COALITION FORMATION IN RIDESHARING WITH WALKING OPTIONS

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ABSTRACT

In this work, we introduce a novel coalition formation mechanism for ridesharing services. We extend on the current literature to integrate walking options within the trip, this allows us to effectively account for a user's value of time when walking, which is not negligible. Additionally, we propose a price allocation method that ensures proportionality for sharing the costs. We formally evaluate the efficacy of our pricing method and discuss its desirable properties. Our study is a step towards developing smart mobility systems that recommend optimal ridesharing coalitions to users and suggest cost allocations that are fair and stable.

1 INTRODUCTION

Ridesharing services allow passengers with similar itineraries to share a car and split the cost of the trip. As a mobility option, ridesharing presents advantages on several fronts. From a social point of view, it contributes to lower congestion, bringing a positive externality to traffic (Santos & Xavier, 2015; Stiglic et al., 2015). From the user's standpoint, ridesharing is usually more affordable than travelling alone (Li et al., 2021). For these reasons, ridesharing services have expanded their penetration rate in several cities, becoming a popular mobility service. In recent times, several commercial platforms have started to offer ridesharing services such as Uber [1], Lyft [2] and DiDi [3] as a sign of the changing landscape in mobility services. In a survey by the *US Census Bureau* (US Census Bureau, 2020), by 2019, ridesharing represented approximately 8–11% of the transportation modality in Canada and the USA.

The surge in ridesharing modalities has motivated several academic studies on the problem of ridesharing formation and cost allocation. For the problem of finding the optimal pool of riders, there have been many approaches using game-theoretic solutions. The work by (Bistaffa et al., 2015) and (Bistaffa et al., 2017) uses cooperative game theory to propose a mechanism to form travellers coalitions and allocate the travel cost. They consider a specific setting where users want to find other travellers who want to travel at a specific time of the day. To reduce the dimensionality of the search space, the work defines the concept of *feasible coalitions* in which commuters are pooled from a proximity graph. For the payment distribution, they propose to use the *kernel* as a solution concept and discuss the disadvantages of using the core on ridesharing problems, in particular, the exponential computation time and the fact that its existence is not guaranteed. Recently, (Li et al., 2021) focus on the ridesharing aspect of route selection and cost allocation using cooperative game theory. Their work focuses in finding the minimal cost route that covers all participants, modelling it similar to a TSP problem.

Focusing now on the walking component, two works include some form of walking in the ridesharing problem. The paper by (Stiglic et al., 2015) includes walking in a problem of matching riders with drivers. They show that when users are willing to walk to a pre-set meeting point, it becomes easier for drivers to pick them up en-route, and increases the matches between passengers and feasible drivers. Similarly to us, they also consider a situation where there is a single pick up and drop off point for all passengers, with no additional stops or detours. However, their focus is on solving a bipartite matching problem between taxi cabs and passengers. Their work does not factor-in the

[1] https://www.uber.com
[2] https://www.lyft.com
[3] http://www. didiglobal.com

walking time as a cost incurred by passengers, and there is no monetary compensation for the walking time nor accounting for the different walking times that users might have. The second work by (Kaan & Olinick, 2013) includes meeting and dropp-off points in a context of solving a *van-pool* problem. They consider a scenario in which employees drive from their homes to a common location to board a van and ride together to their workplace. In their cost equation they include the car's operational costs such as gas per mileage, depreciation, maintenance, etc. This work focuses on answering the question of when employees would prefer a company's sponsored van-pool service rather than driving to their workplace on their private car.

We expand the current literature, by explicitly considering a more general setting in which riders are willing to walk in 2 directions: first from their home to the departure point, and then from the car destination to their own destination. The main difference with the existing literature is in that we explicitly model the walking cost and compensate riders for their walking time. Our walking cost is composed by a penalty term that accounts for user's walking aversion. Moreover, to obtain a fair distribution of costs, we compensate users for the hurdle of walking. By using a game-theoretic framework, we propose a methodology to determine the socially optimal coalition structure that is at the same time fair for individuals.

We believe it is important to acknowledge the walking time as a cost, since it is part of the user's *value of time*. There is extensive literature on the relevance of the concept of value of time (VoT) as a measurement of the trade off between a user's time and money (Mayet & Hansen, 2000) and (Ed., 2004). Authors agree on that the specific VoT depends on individual factors such as socioeconomic characteristics (e.g., income), trip purposes (e.g., commuting, leisure, or business), transport mode,etc. These differences may lead to different behavior from the users, even in the same circumstance (see for example (Rezaee & Hickman, 2015) and (Arnott & Kraus, 1998)). In our work, we consider an *average VoT* for all users.

Introductory example. Consider for example a group of friends who want to share a ride after a night out, or colleagues who live close by want to share a ride. In both cases, riders agree at a meeting point either in source or/and destination. Given a set of riders, there are many possible combinations of how to place passengers into available cars. Our premise is that individuals choose with whom to carpool based on the cost of the trip, including riding and walking, these ridesharing groups are also referred as *coalitions*. We illustrate the interplay between walking and riding costs below.

As an example, consider figure 1 below. On the left panel, there are 3 individuals $\{A, B, C\}$ having nearby origin and destination zones. Assume the three of them choose to share a car, so they walk from their individual origins $\{A_o, B_o, C_o\}$ to a common departure point **o** and then join a carpool to a single stop **d**, from where they walk to their individual destinations $\{A_d, B_d, C_d\}$. This will yield a total value of the coalition equal to the total walking of each agent i.e. $dist\,(A_o, 0) + dist\,(B_o, 0) + dist\,(C_o, 0) + dist\,(A_d, d) + dist\,(B_d, d) + dist\,(C_d, d)$ plus the car price from **o** to **d**.

The right panel of figure 1 depicts an alternative arrangement. In this case, $\{A, B\}$ carpool together while $\{C\}$ travels alone (and therefore has no walking component). According to the Bell number, there are 5 possible arrangements from three elements. Our aim is to study which of them is the socially optimal.

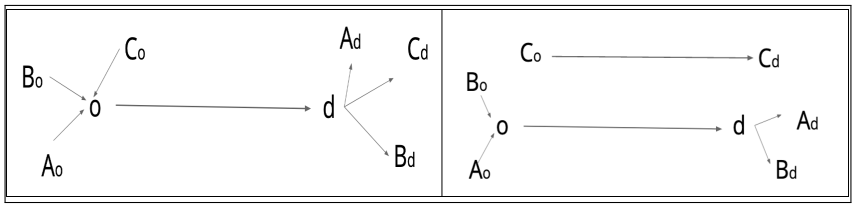

Figure 1: ridesharing with single origin-destination trip and walking.

The formation of these possible travel arrangements, their optimality, and the study of the rationality is the subject of this paper. Our contributions are, first, we propose a methodology based on game-theoretic guarantees for the socially optimal formation of carpools. Our work derives the optimal coalition formation and proposes a cost-sharing mechanism that is individually rational under sensi-

ble conditions, while also accounting for the walking time required to join a carpool. To the best of our knowledge, we are the first to include the cost of walking into the ridesharing problem (in both source and destination) and factor this cost in when performing the cost allocation.

Our proposed methodology is a centralized approach, having the following benefits. First, it preserves privacy within users of the system. Users communicate location and travel preferences to a trusted centralized system, without having to share personal information with others. Additionally, each agent does not need to have observability of all other agents, and communicates only with the central system.

2 FORMAL PRELIMINARIES

Following (Rothe, 2016) and (Nisan et al., 2007), in cooperative games a set of $A = \{a_1, ...a_n\}$ agents work together by forming coalitions, and take joint actions so as to maximize their utility. The goal is to find what is the best coalitional structure to form. A coalitional game can be described in terms of its *characteristic function*, which expresses the value of each coalition.

Definition 1 *A coalitional game with transferable utilities (TU) is the tuple (N, C) where N is the set of players and $C(S)$ is a characteristic function defined as: $C : 2^S \rightarrow \mathcal{R}$ representing the inverse of the value (i.e. the cost) that a subset S of agents can achieve on their own regardless of what the remaining agents do (i.e. without any externality present).*

The Coalition Structure Generation (CSG) problem takes as input the set of n agents together with their characteristic function and yields the minimum cost coalitional structure. In our case, we want to know, who should share a ride with whom, in order to maximize the social optimum (i.e. minimize the social cost).

Definition 2 *A coalition structure $\mathbb{C} = \{S_1, ., S_k, .., S_m\}$, is a partition of A. That is, $S_k \subset A, S_k \cap S_j = \varnothing, (j \neq k), \cup_{k=1}^{K} S_k S_k = A$.*

In our case, each partition, represents the different ridesharing options for riders to join.

3 RIDESHARING AS A COALITIONAL COST GAME

The ridesharing problem considers a set of riders $A = \{a_1, ...a_n\}$ where n is the total number of riders. We consider an unbounded supply of cars, where every car can host up to a determined number of seats, n_k. In our problem, we want to determine the optimal allocation of passengers on these cars to minimize the system-wide travel costs. We formulate the optimal allocation problem as a coalition formation problem using a game-theoretic approach. We require ridesharing coalitions to be *socially optimal*, that is, the combined travel cost for its member is the lowest possible.

3.1 SOCIAL COST OPTIMIZATION PROGRAM

Our aim is to find the optimal coalition structure that minimizes the social cost. For this, we calculate the cost of each coalition as the sum of the walking per member plus the car cost. Below we explain how to calculate each cost and then we explain how the optimal coalition structure is obtained.

3.1.1 COST CALCULATION

The central point of our cost calculation methodology is the addition of walking costs W_i. In a sense, the walking time can be viewed as the *cost of entry* into a coalition. The walking distance depends on the selection methodology for the the pick up and drop-off points, which will explained below.

Determination of the central O-D points. the first step in determining the walking time, is to calculate the meeting and drop-off points. In our approach, we determine these points using the geometric median of the individual's locations. [4]

[4] The geometric median of a set of points is defined as the point minimizing the sum of Euclidean distances within this set of points (Cohen et al., 2016).

As an example, consider the case where there are only 2 individuals in the cluster $\mathcal{A} = \{a_1, a_2\}$, each reports their current departure coordinates $\left(x_i^{origin}, y_i^{origin}\right)$ and arrival coordinates $\left(x_i^{dest}, y_i^{dest}\right), i = 1, 2$. The car will meet them at a mid-point of their origin coordinates and drop them off in the mid point of their destination [5]. In this example: car pick-up point = (x_o, y_o) = $\left(\frac{1}{2} \cdot \sum_{i=1}^{2} x_i^{origin}, \frac{1}{2} \cdot \sum_{i=1}^{2} y_i^{origin}\right)$; car drop-off point = (x_d, y_d) = $\left(\frac{1}{2} \cdot \sum_{i=1}^{2} x_i^{dest}, \frac{1}{2} \cdot \sum_{i=1}^{2} y_i^{dest}\right)$, car trip = $dist\left((x_o, y_o), (x_d, y_d)\right)$; and walking for rider $i, dist\left((x_i^{origin}, y_i^{origin}), (x_o, y_o)\right)$ + $dist\left((x_i^{dest}, y_i^{dest}), (x_d, y_d)\right)$.

Once we have identified the meet-up and drop-off points, we need to assign a monetary value to the walking time.

Cost of Walking. In order to account for the time spent walking, we use the concept of value of time (VoT), this allows us to express an individual's walking time in monetary terms. We assume the VoT can be expressed as a function proportional to the distance walked and assume that this proportion increases with the time walked [6]. The justification for the over-proportionality of the walking cost is inspired by the idea of a threshold distance for reasonable walking, over which individuals see walking as a burden (see for example (Shrestha & Zolnik, 2013)). To capture the marginal aversion to walking into a cost, the VoT of walking from a to b for the individual i is modeled as:

$$w_i(a, b) = dist(a, b) + \frac{1}{\alpha} \cdot dist(a, b) * \log_{\alpha} dist(a, b) \tag{1}$$

where α is a hyper-parameter selected according to the dimensions of the plane and is determined in concordance with ϵ [7]. The graph in the appendix A illustrates the convexity of the $w_i(\cdot)$ function and a comparison against the simple Euclidean distance function.

After we obtain the cost of walking for each member, we can add the cost of the car and get the total cost of the trip for the coalition S_k.

$$C(S_k) = \sum_{i=1}^{n_k} W_i(S_k) + P_{car}(S_k) \tag{2}$$

where W_i is the cost of walking explained before, n_k is the maximum capacity of the car and $P_{car}(S_k)$ represents the taxi fare. In our examples, we consider that the maximum capacity of a car is four members, meaning that coalitions can have *up to* four individuals.

Optimal Coalition Structure. After calculating the total cost of riding for each coalition, we want to find the coalition structure that minimizes this cost. Formally:

$$\arg\min_{S_k} C(S_k) \tag{3}$$

3.2 Cost Allocation Problem

After obtaining the socially optimal coalition structure the next question is how to split the price of the car. This aspect is important as it will determine whether individuals would prefer ridesharing rather than traveling alone, and thus a coalition can be formed.

Formally, the goal is to calculate a payoff vector (x), which is the share of the coalition cost allocated to player i after joining coalition S_k. For the price sharing phase, the challenge is that the walking part of the cost is an NTU (non-transferable utility) component and the only part of the cost that has transferable utilities is the cost of the car. According to Nisan et al. (2007), two common desirable properties are required over the payoff redistribution , *efficiency* (i.e., the entire value of S is split among the members of S) and *individual rationality* (i.e., each agent receives a payoff that is at least the value of its singleton).

[5] When $n = 2$, the convention is that the geometric median matches the mean.

[6] In other words, the VoT function considered is continuous and convex, with positive first and second order derivatives

[7] In our studies, in order to be conservative, we used $\alpha = 10$, which is approx 0.1 the size of the plane.

Definition 3 *A cost allocation is a vector $x = (x_1, ..., x_n)$, such that $x_i \geq 0 \ \forall i$. x is individually rational if $x_i \leq C(i)$. Moreover, if $\sum_{i=1}^{n} x_i = C(A) \ \forall i \in A$, then x is efficient.*

3.2.1 Inversely Proportional Cost Allocation

The idea of our inversely-proportional cost allocation method is that those who walk less, pay a greater share of the price. Formally, let i be a rider in the coalition S, let w_i be the proportion of walking done by rider i, and let wi_i the *proportion* of the car price allocated to rider i, then:

$$w_i = \frac{1}{w_i} \cdot \kappa \tag{4}$$

where $\kappa = \frac{\prod_{i=1}^{n} w_i}{\sum_{i=1}^{n} w_i \cdot w_j}, \forall i \in S$.

The formal results on desired properties of our approach are further described in Section 5.

This cost allocation method is fair in the sense that it compensates individuals for the cost for *entering* a coalition. Fairness in this sense would not be captured by an equal allocation or traditional metrics such as the Shapley value [8].

4 Coalition Formation in Ridesharing

Our methodology consists on three simple steps: *(a)* dimensionality reduction, *(b)* determination of the socially optimal coalitional value and *(c)* cost sharing among ridesharing participants. We first address the optimisation problem of minimising the total cost across the coalitions formed by the system. Given this, we define the value of each coalition as the travel cost of the associated car, as the aggregated cost of walking plus riding. Lastly we propose a methodology to split the travel costs which compensates for the different travel incurred by car-poolers.

4.1 Dimensionality reduction

The problem of searching optimal structures is NP-hard because the size of different possible coalition structures is exponential in the number of coalition members. There are 2^n possible coalitions for a set of n individuals (Rahwan T. & Giovannucci, 2009). The common practice in the ridesharing and ridesharing literature is to reduce the dimensionality of the problem in a sensible way. A popular technique is the use of DBSCAN (Ester et al., 1996) to find proximity clusters among riders with similar origin and destination. For example, the work of (Samy & Elkorany, 2018) apply DBSCAN to cluster individuals based on similarity of their trip profile (routes, pick-up zone, time, etc). Recently, (Syafira et al., 2021) used DBSCAN in a ridesharing problem to match riders and drivers within a certain radius of proximity.

Similar to these works, we use DBSCAN to reduce the partitions space by considering only individuals who are within a certain walking range (considering the combined walking at origin and destination). This allow us to narrow down the space of possible coalitions into a selection of *feasible coalitions*, from a walking standpoint.

Definition 4 (Feasible coalition) *A coalition S_k is ϵ-feasible if all of its members are are interior points of the closed ball $N_\epsilon(p)$ where p is an arbitrary point in the plane. I.e.: $N_\epsilon(p) = \{q \in \mathbb{R}^p \mid dist(p, q) \leq \epsilon\}$. Where the $dist(\cdot)$ function is the Euclidean distance of points in a four dimensional space composed by $(x_{origin}, y_{origin}, x_{destination}, y_{destination})$ and ϵ is an arbitrary radius in 4D.*

Calculation Example. To illustrate, we present an application example. We start by simulating an initial set of 30 random points on a plane of $[0 - 0.250], [0 - 0.250]]$ inches long. Then we perform DBSCAN clustering, for the choice of the $epsilon$ parameter it was set as proportional to the dimensions of the space. Results are shown below, note that the clustering zones depicted in figure 4.1 below are only indicative, as we actually performed a DB-Scan in four dimensions (corresponding to the origin and destination coordinates combined).

[8] Note that the Shapley value is only guaranteed to produce stable results if the characteristic function is convex, which in our case it is not guaranteed.

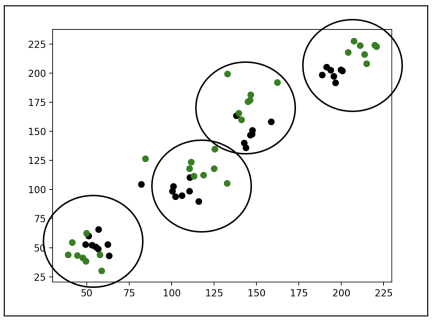

Figure 2: Origin and destination of commuters on the plane with clustering zones. Simplification of clusters in 4D space. The black dots correspond to individual's origin and green dots to their destination.

After clustering individuals by proximity, we obtain the set of feasible coalitions an individual can join. The next step is to determine which is the socially optimal coalition by following algorithm 1 below.

4.2 Coalition Formation Algorithm

Starting from the set of feasible coalitions, the next step is to calculate the characteristic function of each coalition which gives us the total cost of the trip. The cost of the trip is defined as the aggregate cost of walking plus the car cost, as in equation 2. With this, we can find the socially optimal coalition as in equation 3.

After clustering the space, the method can be summarize as follows

Algorithm 1: Determination of the socially optimal coalition per cluster

foreach *cluster* **do**
  get all possible coalition structures S_k;
  foreach *coalition* S_k *in cluster* **do**
    medianCoords $\leftarrow median((x_i^o, y_i^o), (x_i^d, y_i^d))$;
    foreach *i in coalition* **do**
      $W_i(S_k) \leftarrow dist((x_i, y_i), medianCoords))$;
    end
  end
  $C(S_k) \leftarrow \sum_{i=1}^{n_k} W_i(S_k) + P_{car}(S_k)$; `// coalition cost`
end
$SO \leftarrow$ min $\{C(S_k)\}$; `// save the SO per cluster in a container`
return SO per cluster

The table below shows an example composition of the socially optimal coalitions per cluster. Individuals are arranged in cars of up to four passengers such that the total cost of travel (walking time plus trip cost) is minimized. The column **Cost** represents the total cost of the coalitional structure (i.e. the sum of the cost of the car for each coalition plus the total walking cost for each individual on the cluster).

The graph A.2 on the appendix A shows a visual representation of the socially optimal coalition structure within each cluster.

4.3 Ensuring Efficiency and Fairness

Achieving the socially optimum coalitional structure is a desirable property at the aggregate level; however, at the individual level, coalitional costs should be allocated on an individually rational level.

Table 1: Optimal coalition structure per cluster

Cluster	Optimal Coalition Structures	Cost
1	$\{\{a_0, a_6, a_{13}, a_{14}\}, \{a_7, a_{11}, a_{24}, a_{27}\}\}$	351.03
2	$\{\{a_3, a_8, a_{17}\}, \{a_1, a_{18}, a_{19}, a_{22}\}\}$	685.34
3	$\{\{a_4, a_5, a_{10}, a_{16}\}, \{a_2, a_{12}, a_{21}\}\}$	555.5
4	$\{\{a_9, a_{20}, a_{25}\}, \{a_{23}, a_{26}, a_{28}, a_{29}\}\}$	613.43
5	$\{a_{15}\}$	220.85

After the socially optimal coalition is found, we calculated the allocation of the car cost among its members using equation 4. To test for individual rationality, we used definition 3. The graph below shows how much each agent saves by ridesharing compared to riding alone. In no case the individual rationality is broken.

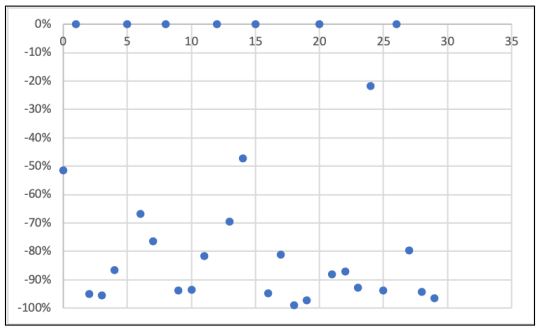

Figure 3: Cost savings of ridesharing versus travelling alone, using the inversely proportional cost allocation method. The horizontal axis represent the agents and the vertical axis represents the savings percentage when ridesharing vs traveling alone.

For this example, the average cost savings across all individuals is 63%, this means, ridesharing produces a saving of more than 60 percent compared to traveling in a taxi alone. The median savings is 81%.

Below we present conditions under which individual rationality can be guaranteed.

5 Properties

Below we analyze the properties of the cost-allocation algorithm based on the inverse porportion of walking.

5.1 Efficiency

Below we show how the formula for distribution of cost is efficient in the sense of definition 3.

Let $P_{car}(a, b)$ be the cost of the car to distribute across members. To allocate this price, we define the share of the payment as inversely proportional to the *proportion* of walking done by each member.

Let w_i be the percentage of walking done by individual $i \in S$. We have the usual condition for proportions:

$$\sum_{i \in S} w = 1 \tag{5}$$

and $0 \leq w_i \leq 1$. Let w_1' be the proportion of the price paid by individual i. The goal is to find a proportional split such that the usual condition holds:

$$\sum_{i \in S} w' = 1 \tag{6}$$

As an additional condition, we want the w'_i to be inversely proportional to walking, i.e.:

$$w'_i = \frac{1}{w_i} \cdot \kappa \tag{7}$$

where the κ term is needed for equation 6 to hold.

To determine κ we plug equation 7 into equation 6, obtaining $\kappa = \frac{\prod_{i=1}^{n} w_i}{\sum_{i=1}^{n} w_i \cdot w_j}$.

The share of the car paid by each member is: $c_i = P_{car}(a, b) \cdot w'_i$. Equation 6 guarantees that this payment is individually rational. Additionally, it is easy to see that the distribution in 7 has the following properties: (a) if two individuals have walked the same proportion, their payment allocation is equal (i.e. if $w_i = w_j$ then $w'_i = w'_j$) and also (b) if individual i has walked p times more (for example) than individual j, then $w'_i = \frac{1}{p} w_j$.

5.2 Study on the Conditions for Individual Rationality

There are certain conditions that have to be met for the socially-optimal coalition to be individually rational. In short, the requirement is that the distance travelled by car in ridesharing should be less than the distance travelled by individual ride, to compensate for the cost of walking. This is obtained when the origin and destination of ridesharing individuals are close enough. in Other words, similar to our rationale for the chose of $epsilon$, there is a tolerance threshold in the density required for ridesharing to be individually rational. Below we formalize this idea.

Let the individual cost of travel be $c_i = P_{car}(a, b)$, where $P_{car} = f(dist(a, b))$ is the cost of the taxi for travelling from coordinate points a to b. Let the individual cost of riding in coalition S_k be $c_i(S_k) = W_i(S_k) + \Psi(S_k)$, with: $\Psi(S_k) = P_{car}(a', b') \cdot w'_i(S_k)$, where $w'_i(\cdot)$ is defined in equation 7. We want to analyze under which conditions $c_i(S_k) \leq c(i)$.

Assumption 1 *We assume that the cost of walking (expressed as the VoT in equation 1 is much smaller than the monetary cost of the car for the same unit of time.*

Under the above assumption, we have that $P_{car} \leq\leq W_i(\cdot)$, both measured in monetary terms. Then we analyze the term $\Psi(\cdot)$, first note that $0 \leq \kappa \leq 1$, then, on eq. 7, for a fixed w_i this equation reaches its maximum when $\kappa = 1$ and $w_i = 1$, which is impossible by the definition of geometric mean [9].

Now we only need to study the conditions for $P_{car}(a, b) \leq P_{car}(a', b')$, which is basically asking that $dist(a, b) \leq dist(a', b')$. This inequality depends on the calculation of the points (a', b'). These points are the result of taking the geometric median across the origin and destination coordinates of the individuals on the cluster. If individuals are too far apart the ridesharing scheme is no longer convenient. This distance is controlled by the ϵ parameter of the DBSCAN.

6 Conclusions

In this work we show that the socially-optimal ridesharing problem can be modelled as a Coalition Formation problem and solved within the framework of game theory. We proposed a methodology to include the cost of walking into the coalitional cost structure. Additionally, we propose a methodology for the cost allocation among coalition members, that is fair in the sense that it compensates members for the cost of walking which is the cost of participation on each coalition. Our empirical evaluation shows that the proposed method is computationally tractable and leads to results that are socially optimal.

For future work we would like to explore and compare additional cost allocation methods, and test stability concepts like the *core, nucleolus* and *kernel*.

[9] The condition $w_i = 1$ is not possible as it would require no walking by all individuals except one.

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A APPENDIX

This section provides additional content to improve the clarity on some of the concepts presented in the main body.

A.1 ADDITIONAL GRAPH FOR THE WALKING COST

The graph below provides a visualization of the weighted walking function against a linear distance function. The graph depicts the convexity of the walking cost.

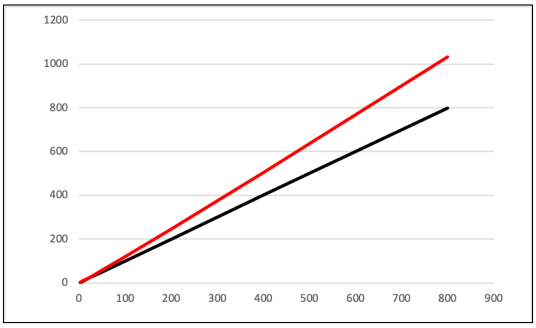

Figure 4: Comparison between the weighted walking (in red) function and linear distance (in black)

A.2 ADDITIONAL GRAPH FOR THE OPTIMAL COALITIONS

The figure below shows an example of the socially optimal coalitions within each cluster. The different colors show how individuals are sub divided into different ridesharing structures (i.e. coalitional structures).

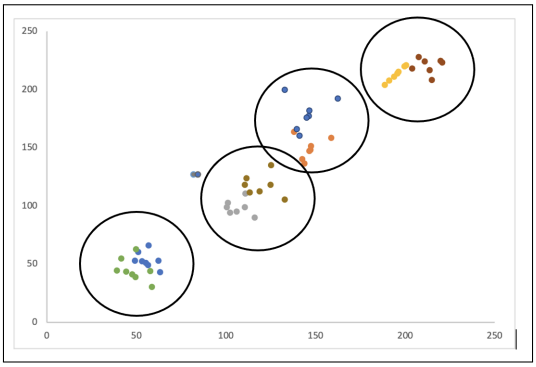

Figure 5: Socially optimal coalition structures per clusters. Simplification of clusters in 4D