An end-to-end Complex-valued Neural Network approach for *k-space* interpolation in Parallel MRI

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Abstract

Parallel MRI techniques in the *k-space*, like GRAPPA are widely used in accelerated MRI. Recently neural-network based non-linear approaches have shown improved performance over linear methods like GRAPPA. But present day neural networks are largely tailored to process real data, hence the complex-valued *k-space* data is processed as two-dimensional real data in these. In this work, we study the performance of an end-to-end complex-valued architecture trained using complex-valued optimization, for interpolating missing values in the *k-space* for parallel MR which we call the *Complex rRAKI*. We propose a generalized version of the ReLU activation function on the complex plane called the *PlaneReLU*. The performance of the Complex rRAKI is evaluated on two publicly-available *k-space* MRI datasets, the fastmri multicoil brain dataset and the fastmri multicoil knee dataset. Comparison of obtained results with those on the baseline rRAKI are also presented. The proposed Complex rRAKI achieves improved performance over the baseline with respect to standard metrics SSIM and NRMSE with 50% fewer parameters.

Keywords: Complex-valued neural network, Complex ReLU, Parallel MRI, GRAPPA

1. Introduction

Parallel MRI uses multiple acquisition coils to acquire partial *k-space* data and then exploit the position information of the coils to obtain high quality reconstructions from the data. The Generalized Auto calibrating Partial Parallel Acquisition (GRAPPA) (Griswold et al., 2002) method estimates the missing values in the *k-space* before computing the inverse Fourier transforms of the individual coil images and combining them to get the final reconstruction. Recently, scan-specific neural network approaches (Zhang et al., 2019; Arefeen et al., 2021) have been developed to learn a potentially non-linear relationship instead. However real-valued neural networks may not be able to exploit information in inherently complex-valued datasets. As MRI sensor data is complex-valued, few works have adapted complex-valued neural networks to MRI reconstruction (Cole et al., 2020; Chatterjee et al., 2021; Vasudeva et al., 2020). But most of these works denoise poor quality zero-filled reconstructions. When the zero-filled reconstructions have a lot of artefacts, there may be loss of important details which cannot be reconstructed back. Thus, it is important to work with *k-space* directly. Also to the best of our knowledge, none of the previous works explores a scan-specific complex-valued neural network, thus relying on huge datasets for training. In this work, we implement an end-to-end complex-valued neural network for the scan-specific parallel MRI approach - the Residual RAKI (Zhang et al., 2019). The major contributions in this work are -

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1. We implement an end-to-end complex-valued neural network trained using complex-valued optimization, called the *Complex rRAKI* for a scan-specific approach for parallel MRI called the Residual RAKI (rRAKI)
2. We propose a generalized version of the ReLU activation function on the complex plane which we call the *PlaneReLU*

2. Methods

The Complex rRAKI architecture outputs the sum of a linear component G_j and a non-linear component F_j consisting of a composition of complex convolution and *PlaneReLU* activation function operations. Let $I \in \mathbb{C}^{H \times W \times C}$ be an input *k-space* of shape (H, W, C) where H and W refer to the height and width and C is the number of coils, also known as number of channels. Let AC_j denote the autocalibration region in channel j, $j = 1, 2, ..\bar{C}$. Let y_j denote the vector containing target elements at locations $(k_x, k_y) \in AC_j$ and y_{source} denote the vector containing the corresponding neighboring elements in the autocalibration regions across all channels. Then Complex rRAKI performs learning over the autocalibration region by minimizing the cost function

$$\mathcal{L}(\gamma_j, \theta_j) = \min_{\gamma_j, \theta_j} \|y_j - G_j(y_{source}; \gamma_j) - F_j(y_{source}; \theta_j)\|_2 + \lambda \|y_j - G_j(y_{source}; \gamma_j)\|_2 \tag{1}$$

where $G_j \colon \mathbb{C}^n \to \mathbb{C}^m$ is a linear complex convolution operator having parameters γ_j and $F_j \colon \mathbb{C}^n \to \mathbb{C}^m$ is a complex-valued CNN parameterized by θ_j, having two blocks of complex convolution and *PlaneReLU* 2.1 activation function, followed by a single complex convolution operation. $\mathcal{L}(\gamma_j, \theta_j)$, $\mathbb{C} \to \mathbb{R}$ is real-valued as we compute the $L2$ norm between the complex output and target. After learning G_j and F_j for each channel j, the interpolation for the vector of missing values s in the channel j is performed as

$$s(k_x, k_y, j) = G_j(\mathcal{N}(k_x, k_y)) + F_j(\mathcal{N}(k_x, k_y)) \tag{2}$$

where $\mathcal{N}(k_x, k_y)$ denotes the neighborhood for the corresponding missing point (k_x, k_y) across all channels.

2.1. Proposed Complex-valued Activation Function : the *PlaneReLU*

We propose a version of ReLU defined over the complex plane called the *PlaneReLU*. For an input $z = x + iy \in \mathcal{C}$ i.e., $x, y \in \mathcal{R}$, the *PlaneReLU* is defined as follows

$$PlaneReLU(x + iy) = \begin{cases} x + iy, & \text{if } ax + by + c \geq 0 \\ \frac{a+b+c}{\alpha}(x + iy), & \text{otherwise} \end{cases} \tag{3}$$

where $a, b, c \in \mathcal{R}$ are learnable parameters and α is a hyperparameter that we set to 3. The *PlaneReLU* activation function divides the complex plane into two halves about the line $ax + by + c = 0$. It fires the input as is in one half of the plane and fires a scaled version of the input in the other half. The parameters a, b and c are learnt to define a suitable line according to the training dataset. The *PlaneReLU* considers both magnitude and phase information while firing without any bias towards either, unlike other ReLU-inspired complex-valued activation functions like the *zReLU* and *modReLU* (Trabelsi et al., 2018) which take the firing decision with bias towards either the phase or magnitude information.

Table 1: Average PSNR, NRMSE and SSIM metrics for *k-space* data from fastmri multicoil brain and knee datasets at an acceleration factor of 5 with cartesian undersampling

fastmri multicoil Brain dataset			
Metric	PSNR	NRMSE	SSIM
rRAKI	31.51 ± 1.3	0.20 ± 0.041	0.84 ± 0.036
Complex rRAKI	31.83 ± 0.79	0.23 ± 0.08	**0.87 ± 0.027**
fastmri multicoil Knee dataset			
rRAKI	28.7 ± 0.73	0.45 ± 0.09	0.60 ± 0.07
Complex rRAKI	29 ± 0.49	**0.35 ± 0.047**	**0.67 ± 0.05**

3. Results, Discussion and Conclusions

The performance of rRAKI and Complex rRAKI architectures on the fastmri multicoil brain dataset and fastmri multicoil knee dataset (Zbontar et al., 2018) are presented in Table 1 and Figure 1. By achieving comparable performance with the SOTA methodology w.r.t. SSIM, PSNR and NRMSE metrics, with 50% fewer parameters (the complex convolution layer has 50% fewer parameters than the corresponding real layer (Jain et al., 2022)). Complex rRAKI along with its new generalized ReLU activation function on the complex plane, the *PlaneReLU*, shows promising potential for exploring complex-valued optimization in the *k-space* as well as other such complex-valued domains. The improved performance of the Complex rRAKI w.r.t. the SSIM and NRMSE metrics is attributed to the structure of its network which respects the complex-valued algebraic structure of the input, thus constraining the degrees of freedom in the neural network and assisting improved learning. **Source code:** *https://github.com/jain-p9/Complex-rRAKI*

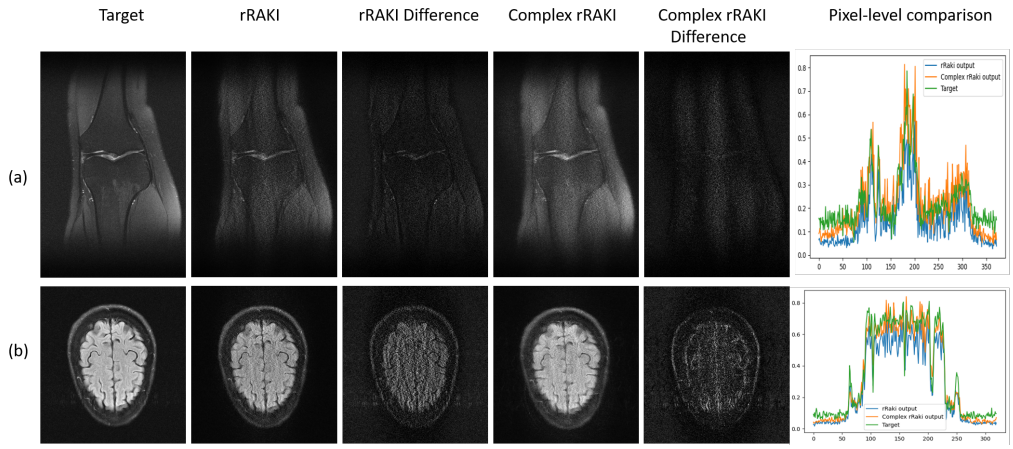

Figure 1: Sample reconstructions and difference images of baseline rRAKI and Complex rRAKI using (a) fastMRI Knee dataset and (b) fastMRI Brain dataset. The pixel-level comparison of the 300th line in the output scan is also shown.

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