Standard-Deviation-Inspired Regularization for Improving Adversarial Robustness

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Abstract

Adversarial Training (AT) has been demonstrated to improve the robustness of deep neural networks (DNNs) to adversarial attacks. AT is a min-max optimization procedure wherein adversarial examples are generated to train a robust DNN. Generally, AT utilize un-targeted adversarial examples because the inner maximization step maximizes the losses of inputs w.r.t their actual classes, ensuring misclassifications into *any* other incorrect classes. The outer minimization involves minimizing the losses on the adversarial examples obtained from the inner maximization. This work proposes a standard-deviation-inspired (SDI) regularization term for improving adversarial robustness and generalization. We argue that the inner maximization is akin to minimizing a modified standard deviation of a model's output probabilities. Moreover, we argue that maximizing the modified standard deviation measure may complement the outer minimization of the AT framework. To corroborate our argument, we experimentally show that the SDI measure may be utilized to craft adversarial examples. Furthermore, we show that combining the proposed SDI regularization term with existing AT variants improves the robustness of DNNs to stronger attacks (e.g., CW and Auto-attack) and improves robust generalization.

1 INTRODUCTION

The vulnerability of deep neural networks (DNNs) to adversarial perturbations is well documented in machine learning literature (Goodfellow et al., 2014; Moosavi-Dezfooli et al., 2016; Papernot et al., 2016; Szegedy et al., 2013), prompting concerns about the deployment of DNNs into safety-critical domains. Hence, for the safe deployment of DNNs, improving their robustness to adversarial perturbations is imperative.

Adversarial Training (AT) Goodfellow et al. (2014); Madry et al. (2018) has been demonstrated to be effective in improving the robustness of DNNs to adversarial attacks. AT is a min-max optimization procedure, where the inner maximization step corresponds to finding adversarial examples in the direction of worst-case loss. The outer minimization minimizes the loss on the crafted adversarial examples. The success of AT in improving the robustness of DNNs to adversarial perturbations has inspired a myriad of variants that have yielded better robustness or computational efficiency (Zhang et al., 2019; Wang et al., 2019; Li et al., 2019; Andriushchenko & Flammarion, 2020; Wong et al., 2020; Shafahi et al., 2019). Furthermore, recent works have employed methods such as adversarial weight perturbation Wu et al. (2020), integration of hypersphere embedding into AT (Pang et al., 2020; Fakorede et al., 2023a), and loss re-weighting Zhang et al. (2020); Liu et al. (2021); Fakorede et al. (2023b); Zhang et al. (2023) to improve the performance of existing AT variants.

In this work, we delve into a standard-deviation-inspired (SDI) measure proposed in a recent study in (Fakorede et al., 2024) for estimating the vulnerability of an input example. Drawing inspiration from the concept of standard deviation, which quantifies the dispersion of data points around the mean of a distribution, the SDI measure aims to capture the dispersion of logits associated with incorrect classes in relation to the logits corresponding to the true class.

We draw a parallel between the inner maximization step of the AT process and minimizing the SDI loss function. Moreover, we argue that the outer minimization step of the AT process, which seeks model param-

eters minimizing the loss on adversarial examples, is conceptually similar to maximizing the SDI measure. Both endeavors aim to enhance the likelihood of correctly classifying individual adversarial examples.

Unlike prevalent information-theoretic losses utilized in min-max *AT* optimization, the *SDI* measure operates independently from concepts like cross-entropy, entropy, or Kullback–Leibler divergence. Consequently, integrating the *SDI* measure into existing *AT* variants could complement information-theory-inspired losses and potentially enhance the performance of these variants. Therefore, we propose adding the *SDI* loss as a regularization term to prominent *AT* variants such as the *standard AT* (Madry et al., 2018) and *TRADES* (Zhang et al., 2019). The proposed regularization term is applied to outer minimization of the respective *AT* variants to maximize the *SDI* measure.

Our contributions are summarized as follows:

1. We propose utilizing the *SDI* measure as a regularization term to existing *AT* variants. Our extensive experiments show that our proposed approach further improves the robustness of existing *AT* variants on strong attacks Auto attacks and CW attack and strong query-based black-box attack SPSA.

2. We argue and experimentally show that the proposed *SDI* regularization on existing *AT* variants improves the generalization to other attacks not seen during adversarial training.

3. In addition, we establish a connection between minimizing the SDI measure and the inner maximization of the min-max *AT* procedure. Specifically, we experimentally show that adversarial examples may be obtained from adversarial perturbations that minimize the *SDI* metric. Furthermore, we compare the success rates of adversarial examples obtained using the *SDI* metric with those obtained using cross-entropy loss and KL divergence on adversarial trained models.

2 BACKGROUND AND RELATED WORK

2.1 NOTATION

We use bold letters to represent vectors. We denote $\mathcal{D} = \{\mathbf{x}_i, y_i\}_{i=1}^n$ a data set of input feature vectors $\mathbf{x}_i \in \mathcal{X} \subseteq \mathbf{R}^d$ and labels $y_i \in \mathcal{Y}$, where $\mathcal{X}$ and $\mathcal{Y}$ represent a feature space and a label set, respectively.

Let $f_\theta : \mathcal{X} \to R^C$ denote a deep neural network (DNN) classifier with parameters θ, and $|C|$ represents the number of output classes. For any $\mathbf{x} \in \mathcal{X}$, let the class label predicted by f_θ be $F_\theta(\mathbf{x}) = \arg\max_k f_\theta(\mathbf{x})_k$, where $f_\theta(\mathbf{x})_k$ denotes the k-th component of $f_\theta(\mathbf{x})$. $f_\theta(\mathbf{x})_y$ is the probability of $\mathbf{x}$ having label y.

We denote $\|\cdot\|_p$ as the l_p- norm over $\mathbf{R}^d$, that is, for a vector $\mathbf{x} \in \mathbf{R}^d, \|\mathbf{x}\|_p = (\sum_{i=1}^d |\mathbf{x}_i|^p)^{\frac{1}{p}}$. An ϵ-neighborhood for $\mathbf{x}$ is defined as $B_\epsilon(\mathbf{x}) : \{\mathbf{x}' \in \mathcal{X} : \|\mathbf{x}' - \mathbf{x}\|_p \leq \epsilon\}$. An adversarial example corresponding to a natural input $\mathbf{x}$ is denoted as $\mathbf{x}'$. We often refer to the loss resulting from the adversarial attack (inner maximization) as adversarial loss.

2.2 ADVERSARIAL ROBUSTNESS.

Adversarial robustness is a machine learning model's capability to resist adversarial attacks. Over the past years, many methods (Guo et al., 2018; Buckman et al., 2018; Dhillon et al., 2018; Madry et al., 2018; Goodfellow et al., 2014; Zhang et al., 2019) have been proposed to improve adversarial robustness of neural networks. However, some of these defenses have been shown to provide a false sense of defense because they intentionally or inadvertently used obfuscated gradients in their defenses (Athalye et al., 2018). In a seminal work, Madry et al. (2018) proposed Adversarial training (AT), which involves training the model with adversarial examples obtained under worst-case loss to improve robustness. Formally, *AT* involves solving a min-max optimization as follows:

$$\min_{\boldsymbol{\theta}} \mathbb{E}_{(\mathbf{x},y)\sim\mathcal{D}} \left[\max_{\mathbf{x}' \in B_\epsilon(\mathbf{x})} L(f_{\boldsymbol{\theta}}(\mathbf{x}'), y) \right] \tag{1}$$

where $L()$ represents the loss function, y is the true label of input feature $\mathbf{x}$, and $\boldsymbol{\theta}$ are the model parameters. The inner maximization in Eq. (1) aims to obtain a worst-case adversarial version of the input $\mathbf{x}$ that increases the loss. The outer minimization then tries to find model parameters that would minimize this worst-case adversarial loss. The efficacy of *AT* has spurred the development of numerous variants (Zhang et al., 2019; Wang et al., 2019; Wu et al., 2020; Pang et al., 2020). A prominent variant *TRADES* Zhang et al. (2019) proposed a principled regularization term that trades off adversarial robustness against natural accuracy. Wang et al. (2019) proposed *MART*, an *AT* variant that differentiates between naturally misclassified examples that are used for the inner maximization of the *AT* process, using this information to improve adversarial robustness. Wu et al. (2020) proposed adversarial weight perturbation, a double perturbation mechanism that employs the perturbation of inputs and weights to improve adversarial robustness. More recent *AT* methods improve existing *AT* variants by employing reweighting (Zhang et al., 2021; Liu et al., 2021; Fakorede et al., 2023b) or incorporating hypersphere embedding (Pang et al., 2020; Fakorede et al., 2023a).

To further improve the robustness of existing *AT* variants, this work proposes a non-information-theory regularization term that maximizes the gap between the estimated probabilities corresponding to the incorrect classes and the estimated probability of the correct class.

2.3 STANDARD DEVIATION AS A RISK MEASURE

The standard deviation measures the spread of a distribution around the mean of that distribution. The standard deviation of a distribution is given as:

$$SD = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N-1}} \tag{2}$$

where x_i is a data-point, μ is the population mean, and $\mathbf{N}$ is the number of data-points in the distribution. A smaller *SD* value suggests that data points are more clustered, whereas a larger *SD* value indicates that data points are farther from the mean. The properties of standard deviation have made it a useful measure of risks in various domains. For instance, the standard deviation is used as a measure of risk in finance for measuring market volatility and asset and portfolio risk by indicating how much the returns of an asset deviate from the mean return (Artzner et al., 1999; Hull, 2012; Ross et al., 2019).

Inspired by recent work by (Fakorede et al., 2024), which proposes a standard-deviation-based vulnerability measure of input samples, this paper argues a standard-deviation-inspired measure to capture the risk of misclassification of training samples. Hence, the standard deviation-inspired measure may be used to obtain adversarial examples and as a regularization term to improve adversarial training.

3 PROPOSED METHOD

Here, we justify the introduction of a Standard-Deviation-Inspired (*SDI*) measure as a regularization term into existing adversarial training approaches. The *SDI* measure was originally proposed in Fakorede et al. (2024) for the purpose of estimating the vulnerability of natural examples. In this paper, we connect the *SDI* measure to the min-max optimization concept in adversarial training and use it as a regularization term.

3.1 THE *SDI* MEASURE

The *SDI* measure adopts an idea similar to standard deviation to characterize the spread of output probability vectors of DNN models for individual training examples. Specifically, the approach measures the variation of a model's estimated probabilities for incorrect classes around the model's estimated probability for the true label of individual input examples $\mathbf{x}$. Formally, given an input-label pair $(\mathbf{x}_i, y_i)$ and the output probabilities of a DNN model on input sample $\mathbf{x}_i$ denoted as $f_\theta(\mathbf{x}_i)$, the *SDI* measure is given as:

$$M_{SDI}(\mathbf{x}_i, y_i, \theta) = \sqrt{\frac{\sum_{k=1}^{|C|}(f_\theta(\mathbf{x}_i)_k - f_\theta(\mathbf{x}_i)_{y_i})^2}{|C|-1}} \tag{3}$$

where $|C|$ is the number of output classes, $f_\theta(\mathbf{x}_i)_k$ is the model's estimated probability corresponding to class k, $f_\theta(\mathbf{x}_i)_{y_i}$ is the model's estimated probability of the true class, and θ denote the model parameters.

Under the condition where $f_\theta(\mathbf{x}_i)_{y_i} \geq \max_{k,k\neq y_i} f_\theta(\mathbf{x}_i)_k$, the $M_{SDI}(\mathbf{x}_i, y_i, \theta)$ measure serves to capture the vulnerability and risk of misclassification of individual examples $\mathbf{x}_i$. A smaller value of $M_{SDI}(\mathbf{x}_i, y_i, \theta)$ suggests that the output probabilities returned for sample $\mathbf{x}_i$ are more evenly distributed among classes, indicating a higher likelihood of misclassification as the model may misclassify it into any of the $k-1$ incorrect classes.

3.2 AN *SDI*-ORIENTED PERSPECTIVE TO ADVERSARIAL TRAINING

In this section, we connect the M_{SDI} measure with the min-max optimization of adversarial training.

AT methods are typically formulated as min-max optimization procedures. The inner maximization step of the AT approach involves generating adversarial examples $\mathbf{x}'_i$ from natural examples $\mathbf{x}_i$ by iteratively adjusting the input data in directions that maximize the loss, using projected gradient descent (PGD) algorithm as follows:

$$\mathbf{x}_i'^{t+1} \leftarrow \prod_{\mathbf{x}'_i \in B_\epsilon(\mathbf{x}_i)} (x_i'^t + \alpha \cdot sign(\nabla_{\mathbf{x}_i'^t} L(\mathbf{x}_i'^t, y_i))). \tag{4}$$

where $\prod$ is the projection operator and $\mathbf{L}$ is a loss function.

Essentially, the adversarial examples produced during the inner maximization process are tailored to reduce the model's confidence in correctly classifying them into their true classes. Moreover, the resulting adversarial examples are untargeted, i.e., the inner maximization misclassifies the adversarial examples without prioritizing any particular incorrect class.

The $M_{SDI}(\mathbf{x}_i, y_i, \theta)$ measure estimates the vulnerability of individual inputs into a DNN model, using the spread of the model's estimated probabilities w.r.t. the model's estimated probability of the actual class of each input. The smaller the $M_{SDI}(\mathbf{x}_i, y_i, \theta)$ estimate on the output probability vector of a model, the more clustered or similar the estimated probabilities of the model on the input, and the less confident the model is about the input's true class. Therefore, the magnitude of $M_{SDI}(\mathbf{x}_i, y_i, \theta)$ value for an input-label pair $(\mathbf{x}_i, y_i)$ is indicative of the degree of risk in misclassifying $\mathbf{x}_i$. In contrast, when $M_{SDI}(\mathbf{x}_i, y_i, \theta)$ returns a large value, it suggests that the model estimates that the probability of $\mathbf{x}$ belonging to class y is very high and that $\mathbf{x}$ has a low risk of misclassification. This observation suggests that adversarial examples may be generated simply by finding adversarial perturbation along the gradient direction that minimizes the *SDI* metric. Formally, we formulate an adversarial example using the SDI measure as follows:

$$\mathbf{x}_i'^{t+1} \leftarrow \prod_{\mathbf{x}'_i \in B_\epsilon(\mathbf{x}_i)} (x_i'^t - \alpha \cdot sign(\nabla_{\mathbf{x}_i'^t} M_{SDI}(\mathbf{x}_i'^t, y_i, \boldsymbol{\theta}))). \tag{5}$$

The above adversarial example generation is achieved using the widely adopted PGD algorithm (Madry et al., 2018), with the notable difference that the sign of the gradient is inverted to move in the opposite direction. For most AT variants, adversarial examples in the inner maximization step are obtained by finding perturbations that maximize a cross-entropy loss function or a Kullback-Leibler divergence. The *SDI* measure does not rely on information-theoretic measures. Therefore, it offers a complementary approach for finding adversarial examples. We provide experimental evidence for our claim in Sec. 4.5.

The outer minimization seeks model parameters that minimize the loss on the adversarial examples generated during the inner maximization step. Essentially, the outer minimization process aims to maximize the likelihood of correctly classifying individual adversarial training examples. Invariably, the outer minimization minimizes the likelihood of incorrect classification by increasing the probability gap between the example belonging to the label and belonging to incorrect classes. This conceptually aligns with the goal of maximizing the *SDI* measure. Maximizing the *SDI* metric encourages the model to correctly classify the input to its true class by widening the probability gap between the estimated probability for the true class and the estimated probabilities for other incorrect classes. Moreover, when $f_\theta(\mathbf{x}_i)_y \geq \max_{k,k\neq y} f_\theta(\mathbf{x}_i)_k$, maximizing the M_{SDI} measure maximizes $f_\theta(\mathbf{x}_i)_y$.

3.3 SDI REGULARIZATION

Here, we propose the *SDI* regularization term for improving adversarial training.

Typically, adversarial training techniques involve training models using adversarial examples generated by various forms of PGD attacks. However, this approach may lead to overly specialized models defending against PGD attacks, potentially causing poor generalization to different attack types. As discussed earlier, the M_{SDI} metric introduced in the preceding section possesses favorable attributes that complement the objectives of adversarial training. Hence, to improve the robust generalization and performance of existing AT methods, we propose adding a regularization term that maximizes the M_{SDI} measure on each training example.

Maximizing the M_{SDI} metric as a regularization term encourages the model to maximize the output probability of a training example belonging to its actual class, thus improving training. Moreover, since existing AT variants depend on information-theoretic measures for both the inner maximization step and the outer minimization step, applying the M_{SDI} metric as a regularization term offers a complementary addition to AT methods that does not depend on the information-theoretic measures that these AT methods are based. Lastly, maximizing the M_{SDI} metric facilitates the widening of the probability gaps between the probability of the actual class of individual adversarial examples and the probabilities corresponding to incorrect classes, thus improving the discriminability of the model.

Note that maximizing the M_{SDI} measure to improve $f_\theta(\mathbf{x}_i)_y$ is only valid when $f_\theta(\mathbf{x}_i)_y \geq \max_{k,k\neq y} f_\theta(\mathbf{x}_i)_k$. When $f_\theta(\mathbf{x}_i)_y < \max_{k,k\neq y} f_\theta(\mathbf{x}_i)_k$, maximizing M_{SDI} may further minimize $f_\theta(\mathbf{x}_i)_y$, since the probability gap between each $f_\theta(\mathbf{x}_i)_{k,k\neq y}$ and $f_\theta(\mathbf{x}_i)_y$ is further increased to maximize the M_{SDI} measure. Therefore, we propose a regularization term L_{SDI} that selectively maximizes the M_{SDI} measure on samples whose output probabilities satisfies $f_\theta(\mathbf{x}_i)_y \geq \max_{k,k\neq y} f_\theta(\mathbf{x}_i)_k$.

We utilize the *multi-class margin* from (Koltchinskii & Panchenko, 2002) to determine input samples satisfying the desired conditions. For a DNN denoted by f_θ and the input-label pair $(\mathbf{x}_i, y_i)$, the margin $d_m(\mathbf{x}_i, y_i; \theta)$ is given as follows:

$$d_m(\mathbf{x}_i, y_i; \theta) = f_\theta(\mathbf{x}_i)_{y_i} - \max_{k,k\neq y_i} f_\theta(\mathbf{x}_i)_k \tag{6}$$

where $f_\theta(\mathbf{x}_i)_{y_i}$ is the model's predicted probability of the correct label y_i, and $\max_{k,k\neq y_i} f_\theta(\mathbf{x}_i)_k$ is the maximum prediction of the remaining classes.

We formally describe the proposed *SDI* regularization term as follows:

$$L_{SDI}(\mathbf{x}_i, y_i; \theta) = \begin{cases} M_{SDI}(\mathbf{x}_i, y_i; \theta), & \text{if } d_m(\mathbf{x}_i, y_i; \theta) \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

In this paper, we apply the $L_{SDI}(\mathbf{x}_i, y_i; \theta)$ regularization term to two prominent adversarial training methods: *standard AT* (Madry et al., 2018) and *TRADES (Zhang et al., 2019)*. We refer to the *SDI*-regularized *standard AT* and *TRADES* as *AT-SDI* and *TRADES-SDI* respectively. The regularized training objectives are stated as follows:
AT-SDI:

$$\sum_i L_{CE}(f_\theta(\mathbf{x}'_i), y_i) - \beta \cdot L_{SDI}(\mathbf{x}'_i, y_i, \theta) \tag{7}$$

TRADES-SDI:

$$\sum_i L_{CE}(f_\theta(\mathbf{x}_i), y) + \frac{1}{\lambda} \cdot KL(f_\theta(\mathbf{x}_i) \| f_\theta(\mathbf{x}'_i)) - \beta \cdot L_{SDI}(\mathbf{x}'_i, y_i, \theta) \tag{8}$$

where β in Eq. (7) or (8) represents the regularization hyperparameter for controlling the weight of the *SDI* regularization term, and KL in Eq. (8) represents Kullback–Leibler divergence. In the proposed *AT-SDI* and *TRADES-SDI*, the L_{SDI} regularization term is selectively applied. The regularization term is only applied to adversarial training instances satisfying:$f_\theta(\mathbf{x}'_i)_y \geq \max_{k,k\neq y} f_\theta(\mathbf{x}'_i)_k$. If $f_\theta(\mathbf{x}'_i)_y < \max_{k,k\neq y} f_\theta(\mathbf{x}'_i)_k$ on a sample $\mathbf{x}'_i$, the normal AT or$TRADES$ adversarial training is applied on $\mathbf{x}'_i$.

As an example, the proposed *AT-SDI* algorithm for adversarial training is presented in the following.

Algorithm 1 AT-SDI Algorithm.

Input: a neural network model with the parameters θ, step size κ, T PGD steps, a training dataset $\mathcal{D}$ of size n, and $|C|$ is the number of classes.

Output: a robust model with parameters θ^*

for *epoch* $= 1$ to num_epochs **do**
  for *batch* $= 1$ to num_batchs **do**
    sample a mini-batch $\{(x_i, y_i)\}_{i=1}^{M}$ from $\mathcal{D}$; ▷ mini-batch of size M.
    for i = 1 to M **do**
      $\mathbf{x}_i \leftarrow \mathbf{x}_i + 0.001 \cdot \mathcal{N}(0, 1)$; ▷ $\mathcal{N}(0, I)$ is a Gaussian distribution with zero mean and identity variance.
      for $t = 1$ to T **do**
        $\mathbf{x}'_i \leftarrow \prod_{B_{\epsilon_i}(\mathbf{x}_i)}(x_i + \kappa \cdot sign(\nabla_{\mathbf{x}'_i} L_{CE}(f_\theta(\mathbf{x}'_i), y_i))$
      end for
    end for
    $M_{SDI}(\mathbf{x}'_i, y_i; \theta) = \{\sum_{k=1}^{|C|} \frac{(f_\theta(\mathbf{x}'_i)_k - f_\theta(\mathbf{x}'_i)_{y_i})^2)}{|C|-1}\}^{0.5}$
    $d_m(\mathbf{x}'_i, y_i; \theta) = f_\theta(\mathbf{x}'_i)_{y_i} - \max_{k, k \neq y_i} f_\theta(\mathbf{x}'_i)_k$
    if $d_m(\mathbf{x}'_i, y_i; \theta) \geq 0$ **then**
      $L_{SDI}(\mathbf{x}'_i, y_i; \theta) \leftarrow M_{SDI}(\mathbf{x}'_i, y_i; \theta)$
    else
      $L_{SDI}(\mathbf{x}'_i, y_i; \theta) \leftarrow 0$
    end if
    $\theta \leftarrow \theta - \eta \nabla_\theta \frac{1}{|M|}(\sum_{i=1}^{M} L_{CE}(f_\theta(\mathbf{x}'_i), y_i) - \beta \cdot L_{SDI}(\mathbf{x}'_i, y_i; \theta))$
  end for
end for

4 EXPERIMENTS

In this section, we conduct an extensive evaluation of the proposed method. To assess its versatility, we test it on various datasets, including CIFAR-10 (Krizhevsky et al., 2009), CIFAR-100 (Krizhevsky et al., 2009), SVHN (Netzer et al., 2011), and Tiny ImageNet Deng et al. (2009). We apply simple data augmentations, such as 4-pixel padding with 32 × 32 random crop and random horizontal flip, to each of the datasets. Additionally, we employ ResNet-18 (He et al., 2016) and WideResNet-34-10 (He et al., 2016) as the backbone models.

4.1 EXPERIMENTAL SETUP

4.1.1 Training Parameters.

We train the backbone networks using mini-batch gradient descent for 110 epochs, with a momentum of 0.9 and a batch size of 128. For training CIFAR-10, we used a weight decay of 5e-4, and for CIFAR-100, SVHN, and TinyImageNet, we used a weight decay of 3.5e-3. The initial learning rate was set to 0.1 (0.01 for CIFAR-100, SVHN, and TinyImageNet), and it was divided by 10 at the 75th epoch and then again at the 90th epoch.

4.1.2 Hyperparameters.

We set the value of β to 3.0 for training AT-SDI and *TRADES-SDI* on CIFAR-10, SVHN, and TinyImagenet. For CIFAR-100 using AT-SDI and *TRADES-SDI*, we set β to 3.0. When incorporating AWP (Wu et al., 2020) into *AT-SDI* and *TRADES-SDI*, we respectively set β to 3.0 and 1.0. The hyperparameters are tuned using a validation set. We provide ablation studies on changing β values for *AT-SDI* and *TRADES-SDI* for CIFAR-10 using Wideresnet-34-10 in Tables 7 and 8.

4.2 BASELINES

We use prominent methods Standard *AT* (Madry et al., 2018) and *TRADES* (Zhang et al., 2019) as our baselines. In addition, we compare our results to other popular works *MART* (Wang et al., 2019), *AWP* (Wu et al., 2020), *MAIL* (Liu et al., 2021), and *ST-AT* (Li et al., 2023). All hyperparameters of the baseline

methods remain consistent with those in their original papers. Nevertheless, we maintain consistency by using the same learning rate, batch size, and weight decay values as those utilized during the training of our proposed method.

4.3 THREAT MODELS

We evaluate the performance of the proposed method against strong attacks under *white-box* and *black-box* settings, as well as the Auto attack.

White-box attacks. These attacks have access to model parameters. To assess robustness on CIFAR-10 using Resnet-18 and Wideresnet-34-10, we employ the *PGD* attack with $\epsilon = 8/255$, step size $\kappa = 1/255$, and $K = 20$ iterations (PGD-20). Additionally, we utilize the *CW* attack (CW loss (Carlini & Wagner, 2017) optimized by *PGD-20*) with $\epsilon = 8/255$ and step size $1/255$. On SVHN and TinyImageNet, we use the *PGD* attack with $\epsilon = 8/255$, step size $\kappa = 1/255$, and $K = 20$ iterations.

Black-box attacks. In black-box settings, the adversarial attack method does not have access to the model parameters. We evaluate robust models trained on CIFAR-10 against strong black-box attack, SPSA (Uesato et al., 2018), with 100 iterations. These attacks use a perturbation size of 0.001 for gradient estimation, a learning rate of 0.01, and 256 samples for each gradient estimation. All black-box evaluations are conducted on trained Wideresnet-34-10.

Auto attacks (AA). Lastly, we assess the robustly trained models using *Autoattack* (l_∞ and l_2) (Croce & Hein, 2020b), which is a powerful ensemble of attacks consisting of APGD-CE (Croce & Hein, 2020b), APGD-T (Croce & Hein, 2020b), FAB-T (Croce & Hein, 2020a), and Square (a black-box attack) (Andriushchenko et al., 2020).

4.4 PERFORMANCE EVALUATION

We present our experimental results and comparisons on various datasets using ResNet-18 and WideResNet-34-10 architectures. Specifically, results for CIFAR-10 on ResNet-18 and WideResNet-34-10 are summarized in Tables 1 and 2, respectively, while results for CIFAR-100, SVHN, and Tiny ImageNet using ResNet-18 are presented in Tables 3, 4, and 5, respectively. Additionally, comparisons with other prominent baselines are provided in Table 6. Finally, we compare the perfomance of adversarial examples generated using the *SDI* metric approach described in Eq. (5) to adversarial examples crafted using cross-entropy and KL-divergence losses.

All experimental results are averaged over three runs with random seeds, and standard deviations are omitted as they are deemed insignificant (< 0.3).

4.4.1 Comparing *AT* and *TRADES* with their *SDI*-regularized variants.

In this comparison, we evaluate the performance of *AT* and *TRADES* against their respective variants with the *SDI* regularization term, *AT-SDI* and *TRADES-SDI*. Experimental findings indicate that the proposed regularization term enhances robustness against various adversarial attacks, including *Autoattacks* and *CW*. Specifically, when applied to ResNet-18 and WideResNet-34-10 architectures on CIFAR-10, *AT-SDI* demonstrates improvements over *AT* across all evaluated attacks (see Tables 1 and 2). For example, on WideResNet-34-10, *AT-SDI* outperforms *AT* in robustness against *PGD-20* (+0.45 %), *CW* (+2.54 %), and *Autoattacks* (+1.65 %). The improvement in robustness are achieved without a significant reduction in the natural accuracy.

Similarly, *TRADES-SDI* exhibits superior performance compared to *TRADES* on *PGD-20* (+1.19 %), *CW* (+2.06 %), and *Autoattacks* (+1.14%). Training with *TRADES-SDI* also exhibit a noticeable improvement of 0.67 % on the natural accuracy. Overall, *AT-SDI* achieves greater improvement in robustness against *CW* attacks compared to *TRADES-SDI*, while *TRADES-SDI* demonstrates better enhancement against *PGD-20* attacks compared to *AT* across ResNet-18 and WideResNet-34-10 architectures.

The proposed SDI regularization term also enhances robustness on CIFAR-100 when applied to ResNet-18 across all evaluated adversarial attacks (see Table (3)). The margin of improvement in robustness against

adversarial attacks on CIFAR-100 appears to be larger than that observed on CIFAR-10 for both *AT-SDI* and *TRADES-SDI*. Similar improvements in robustness are observed when *AT-SDI* and *TRADES-SDI* are utilized to train Resnet-18 on SVHN dataset. Results in Table (4) show that *AT-SDI* outperforms *AT* on *CW* (+5.23%), *Autoattack* (+ 1.20%), and *PGD-20* (+2.43%).

Finally, Table 5 also clearly shows that the proposed training objective improves the robustness of Resnet-18 against all the evaluated attacks on Tiny Imagenet. The consistent improvement of performance across all the datasets tested validates the efficacy of the proposed *SDI* regularization term.

Table 1: White-box attack robust accuracy for ResNet-18 on CIFAR-10.

Defense	Natural	PGD-20	*CW*	AA
Standard-AT	**84.10**	52.78	51.80	47.95
AT - SDI (Ours)	83.88	**53.43**	**53.71**	**49.56**
TRADES	**82.65**	52.82	51.82	48.96
TRADES - SDI (Ours)	82.04	**53.87**	**52.61**	**50.80**

Table 2: White-box attack robust accuracy for Wideresnet-34-10 on CIFAR-10.

Defense	NATURAL	PGD-20	CW	AA
Standard AT	**86.23**	56.32	54.95	51.92
AT-SDI (Ours)	86.11	**56.78**	**57.49**	**53.57**
TRADES	84.70	56.30	54.51	53.07
TRADES-SDI (Ours)	**85.37**	**57.49**	**56.57**	**54.21**

Table 3: White-box attack robust accuracy for ResNet-18 on CIFAR-100.

Defense	Natural	$PGD-20$	*CW*	AA
Standard-AT	56.59	28.18	25.64	24.07
AT - SDI (Ours)	**57.96**	**30.78**	**29.37**	**26.38**
TRADES	56.96	29.21	25.57	24.65
TRADES - SDI (Ours)	**60.68**	**31.21**	**28.73**	**26.45**

Table 4: White-box attack robustness accuracy for ResNet-18 on SVHN.

Defense	Natural	$PGD-20$	*CW*	AA
Standard-AT	**92.57**	55.67	52.92	45.95
AT-SDI (Ours)	92.10	**58.10**	**58.15**	**47.15**
TRADES	**90.83**	57.27	53.59	46.45
TRADES-SDI (Ours)	90.54	**59.21**	**56.39**	**49.21**

Table 5: White-box attack robustness accuracy for ResNet-18 on Tiny Imagenet.

Defense	Natural	$PGD-20$	*CW*	AA
Standard-AT	48.83	23.96	21.85	17.91
AT-SDI (Ours)	**49.73**	**24.79**	**23.16**	**20.01**
TRADES	49.11	22.82	17.79	16.82
TRADES-SDI (Ours)	**51.77**	**25.11**	**21.42**	**19.71**

4.4.2 Comparison with other prominent baselines.

Here, we compare our approach with other prominent methods from existing works, including *MART* (Wang et al., 2019), adversarial weight perturbation (*AWP*) (Wu et al., 2020), and ST-AT (Li et al., 2023). Additionally, for a fair comparison with *AWP*, we combine *AT-SDI* and *TRADES-SDI* with *AWP* and denote them

as *AT-SDI + AWP* and *TRADES-SDI + AWP*, respectively. In both *AT-SDI + AWP* and *TRADES-SDI + AWP*, the *SDI* regularization term is employed for perturbing the network weights.

Experimental results displayed in Table 6 show that *AT-SDI* outperforms all existing baselines in robustness against *CW* attacks. Additionally, *AT-SDI* outperforms *TRADES* and *MART* against *Autoattacks*. However, *AT-SDI* slightly underperforms compared to *MART* against *PGD-20*. Furthermore, *AT + AWP* also marginally outperforms *AT-SDI* against Autoattacks. On the other hand, *TRADES-SDI* achieves better performance than all baselines on *CW* and *Autoattacks*.

Combining our approach with *AWP* further improves robustness against the evaluated attacks. Specifically, *AT-SDI + AWP* and *TRADES-SDI + AWP* outperform *AWP + AT* and *TRADES + AWP* against every adversarial attack. *AT-SDI + AWP* and *TRADES-SDI + AWP* demonstrate improved performance across all attacks. *AT-SDI + AWP* enhances robustness to *PGD-20*, *CW* attacks, and Autoattacks over *AWP* by 2.11%, 4.38%, and 2.79%, respectively. *AT-SDI+AWP* also considerably outperforms *AWP* robustness against *SPSA*, a strong query-based blackbox attack, by 2.99%. Additionally, *AT-SDI + AWP* achieves superior performance on natural samples. *TRADES-SDI + AWP* improves performance over *AWP* on all the attacks but dips by 0.15% in performance on natural examples.

Table 6: Comparison with other baselines under white-box and black-box attacks on CIFAR-10 for Wideresnet-34-10.

Defense	NATURAL	PGD-20	CW	AA	SPSA
Standard AT	86.23	56.32	54.95	51.92	61.05
TRADES	84.70	56.30	54.51	53.07	61.15
MART	84.17	58.10	54.51	51.11	58.91
MAIL ((Liu et al., 2021))	86 .81	60.49	51.45	47.11	59.25
ST-AT ((Li et al., 2023))	84.91	57.52	55.11	53.54	61.34
AWP ((Wu et al., 2020))	85.36	58.04	55.92	53.92	62.57
AT-SDI (ours)	86.11	56.78	57.49	53.57	62.46
TRADES-SDI (ours)	85.37	57.49	56.57	54.21	62.31
AT-SDI + AWP (ours)	**88.21**	60.15	**60.30**	56.71	**65.56**
TRADES-SDI + AWP (ours)	85.21	**60.72**	58.15	**56.82**	63.41

4.4.3 SDI-regularization Improves Generalization of Adversarial Training.

Most *AT* methods involve training with a specific type of adversarial examples crafted by maximizing either the cross-entropy or KL-divergence measure using *PGD*. Therefore, the adversarial examples utilized for adversarial training do not entirely reflect the universe of all possible adversarial attacks that a robust model may encounter. This limitation can lead to poor generalization of adversarially trained models to other types of adversarial examples (Song et al., 2018).

Typically, adversarial training methods exhibit significantly higher performance on *PGD* attacks, as evident from the experimental results tables. However, when subjected to other types of attacks, the performance of robust models tends to diminish. For example, it can be observed from the tables that the robust accuracy on *CW* and *AA* attacks are notably lower compared to *PGD-20*.

The introduction of the L_{SDI} regularization term to the standard *AT* and *TRADES* improves their performances against other attacks. Unlike other *AT* methods, *AT-SDI* considerably improves the robustness against *CW* and *AA* on all the datasets evaluated. In fact, training a model using the proposed *AT-SDI* consistently improves the performance of the resulting robust model to *CW* attack and achieve better performance over *PGD-20* on *CIFAR-10* dataset, as may be observed in Tables 1, 2, and 6. Note that *CW* adversarial examples are not used for training, yet better robust accuracies are recorded compared to *PGD* adversarial examples, which are typically used for adversarial training. Significant improvement in robustness against *CW* and *AA* can also be observed on other datasets, CIFAR-100, SVHN, and Tiny Imagenet. The SDI regularization term also improves the performance of *TRADES* on *CW* and *AA* attacks. Experiments in Tables 1 - 5 show that the SDI regularization reduces the performance gap between PGD-20 and the other attacks.

Combining *AWP* (Wu et al., 2020) with *AT-SDI* achieves a high robustness of 60.30% on CW, improving *AWP* by 4.38% on CIFAR-10. Further, the robustness performance on *CW* is better than the robustness performance on *PGD-20*. Also in Table 6, the improvement in performance is noticeable against *AA* and *SPSA* attacks. Similarly, the improvements in robustness to *CW AA* can be observed in Table 6, when the L_{SDI} regularization is applied to *TRADES + AWP* .

Overall, the proposed L_{SDI} regularization term consistently minimizes the performance gaps between robustness to *PGD-20* adversarial examples and other types of adversarial examples. This supports our argument that the L_{SDI} regularization improves the generalization of adversarial training. An intuitive explanation for this observation is that the L_{SDI} regularization is not dependent on the specific algorithmic nuances of individual adversarial attacks and defenses. Instead, it explicitly maximizes the probability gaps between the probability of the true class of each adversarial example and the probabilities corresponding to incorrect classes.

4.4.4 Sensistivity Analysis of the hyper-parameter β

Here, we study the influence of the regularization hyper-parameter β on *AT - SDI* and *TRADES-SDI* performance.

Experimental results in Tables 7 and 8 show that increasing the value of β leads to moderate reduction in the natural accuracy of *AT-SDI* and *TRADES-SDI*. The robust accuracy of *PGD-20* remains relatively stable for various values of β on *AT-SDI* and *TRADES-SDI*. Nevertheless, *AT-SDI* exhibits noticeable improvement in robustness against *CW* attack as β increases. The performance of *AT-SDI* against *Autoattack* also improves with increasing β value but diminishes as β gets too large. We selected $\beta = 3.0$ since it maintains a good trade-off between the natural and robust accuracy.

Like *AT-SDI*, *TRADES-SDI* is also sensitive to β. The variations in natural accuracy are moderate when β is varied. The robustness to *PGD-20* remains relatively stable as β varies. It can be observed from 8 that the robust performance tp *CW* attack and *AA* increases as β is increased, but both slightly reduce when β is set to 5.0.

Table 7: Ablation studies for AT-SDI for Wideresnet-34-10 on CIFAR-10.

β	NATURAL	PGD-20	CW	AA
1.0	86.75	56.77	57.07	53.16
2.0	86.52	56.84	57.19	53.44
3.0	86.11	56.78	57.49	53.57
4.0	85.78	56.69	57.49	53.39
5.0	85.59	56.37	57.81	52.73

Table 8: Ablation studies for *TRADES-SDI* for Wideresnet-34-10 on CIFAR-10.

β	NATURAL	PGD-20	CW	AA
1.0	85.51	57.27	55.66	53.43
2.0	85.43	57.29	56.01	53.61
2.5	85.39	57.36	56.28	53.88
3.0	85.37	57.49	56.57	54.21
4.0	85.24	57.33	56.66	54.11
5.0	85.21	57.31	56.59	54.05

4.5 ADVERSARIAL ATTACK USING THE *SDI* METRIC

In Section 3, we argue that the proposed *SDI* metric in Eq. (3) can be utilized for crafting adversarial examples.

Figure 1: Comparison of natural CIFAR-10 images with the adversarial perturbations and adversarial examples obtained by SDI-PGD attack defined in Eq. (5). Images in the first row represent natural CIFAR-10 images and their correct labels. The second and third rows represent the corresponding l_∞ adversarial perturbations, with $\epsilon = 8/255$ and 0.2 respectively. The fourth row represents the corresponding adversarial examples and their incorrect labels for each image in row one.

Here, we compare PGD-based adversarial examples optimized using the *SDI* metric with existing popular PGD-based adversarial examples crafted using information-theoretic measures such as cross-entropy (Madry et al., 2018) and KL-divergence (Zhang et al., 2019). The M_{SDI}-measure-optimized adversarial examples are crafted using the approach described in Eq. (5). We compare the performances of each approach under *AT* and *TRADES*. In each case, the attack is obtained using the conventional attack settings: 20 PGD iterations, perturbation bound ϵ 0.031, and step size 0.003. The algorithm for obtaining adversarial examples using the M_{SDI} measure is provided in Algorithm 2.

Algorithm 2 SDI-PGD Algorithm.

Input: a neural network model with the parameters θ, step size κ, natural examples $\mathbf{x}_i$ in a labelled dataset $\mathcal{D}$ of size n and $|C|$ is the number of classes.

Output: Adversarial examples $\mathbf{x}'_i$

Sample $(\mathbf{x}_i, y_i)$ from $\mathcal{D}$;
$\mathbf{x}_i^{'} \leftarrow \mathbf{x}_i + 0.001 \cdot \mathcal{N}(0,1)$; ▷ $\mathcal{N}(0, I)$ is a Gaussian distribution with zero mean and identity variance.
for $t = 1$ to T **do** ▷ T is the number of PGD iteration steps.
  $M_{SDI}(\mathbf{x}'_i, y_i; \theta) = \{\sum_{k=1}^{|C|} \frac{(f_\theta(\mathbf{x}'_i)_k - f_\theta(\mathbf{x}'_i)_{y_i})^2)}{|C|-1}\}^{0.5}$
  $\mathbf{x}'_i \leftarrow \prod_{B_\epsilon(\mathbf{x}_i)}(x_i - \kappa \cdot sign(\nabla_{\mathbf{x}'_i} M_{SDI}(\mathbf{x}'_i, y_i; \theta))$ ▷ $\prod$ denotes the projection operator.
end for
return $\mathbf{x}'_i$ ▷ Return adversarial example.

We display some adversarial examples obtained using the proposed M_{SDI}-measure-optimized approach and their corresponding misclassified labels in the last row of Figure 1. Additionally, we display the corresponding adversarial perturbations of the original images in rows two and three.

We compare the robustness of two prominent adversarial training variants, *AT* and *TRADES*, against *PGD*-based adversarial examples generated using cross-entropy loss, KL-divergence, and the SDI measure. The experimental results are presented in Table 9. It is evident from the table that adversarial examples crafted using the SDI metric exhibit significantly greater strength compared to those crafted using KL-divergence. Nevertheless, adversarial examples generated with the cross-entropy loss demonstrate a marginally greater strength than those obtained using the proposed SDI metric.

The results show that the *SDI* metric produces 'useful gradients' for generating adversarial examples, a desirable quality as discussed in prior studies (Athalye et al., 2018; Papernot et al., 2017). This suggests that optimizing the *SDI* metric for improving adversarial robustness would not lead to gradient obfuscation (Athalye et al., 2018).

Table 9: Comparison of successes of PGD attacks crafted using cross-entropy, KL-divergence, and *SDI* measure on defences *AT* (Madry et al., 2018) and *TRADES* (Zhang et al., 2019) on CIFAR-10 for Resnet-18.

PGD Attack Measure	*AT*	*TRADES*
Cross-entropy	**52.84**	**52.82**
KL-divergence	68.03	68.87
SDI	53.95	54.32

5 CONCLUSION

We introduce a novel regularization term based on a standard deviation-inspired (*SDI*) measure to improve adversarial robustness. The *SDI* measure captures the spread of a model's estimated probabilities with respect to the true class of each input. We establish a connection between optimizing the *SDI* measure and the min-max optimization procedure in adversarial training. Specifically, we illustrate that the SDI measure may be optimized for generating adversarial examples by seeking perturbations that minimize the SDI measure.

We demonstrate with experimental study that maximizing the SDI measure on adversarial training examples contributes to improving the robustness of existing adversarial training methods. Empirical results indicate that the proposed regularization significantly improves existing adversarial training variants' robustness and generalization capabilities.

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