarial examples $\hat{\mathcal{D}}_s$ and updating the local model parameters w.

Algorithm 1 Fed_BVA

1: **Input:** K (number of clients, with local data sets $\{\mathcal{D}_k\}_{k=1}^{K}$ indexed by k); f (learning model), E (number of local epochs); F (fraction of clients selected on each round); B (batch size of local client); η (learning rate); $\mathcal{D}_s$ (data set on server); ϵ (perturbation magnitude).
2: **Initialization:** Initialize w_G^0 and $\hat{\mathcal{D}}_s = \emptyset$
3: **for** each round $r = 1, 2, \cdots$ **do**
4: $\quad m = \max(F \cdot K, 1)$
5: $\quad S_r \leftarrow$ randomly sampled m clients
6: $\quad$ **for** each client $k \in S_r$ in parallel **do**
7: $\quad\quad w_k^r \leftarrow$ **ClientUpdate**$(w_G^{r-1}, \hat{\mathcal{D}}_s, k)$
8: $\quad$ **end for**
9: $\quad \hat{\mathcal{D}}_s \leftarrow$ **BVAttack**$(\mathcal{D}_s, \{w_k^r\}|k \in S_r)$
10: $\quad w_G^r \leftarrow$ **Aggregate**$(w_k^r|k \in S_r)$
11: **end for**
12: **return** w_G

Algorithm 2 ClientUpdate$(w, \hat{\mathcal{D}}_s, k)$

Initialize k^{th} client's model with w
$\mathcal{B} \leftarrow$ split $\mathcal{D}_k \cup \hat{\mathcal{D}}_s$ into batches of size B
for each local epoch $i = 1, 2, \cdots, E$ **do**
$\quad$ **for** local batch $(x, t) \in \mathcal{B}$ **do**
$\quad\quad w \leftarrow w - \eta\nabla L(f_{\mathcal{D}_k}(x; w), t)$
$\quad$ **end for**
end for
return w

Algorithm 3 BVAttack$(\mathcal{D}_s, \{w_k^r\}|k \in S_r)$

Initialize $\hat{\mathcal{D}}_s = \emptyset$
for $(x, t) \in \mathcal{D}_s$ **do**
$\quad$ Estimate the gradients $\nabla_x B(x)$ and $\nabla_x V(x)$ using Theorem 4.1
$\quad$ Calculate $\hat{x}$ using Eq. (6) or (7) and add to $\hat{\mathcal{D}}_s$
end for
return $\hat{\mathcal{D}}_s$

The proposed algorithm is summarized in Alg. 1. Given the server's $\mathcal{D}_s$ and clients' training data $\{\mathcal{D}_k\}_{k=1}^{K}$ as input, the output is a robust global model on the server. First, it initializes the server's model parameter w_G and perturbed data $\hat{\mathcal{D}}_s$, and then assigns to the randomly selected clients (Steps 4-5). Next, each client optimizes its own local model with the received parameters w and $\hat{\mathcal{D}}_s$ (Steps 6-8), and uploads the updated parameters back to the server. A last, the server generate the perturbed data $\hat{\mathcal{D}}_s$ (Step 9) using the proposed bias-variance attack algorithm (see Alg. 3) and global model parameters (Step 10) using an aggregation method, e.g., FedAvg (McMahan et al., 2017).

Specifically, for client update (in Alg. 2), each client initializes its local model $f_{\mathcal{D}}(\cdot)$ with the received global parameters w_G, and then optimizes the local model with its own clean data $\mathcal{D}_k$ and the received data $\hat{\mathcal{D}}_s$. Following (McMahan et al., 2017; Xie et al., 2019), considering the learning model $f(\cdot)$, we train each client with E local epochs, learning rate η, and batch size B.

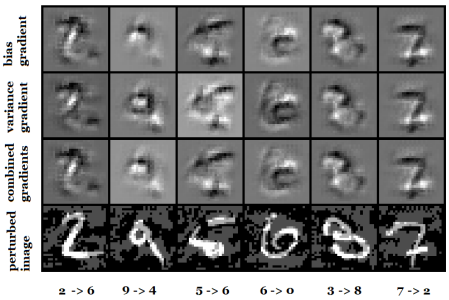

Figure 1: Visualizations of bias, variance, bias+variance, and perturbed images for MNIST.

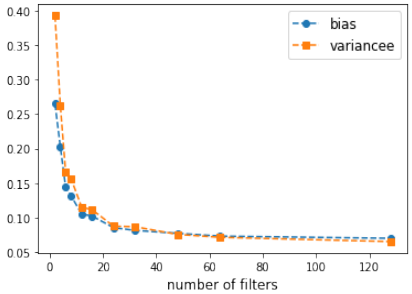

Figure 2: Bias-variance curve w.r.t. the CNN model complexity on MNIST.

5 Experiments

5.1 Settings

In this section, we evaluate the adversarial robustness of our proposed algorithm on four benchmark data sets: MNIST[4], Fashion-MNIST[5], CIFAR-10[6] and CIFAR-100[6]. The baseline models we used include: (1). `FedAvg`: the classical federated averaging model (McMahan et al., 2017). (2). `FedAvg_AT`: The simplified version of our proposed method where the local clients performs adversarial training with the asymmetrical transmitted perturbed data generated on top of `FedAvg`'s aggregation. (3) - (5). `Fed_Bias, Fed_Variance, Fed_BVA`: Our proposed methods where the asymmetrical transmitted perturbed data is generated using the gradients of bias-only attack, variance-only attack, and bias-variance attack respectively. (6). `EAT`: Ensemble adversarial training (Tramèr et al., 2018), where each client performs local adversarial training using Eq. (5) and their model updates are aggregated on server using `FedAvg`. For fair comparison, all baselines are modified to the asymmetrical communications setting (`FedAvg` and `EAT` has clean $\mathcal{D}_s$ received), and all their initializations are set to be the same. (7). `EAT+Fed_BVA`: A combination of baselines (5) and (6). Note that baselines (5) and (6) have high computational requirements on client devices and are usually not preferred in real scenarios.

For the defense model, we use a 4-layer CNN model for MNIST and Fashion-MNIST, and VGG9 architecture for CIFAR-10 and CIFAR-100. Regarding blackbox attacks, we apply ResNet18, VGG11, Xception, and MobileNetV2 for CIFAR data, and provide a variety of models for MNIST and Fashion-MNIST by following the design of (Tramèr et al., 2018). The training is performed using SGD optimizer with fixed learning rate of 0.01 and momentum of value 0.9. All hyper-parameters of federated learning are presented in Table 5. We empirically demonstrate that these hyper-parameter settings are preferable in terms of both training accuracy and robustness. Please see the details in Fig. 6 - Fig. 14 in the Appendix. To evaluate the robustness of our federated learning algorithm against existing adversarial attacks, except for the clean model training, we perform FGSM (Goodfellow et al., 2015), PGD (Kurakin et al., 2017) with 10 and 20 steps towards the aggregated server model on the test set $\mathcal{D}_{test}$. Following (Goodfellow et al., 2015; Wang et al., 2019), the maximum perturbations allowed are $\epsilon = 0.3$ for MNIST and Fashion-MNIST, and $\epsilon = \frac{16}{255}$ for CIFAR-10 and CIFAR-100. For IID sampling, the data is shuffled and uniformly partitioned into each client; For non-IID setting, data is divided into $2F \cdot K$ shards based on sorted example labels, then assigns each client with 2 shards. Thereby, each client will have data with at most two classes.

5.2 Result Analysis

To analyze the properties of our proposed `Fed_BVA` framework, in Fig. 1, we visualize the extracted gradients using adversarial attack from bias, variance, and bias-variance. Notice that the gradients of bias and variance are similar but with subtle differences in local pixel areas. However, according to Theorem 4.1, the gradient calculation of these two are quite different: bias requires the target label as input, but variance only needs the model output and main prediction. From another perspective, we also investigate the bias-variance magnitude relationship with varying model complexity. As shown in Fig. 2, with increasing model complexity (more convolutional filters in CNN), both bias and variance decrease. This result is different from the double-descent curve or bell-shape variance curve claimed in (Belkin et al., 2019; Yang et al., 2020). The reasons are twofold: First, their

[4] http://yann.lecun.com/exdb/mnist
[5] https://github.com/zalandoresearch/fashion-mnist
[6] https://www.cs.toronto.edu/~kriz/cifar.html

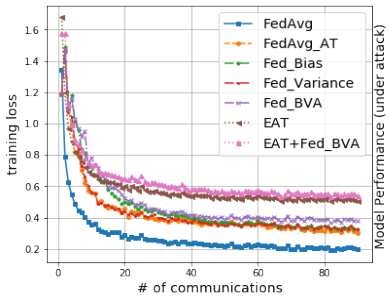

Figure 3: Convergence on Fashion-MNIST

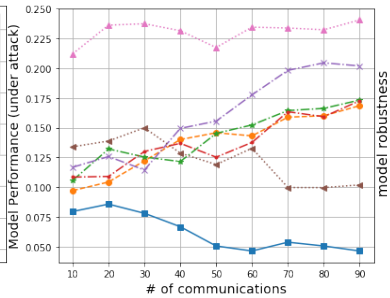

Figure 4: Performance on Fashion-MNIST

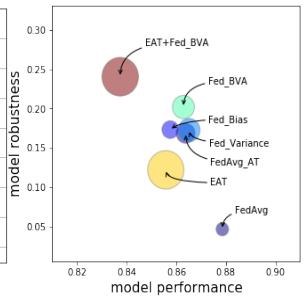

Figure 5: Efficiency on Fashion-MNIST

	IID				non-IID			
	Clean	FGSM	PGD-10	PGD-20	Clean	FGSM	PGD-10	PGD-20
FedAvg	**0.9885**	0.6610	0.5717	0.2513	**0.9776**	0.4910	0.4678	0.1409
FedAvg_AT	0.9876	0.8008	0.7383	0.5009	0.9756	0.7043	0.6423	0.3929
Fed_Bias	0.9855	0.8033	0.7814	0.5794	0.9711	0.6564	0.5750	0.3271
Fed_Variance	0.9843	0.8090	0.7906	0.5826	0.9741	0.6798	0.6067	0.3919
Fed_BVA	0.9859	0.8204	0.8105	0.6253	0.9701	0.7130	0.6747	0.4842
EAT	0.9808	**0.9015**	0.9084	0.8130	0.9737	**0.8068**	0.7281	0.3875
EAT+Fed_BVA	0.9805	0.8991	**0.9116**	**0.8258**	0.9606	0.7947	**0.8194**	**0.6779**

Table 1: Accuracy (in %) of MNIST under white-box attacks in IID and non-IID settings

bias-variance definitions are from the MSE regression decomposition perspective, whereas our decomposition utilizes the concept of main prediction and the generalization error is decomposed from the classification perspective; Second, their implementations only evaluate the bias and variance using training batches on one central model and thus is different from the definition which requires the variance to be estimated from multiple sub-models (in our scenario, client models).

The convergence plot of all baselines is presented in Fig. 3. We observe that `FedAvg` has the best convergence, and all robust training will have a slightly higher loss upon convergence. This matches the observations in (Madry et al., 2018) which states that training performance may be sacrificed in order to provide robustness for small capacity networks. For the model performance shown in Fig. 4, we observe that the aggregation of federated learning is vulnerable to adversarial attacks since both `FedAvg` and `EAT` have decreased performance with an increasing number of server-client communications. Other baselines that utilized the asymmetrical communications have increasing robustness with more communication rounds although only a small number of perturbed examples ($n_s = 64$) are transmitted. We also observe that when communication rounds reach 70, `Fed_BVA` starts to have similar performance as `EAT` while the latter is more resource-demanding as shown in Fig. 5, where the pie plot size represents the running time. Overall, bias-variance based adversarial training via asymmetric communication is both effective and efficient for robust federated learning.

For the comprehensive experiments shown in Table 1 and Table 2, it is easy to verify that our proposed model outperforms all other baselines regardless of the source of the perturbed examples (i.e., locally generated like `EAT+Fed_BVA` or asymmetrically transmitted from the server like `Fed_BVA`). Comparing with standard robust federated learning `FedAvg_AT`, the performance of `Fed_BVA` against adversarial attacks still increases $4\% - 13\%$ and $2\% - 9\%$ on IID and non-IID settings respectively, although `Fed_BVA` is theoretically suitable for the cases that clients have IID samples. In Table 3, we observe a similar trend where `Fed_BVA` outperforms `FedAvg_AT` on CIFAR-10 and CIFAR-100 (with $0.2\% - 10\%$ increases) when defending different types of adversarial examples. Comparing with strong local adversarial training baseline `EAT`, we also observe maximum 13% accuracy increase when applying its bias-variance oriented baseline `EAT+Fed_BVA`.

We also conducted various additional experiments in Appendix A.5 which includes: (1) Comparison of efficiency and effectiveness of `Fed_BVA` using cross-entropy loss and MSE loss; (2) Comparison of single-step `Fed_BVA` and multi-step `Fed_BVA` in terms of the generation of $\hat{\mathcal{D}}_s$; (3) Three training scenarios of `Fed_BVA` that use client-specific adversarial examples or universal adversarial examples; (4) Ablation study in terms of number of shared perturb examples n_s, optimizer's momentum, and number of local epochs E; (5) Blackbox attacking transferability between various models on all four data sets under multiple settings.

	IID				non-IID			
	Clean	FGSM	PGD-10	PGD-20	Clean	FGSM	PGD-10	PGD-20
FedAvg	**0.8786**	0.3103	0.0851	0.0463	**0.8157**	0.2093	0.0587	0.0166
FedAvg_AT	0.8638	0.4995	0.198	0.1685	0.7355	0.4955	0.1811	0.1283
Fed_Bias	0.8576	0.5115	0.2161	0.1732	0.719	0.4506	**0.2256**	**0.1745**
Fed_Variance	0.8652	0.4878	0.2095	0.1726	0.6804	0.4365	0.2159	0.1642
Fed_BVA	0.8629	0.5300	0.2431	0.2019	0.6624	0.4925	0.1504	0.0876
EAT	0.8558	**0.8055**	0.1879	0.1019	0.7841	0.5605	0.0774	0.0273
EAT+Fed_BVA	0.8324	0.7149	**0.3668**	**0.2406**	0.718	**0.6194**	0.1528	0.1022

Table 2: Accuracy (in %) of Fashion-MNIST under white-box attacks in IID and non-IID settings

	CIFAR-10				CIFAR-100			
	Clean	FGSM	PGD-10	PGD-20	Clean	FGSM	PGD-10	PGD-20
FedAvg	**0.7887**	0.1477	0.0337	0.0254	**0.7298**	0.1497	0.1224	0.0422
FedAvg_AT	0.7793	0.2079	0.0866	0.0753	0.7039	0.1663	0.0721	0.0518
Fed_Bias	0.7760	0.2043	0.0894	0.079	0.7036	0.1601	0.1654	0.0628
Fed_Variance	0.781	0.1969	0.0799	0.0693	0.7121	0.1672	0.1683	0.0643
Fed_BVA	0.7812	0.2119	0.0890	0.0771	0.7074	0.1677	0.1717	0.0648
EAT	0.7672	0.2796	0.1929	**0.1892**	0.6613	0.2648	0.2033	0.1865
EAT+Fed_BVA	0.7676	**0.2803**	**0.1935**	**0.1892**	0.6588	**0.2708**	**0.3316**	**0.2092**

Table 3: Accuracy (in %) of CIFAR-10 and CIFAR-100 under white-box attacks

6 Related Work

Adversarial Machine Learning: While machine learning models have achieved remarkable performance over clean inputs, recent work (Goodfellow et al., 2015; Szegedy et al., 2014) showed that those trained models are vulnerable to adversarially chosen examples by adding the imperceptive noise to the clean inputs. In general, the adversarial robustness of centralized machine learning models have been explored from the following aspects: adversarial attacks (Biggio et al., 2012; Mei & Zhu, 2015; Carlini & Wagner, 2017; Shafahi et al., 2018; Guo et al., 2019; Athalye et al., 2018; Zhu et al., 2019), defense (or robust model training) (Madry et al., 2018; Tramèr et al., 2018; Wong & Kolter, 2018; Cohen et al., 2019; Shafahi et al., 2019; Tramèr & Boneh, 2019; Carlini et al., 2019) and interpretable adversarial robustness (Schmidt et al., 2018; Cullina et al., 2018; Gilmer et al., 2019; Yin et al., 2019; Tsipras et al., 2018).

Decentralized Learning: Decentralized learning with preserved privacy (Konečnỳ et al., 2016; McMahan et al., 2017; Hard et al., 2018) has become prevalent in recent years. Meanwhile, the vulnerability of federated learning to backdoor attacks has also been explored by (Bagdasaryan et al., 2018; Bhagoji et al., 2019; Xie et al., 2019). Following their work, multiple robust federated learning models (Fang et al., 2019; Pillutla et al., 2019; Portnoy & Hendler, 2020; Mostafa, 2019) are also proposed and studied. In this paper, we studied the underlying true cause of federated learning's vulnerability from the perspective of bias-variance analysis. This is in sharp contrast to the existing work that focused on performing client-level model poisoning attacks or designing server-level aggregation variants with hyper-parameter tuning.

Bias-Variance Decomposition: Bias-variance decomposition (Geman et al., 1992) was originally introduced to analyze the generalization error of a learning algorithm. Then, a generalized bias-variance decomposition (Pedro, 2000; Valentini & Dietterich, 2004) was studied in the classification setting which enabled flexible loss functions (e.g., squared loss, zero-one loss). More recently, bias-variance trade-off was experimentally evaluated on modern neural network models (Neal et al., 2018; Belkin et al., 2019; Yang et al., 2020).

7 Conclusion

In this paper, we proposed a novel robust federated learning framework, in which the aggregation incurred loss during the server's aggregation is dissected into a bias part and a variance part. Our approach improves the model robustness through adversarial training by supplying a few bias-variance perturbed samples to the clients via asymmetrical communications. Extensive experiments have been conducted where we evaluated its performance from various aspects on several benchmark data sets. We believe the further exploration of this direction will lead to more findings on the robustness of federated learning.

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