Anticipating Performativity by Predicting from Predictions

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Abstract

Predictions about people, such as their expected educational achievement or their credit risk, can be performative and shape the outcome that they are designed to predict. Understanding the causal effect of predictions on the eventual outcomes is crucial for foreseeing the implications of future predictive models and selecting which models to deploy. However, this causal estimation task poses unique challenges: model predictions are usually deterministic functions of input features and highly correlated with outcomes, which can make the causal effects of predictions on outcomes impossible to disentangle from the direct effect of the covariates. We study this problem through the lens of causal identifiability. Despite the hardness of this problem in full generality, we highlight three natural scenarios where the causal effect of predictions can be identified from observational data: randomization in predictions, overparameterization of the predictive model deployed during data collection, and discrete prediction outputs. Empirically we show that given our identifiability conditions hold, standard variants of supervised learning that predict from predictions by treating the prediction as an input feature can find transferable functional relationships that allow for conclusions about newly deployed predictive models. These positive results fundamentally rely on *model predictions being recorded during data collection*, bringing forward the importance of rethinking standard data collection practices to enable progress towards a better understanding of social outcomes and performative feedback loops.

1 Introduction

Predictions can impact sentiments, alter expectations, inform actions, and thus change the course of events. Through their influence on people, predictions have the potential to change the regularities in the population they seek to describe and understand. This insight underlies the theories of performativity [38] and reflexivity [62] that play an important role in modern economics and finance. Recently, Perdomo et al. [51] pointed out that the social theory of performativity has important implications for machine learning theory and practice. Prevailing approaches to supervised learning assume that features X and labels Y are sampled jointly from a fixed underlying data distribution that is unaffected by attempts to predict Y from X. Performativity questions this assumption and suggests that the deployment of a predictive model can disrupt the relationship between X and Y. Hence, changes to the predictive model can induce shifts in the data distribution. For example, consider a lender with a predictive model for risk of default – performativity could arise if individuals who are predicted as likely to default are given higher interest loans, which make default even more likely [41], akin to a self-fulfilling prophecy. In turn, a different predictive model that predicts smaller risk and suggests offering more low-interest loans could cause some individuals who previously looked risky

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to be able to pay the loans back, which would appear as a shift in the relationship between features X and loan repayment outcomes Y. This performative nature of predictions poses a challenge to using historical data to predict the outcomes that will arise under the deployment of future models.

1.1 Our work

In this work, we aim to understand under what conditions observational data is sufficient to identify the performative effects of predictions. Only when causal identifiability is established can we rely on data-driven strategies to anticipate performativity and reason about the downstream consequences of deploying new models. Towards this goal, we focus on a subclass of performative prediction problems in this paper where performative effects of predictions solely surface as a shift in the outcome variable, and the distribution over covariates X is unaffected by the prediction $\hat{Y}$. Our goal is to identify the expected counterfactual outcome

$$\mathcal{M}_Y(x, \hat{y}) \triangleq \mathbb{E}[Y|X = x, \text{do}(\hat{Y} = \hat{y})].$$

Understanding the causal mechanism $\mathcal{M}_Y$ is crucial for model evaluation, as well as model optimization. In particular, it allows for offline evaluation of the potential outcome Y of an individual X subject to a predictive model f_{new} with the prediction $\hat{Y} = f_{\text{new}}(X)$ before actually deploying it.

The need for observing predictions. We start by illustrating the hardness of performativity-agnostic learning by relating performative prediction to a concept shift problem. Using the specifics of the performative shift, we establish a lower bound on the extrapolation error of predicting Y from X under the deployment of a new model f_{new} that is different from the model f_{train} deployed during data collection. In particular, the extrapolation error grows with the distance between the prediction functions of the two models and the strength of performativity. This lower bound on the extrapolation error demonstrates the necessity to take performativity into account for reliably predicting Y.

Predicting from predictions. We then explore the feasibility of learning performative effects when the training data recorded the predictions and training data samples $(X, Y, \hat{Y})$ are available. As an identification strategy for learning $\mathcal{M}_Y$, we focus on building a meta machine learning model that predicts Y for an individual with features X, subjected to a prediction $\hat{Y}$. We term this data-driven strategy *predicting from predictions*; it treats the predictions as an input to the meta machine learning model. The meta model seeks to answer "what would the outcome be if we were to deploy a different prediction model?" Crucially, this "what if" question is causal in nature; it aims to understand the potential outcome under an intervention which is different from merely estimating the outcome variable in previously seen data. Whether such a transferable model is learnable depends on whether the training data provides causal *identifiability* [49] Only after causal identifiability is established can we rely on observational data to select and design optimal prediction models under performativity.

Establishing identifiability. For our main technical results, we first show that, in general, observing $\hat{Y}$ is *not* sufficient for identifying the causal effects of predictions. In particular, if the training data was collected under the deployment of a deterministic prediction function, the mechanism $\mathcal{M}_Y$ can not be uniquely identified. The reason is a lack of coverage in the training data as X and $\hat{Y}$ are deterministically bound. Next, we establish several conditions under which observing $\hat{Y}$ is sufficient for identifying $\mathcal{M}_Y$. The first condition exploits the presence of randomness in the prediction. This randomness could be purposely built into the prediction for individual fairness, differential privacy, or other considerations. The second condition exploits the property that predictive models are often over-parameterized, which leads to incongruence in functional complexity between different causal paths, enabling the effects of predictions to be separated from other variables' effects. The third condition takes advantage of discreteness in predictions such that performative effects can be disentangled from the continuous relationship between covariates and outcomes. Together, these results reveal that particularities of the performative prediction problem can enable us to recover the causal effect of predictions from observational data. In particular, we show that, under these conditions, standard supervised learning techniques can be used to find these transferable functional relationships by treating predictions as model inputs. Empirically, we demonstrate that supervised learning succeeds in finding $\mathcal{M}_Y$ even in finite samples.

We conclude with a discussion of limitations and extensions of our work, pointing out potential violations of the modeling assumptions underlying our causal analysis and proposing directions for future work.

1.2 Broader context and related work

The work by Perdomo et al. [51], initiated the discourse of performativity in the context of supervised learning by pointing out that the deployment of a predictive model can impact the data distribution we train our models on. Existing scholarship on performative prediction [c.f., 51, 42, 12, 44, 24, 26, 68, 45, 52, 31] has predominantly focused on achieving a particular solution concept with a prediction function that maps X to Y in the presence of unknown performative effects. We are interested in understanding the underlying causal mechanism of the performative distribution shift. Our work is motivated by the seemingly natural approach of lifting the supervised-learning problem and incorporating the prediction as an input feature when building a meta machine learning model for explaining Y. By establishing a connection to causal identifiability, our goal is to understand when such a data-driven strategy can help anticipate the down stream effects of predictions

This work focuses on the setting where predictions lead to changes in the relationship between covariates X and label Y, while the marginal distribution $P(X)$ over covariates is assumed to be fixed. This setting where performativity only surfaces in the label describes an interesting subclass of problems falling under the umbrella of performative (aka. model-induced or decision-dependent) distribution shifts [51, 37, 12]. Our assumptions are complementary to the strategic classification framework [7, 20] that focuses on a setting where performative effects concern $P(X)$, while $P(Y|X)$ is assumed to remain stable. Consequently, causal questions in strategic classification [e.g., 22, 3, 59] are concerned with identifying stable causal relationships between X and Y. Since we assume $P(Y|X)$ can change (i.e. the true underlying 'concept' determining outcomes can change), conceptually different questions emerge in our work. Similar in spirit to strategic classification, the work on algorithmic recourse and counterfactual explanations [32, 28, 65] focuses on the causal link between features and predictions, whereas we focus on the down-stream effects of predictions.

There are interesting parallels between our work and related work on the offline evaluation of online policies [e.g., 35, 63, 36, 58]. In particular, [63] explicitly emphasize the importance of logging propensities of the deployed policy during data collection to be able to mitigate selection bias. In our work the deployed model can induce a concept shift. Thus, we find that additional information about the predictions of the deployed model needs to be recorded to be able to foresee the impact of a new predictive model on the conditional distribution $P(Y|X)$, beyond enabling propensity weighting [55]. A notable work by [66] investigates how predictions at one time step impact predictions in future time steps. Complementary to these existing works we show that randomness in the predictive model is not the only way causal effects of predictions can be identified.

For our theoretical results, we build on classical tools from causal inference [48, 57, 64]. In particular, we distill unique properties of the performative prediction problem to design assumptions for the identifiability of the causal effect of predictions.

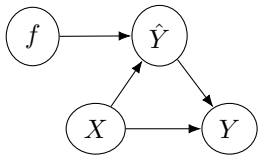
2 The causal force of prediction

Predictions can be performative and impact the population of individuals they aim to predict. Formulized it in the language of causal inference [48]: the deployment of a predictive model represents an intervention on a causal diagram that describes the underlying data generation process of the population. We will expand on this causal perspective to study an instance of ths performative prediction problem described below.

2.1 Prediction as a partial mediator

Consider a machine learning application relying on a predictive model f that maps features X to a predicted label $\hat{Y}$. We assume the predictive model f is performative in that the prediction $\hat{Y} = f(X)$ has a direct causal effect on the outcome variable Y of the individual it concerns. Thereby the prediction impacts how the outcome variable Y is generated from the features X. The causal diagram illustrating this setting is visualized in Figure 1.

The features $X \in \mathcal{X} \subseteq \mathbb{R}^d$ are drawn i.i.d. from a fixed underlying continuous distribution over covariates $\mathcal{D}_X$ with support $\mathcal{X}$. The outcome $Y \in \mathcal{Y} \subseteq \mathbb{R}$ is a function of X, partially mediated by the prediction $\hat{Y} \in \mathcal{Y}$. The prediction $\hat{Y}$ is determined by the deployed predictive model $f : \mathcal{X} \to \mathcal{Y}$. For a given prediction function f, every individual is assumed to be sampled i.i.d. from the data generation process described by the causal graph in Figure 1. We assume the exogenous noise ξ_Y is zero mean, and ξ_f allows the prediction function to be randomized.



$$X = \xi_X \qquad \xi_X \sim \mathcal{D}_X \tag{1}$$
$$\hat{Y} = f(X, \xi_f) \qquad \xi_f \sim \mathcal{D}_f \tag{2}$$
$$Y = g(X, \hat{Y}) + \xi_Y \qquad \xi_Y \sim \mathcal{D}_Y \tag{3}$$

Figure 1: Performative effects mediated by predictions for a given f

Note that our model is not meant to describe performativity in its full generality (which includes other ways f may affect $P(X, Y)$). Rather, it describes an important and practically relevant class of performative feedback problems that are characterized by two properties: 1) performativity surfaces only in the label Y, and 2) performative effects are mediated by the prediction, such that $Y \perp\!\!\!\perp f \mid \hat{Y}$, rather than dependent on the specifics of the decision rule.

Application examples. Causal effects of predictions on outcomes have been documented in multiple contexts: A bank's prediction about the client (e.g., his or her creditworthiness in applying for a loan) determines the interest rate assigned to them, which in turn changes a client's financial situation [41]. Mathematical models that predict stock prices inform the actions of traders and thus heavily shape financial markets and economic realities [38]. Zillow's housing price predictions directly impact sales prices [39]. Predictions about the severity of an illness play an important role in treatment decisions and hence the very chance of survival of the patient [34]. Another prominent example from psychology is the Pygmalion effect [56]. It refers to the phenomenon that high expectations lead to improved performance, which is widely documented in the context of education [6], sports [61], and organizations [16]. Examples of such performativity abound, and we hope to have convinced the reader that the performative effects in the label are important for algorithmic prediction.

2.2 Implications for performativity-agnostic learning

Begin with considering the classical supervised learning task where $\hat{Y}$ is unobserved. The goal is to learn a model $h : \mathcal{X} \to \mathcal{Y}$ for predicting the label Y from the features X. To understand the inherent challenge of classical prediction under performativity, we investigate the relationship between X and Y more closely. Specifically, the data generation process (Figure 1) implies that

$$P(Y|X) = \int P(Y|\hat{Y}, X) P(\hat{Y}|X) \mathrm{d}\hat{Y}. \tag{4}$$

This expression makes explicit how the relationship between X and Y that we aim to learn depends on the predictive model governing $P(\hat{Y}|X)$. As a consequence, when the deployed predictive model at test time differs from the model at training time, performative effects surface as concept shift [17]. Such distribution shift problems are known to be intractable without structural knowledge about the shift, implying that we can not expect h to generalize to distributions induced by future model deployments. Let us inspect the resulting extrapolation gap in more detail and put existing positive results on performative prediction into perspective.

Extrapolation loss. We illustrate the effect of performativity on predictive performance using a simple instantiation of the structural causal model from Figure 1. Therefore, assume a linear performative effect of strength $\alpha > 0$ and a base function $g_1 : \mathcal{X} \to \mathcal{Y}$

$$g(X, \hat{Y}) := g_1(X) + \alpha \hat{Y}. \tag{5}$$

Now, assume we collect training data under the deployment of a predictive model f_θ and validate our model under the deployment of f_ϕ. We adopt the notion of a distribution map from Perdomo et al. [51] and write $\mathcal{D}_{XY}(f)$ for the joint distribution over (X, Y) surfacing from the deployment of a model f. We assess the quality of our predictive model $h : \mathcal{X} \to \mathcal{Y}$ over a distribution $\mathcal{D}_{XY}(f)$ induced by f via the loss function $\ell : \mathcal{Y} \times \mathcal{Y} \to \mathbb{R}$ and write $\mathrm{R}_f(h) := \mathrm{E}_{x,y \sim \mathcal{D}_{XY}(f)} \ell(h(x), y)$ for the risk of h on the distribution induced by f. We use h_f^* for the risk minimizer $h_f^* := \mathrm{argmin}_{h \in \mathcal{H}} \mathrm{R}_f(h)$, and $\mathcal{H}$ for the hypothesis class we optimize over. Proposition 1 bounds the extrapolation loss and can be viewed as a concrete instantiation of the more general extrapolation bounds for performative prediction discussed in [37] within the feedback model from Figure 1.

Proposition 1 (Hardness of performativity-agnostic prediction)**.** *Consider the data generation process in Figure 1 with g given in* (5) *and f_θ, f_ϕ being deterministic functions. Take a loss function*

$\ell : \mathcal{Y} \times \mathcal{Y} \to \mathbb{R}$ that is γ-smooth and μ-strongly convex in its second argument. Let $h^*_{f_\theta}$ be the risk minimizer over the training distribution and assume the problem is realizable, i.e., $h^*_{f_\theta} \in \mathcal{H}$. Then, we can bound the extrapolation loss of $h^*_{f_\theta}$ on the distribution induced by f_ϕ as

$$\frac{\gamma}{2}\alpha^2 d^2_{\mathcal{D}_X}(f_\theta, f_\phi) \geq \Delta R_{f_\theta \to f_\phi}(h^*_{f_\theta}) \geq \frac{\mu}{2}\alpha^2 d^2_{\mathcal{D}_X}(f_\theta, f_\phi) \tag{6}$$

where $d^2_{\mathcal{D}_X}(f_\theta, f_\phi) := \mathrm{E}_{x\sim\mathcal{D}_X}(f_\theta(x) - f_\phi(x))^2$ and $\Delta R_{f_\theta \to f_\phi}(h) := \mathrm{R}_{f_\phi}(h) - \mathrm{R}_{f_\theta}(h)$.

The extrapolation loss $\Delta R_{f_\theta \to f_\phi}(h^*_{f_\theta})$ is zero if and only if either the strength of performativity tends to zero ($\alpha \to 0$), or the predictions of the two predictors f_θ and f_ϕ are identical over the support of $\mathcal{D}_X$. If this is not the case, an extrapolation gap is inevitable. This elucidates the fundamental hardness of performative prediction from feature, label pairs (X, Y) when performative effects disrupt the causal relationship between X and Y.

The special case where $\alpha = 0$ aligns with the assumption of classical supervised learning, in which there is no performativity. This may hold in practice if the predictive model is solely used for descriptive purposes, or if the agent making the prediction does not enjoy any economic power [21]. The second special case where the extrapolation error is small is when $d^2_{\mathcal{D}_X}(f_\theta, f_\phi) \to 0$. In which case $\mathcal{D}_{XY}(f_\theta)$ and $\mathcal{D}_{XY}(f_\phi)$ are equal in distribution and hence exhibit the same risk minimizer. Such a scenario can happen, for example, if the model f_ϕ is obtained by retraining f_θ on observational data and a fixpoint is reached ($f_\theta = h^*_{f_\theta}$). The convergence of policy optimization strategies to such fixpoints (perfromative stablity) has been studied in prior work [e.g., 51, 42, 12] and enabled optimality results even in the presence of performative concept shifts, relying on the target model f_ϕ not being chosen arbitrarily, but based on a pre-specified update strategy.

3 Identifying the causal effect of prediction

Having illustrated the hardness of performativity-agnostic learning, we explore under what conditions incorporating the presence of performative predictions into the learning task enables us to anticipate the perfromative effects of $\hat{Y}$ on Y. Towards this goal, we assume that the mediator $\hat{Y}$ in Figure 1 is observed—the prediction takes on the role of the treatment in our causal analysis and we can not possibly hope to estimate the treatment effect of a treatment that is unobserved.

3.1 Problem setup

Assume we are given access to data points $(x, \hat{y}, y)$ generated i.i.d. from the structural causal model in Figure 1 under the deployment of a prediction function f_θ. From this observational data, we wish to estimate the expected potential outcome of an individual under the deployment of an unseen (but known) predictive model f_ϕ. We note that given our causal graph, the implication of intervening on the function f can equivalently be explained by an intervention on the prediction $\hat{Y}$. Thus, we are interested in identifying the causal mechanism:

$$\mathcal{M}_Y(x, \hat{y}) := \mathbb{E}[Y|X = x, \mathrm{do}(\hat{Y} = \hat{y})]. \tag{7}$$

Unlike $P(Y|X)$, the mecahnism $\mathcal{M}_Y$ is invariant to the changes in the predictive model governing $P(\hat{Y}|X)$. Thus, being able to identify $\mathcal{M}_Y$ will allow us to make inferences about the potential outcome surfacing from planned model updates beyond explaining patterns in historical data. We can evaluate $\mathcal{M}_Y$ to infer y for any x at $\hat{y} = f_\phi(x)$ for f_ϕ being the model of interest.

For simplicity of notation, we will write $\mathcal{D}(f_\theta)$ to denote the joint distribution over $(X, \hat{Y}, Y)$ of the observed data collected under the deployment of the predictive model f_θ. We say $\mathcal{M}_Y$ can be identified, if it can uniquely be expressed as a function of observed data. More formally:

Definition 1 (identifiability)**.** *Given a predictive model f, the causal graph in Figure 1, and a set of assumptions A. We say $\mathcal{M}_Y$ is identifiable from $\mathcal{D}(f)$, if for any function h that complies with assumptions A and $h(x, \hat{y}) = \mathcal{M}_Y(x, \hat{y})$ for pairs $(x, \hat{y}) \in \mathrm{supp}(\mathcal{D}_{XY}(f))$ it must also hold that $h(x, \hat{y}) = \mathcal{M}_Y(x, \hat{y})$ for all pairs $(x, \hat{y}) \in \mathcal{X} \times \mathcal{Y}$.*

Without causal identifiability, there might be models $h' \neq \mathcal{M}_Y$ that explain the training distribution equally well but do not transfer to the distribution induced by the deployment of a new model. Causal identifiability is crucial for enabling extrapolation. It quantifies the limits of what we can infer given access to the training data distribution, ignoring finite sample considerations.

Identification with supervised learning. Identifiability of $\mathcal{M}_Y$ from samples of $\mathcal{D}(f_\theta)$ implies that the historical data collected under the deployment of f_θ contains sufficient information to recover the invariant relationship (7). As a concrete identification strategy, consider the following standard variant of supervised learning that takes in samples $(x, \hat{y}, y)$ and builds a meta-model that predicts Y from $X, \hat{Y}$ by solving the following risk minimization problem

$$h_{\text{SL}} := \operatorname*{argmin}_{h \in \mathcal{H}} \ \mathrm{E}_{(x,\hat{y},y)\sim\mathcal{D}(f_\theta)}\big[\,(h(x,\hat{y}) - y)^2\,\big]. \tag{8}$$

where $\mathcal{H}$ denotes the hypothesis class. We consider the squared loss for risk minimization because it pairs well with the exogeneous noise ξ_Y in (3) being additive and zero mean. The strategy (8) is an instance of what we term *predicting from predictions*. Lemma 2 provides a sufficient condition for the supervised learning solution h_{SL} to recover the invariant causal quantity $\mathcal{M}_Y$.

Lemma 2 (Identification strategy)**.** *Consider the data generation process in Figure 1 and a set of assumptions A. Given a hypothesis class $\mathcal{H}$ such that every $h \in \mathcal{H}$ complies with A and the problem is realizable, i.e., $\mathcal{M}_Y \in \mathcal{H}$. Then, if $\mathcal{M}_Y$ is causally identifiable from $\mathcal{D}(f_\theta)$ given A, the risk minimizer h_{SL} in (8) will coincide with $\mathcal{M}_Y$.*

3.2 Challenges for identifiability

The main challenge for identification of $\mathcal{M}_Y$ from data is that in general, the prediction rule f_θ which produces $\hat{Y}$ is a deterministic function of the covariates X. This means that, for any realization of X, we only get access to one $\hat{Y} = f_\theta(X)$ in the training distribution, which makes it challenging to disentangle the direct and the indirect effects of X on Y. To illustrate this challenge, consider the function $h(x, \hat{y}) := \mathcal{M}_Y(x, f_\theta(x))$ that ignores the input parameter $\hat{y}$ and only relies on x for explaining the outcome. This function explains y equally well and can not be differentiated from $\mathcal{M}_Y$ based on data collected under the deployment of a deterministic prediction rule f_θ. The problem is akin to fitting a linear regression model to two perfectly correlated covariates. More broadly, this ambiguity is due to what is known as a *lack of overlap* (or lack of positivity) in the literature of causal inference [47, 23]. In the covariate shift literature, the lack of overlap surfaces when the covariate distribution violates the common support assumption and the propensity scores are not well-defined (see e.g., Pan and Yang [46]). This problem renders causal identification and thus data-driven learning of performative effects from deterministic predictions fundamentally challenging.

Proposition 3 (Nonidentifiability from deterministic predictions)**.** *Consider the structural causal model in Figure 1. Assume Y non-trivially depends on $\hat{Y}$, and the set $\mathcal{Y}$ is not a singleton. Then, given a deterministic prediction function f, the mechanism $\mathcal{M}_Y$ is not identifiable from $\mathcal{D}(f)$.*

The identifiability issue persists as long as the two variables $X, \hat{Y}$ are deterministically bound and there is no incongruence or hidden structure that can be exploited to disentangle the direct effect of X on Y from the indirect effect mediated by $\hat{Y}$. In the following, we focus on particularities of prediction problems and show how they allow us to identify $\mathcal{M}_Y$.

3.3 Identifiability from randomization

We start with the most natural setting that provides identifiability guarantees: randomness in the prediction function f_θ. Using standard arguments about overlap [47] we can identify $\mathcal{M}_Y(x, \hat{y})$ for any pair $x, \hat{y}$ with positive probability in the data distribution $\mathcal{D}(f_\theta)$ from which the training data is sampled. To relate this to our goal of identifying the outcome under the deployment of an unseen model f_ϕ we introduce the following definition:

Definition 2 (output overlap)**.** *Given two predictive models f_θ, f_ϕ, the model f_ϕ is said to satisfy output overlap with f_θ, if for all $x \in \mathcal{X}$ and any subset $\mathcal{Y}' \subseteq \mathcal{Y}$ with positive measure, it holds that*

$$\frac{\mathrm{P}[f_\phi(x) \in \mathcal{Y}']}{\mathrm{P}[f_\theta(x) \in \mathcal{Y}']} > 0. \tag{9}$$

In particular, output overlap requires the support of the new model's predictions $f_\phi(x)$ to be contained in the support of $f_\theta(x)$ for every potential $x \in \mathcal{X}$. The following proposition takes advantage of the fact that the joint distribution over (X, Y) is fully determined by the deployed model's predictions to relate output overlap to identification:

Proposition 4. *Given the causal graph in Figure 1, the mechanism $\mathcal{M}_Y(x, \hat{y})$ is identifiable from $\mathcal{D}(f_\theta)$ for any pair $x, \hat{y}$ with $\hat{y} = f_\phi(x)$, as long as f_ϕ is a prediction function that satisfies output overlap with f_θ.*

Proposition 4 allows us to pinpoint the models f_ϕ to which we can extrapolate to from data collected under f_θ. Furthermore, it makes explicit that for collecting data to learn about performative effects, it is ideal to deploy a predictor f_θ that is randomized so that the prediction output has full support over $\mathcal{Y}$ for any x. Such a model would generate a dataset that guarantees global identification of $\mathcal{M}_Y$ over $\mathcal{X} \times \mathcal{Y}$ and thus robust conclusions about any future deployable model f_ϕ. One interesting and relevant setting that satisfies this property is the differentially private release of predictions through an additive Laplace (or Gaussian) noise mechanism applied to the output of the prediction function [13].[1]

While standard in the literature, a caveat of identification from randomization is that there are several reasons a decision-maker may choose not to deploy a randomized prediction function in performative environments, including negative externalities and concerns about user welfare [29], but also business interests to preserve consumer value of the prediction-based service offered. In the context of our credit scoring example, random predictions would imply that interest rates are randomly assigned to applicants in order to learn how the rates impact their probability of paying back. We can not presently observe this scenario, given regulatory requirements for lending institutions.

3.4 Identifiability through overparameterization

The following two sections consider situations where we can achieve identification, without randomization, from data collected under a deterministic f_θ. Our first result exploits incongruences in functional complexity arising from machine learning models that are overparameterized [e.g. 30]. By overparameterization, we refer to the fact that the representational complexity of the model is larger than the underlying concept it needs to describe.

Assumption 1 (overparameterization)**.** *We say a function f is overparameterized with respect to $\mathcal{G}$ over $\mathcal{X}$ if there is no function $g' \in \mathcal{G}$ and $c \in \mathbb{R}$ such that $f(x) = c \cdot g'(x)$ for all $x \in \mathcal{X}$.*

A challenge for identification is that for deterministic f_θ the prediction can be reconstructed from X without relying on $\hat{Y}$, and thus $h(x, \hat{y}) = \mathcal{M}_Y(x, f_\theta(x))$ can not be differentiated from $\mathcal{M}_Y$ based on observational data. However, note that this ambiguity relies on there being an admissable h such that $h(\cdot, \hat{y})$ for a fixed $\hat{y}$ can represent f_θ. If f_θ is overparameterized with respect to the hypothesis class $\mathcal{H}$, this ambiguity is resolved. Let us make this intuition concrete with an example:

Example 3.1. *Assume the structural equation for y in Figure 1 is $g(x, \hat{y}) = \alpha x + \beta \hat{y}$ for some unknown α, β. Consider prediction functions f_θ of the following form $f_\theta(x) = \gamma x^2 + \xi x$ for some $\gamma, \xi \geq 0$. Consider $\mathcal{H}$ be the class of linear functions. Then, any admissable estimate $h \in \mathcal{H}$ takes the form $h(x, \hat{y}) = \alpha' x + \beta' \hat{y}$. For h to be consistent with observations we need $\alpha' + \beta' \xi = \alpha + \beta \xi$ and $\beta' \gamma = \beta \gamma$. This system of equations has a unique solution as long as $\gamma > 0$ which corresponds to the case where f_θ is overparameterized with respect to $\mathcal{H}$. In contrast, for $\gamma = 0$ the function $h(x, \hat{y}) = (\alpha + \beta \xi) x$ would explain the training data equally well.*

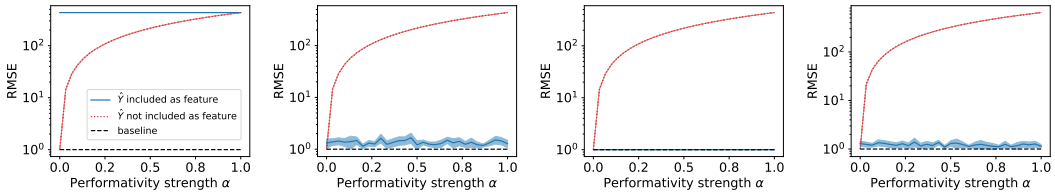
The following result generalizes this argument to separable functions.

Proposition 5. *Consider the structural causal model in Figure 1 where f_θ is a deterministic function. Assume that g can be decomposed as $g(X, \hat{Y}) = g_1(X) + \alpha \hat{Y}$ for some $\alpha > 0$ and $g_1 \in \mathcal{G}$, where the function class $\mathcal{G}$ is closed under addition (i.e. $g_1, g_2 \in \mathcal{G} \Rightarrow a_1 \cdot g_1 + a_2 \cdot g_2 \in \mathcal{G} \quad \forall a_1, a_2 \in \mathbb{R}$). Let $\mathcal{H}$ contain functions that are separable in X and $\hat{Y}$, linear in $\hat{Y}$, and $\forall h \in \mathcal{H}$ it holds that $h(\cdot, \hat{y}) \in \mathcal{G}$ for a fixed $\hat{y}$. Then, if f_θ is overparameterized with respect to $\mathcal{G}$ over the support of $\mathcal{D}_X$, $\mathcal{M}_Y$ is identifiable from $\mathcal{D}(f_\theta)$.*

3.5 Identifiability from classification

A second ubiquitous source of incongruence that we can exploit for identification is the *discrete* nature of predictions in the context of classification. The resulting discontinuity in the relationship between X and $\hat{Y}$ enables us to disentangle $\mathcal{M}_Y$ from the direct effect of X on Y. This identification strategy is akin to the popular regression discontinuity design [33] and relies on the assumption that all other variables in X are continuously related to Y around the discontinuities in $\hat{Y}$.

[1] In Appendix B we discuss two additional natural sources of randomness (randomized decisions and noisy measurements of covariates) that can potentially help identification with appropriate side-information.



(a) Non-identifiable setting (b) randomized f_θ (c) overparameterized f_θ (d) discrete f_θ

Figure 2: Extrapolation error of supervised learning with and without access to $\hat{Y}$. (a) In the non-identifiable setting, adding $\hat{Y}$ as a feature harms generalization performance. (b)-(d) Randomization, overparameterization, and discrete predictions are each sufficient for avoiding this failure mode.

Proposition 6. *Consider the structural causal model in Figure 1 where f_θ is a deterministic function. Assume that the structural equation for Y is separable $g(X, \hat{Y}) = g_1(X) + g_2(\hat{Y}), \forall X, \hat{Y}$ for some differentiable functions g_1 and g_2. Further, suppose X is a continuous random variable and $\hat{Y}$ is a discrete random variable that takes on at least two distinct values with non-zero probability. Then, $\mathcal{M}_Y$ is identifiable from $\mathcal{D}(f_\theta)$.*

Similar to Proposition 5, the separability assumption together with incongruence provides a way to disentangle the direct effect from the indirect effect of X on Y. Separability is necessary in order to achieve global identification guarantees without randomness, the identification of entangled components without overlap is fundamentally hard. Thus, under violations of the separability assumptions, we can only expect the separable components of g to be correctly identified. Similarly, a regression discontinuity design only enables the identification of the causal effect locally around the discontinuity. Extrapolation away from the decision boundary to models f_ϕ that are substantially different from f_θ increasingly relies on separability to hold true.

4 Empirical evaluation

We investigate empirically how well the supervised learning solution h_{SL} in (8) is able to identify the causal mechanism $\mathcal{M}_Y$ from observational data in practical settings with finite data.

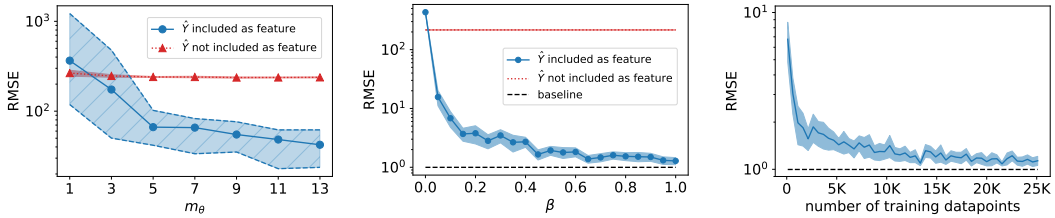
Methodology. We generated semi-synthetic data for our experiments, using a Census income prediction dataset from `folktables.org` [11]. Using this dataset as a starting point, we simulate a training dataset and test dataset with distribution shift as follows: First, we choose two different predictors f_θ and f_ϕ to predict a target variable of interest (e.g. income) from covariates X (e.g. age, occupation, education, etc.). If not specified otherwise, f_θ is fit to the original dataset to minimize squared error, while f_ϕ is trained on randomly shuffled labels. Next, we posit a function g for simulating the performative effects. Then, we generate a *training* dataset of $(X, \hat{Y}, Y)$ tuples from the causal model in Figure 1, using the covariates X from the original data, g, and f_θ to generate $\hat{Y}$ and Y. Similarly, we generate a *test* dataset of $(X, \hat{Y}, Y)$ tuples, using X, g, f_ϕ. We assess how well supervised methods learn transferable functional relationships by fitting a model h_{SL} to the training dataset and then evaluating the root mean squared error (RMSE) for regression and the accuracy for classification on the test dataset. In our figures, we visualize the standard error from 10 replicates with different random seeds and we compare it to an in-distribution baseline trained and evaluated on samples of $\mathcal{D}(f_\phi)$. If not specified otherwise we use $N = 200,000$ samples.

4.1 Necessity of identification guarantees for supervised learning

We start by illustrating why our identification guarantees are crucial for supervised learning under performativity. Therefore, we instantiate the structural equation g in Figure 1 as

$$g(X, \hat{Y}) = g_1(X) + \alpha \hat{Y} \tag{10}$$

with $g_1(X) = \beta^\top X$ and $\xi_Y \sim \mathcal{N}(0, 1)$. The coefficients β are determined by linear regression on the original dataset. The hyperparameter α quantifies the performativity strength that we vary in our experiments. The predictions $\hat{Y}$ are generated from a linear model f_θ that we modify to illustrate the resulting impact on identifiability. We optimize h_{SL} in (8) over $\mathcal{H}$ being the class of linear functions.



(a) Varying functional complexity of f_θ

(b) Varying degree of noise in f_θ.

(c) Varying N for randomized f_θ

Figure 3: Ablation study of extrapolation performance. (a) We vary m_θ. Adding $\hat{Y}$ as a feature helps as soon as f_θ is overparameterized with respect to g_1 ($m_\theta > 3$). (b) We vary the noise in the predictions of f_θ. A small amount of noise is sufficient for identifiability. (c) We vary number of datapoints for training h_{SL}.

We start by illustrating a failure mode of supervised learning in a non-identifiability setting (Proposition 3). Therefore, we let f_θ be a deterministic linear model fit to the base dataset ($f_\theta(X) \approx \beta^\top X$). This results in $\mathcal{M}_Y$ not being identifiable from $\mathcal{D}(f_\theta)$. In Figure 2(a) we can see that supervised learning indeed struggles to identify a transferable functional relationship from the training data. The meta model returns $h_{\text{SL}}(X, \hat{Y}) = (1+\alpha)\hat{Y}$, instead of identifying g, which leads to a high extrapolation error independent of the strength of performativity. While we only show the error for one f_ϕ in Figure 2(a), the error grows with the distance $d^2_{\mathcal{D}_x}(f_\theta, f_\phi)$. In contrast, when the feature $\hat{Y}$ is not included, the supervised learning strategy returns $h_{\text{SL}}(X) = (1+\alpha)\beta^\top X$. The extrapolation loss of this performativity-agnostic model scales with the strength of performativity (Proposition 1) and is thus strictly smaller than the error of the model that predicts from predictions.

Next, we move to the regime of our identification results (Proposition 4-6). Therefore, we modify the way the predictions in the training data are generated. In Figure 2(b) we use additive Gaussian noise to determine the predictions as $\hat{Y} = f_\theta(X) + \eta$ with $\eta \in \mathcal{N}(0, \sigma^2)$. In Figure 2(c) we augment the input to f_θ with second-degree polynomial features to achieve overparameterization. In Figure 2(d) we round the predictions of f_θ to obtain discrete values. In all three cases, including $\hat{Y}$ as a feature is beneficial and allows the model to match in-distribution accuracy baselines, closing the extrapolation gap that is inevitable for performativity-agnostic prediction.

4.2 Strength of incongruence and finite samples

We next conduct an ablation study and investigate how the degree of overparameterization and the noise level for randomized f_θ impacts the extrapolation performance of supervised learning. Therefore, we consider the setup in (10) with a general function g_1. We fix the level of performativity at $\alpha = 0.5$ for this experiment. We optimize h_{SL} in (8) over $\mathcal{H}$ (which we vary).

In Figure 3(a) we investigate the effect of overparameterization of f_θ on the extrapolation error of h_{SL}. We choose fully connected neural networks with a single hidden layer to represent the functions g_1, f_θ and h_{SL}. For g_1 and $\mathcal{H}$ we take a neural network with $m = 3$ units in the hidden layer. The model g_1 is fit it to the original dataset. We vary the number of units in the hidden layer of f_θ, denoted m_θ. As expected, the extrapolation error decreases with the complexity of f_θ. As soon as $m_\theta > m_\phi$ there is a significant benefit to including predictions as features. In this regime, $\mathcal{M}_Y$ becomes identifiable as Proposition 5 suggests. In turn, without access to $\hat{Y}$ the model suffers an inevitable extrapolation gap due to a concept shift that is independent of the properties of f_θ. In Figure 2(b) we investigate the effect of the *magnitude of additive noise* added to the predictions. Here $\mathcal{H}$ and g_1 are linear functions. We have $\hat{Y} = f_\theta(X) + \beta\eta$ with $\eta \in \mathcal{N}(0, 1)$ and we vary the noise level β. We see that even small amounts of noise are sufficient for identification and adding $\hat{Y}$ as a feature to our meta-machine lenaring model is effective as soon as the noise in f_θ is non-zero. In Figure 2(c) we fix the noise level at $\alpha = 0.5$ and vary the number of samples N. We find that only moderate dataset sizes are necessary for predicting from predictions to approximate $\mathcal{M}_Y$ in our identifiable settings.

5 Discussion

This paper focused on identifying the causal effect of predictions on outcomes from observational data. We point out several natural situations where this causal question can be answered, but we

also highlight situations where observational data is not sufficiently informative to reason about performative effects. By establishing a connection between causal identifiability and the feasibility of anticipating performative effects using data-driven techniques, this paper contributes to a better understanding of the suitability of supervised learning techniques for explaining social effects arising from the deployment of predictive models in economically and socially relevant applications.

We hope the positive results in this work serve as a *message for data-collection*: only if predictions are observed, they can be incorporated to anticipate the performative effects of future model deployments. Thus, access to this information is crucial for an analyst hoping to understand the effects of deployed predictive models, an engineer hoping to foresee consequences of model updates, or a researcher studying performative phenomena. To date, such data is scarcely available in benchmark datasets, hindering the progress towards a better understanding of performative effects, essential for the reliable deployment of algorithmic systems in the social world.

At the same time we have shown that the deterministic nature of prediction poses unique challenges for causal identifiability even if $\hat{Y}$ is observed. Thus, the success of observational designs (as shown in our empirical investigations) is closely tied to the corresponding identifiability conditions being satisfied. Our results must not be understood as a green-light to justify the use of supervised learning techniques to address performativity in full generality beyond the scope of our theoretical results.

Limitations and Extensions. The central assumption of our work is the causal model in Figure 1. While carving out a rich and interesting class of performative prediction problems that allows us to articulate the challenges of covariates and predictions being coupled, it can not account for all mechanisms of performativity. This in turn gives rise to interesting questions for follow-up studies.

A first neglected aspect is *performativity through social influence*. Our causal model, relies on the stable unit treatment value assumption (SUTVA) [23]. There is no possibility for the prediction of one individual to impact the outcome of his or her peers. Such an individualistic perspective is not unique to our paper but prevalent in existing causal analyses and model-based approaches to performative prediction and strategic classification [e.g., 20, 25, 43, 3, 18, 22]. Spillover effects [cf. 60, 64, 1, 40] are yet unexplored in the context of performative prediction. Nevertheless, they have important implications for how causal effects should be estimated and interpreted. In the context of our work they imply that an intervention on f can no longer be explaind solely by changing an individual's prediction. As a result, approaches for microfounding performative effect based on models learned from simple, unilateral interventions on an individual's prediction result in different causal estimates than supervised learning based methods for identification as studied in this work. A preliminary study included in Appendix C shows that data-driven techniques can pick up on interference patterns in the data and benefit from structural properties such as network homophily [19], whereas individualistic modeling misses out on the indirect component arising from neighbors influencing each other.

A second aspect is *performativity in non-causal prediction.* Our model posits that prediction is solely based on features X that are causal for the outcome Y. This is a desirable situation in many practical applications because causal predictions disincentivize gaming of strategic individuals manipulating their features [43, 3] and offer explanations for the outcome that persist across environments [54, 8]. Nevertheless, non-causal variables are often included as input features in practical machine learning prediction tasks. Establishing a better understanding for the implications of the resulting causal dependencies due to performativity could be an important direction for future work.

Finally, performative effect can also lead to *covariate shift* and impact the joint distribution $P(X, Y) = P(Y|X)P(X)$ over covariates and labels. We assumed that performative effects only surface in $P(Y|X)$. For our theoretical results, this implied that overlap in the X variable across environments is trivially satisfied, which enabled us to pinpoint the challenges of learning performative effects due to the coupling between X and $\hat{Y}$. For establishing identification in the presence of a causal arrow $f_\theta \to X$ additional steps are required to ensure identifiability.

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1. For all authors...
   (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [Yes]
   (b) Did you describe the limitations of your work? [Yes]
   (c) Did you discuss any potential negative societal impacts of your work? [Yes] see Appendix F.
   (d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [Yes]
2. If you are including theoretical results...
   (a) Did you state the full set of assumptions of all theoretical results? [Yes]
   (b) Did you include complete proofs of all theoretical results? [Yes]
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   (a) Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? [Yes]
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   (a) Did you include the full text of instructions given to participants and screenshots, if applicable? [N/A]
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   (c) Did you include the estimated hourly wage paid to participants and the total amount spent on participant compensation? [N/A]

A The extrapolation problem

This section provides additional motivation for the setup from Section 3.1. A general goal in the context of performativity is to evaluate a statistic $\kappa(X, Y)$ over the hypothetical distribution $\mathcal{D}(f_\phi)$ surfacing from the deployment of f_ϕ in order to decide whether we want to deploy f_ϕ or not. This can be summarized with the following extrapolation problem:

Extrapolation Problem $\mathsf{EP}(\kappa, f_\theta, f_\phi)$

Input:	Sample access to a training distribution $\mathcal{D}_{X,Y}(f_\theta)$ for a known predictor f_θ. Test-time predictor: f_ϕ, Statistic to evaluate: κ
Assumption:	Data is generated from the structural causal model described in Figure 1
Output:	$\mathbb{E}_{(x,y)\sim\mathcal{D}_{XY}(f_\phi)}[\kappa(x, y)]$

Being able to solve this extrapolation problem will enable both policy evaluation and policy optimization. The statistic κ can be a measure of accuracy, but could also be a measure of social welfare, individual improvement, and other goals for socially beneficial machine learning. In the main body of our work, we have focused on $\kappa(X, Y) = Y$ for simplicity of exposition. However, the criteria for identifiability would remain the same for any continuous function κ, only the target in the supervised learning objective (8) would need to be adapted accordingly.

We say $\mathsf{EP}(\kappa, f_\theta, f_\phi)$ can be solved if the output can be identified from the input data. Relating this learning problem to classical results from causal inference [2] reveals the following sufficient condition for solvability that we target in this work.

Lemma 7. *For any continuous statistic κ and predictive model f_ϕ, a sufficient condition for $\mathsf{EP}(\kappa, f_\theta, f_\phi)$ being solvable is identifiability of*

$$\mathcal{M}_Y^\kappa(x, \hat{y}) = \mathbb{E}[\kappa(X, Y)|X = x, \mathrm{do}(\hat{Y} = \hat{y})]$$

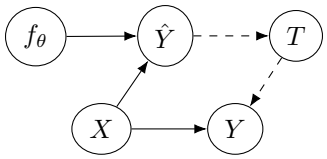
from $\mathcal{D}(f_\theta)$.

Proof. If we can identify $\mathcal{M}_Y^\kappa(x, \hat{y})$ we can evaluate the output for any $x, \hat{y}$ and hence $\mathsf{EP}(\kappa, f_\theta, f_\phi)$ as $\mathbb{E}_{x\sim\mathcal{D}_X}[\mathcal{M}_Y^\kappa(x, f_\phi(x))]$, where the distribution over covariates $\mathcal{D}_X$ is obtained by marginalizing the input distribution $\mathcal{D}_{XY}(f_\theta)$ over X. □

B Identification with side-information from alternate sources of randomness

In Section 3.3 we have investigated how randomness in the prediction function leads to identification. In this section we demonstrate that identification can also be achieved from other natural sources of randomness if additional side-information, observations, or more fine grained knowledge about the causal graph structure is available. However, incorporating such side-information requires going beyond standard ERM which is not the main focus of this work. Therefore we include this discussion in the appendix.

Randomized prediction-based decision. Assume the performative effect of predictions is mediated by a down-stream decision $T \in \{0, 1\}$ based on the prediction $\hat{Y}$ such that $Y \perp\!\!\!\perp \hat{Y} \mid T, X$. This scenario is illustrated in the following causal graph:



In this case, randomness in the discrete decision function T (instead of the continuous prediction $\hat{Y}$) is sufficient for identification of the causal graph. Randomness in prediction-based decisions can be a deliberate part of an algorithmic system for a number of reasons, including designing individually fair decision rules [14, 5, 4].

Noisy measurement of covariates A second natural source of randomness in performative prediction is noise in the measurement of the covariates X, representing the unobserved true underlying attributes U. This scenario is illustrated in the following graph

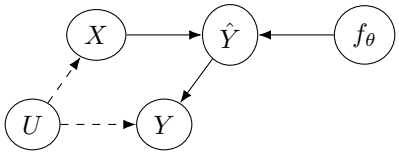
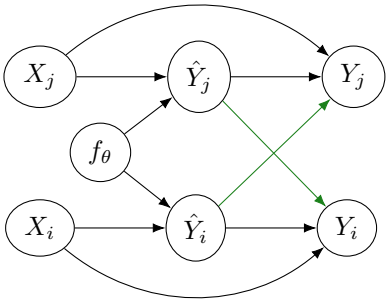


Figure 4: X is a noisy measurement of U.

For example, a student's college performance depends on their underlying scholastic ability, but predictions of performance (and perhaps admissions decisions) are made based on a noisy proxy like SAT score. In this case, side-information about the structure of the measurement noise enables identification [15] without precise knowledge of U. The intuition is that the attributes U that are causal for the outcome Y enter the prediction through the noisy measurements X, which adds independent variation to the indirect causal path.

C Social influence

We have mentioned that the stable unit treatment value assumption (SUTVA) [23] underlying our causal analysis could be violated in certain performative prediction settings due to social influence and spill-over effects. We want to use this section to discuss the following simple interference patterns that generalizes our causal graph from Figure 1:



$$
\begin{aligned}
X_i &= \xi_X && \xi_X \sim \mathcal{D}_X \\
\hat{Y}_i &= f_\theta(X_i) \\
Y_i &= g'(X_i, \hat{Y}_i, (\hat{Y}_j)_{j\in[n]}) + \xi_Y && \xi_Y \sim \mathcal{D}_Y
\end{aligned}
$$

In particular, it allows for predictions of individual j to impact the outcome of individual $i \neq j$, which implies that

$$\mathbb{E}[Y_i | X_i = x, \text{do}(f_\theta = f_{\theta^*}))] \;\neq\; \mathbb{E}[Y_i | X_i = x, \text{do}(\hat{Y}_i = f_{\theta^*}(x))] \tag{11}$$

Such effects could arise due to information flow about predictions in the population through social media platforms [9] or verbal sharing. This in turn leads to indirect exposure that can bring forward phenomena of social comparison such as envy or encouragement [10]. In the presence of such interference effects the causal effect of intervening on the predictive model is no longer the same as the causal effect of intervening on an individual's prediction, as stated in (11); on the left-hand side the predictions of all individuals are changed, whereas on the right-hand side only the prediction of individual i changed.

Exposure modeling. In the broadest generality the outcome Y_i in the above causal graph could depend on the predictions assigned to any other individual. This results in 2^{n-1} possible interference patterns. Thus, to formally reason about interference effects through predictions, it is common to assume ther is a sufficient statistic G_i that mediates the dependency among units, such that $Y_i \perp\!\!\!\perp \{\hat{Y}_j\}_{j\neq i} | G_i$ for all $i \in [n]$. The statistic G_i could encode the exposure of the entire population, the average prediction across the population, relevant predictions of the closest neighbors in a social

network, or the relative value of $\hat{Y}_i$ compared to peers in a group. G_i is typically constructed based on domain knowledge and is often assumed to be low-dimensional, limiting the complexity of interference among units and making the problem more tractable. What is unique to the prediction setting studied in this work, compared to randomized treatment assignments, is that predictions (and hence G_i) are typically correlated with the covariates and thus inherit structures present in the population, such as network homophily.

Homophily. *Homophily* refers to the tendency for individuals to be similar to their neighbors which surfaces in our setting as correlations between the features of neighboring units [19]. In the context of prediction, this further implies that a smooth prediction function f_θ will also exhibit correlations between predictions assigned to neighbors. We formalize it through the following property:

$$|\mathbb{E}_{j\in N(i)}\hat{Y}_j - \hat{Y}_i| < \delta \text{ for every } i \in [n] \text{ and some small } \delta \geq 0. \tag{12}$$

In the following, we want to highlight that in the presence of homophily the data-driven strategy (8) (that builds a model based on data collected under a population intervention) is able to implicitly pick up on the interference effects present in the data, whereas this information is not available from data collected under unilateral interventions. More, specifically, assume interference effects are mediated by the average prediction in the neighborhood, i.e., $G_i = \mathbb{E}_{j\in N(i)}\hat{Y}_j$, then the outcome of individual i can (at least partially) be explained by the prediction $\hat{Y}_i$ itself. This results in a machine learning model (8) that will implicitly pick up the interference effects from the training data in order to explain the total causal effect of f_θ on the outcome. This is helpful for prediction, despite a misspecified causal graph. We illustrate this advantageous property over microfoundation models with the following example:

Linear-in-means model. Consider the following model proposed by Manski [40]:

$$Y_i = g(X_i) + \alpha\hat{Y}_i + \beta G_i \quad \text{where} \quad G_i = \frac{1}{|N(i)|}\textstyle\sum_{j\in N(i)}\hat{Y}_j \tag{13}$$

for some $\alpha > \beta > 0$. This structural causal model describes a setting of positive interference where spillover effects are mediated by the average prediction in the neighborhood of an individual and represent a dampened version of the direct effect. We can show that fitting a model h to explain Y as a function of X and $\hat{Y}$ leads to smaller estimation error than learning h from unilateral interventions.

Proposition 8. *Given the structural causal model in* (13)*. Assume the homophily assumption* (12) *holds for $\delta = 0$. Then, under the same identifiability conditions established in Section 3 for the SUTVA case. The supervised leanrning solution h_{SL} will find a transferrable functional relationship even in the presence of interference.*

Without explicitly measuring G_i, fitting a model to explain Y as a function of X and $\hat{Y}$ will result in $h(x, \hat{y}) = g(x) + (\alpha + \beta)\hat{y}$, which identifies the causal quantity of interest, i.e., the effect of an intervention to the predictive model on the outcome Y_i of an individual. In contrast, an estimate based on unilateral interventions would result in $h(x, \hat{y}) = g'(x) + \alpha\hat{y}$ which systematically underestimates the overall strength of performative effects and thus leads to a biased estimator.

D Proofs

Assumption 2 (positivity)**.** *Consider the causal graph in Figure 1. Positivity of $\hat{Y}$ over $\mathcal{Y}$ is satisfied if $P[\hat{Y} \in \mathcal{S}|X = x] > 0$ for all $x \in \mathcal{X}$ and all sets $\mathcal{S} \subseteq \mathcal{Y}$ with positive measure, i.e., $P[\mathcal{S}] > 0$.*

Lemma 9. *If the training distribution satisfies positivity of $\hat{Y}$ over $\mathcal{Y}' \subseteq \mathcal{Y}$, then $\mathbb{E}[Y|X = x, \hat{Y} = \hat{y}]$ is identifiable from the training data for any $\hat{y} \in \mathcal{Y}'$.*

D.1 Proof of Proposition 1

For notational convenience we write $h_{\text{opt}}(f_\theta)$ for $h^*_{f_\theta}$. From realizability it follows that $\mathrm{R}_{f_\theta}(h_{\text{opt}}(f_\theta)) = 0$. Hence, the extrapolation loss is equal to

$$\mathrm{Err}_{f_\theta \to f_\phi}(h_{\text{opt}}(f_\theta)) = \mathrm{R}_{f_\phi}(h_{\text{opt}}(f_\theta)) - \mathrm{R}_{f_\theta}(h_{\text{opt}}(f_\theta)) = \mathrm{R}_{f_\phi}(h_{\text{opt}}(f_\theta))$$

and it remains to bound $\mathrm{R}_{f_\phi}(h_{\text{opt}}(f_\theta))$:

$$\begin{aligned}
\mathrm{R}_{f_\phi}(h_{\text{opt}}(f_\theta)) &= \mathrm{E}_{x,y\sim\mathcal{D}_{\mathrm{X,Y}}(f_\phi)}\mathcal{L}(h_{\text{opt}}(f_\theta)(x), y) && (14)\\
&= \mathrm{E}_{x\sim\mathcal{D}_X}\mathcal{L}(h_{\text{opt}}(f_\theta)(x), g(x, f_\phi(x))) && (15)\\
&= \mathrm{E}_{x\sim\mathcal{D}_X}\mathcal{L}(g(x, f_\theta(x)), g(x, f_\phi(x))) && (16)
\end{aligned}$$

Further assuming that the loss function $\mathcal{L}$ is μ-strongly convex and γ-smooth in the second argument. Then,

$$\begin{aligned}
\mathrm{R}_{f_\phi}(h_{\text{opt}}(f_\theta)) &\geq \frac{\mu}{2}\,\mathrm{E}_{x\sim\mathcal{D}_X}\left(g(x, f_\theta(x)) - g(x, f_\phi(x))\right)^2 && (17)\\
\mathrm{R}_{f_\phi}(h_{\text{opt}}(f_\theta)) &\leq \frac{\gamma}{2}\,\mathrm{E}_{x\sim\mathcal{D}_X}\left(g(x, f_\theta(x)) - g(x, f_\phi(x))\right)^2 && (18)
\end{aligned}$$

and the result follows.

D.2 Proof of Lemma 2

Given that the risk minimization problem (8) is realizabe and $\mathcal{M}_Y$ is uniquely identifiable over $\mathcal{H}$, the risk minimizer of the squared loss corresponds to $\mathbb{E}[Y|X = x, \hat{Y} = \hat{y}]$. Given the graph structure in Figure 1 there is no unobserved confounding and hence

$$\mathbb{E}[Y|X = x, \hat{Y} = \hat{y}] = \mathbb{E}[Y|X = x, \mathrm{do}(\hat{Y} = \hat{y})] \implies h_{\text{SL}}(x, \hat{y}) = \mathcal{M}_Y(x, \hat{y}).$$

D.3 Proof of Proposition 3

Our goal is to show that $\mathcal{M}_Y(X, \hat{Y})$ can not uniquely be identified from $\mathcal{D}(f_\theta)$ if f_θ is a deterministic function. The proof is by construction of a h that fits the training data equally well, but does not generalize to data induced by a new prediction function.

Since f_θ is deterministic it holds that $\hat{y} = f_\theta(x)$ for all pasirs $x, \hat{y}$ in the observed data distribution. Thus, the function h defined as follows

$$h(x, \hat{y}) = \mathcal{M}_Y(x, f_\theta(x))$$

is equally compatible with the observational data. That is

$$\mathbb{E}_{\hat{Y}=f_\theta(X)}\mathbb{E}[Y|X, \hat{Y}] = \mathcal{M}_Y(X, \hat{Y}) = h(X, \hat{Y}).$$

Hence, $\mathcal{M}_Y$ can not be distinguished from h based on observational data. It remains to show that $\mathcal{M}_Y$ and h do not coincide on new data.

We assume that Y non-trivially depends on $\hat{Y}$ and $\mathcal{Y}$ is not a singleton. This means, given some x, for every $\hat{y}$ there exists a $\hat{y}' \in \mathcal{Y}$ such that $g(x, \hat{y}) \neq g(x, \hat{y}')$. Define f_ϕ such that for $\hat{y} = f_\theta(x)$, we set $f_\phi(x) = \hat{y}'$. Then,

$$\mathbb{E}_{\hat{Y}=f_\phi(X)}\mathbb{E}[Y|X, \hat{Y}] = \mathcal{M}_Y(X, \hat{Y}) \neq h(X, \hat{Y})$$

which concludes the proof.

D.4 Proof of Proposition 4

Output overlap guarantees that $P[\hat{Y} = f_\phi(x)|X = x] > 0$ in the training distribution $\mathcal{D}(f_\theta)$ for any $x \in \mathcal{X}$. Identification and extrapolation to models f_ϕ that satisfy output overlap with f_θ follows from positivity (Lemma 9) and the causal graph (Figure 1) which implies that there is no unobserved confounding and $\mathbb{E}[Y|X = x, \hat{Y} = \hat{y}] = \mathbb{E}[Y|X = x, \mathrm{do}(\hat{Y} = \hat{y})]$.

D.5 Proof of Proposition 5

We first note that the overparameterization assumption implies that $g_1 \in \mathcal{G}$ and f_θ are linearly independent. (Proof by contradiction. If not, then there exists $\alpha_1 \neq \alpha_1', \alpha_2 \neq \alpha_2'$ and functions $g_1, g_1' \in \mathcal{G}$ such that $\alpha_1 g_1(x) + \alpha_2 f_\theta(x) = \alpha_1' g_1'(x) + \alpha_2' f_\theta(x)$ for all x; it implies that $\alpha_1 g_1(x) - \alpha_1' g_1'(x) = (\alpha_2 - \alpha_2') f_\theta(x)$ for all x. $\alpha_1 g_1(x) - \alpha_1' g_1'(x) \in \mathcal{G}$ since the class $\mathcal{G}$ is closed under addition. This leads to a contradiction with the fact that $f_\theta(\cdot)$ is overparametrized with respect to $\mathcal{G}$, which requires there exist no function $g \in \mathcal{G}$ such that $g(x) = c f_\theta(x)$ for some $c > 0$.)

Next, since any $h \in \mathcal{H}$ is separable in X and $\hat{Y}$, linear in $\hat{Y}$, and that $h(\cdot, \hat{y}) \in \mathcal{G}$ for any $\hat{y}$, we have that $h(x, \hat{y}) = g_1'(x) + \alpha' \hat{y}$ for some $g_1' \in \mathcal{G}$ and some constant $\alpha' \in \mathbb{R}$. Therefore, finding $\mathcal{M}_Y$ amounts to solve g_1', α' from the observational data relationship $g_1(X) + \alpha \hat{Y} = g_1'(X) + \alpha' \hat{Y}$ subject to the constraint that $\hat{Y} = f_\theta(X)$. Plugging in the constraints gives $g_1(X) + \alpha f_\theta(X) = g_1'(X) + \alpha' f_\theta(X)$. This equations gives a unique solution that $g_1' = g_1$ and $\alpha' = \alpha$ if we have observation from all values of X, hence the identifiability of $\mathcal{M}_Y$.

D.6 Proof of Proposition 6

The proof is inspired by Wang and Blei [67] and Puli et al. [53]. Because $\hat{Y}$ is discrete and $\mathbb{E}[Y|\mathrm{do}(\hat{Y} = \hat{y}), X = x]$ is separable, we have that

$$\frac{\partial}{\partial x}\mathbb{E}[Y|\mathrm{do}(\hat{Y} = \hat{y}), X = x] = \frac{\partial}{\partial x} g_1(x) = \frac{\partial}{\partial x}\mathbb{E}[Y|\hat{Y} = \hat{y}', X = x]$$

for any pair of $\hat{Y}, X$ that are observable, i.e. $\hat{y}' = f_\theta(x)$. This implication is due to $g_2(\hat{y})$ being a piecewise constant function (its partial derivative is zero with respect to x). Therefore, $\mathcal{M}_Y$ is identifiable as

$$\mathcal{M}_Y(x, \hat{y}) = \mathbb{E}[Y|\mathrm{do}(\hat{Y} = \hat{y}), X = x] = \mathbb{E}[Y|\hat{Y} = \hat{y}, X = x'] + \int_{x'}^{x} \frac{\partial}{\partial x}\mathbb{E}[Y|\hat{Y} = \hat{y}', X = x]\mathrm{d}x,$$

for any $x' \neq x$ such that $(x', \hat{y})$ is observed. This equation establishes the identifiability of $\mathcal{M}_Y$. It also implies that the solution of the risk minimization problem (8) must coincides with $\mathcal{M}_Y$ if $\mathcal{H}$ satisfies the identifiability condition, i.e. $\mathcal{H}$ contains only separable functions $g(x, \hat{y})$ with differentiable g_1, g_2; these constraints implies the uniqueness of solution to the risk minimization problem, hence the solution must coincide with $\mathcal{M}_Y$.

E Experiment details and additional experiments

E.1 Data and Licenses

Data in folktables was extracted from Census Bureau databases, which collected data in standardized surveys with consent.[2] The Census Bureau takes care to ensure that through their pre-processing of survey results, personally identifiable information is not included in their data releases.[3]

The income dataset with binary outcome variables used in the main text is the ACSIncome task defined in folktables, with data from the 2018 Census from the state of California. The income dataset with continuous outcome variables is a modified version of ACSIncome that performs the same pre-processing, except it leaves the income target variable as a real number, rather than thresholding to produce a binary outcome. Additional experiments below (referred to as Census travel time) were conducted on the ACSTravelTime task defined in folktables, with data from the 2018 Census from the state of California. The features and preprocessing for these datasets can be found in the code documentation of [11]. The data contains 10 features:

> [RevolvingUtilizationOfUnsecuredLines, age, NumberOfTime30-59DaysPastDueNotWorse, DebtRatio, MonthlyIncome, NumberOfOpenCreditLinesAndLoans, NumberOfTimes90DaysLate, NumberRealEstateLoansOrLines, NumberOfTime60-89DaysPastDueNotWorse, NumberOfDependents],

the target variable is a binary indicator for whether an individual became delinquent on a loan (SeriousDlqin2yrs).

E.2 Experimental details

Machine learning models were trained using functionalities from sklearn [50] with default parameters if not specified otherwise. We use the class `LinearRegression` from `sklearn.linear_model` for the linear models and the class `MLRRegressor` from `sklearn.neural_network` for the fully connected neural network models.

Training data. The Census income dataset composes of 195665 datapoints, if not specified otherwise the full dataset was used for training.

Test dataset. We train f_ϕ on randomized labels. More precisely, we randomly shuffle the labels among data points in the original dataset to obtain f_ϕ. This process leads a model that is different from f_θ which serves to test the extrapolation performance of our meta-machine learning model.

Overparameterization. For the experiment in Figure 2(c) second degree polynomial features were included to achieve overparameterization (using `sklearn.preprocessing`), no further hyperparameters were set; all second-order terms were included in the overparameterization. For the experiments in Figure 3(a) we simulate the degree of overparameterization by working with neural networks and varying the number of neurons in the hidden layers using the parameter `hidden_layer_sizes`.

Randomization. For the randomized decision experiments we use Gaussian noise. If not specified otherwise it is drawn from $\mathcal{N}(0, 1)$.

Discretization. To obtain discrete predicions, we round the prediction outputs of the linear model f_θ so we achieve 4 distinct discrete values.

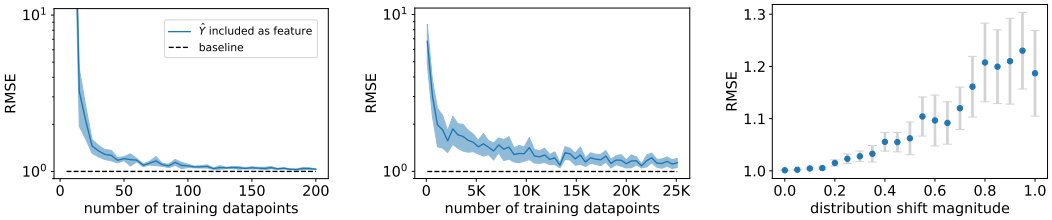
Finite data. The finite data experiments were conducted on datasets with a continuous target variable Y (Census income), and performance was assessed using root mean squared error (RMSE). To simulate the effect of trainingset size we randomly subsample the original data to obtain a smaller training set size for our meta machine learning model.

We simulate different amounts of distibrution shift by choosing $\phi' = \rho\theta + (1 - \rho)\phi$ where θ are the parameters of the model trained on the original data, and ϕ are the parameters of a model trained on randomized labels.

Infrastructure. Experiments were run on 4 CPU cores for a total of 200 hours.

[2] documentation: https://www.census.gov/programs-surveys/acs/microdata/documentation.html.

[3] Licenses and terms of use: https://www.census.gov/data/developers/about/terms-of-service.html.



(a) overparameterized f_θ (b) randomized f_θ (c) randomized f_θ

Figure 5: **Effect of training dataset size.** With a moderate amount of training data, randomized decisions and overparameterization can find transferable functions h_{SL}. The variance in the extrapolation loss increases with the distribution shift magnitude.

Baseline. The baseline in the plots is the RMSE of a model trained on samples from the test set and evaluated on a validation set that is held out from the test set, but from the same distribution. Thus it represents a setting with no distribution shift.

E.3 Additional Experiments: Learning with finite data

Recall that causal identification results are feasibility guarantees. They imply that $\mathcal{M}_Y$ can be recovered from observational data in the limit of infinite data. However, in practical settings, we only get access to a finite set of data points from the training distribution. In the following, we show that supervised learning can successfully learn transferable functions h_{SL} with only a few training data points, given that our identifiability conditions are satisfied.

In Figure 5(a)-(b) we consider the same setup as in Section 4.1; we fix performativity strength at $\alpha = 0.5$, and vary training set size. We find that only moderate dataset sizes are necessary for h_{SL} to identify an accurate model that is robust to performative distribution shifts in our identifiable settings.

In Figure 5(c) we choose $N = 5000$ and investigate the performance of supervised learning as we vary the distance between predictions from f_θ and f_ϕ, i.e. the distribution shift between train and test set. We achieve this by interpolating the parameters of the predictive model in the test set between f_θ and f'_ϕ where the latter is trained on randomized labels as before. We observe that the error and variance of h_{SL} grow with the magnitude of the distribution shift. This can be explained as failures in the meta model to identify the transferable causal model $\mathcal{M}_Y$ become more pronounced as distribution shifts get larger. In addition, the variance is expected to grow further away from the mean of $f_\theta(x)$ due to data scarcity implied by the shape of the noise distribution in the randomized f_θ. This observation supports our recommendation to explore the parameter space gradually for policy optimization under performativity, instead of directly extrapolating to models f_ϕ that are substantially different from f_θ.

E.4 Additional experiments: Robustness to model misspecifications

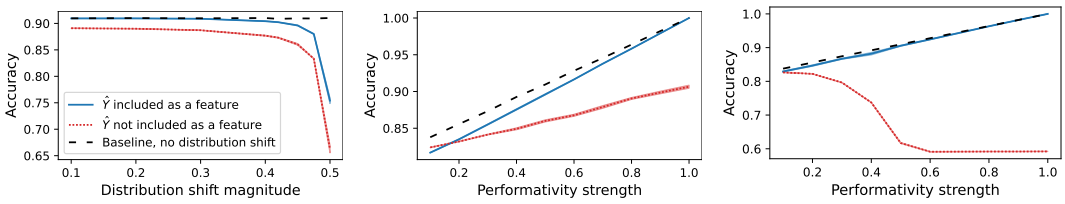
We investigate the robustness of supervised learning to misspecification of g. We performed these experiments for the dataset used in the main body, as well as the Census travel time and Kaggle credit score datasets [27]. We found qualitatively similar results on all datasets (Figures 6, 7 and 8)

We focus on discrete classification where f_θ and f_ϕ are binary predictors. Our theoretical result in Proposition 6 depend on knowing the correct model class $\mathcal{H}$ to optimize h_{SL} on. Here we test the resiliency to model misspecification. Therefore, we define $\mathcal{H}$ to be the class of linear functions but construct a non-linear data generation process as follows

$$g(x, \hat{y}) = \hat{y} \text{ with probability } p, \text{and } g(x, \hat{y}) = g_1(x) \text{ otherwise,} \tag{19}$$

where $g_1(x)$ is a (possibly non-linear) function that maps x to its original label y in the original dataset, and $p \in [0, 1]$ is a hyperparameter for performativity strength that we vary. Like in the finite data experiments, we also vary the distance between predictions from f_θ and f_ϕ, i.e. distribution shift magnitude.

Figure 6(a) shows that despite misspecification, the meta model benefits of having access to $\hat{y}$ and the accuracy of h_{SL} remains close to in-distribution accuracy as long as distribution shifts are not too large (specifically, until f_θ and f_ϕ become uncorrelated). Figure 6(b) highlights that the benefit of



(a) Distance between f_θ and f_ϕ (b) f_ϕ (test) fit to noisy labels (c) Reversed: f_θ (train) fit to noisy labels

Figure 6: **Performance degrades gracefully under model misspecification (Census income prediction dataset).** (a) Accuracy on the test distribution (higher is better) is plotted against distribution shift magnitude; supervised learning remains accurate until the train and test predictors, f_θ and f_ϕ, are uncorrelated (shift magnitude of 0.5). (b) Accuracy is plotted against performativity strength; despite model misspecification, accuracy is higher when $\hat{Y}$ is included as a feature, across most performativity strengths. (c) When the training set predictor f_θ is fit to random labels and is less accurate than f_ϕ, including $\hat{Y}$ as a feature universally improves accuracy.

adding $\hat{y}$ as a feature persists across almost all values for p. However, h_{SL} is more prone to errors from model misspecification when performativity is very weak. This is intuitive, since $\hat{Y}$ is correlated with the outcome Y, and a misspecified h_{SL} might be best off attributing this correlation to the causal link; in such extreme cases, Figure 6(b) suggests that accuracy is slightly improved by dropping $\hat{Y}$ as a feature. However, this improvement for $p \to 0$ is small, and Figure 6(c) shows that if the accuracy of f_θ is reduced (by fitting to noisy labels), this effect disappears. But across all three experiments, including $\hat{Y}$ as a feature outperforms not including it, even when little performativity is present.

Additional experimental details: The model misspecification experiments were conducted on datasets with a binary target variable Y (Census travel time and Kaggle credit), and performance was assessed using classification accuracy. The predictors f_θ and f_ϕ are both always gradient boosted decision trees implemented in sklearn [50] with the default hyperparameters. As in the finite data experiments, f_θ is usually fit to minimize error on the original dataset, while f_ϕ is fit on a noisy version of the original dataset, where the level of noise is tunable. Denoting the noise with γ, we generate noisy labels y' via

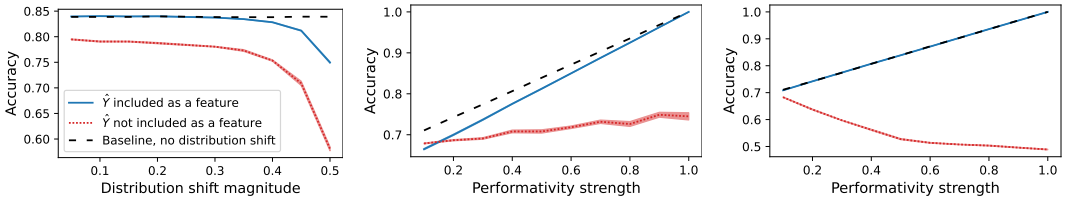
$$y' = y \text{ with probability } 1-\gamma, \text{ and } 1-y \text{ with probability } \gamma.$$

In other words, γ determines the probability that the label is flipped, with 0.5 making y and y' uncorrelated.

Hyperparameters for the experiments were set to the following unless otherwise specified:

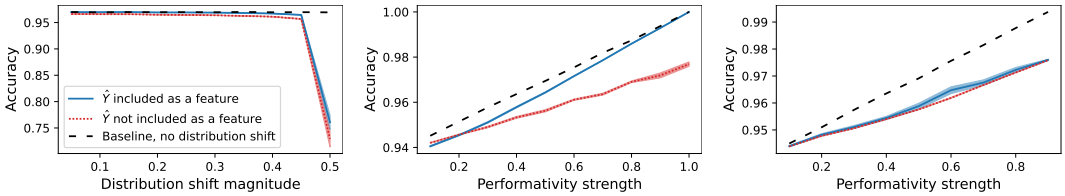
- Training set size: 200,000 datapoints
- Magnitude of distribution shift γ: 0.45
- Strength of performativity α: 0.5

For experiments where we denote that the training predictor f_θ is given noisy labels, this means that f_θ is fit to a random permutation of the original dataset labels.



(a) Effect of larger distribution shifts (b) f_ϕ (test) fit to noisy labels (c) Reversed: f_θ (train) fit to noisy labels

Figure 7: **Performance degrades gracefully under model misspecification (Census travel time dataset).** (a) Accuracy on the test distribution (higher is better) is plotted against distribution shift magnitude; supervised learning remains accurate until the train and test predictors, f_θ and f_ϕ, are uncorrelated (shift magnitude of 0.5). (b) Accuracy is plotted against performativity strength; despite model misspecification, accuracy is higher when $\hat{Y}$ is included as a feature, across most performativity strengths. (c) When the training set predictor f_θ is fit to random labels and is less accurate than f_ϕ, including $\hat{Y}$ as a feature universally improves accuracy.



(a) Effect of larger distribution shifts (b) Standard setup: f_ϕ (test) fit to noisy labels (c) Reversed: f_θ (train) fit to noisy labels

Figure 8: **Performance degrades gracefully under model misspecification (Kaggle credit score dataset).** (a) Accuracy on the test distribution (higher is better) is plotted against distribution shift magnitude; supervised learning remains accurate until the train and test predictors, f_θ and f_ϕ, are uncorrelated (shift magnitude of 0.5). (b) Accuracy is plotted against performativity strength; despite model misspecification, accuracy is higher when $\hat{Y}$ is included as a feature, across most performativity strengths. (c) When the training set predictor f_θ is fit to random labels and is less accurate than f_ϕ, including $\hat{Y}$ as a feature universally improves accuracy.

F Societal impact

The fact that predictions are performative and have an impact on the population they predict is a natural phenomenon observed in various applications. In this work, we discuss one dimension of performativity and investigate how to develop an improved causal understanding of these performative effects from data. Our intent is to develop this understanding from observational data in order to foresee potential negative consequences of a future model deployment before actually deploying it across an entire population. Typical machine learning approaches would not take these consequences into account when training a predictive model. At most, they would observe performative effects in a monitoring step after deploying a model and then decide post-hoc whether the model satisfies a given constraint. At this point, harm might already have been caused, even if unintentional. Naturally, though, any improvement in understanding can also be used with bad intent. Instead of being treated as a potential for harm to mitigate against, performative effects could also be instrumentalized by profit-maximizing firms or self-interested agencies in order to achieve their goals [21]. These goals might not always be aligned with social welfare and if the respective firm has high performative power, i.e. ability to influence performative effects, these actions hold the potential for social harm.