BAYES RISK CTC: CONTROLLABLE CTC ALIGNMENT IN SEQUENCE-TO-SEQUENCE TASKS

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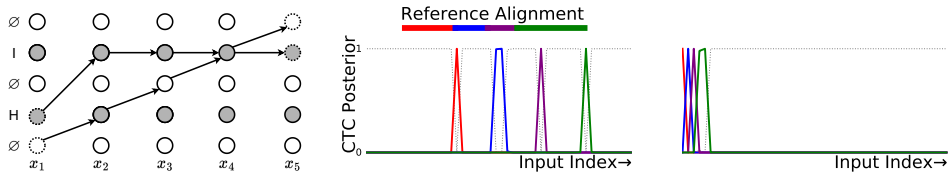
ABSTRACT

Sequence-to-Sequence (seq2seq) is a group of tasks that transcribe the input sequence to the target sequence. Connectionist Temporal Classification (CTC) criterion is widely used in multiple seq2seq tasks, especially automatic speech recognition (ASR). Besides predicting the target sequence, a side product of CTC is to predict the hard alignment between the input and the target. Specifically, there are multiple sequences (called *paths*) in CTC training that explicitly map each input unit to either a target unit or a blank symbol, and the predicted alignment is the most probable path. Although successfully predicting any of these paths can yield the correct target sequence, the choice of which path will be most probable and become the predicted alignment is always uncertain. In addition, it is consistently observed in experiments that the alignment predicted by vanilla CTC drifts away from its reference significantly and rarely provides practical functionalities. Thus, the motivation of this work is to make the CTC alignment prediction controllable and equip CTC with extra functionalities. The Bayes risk CTC (BRCTC) criterion is then proposed in this work, in which a customizable Bayes risk function is adopted to describe the desired characteristics of the predicted alignment. Besides keeping the target prediction ability, the BRCTC with various risk function designs can have different functionalities, two of which have been explored in this work: down-sampling the hidden representations to reduce the inference cost; achieving a better performance-latency trade-off for online models. Experimentally, the proposed BRCTC reduces the inference cost of offline models by up to 47% and cuts down the overall latency of online systems to an unseen level. Code is released[1].

1 INTRODUCTION



(a) CTC paths (b) Vanilla CTC Posterior (c) BRCTC Posterior (ours)

Figure 1: (a) An intuitive explanation of CTC paths using 5 input units ($x_1, ..., x_5$) and 2 output units (characters *HI*). ∅ represents the blank symbol. Dashed circles represent the valid starting and ending nodes of the paths. Each path suggests a hard aligning relationship between the input and target sequences. (b) Posterior of a vanilla CTC ASR system. The dashed line represents the blank symbol while the colored lines represent different non-blank tokens. The predicted alignment significantly drifts away from its reference[3]but the predicted non-blank token sequence is correct. (c) Posterior of a BRCTC ASR system that adopts the method in section 3.3. All non-blank spikes are squeezed to the earlier time-stamps.

[1]Due to the review policy, code will be released after acceptance.

[3]Reference alignment is obtained by a deep neural network-hidden Markov model (DNN-HMM) system.

Many machine learning tasks can be interpreted as sequence-to-sequence (seq2seq) tasks as they aim to transcribe the input sequence into the desired target sequence. Connectionist Temporal Classification (CTC) Graves et al. (2006) is a fundamental criterion for seq2seq tasks. The CTC criterion was initially proposed for automatic speech recognition (ASR) but its usage has been extended to many other tasks like machine translation (MT)Qian et al. (2021), speech translation (ST)Yan et al. (2022), optical character recognition (OCR) Graves & Schmidhuber (2008); Liu et al. (2018), lip reading Assael et al. (2017), hand gesture detection Molchanov et al. (2016) and even robot controlShiarlis et al. (2018). Research on CTC is of wide interest, as many advanced systems for seq2seq tasks are based on CTC Yao et al. (2021), its extension Graves (2012); Higuchi et al. (2020); Qian et al. (2021) and its hybrid with attentionWatanabe et al. (2017); Yan et al. (2022).

In CTC, each input unit is explicitly mapped to either a target unit or a blank symbol. During training, all of these potential input-target mappings (called *paths*) are enumerated and their posteriors are summed and maximized so that the target sequence can be predicted effectively. Fig 1.a gives an intuitive explanation of the paths in CTC. Besides predicting the target sequence, another functionality of CTC is to predict the input-target *alignment*. Unlike the attention-based methods Chan et al. (2016); Vaswani et al. (2017) that softly predict the aligning relationship by attention weights, CTC predicts a hard alignment. Usually, there is a path whose posterior is dominantly larger than the others Zeyer et al. (2021), so this dominant path is considered the predicted hard alignment between the input and the target sequences. In CTC implementation, unit-level classification over all possible target units is conducted for each input unit to obtain the posterior of each path. Fig. 1.b demonstrates the dominant posterior of the predicted alignment by plotting the unit-level posteriors.

Since predicting any path will yield the correct target sequence, the vanilla CTC is designed to treat all paths equally during training. However, this equality for paths will result in uncertainty about which path will be selected as the predicted alignment. Additionally, it is observed in our experiments and literature Sak et al. (2015) that there is a disagreement between the predicted alignment and its reference (see Fig. 1.b), which limits its usage in real applications. Thus, the motivation of this work is to control the CTC alignment prediction, making it certain and functional. Specifically, instead of pursuing the accuracy of alignment prediction, this work intentionally selects the path with customizable characteristics.

This paper proposes a novel Bayes risk CTC (BRCTC) criterion to make CTC alignment prediction controllable. To express our preference for the paths with the desired properties, a Bayes risk function is adopted to weigh all paths during training. To be more detailed, the forward-backward algorithm of the original CTC is revised into a divide-and-conquer manner: the paths are firstly divided into several groups according to the concerned properties, and the path groups with more preferred characteristics will receive larger risk values during training. Compared with the vanilla CTC, BRCTC can still reserve the models' transcription ability since the posteriors of the paths are still optimized. Besides, the alignment prediction from BRCTC will obtain the desired characteristics due to the adoption of the risk function. Note the designs of how the paths are grouped and what risk value is assigned to each path group are left customizable, so the exact functionalities can be tailor-made according to specific applications, like offline and online scenarios.

In applications, the proposed BRCTC provides novel solutions to two long-standing problems of seq2seq tasks. For offline systems, the BRCTC can help to down-sample the intermediate hidden representations so that the mismatch between the input and target lengths is alleviated and the inference cost is significantly reduced. For online systems like streaming ASR, the BRCTC provides a better trade-off between transcription quality and latency. Besides ASR, the proposed BRCTC criterion can also be generalized to other seq2seq tasks like MT and ST. Experimentally, the BRCTC achieves up to 47% inference cost reduction for offline systems; it also allows us to overcome the algorithmic delay of online models to build systems with extremely low overall latency that can hardly be achieved by vanilla CTC.

Our main contributions are listed as follows. (1) Bayes risk CTC, an extension of CTC, is proposed as a customizable approach to achieve controllable CTC alignment prediction. (2) With various designs of the risk functions, the Bayes risk CTC provides practical solutions for two long-standing problems of seq2seq tasks. (3) Strong experimental evidence is provided in this work to argue that, for CTC-based systems, the target prediction can be decoupled with its alignment prediction.

2 REVIEW ON CONNECTIONIST TEMPORAL CLASSIFICATION (CTC)

Seq2seq tasks are to transcribe the input sequence $\mathbf{x} = [\mathbf{x}_1, ..., \mathbf{x}_T]$ to the target sequence $\mathbf{l} = [l_1, ..., l_U]$, where any $\mathbf{x}_t$ is a vector (e.g., features or token embeddings) while any token l_u belongs to a known vocabulary $\mathcal{L}$. T and U are the lengths of input and target respectively. Unless other specified, our discussion is temporarily restricted to ASR. Generalizing the concept of CTC to MT and ST tasks needs more discussion, which is presented in section 5.

2.1 TRAINING PROCESS AND ALIGNMENT PREDICTION

CTC Graves et al. (2006) is a widely used criterion in seq2seq tasks. Following the Bayesian decision theory, CTC tries to maximize the posterior $P(\mathbf{l}|\mathbf{x})$ during training. Instead of maximizing it directly, CTC maximizes the summed probability of all *paths*. Note $\varnothing$ as the blank symbol and extend the vocabulary $\mathcal{L}' = \mathcal{L} \cup \{\varnothing\}$. Any symbol sequence $\pi = [\pi_1, ..., \pi_T] \in {\mathcal{L}'}^T$ is a path if $\mathcal{B}(\pi) = \mathbf{l}$, where $\mathcal{B}$ is a deterministic mapping that removes all $\varnothing$ and repetitive tokens but reserves the repetitive tokens separated by $\varnothing$ (e.g., $\mathcal{B}(\varnothing aa \varnothing abb) = aab$). Thus, to maximize the posterior $P(\mathbf{l}|\mathbf{x})$ is equivalent to maximizing the summed posterior of all paths:

$$P(\mathbf{l}|\mathbf{x}) = \sum_{\pi \in \mathcal{B}^{-1}(\mathbf{l})} p(\pi|\mathbf{x}) \tag{1}$$

where $\mathcal{B}^{-1}(\mathbf{l})$ is the set of all paths. Next, to compute the posterior $p(\pi|\mathbf{x})$ of each path π, unit-level posterior over $\mathcal{L}'$ is computed for each input unit, which yields the CTC posterior $\mathbf{y} = [\mathbf{y}^1, ..., \mathbf{y}^T]$. Here each element $\mathbf{y}^t$ is a distribution over $\mathcal{L}'$ at step t and $y^t_{\pi_t}$ is the posterior for element π_t. So the path posterior is formulated as:

$$p(\pi|\mathbf{x}) = \prod_{t=1}^{T} p(\pi_t|\pi_1^{t-1}, \mathbf{x}) \approx \prod_{t=1}^{T} p_t(\pi_t|\mathbf{x}) = \prod_{t=1}^{T} y^t_{\pi_t} \tag{2}$$

where the context π_1^{t-1} is discarded in the approximation concerning the *conditional independence assumption* of CTC.

As demonstrated in Fig.1.a, any path represents an aligning relationship between the input sequence $\mathbf{l}$ and the target sequence $\mathbf{x}$. Commonly, the CTC posterior is peaky Zeyer et al. (2021) and the posterior of one certain path is dominantly larger than all others. To this end, the path with the dominant posterior is usually considered the predicted alignment during training: $\text{ali}(\mathbf{l}, \mathbf{x}) = \arg\max_{\pi \in \mathcal{B}^{-1}(\mathbf{l})} p(\pi|\mathbf{x})$.

2.2 FORWARD-BACKWARD ALGORITHM

Since the number of possible paths in $\mathcal{B}^{-1}(\mathbf{l})$ will grow exponentially with T and U increasing, directly enumerating all paths and their corresponding posteriors is impractical. As an alternative, the *forward-backward algorithm* enables the training objective of CTC to be computed efficiently.

The first step of the forward-backward algorithm is to extend the label sequence $\mathbf{l} = [l_1, ..., l_U]$ into an extended sequence $\mathbf{l}' = [\varnothing, l_1, ..., \varnothing, l_U, \varnothing]$ by inserting a $\varnothing$ between every two non-blank tokens as well as the start and the end of the sequence so that $|\mathbf{l}'| = 2U + 1$. Then, define the forward variable $\alpha(t, v), (1 \le t \le T, 1 \le v \le 2U + 1)$ as the summed probability of all path prefix π_1^t that are aligned with the prefix of the expanded sequence $\mathbf{l}'^v_1$; symmetrically, define the backward variable $\beta(t, v), (1 \le t \le T, 1 \le v \le 2U + 1)$ as the summed probability of all path suffix π_t^T that are aligned with the suffix of the expanded sequence $\mathbf{l}'^{2U+1}_v$:

$$\alpha(t, v) = \sum_{\substack{\pi : \mathcal{B}(\pi_1^t) = \mathcal{B}(\mathbf{l}'^v_1) \\ \pi_t = l'_v}} \prod_{t'=1}^{t} y^{t'}_{\pi_{t'}}, \qquad \beta(t, v) = \sum_{\substack{\pi : \mathcal{B}(\pi_t^T) = \mathcal{B}(\mathbf{l}'^{2U+1}_v) \\ \pi_t = l'_v}} \prod_{t'=t}^{T} y^{t'}_{\pi_{t'}} \tag{3}$$

With any fixed t and v, the summed probability of all paths whose t-th elements π_t is exactly ${l'}_v$ can be represented by the forward and backward variables as below, which is also termed as the

occupation probability:

$$\sum_{\substack{\pi \in \mathcal{B}^{-1}(\mathbf{l}) \\ \pi_t = \mathbf{l}'_v}} p(\pi|\mathbf{x}) = \sum_{\substack{\pi \in \mathcal{B}^{-1}(\mathbf{l}) \\ \pi_t = \mathbf{l}'_v}} \prod_{t'=1}^{T} y^{t'}_{\pi_{t'}} = \frac{\alpha(t,v) \cdot \beta(t,v)}{y^t_{l'_v}} \tag{4}$$

In addition, for any constant $1 \leq t \leq T$, the choices of π_t for any possible path $\pi \in \mathcal{B}^{-1}(\mathbf{l})$ are restricted to the elements in $\mathbf{l}'$ and these choices are exclusive[4]. Thus, enumerating all index v with Eq.4 will consider all possible paths and then the training objective of CTC can be written as:

$$P(\mathbf{l}|\mathbf{x}) = \sum_{\pi \in \mathcal{B}^{-1}(\mathbf{l})} p(\pi|\mathbf{x}) = \sum_{v=1}^{2U+1} \sum_{\substack{\pi \in \mathcal{B}^{-1}(\mathbf{l}) \\ \pi_t = \mathbf{l}'_v}} p(\pi|\mathbf{x}) = \sum_{v=1}^{2U+1} \frac{\alpha(t,v) \cdot \beta(t,v)}{y^t_{l'_v}} \tag{5}$$

The computation of the $\alpha(t,u)$ and $\beta(t,u)$ is recursive and the CTC gradients can also be computed in close form using the forward and backward variables. Details are presented in Appendix A.

3 Bayes risk CTC

In this part, a general formulation of the proposed Bayes risk CTC (BRCTC) criterion is presented in section 3.1. Examples about how paths can be grouped to fit the forward-backward process are presented in section 3.2. Finally, we demonstrate how the proposed BRCTC with customizable risk designs can be used to tackle two different practical problems in section 3.3 and section 3.4.

3.1 General Formulation

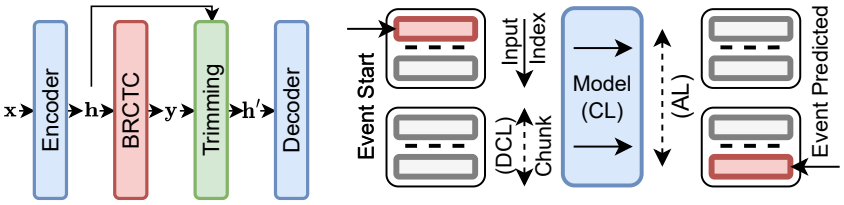
CTC prediction has two functionalities: predicting the target sequence and predicting the input-target alignment. The former is implemented by discriminating all paths $\pi \in \mathcal{B}^{-1}(\mathbf{l})$ from the sequence set ${\mathcal{L}'}^T$ since feeding each path into $\mathcal{B}$ will yield the target $\mathbf{l}$. In vanilla CTC, however, no constraint is posed to the latter since Eq.1 treats all paths equally and the choice of the dominant path, a.k.a. the alignment, is left unpredictable. To make the alignment prediction controllable is exactly to break this equality and intentionally select the desired paths among $\mathcal{B}^{-1}(\mathbf{l})$. To this end, a Bayes risk function $r(\pi)$ is adopted to describe the characteristics of the desired paths. The modified CTC objective can be written as:

$$J_{\text{brctc}}(\mathbf{l}, \mathbf{x}) = \sum_{\pi \in \mathcal{B}^{-1}(\mathbf{l})} [p(\pi|\mathbf{x}) \cdot r(\pi)] \tag{6}$$

The revised objective is termed Bayes risk CTC (BRCTC) due to the adoption of the Bayes risk function. Note when $r(\pi) = 1$, BRCTC becomes equivalent to vanilla CTC. Since BRCTC is still maximizing the posteriors of the paths in $\mathcal{B}^{-1}(\mathbf{l})$, ideally it will still reserve the models' target prediction performance.

Directly applying the risk function to each path is still prohibitive as enumerating all possible paths is still computationally impractical. To this end, the forward-backward algorithm in the vanilla CTC should be inherited to efficiently compute the BRCTC objective. Inheriting the forward-backward process will lead the design of the risk function to a divide-and-conquer paradigm and then designing the risk function is equivalent to specifying two things: 1) how the paths are divided into groups and 2) what risk values are assigned to each path group. Note the former part of grouping requires the summed posterior within a group to be computationally feasible using the forward and backward variables. Formally, the desired characteristics are about some specific properties of these paths, e.g., the largest index of all non-blank elements within the path. Note $f(\pi)$ as the concerned property of path π, then all paths that satisfy $f(\pi) = \tau$ can form a group, where τ is a possible value of the concerned property. Then, all paths in the same path group will receive the same risk value. We note $r_g(\tau)$ as a function of τ, which is the shared risk value within the group as a replacement of $r(\pi)$.

[4]Here we should assume the blank symbols in $\mathbf{l}'$ are different from each other to avoid confusion.



(a) Offline down-sampling (b) Online inference and its latency sources

Figure 2: (a) Down-sampling process using BRCTC criterion. $\mathbf{h}$ is trimmed before being fed into the decoder. (b) Inference process of the online model and its three exclusive sources of latency. DCL: data collecting latency. CL: computational latency. AL: algorithmic latency.

Thus, the BRCTC objective is to enumerate all path groups with the corresponding risk value $r_g(\tau)$ like below. A more detailed explanation is provided in Appendix B.

$$J_{\text{brctc}}(\mathbf{l}, \mathbf{x}) = \sum_{\tau} \sum_{\substack{\pi \in \mathcal{B}^{-1}(\mathbf{l}) \\ f(\pi)=\tau}} [p(\pi|\mathbf{x}) \cdot r(\pi)] = \sum_{\tau} \sum_{\substack{\pi \in \mathcal{B}^{-1}(\mathbf{l}) \\ f(\pi)=\tau}} [p(\pi|\mathbf{x}) \cdot r_g(\tau)] = \sum_{\tau} [r_g(\tau) \cdot \sum_{\substack{\pi \in \mathcal{B}^{-1}(\mathbf{l}) \\ f(\pi)=\tau}} p(\pi|\mathbf{x})] \tag{7}$$

3.2 Examples about how paths are grouped

Here we provide detailed examples of how the paths are grouped to be compatible with the forward-backward process. An off-the-shelf example is in Eq.4, in which paths can be grouped by their choices of t-th element π_t. So the occupation probability is also the summed posterior of a path group, which can be represented by the forward and backward variables.

A more complicated but useful example is to group the paths according to the ending point of a certain non-blank token. Since any path is an aligning relationship between the input and the target, it is common to ask when the prediction of each token l_u is finished within this path. Formally, with the known constant u, the concerned property of π can be defined as $\tau = f_u(\pi) = \arg\max_t(\pi_t = l_u)$ [5]. If so, the summed probability of each path group can be formulated by the forward and backward variables as below. A detailed explanation for this formulation is provided in Appendix C.

$$\sum_{\substack{\pi \in \mathcal{B}^{-1}(\mathbf{l}) \\ f_u(\pi)=\tau}} p(\pi|\mathbf{x}) = \frac{\alpha(\tau, 2u) \cdot \hat{\beta}(\tau, 2u)}{y^{\tau}_{\pi_\tau}}, \text{ s.t. } \hat{\beta}(\tau, 2u) = \begin{Bmatrix} \beta(\tau, 2u) - \beta(\tau+1, 2u) \cdot y^{\tau}_{\pi_\tau}, \text{if } \tau < T \\ \beta(\tau, 2u), \qquad\qquad \text{Otherwise} \end{Bmatrix} \tag{8}$$

Combine Eq.7, 8, for any constant u, path groups with different τ and the corresponding risk values $r_g(\tau)$ are enumerated as below. This strategy of grouping paths are adopted in section 3.3 and 3.4.

$$J_{\text{brctc}}(\mathbf{l}, \mathbf{x}) = \sum_{\tau=1}^{T} r_g(\tau) \cdot \frac{\alpha(\tau, 2u) \cdot \hat{\beta}(\tau, 2u)}{y^{\tau}_{\pi_\tau}} \tag{9}$$

3.3 Application: Down-sample

A long-standing problem for a series of offline seq2seq tasks (like ASR, ST) is that the input sequence is much longer than the output sequence, a.k.a., $|\mathbf{x}| \gg |\mathbf{l}|$[6]. If an encoder is used to process $\mathbf{x}$ into the encoder hidden output $\mathbf{h}$, this can partially be interpreted as $|\mathbf{h}| \gg |\mathbf{l}|$. The length mismatch between $\mathbf{h}$ and $\mathbf{l}$ leads to increased computational cost: for the mainstream encoder-decoder

[5] There might be some repetitions in $\mathbf{l}$, but we still consider all tokens in $\mathbf{l}$ are different for simplicity. The correlation between any non-blank π_t and l_u is clear so this notation will not lead to confusion.

[6] $|\cdot|$ is the length function.

architecture, the inference cost of the decoder is highly correlated with the length of $\mathbf{h}$. The proposed BRCTC is capable of reducing the length of the $\mathbf{h}$ to save inference costs. The workflow is shown in Fig.2.a. The non-blank spikes of CTC posterior $\mathbf{y}$ are pushed to the earlier time-stamps using BRCTC, then the posterior $\mathbf{y}$ is adopted as a reference to trim the $\mathbf{h}$ into the shorter $\mathbf{h}'$.

Pushing all non-blank spikes to the early input units requires the predicted alignment to finish its prediction of l_U as early as possible. Consider the latter situation in sec.3.2 and set $u = U$, the concerned property τ is the input index where all non-blank elements have completed in the paths. Subsequently, paths with smaller τ are more preferred so the risk values should be larger. We adopt $r_g(\tau) = e^{-\lambda \cdot \tau / T}$ as the risk function in this application, where λ is an adjustable hyper-parameter called *risk factor*. Finally, the training objective is updated as:

$$J_{\text{brctc}}(\mathbf{l}, \mathbf{x}) = \sum_{\tau=1}^{T} e^{-\lambda \cdot \tau / T} \cdot \frac{\alpha(\tau, 2U) \cdot \hat{\beta}(\tau, 2U)}{y^{\tau}_{\pi_\tau}} \tag{10}$$

Provided the continuous and highly confident blank predictions in $\mathbf{y}$ like in Fig.1.c, it is reasonable to assume the corresponding elements in $\mathbf{h}$ contain nearly no useful semantics and can be trimmed[7]. Formally, $\mathbf{h}$ is trimmed to $\mathbf{h}' = [\mathbf{h}_1, ..., \mathbf{h}_{m+D}]$, where m is the maximum value of t s.t. $\forall\ t' > t, \mathbf{y}^{t'}_{\varnothing} > 99\%$; $D = 5$ is a small integer to keep a safety margin for the trimming. The trimmed hidden output $\mathbf{h}'$ is fed into downstream architectures as a replacement of $\mathbf{h}$.

3.4 Application: performance-latency trade-off

Another problem for online seq2seq systems (e.g., streaming ASR) is the trade-off between the transcription performance and the system latency. For online systems with constrained context, better transcription quality requires more context, which, however, will result in longer latency. Assume the input sequence is fed into the system chunk-by-chunk, this paper defines the total latency as the sum of three exclusive resources as shown in Fig.2.b: **Data collecting latency (DCL)**: the time to wait before the input signal forms a chunk. This depends on the model design. **Computational latency (CL)**: the time consumed by model inference. Only this latency depends on the hardware performance. **Algorithmic latency (AL)**: the difference between the input-indexes when an event starts and when it is predicted. This is learned during the model training.

The proposed BRCTC can guide the model to emit non-blank spikes at early time-stamps so that the algorithmic latency (AL) can be reduced. As a benefit, it provides a better overall performance-latency trade-off than the vanilla CTC systems (more discussion is in section 4.3). Formally, if a non-blank token l_u is required to be emitted earlier, the concerned property τ is exactly the ending point of its prediction within the path. Thus, a tailor-made training objective for the token l_u is:

$$J'_{\text{brctc}}(\mathbf{l}, \mathbf{x}, u) = \sum_{\tau=1}^{T} e^{-\lambda \cdot (\tau - \tau') / T} \cdot \frac{\alpha(\tau, 2u) \cdot \hat{\beta}(\tau, 2u)}{y^{\tau}_{\pi_\tau}}, \text{ where } \tau' = \arg\max_{\tau} \frac{\alpha(\tau, 2u) \cdot \hat{\beta}(\tau, 2u)}{y^{\tau}_{\pi_\tau}} \tag{11}$$

The design of the group risk function $r_g(\tau)$ is still the exponential decay function but with an extra bias τ'. Without this bias, the absolute values of $J'_{\text{brctc}}(\mathbf{l}, \mathbf{x}, u)$ will be unbalanced for different token l_u.[8] To guide every token l_u to emit earlier requires the enumeration of all u. So the global training objective to maximize is then transformed into:

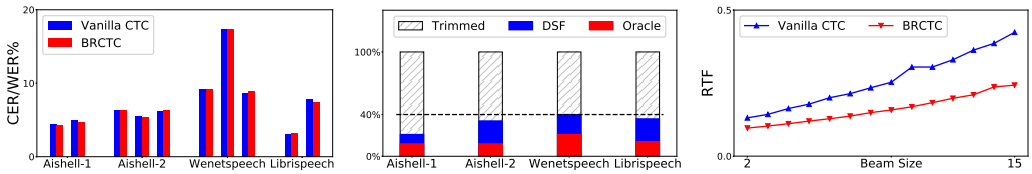
$$J_{\text{brctc}}(\mathbf{l}, \mathbf{x}) = \frac{1}{U} \cdot \sum_{u=1}^{U} \log J'_{\text{brctc}}(\mathbf{l}, \mathbf{x}, u) \tag{12}$$

4 Experiments

Our experiments are mainly designed to examine the two applications of the proposed BRCTC criterion. The experimental setup is introduced in section 4.1. The BRCTC down-sampling method

[7] Transforming $\mathbf{h}$ into $\mathbf{y}$ only adopts a simple linear classifier and the softmax function

[8] Usually, the path groups with τ being equal or close to τ' make most of the contributions in Eq.11. For different u, the τ', along with the risk values for these major path groups, is different. So different τ' leads to unbalance in the absolute values of $J'_{\text{brctc}}(\mathbf{l}, \mathbf{x}, u)$. As a remedy, taking τ' as a bias ensures that the path group with maximum posterior always receives the risk of 1.0 regardless of u.



(a) CER / WER (↓%) (b) DSF (↓) & Oracle (c) RTF (↓) vs. Beam Size

Figure 3: Transducer performance on various ASR tasks w/o BRCTC.

and performance-latency trade-off are validated in section 4.2 and section 4.3 respectively. The BRCTC is generalized to MT and ST in section 4.4. The visualization results are analyzed in section 4.5.

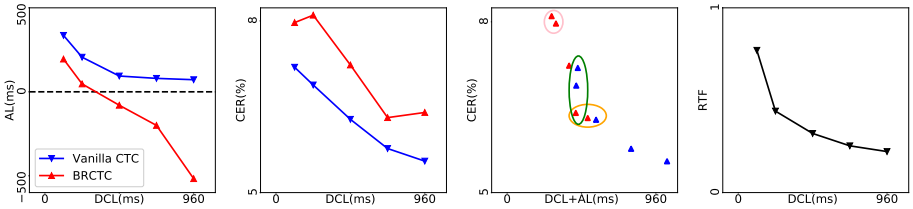
4.1 Experiment Setup

Datasets: For ASR, the experiments are conducted on Aishell-1Bu et al. (2017), Aishell-2Du et al. (2018), WenetspeechZhang et al. (2022) and LibrispeechPanayotov et al. (2015). The volumes of these datasets range from 178 hours to 10k hours. Librispeech is in English and the others are in Mandarin. For MT and ST, IWSLT14Cettolo et al. (2012) and MuST-C-v2 En-DeDi Gangi et al. (2019) are adopted.
Models: For all tasks, the Hybrid CTC/Attention architectureWatanabe et al. (2017) is evaluated. For offline ASR, the TransducerGraves (2012) plus CTC criterion is additionally evaluated.
Training and Decoding: For training, a two-stage method is proposed for the offline down-sampling application and vanilla CTC is directly replaced by the BRCTC in the online systems. For offline decoding, default algorithms with decoder recurrence Watanabe et al. (2017); Graves (2012); Yan et al. (2022) are used to demonstrate the reduction of inference cost; for online decoding, CTC greedy search is adopted to better measure the transcription performance and latency. Our BRCTC implementation depends on the differentiable finite-state transducer Hannun et al. (2020).
Evaluation Metrics: To compare the transcription performance, CER (for Mandarin) and WER (for English) are reported for ASR task; detokenized case-sensitive BLEUPost (2018) is reported for MT and ST tasks. To compare the computational cost of offline models, the real-time factor (RTF), the down-sampling factor (DFS, a.k.a., $|\mathbf{h}'|/|\mathbf{h}|$) and its oracle (a.k.a., $|\mathbf{l}|/|\mathbf{h}|$) are reported. To analyze the latency, time is measured for hardware-independent latency while the RTF is the indicator of hardware latency. All details of the experimental setup are in appendix D and E for reproducibility.

4.2 Results on BRCTC down-sampling

This part evaluates the effectiveness of BRCTC down-sampling method on ASR task. Our main results of the Transducer plus CTC model are plotted in Fig.3 and the tendency for the hybrid CTC/attention model is similar. The complete results are in appendix F.

Firstly, the transcription performance of the Transducers plus CTC or BRCTC is reported in Fig 3.a. As suggested, with examining datasets in varying volumes and different languages, the adoption of BRCTC does not result in noticeable variance in the transcription quality. Secondly, Fig 3.b. demonstrates the effectiveness of the down-sampling process. The encoder hidden output $\mathbf{h}$ is trimmed by at least 60% and the trimmed $\mathbf{h}'$ is roughly as 2∼2.5 times long as the $\mathbf{l}$ only. So the length mismatch of input and output sequences is significantly alleviated. Thirdly, since the inference cost of decoder recurrence highly depends on the length of $\mathbf{h}$, replacing $\mathbf{h}$ by $\mathbf{h}'$ will reduce the inference cost remarkably. For the label-synchronous decoding of hybrid CTC/AttentionWatanabe et al. (2017), the computational cost of each decoding step is reduced since the expense of attention computation will be smaller. For the frame-synchronous decoding of TransducersGraves (2012), the inference cost is reduced since the number of frames in decoding is reduced linearly along with $|\mathbf{h}|$. Since the inference cost of the decoder is sensitive to the beam size, we sweep the beam size from 2 to 15 on Librispeech test-other set. As Fig.3.c. suggests, the relative inference cost reduction becomes larger with the beam size growing.



(a) DCL - AL (b) DCL - CER (c) DCL+AL - CER (d) DCL - RTF

Figure 4: Aishell-1 trade-off between transcription performance and latency w/o BRCTC.

4.3 Results on BRCTC performance-latency trade-off

This part evaluates the trade-off between the transcription performance and the latency w/o the adoption of BRCTC on streaming ASR task. Our main observation is shown in Fig.4. Fig.4.a reflects the trade-off between the DCL and AL. Firstly, the models designed with lower latency (smaller DCL) will encounter larger AL since the models are not confident with the highly restricted context and will wait for longer input before decisions. This DCL-AL relationship suggests the system with extremely low latency is not feasible by only designing low DCL. Secondly, the adoption of BRCTC achieves consistent AL reduction since it tries to push the spikes of all non-blank tokens to the left. Thirdly, the AL gap between vanilla CTC and BRCTC increases along with DCL increasing, since a longer chunk provides a higher performance ceiling for BRCTC while the AL for vanilla CTC is always larger than 0ms.

Fig.4.b reflects the trade-off between the DCL and CER. As expected, models with smaller receptive fields (smaller DCL) will degrade more in their transcription quality. Also, the adoption of BRCTC results in consistent CER reduction compared with its baseline, since earlier emissions will also restrict the accessible context.

Summarizing Fig.4.a&b leads to Fig.4.c, in which the trade-off between all hardware-independent latency (DCL + AL) and the transcription quality (CER) is demonstrated. The adoption of BRCTC provides several benefits: 1) With similar CER, BRCTC leads to lower latency (the orange circle); 2) with a similar latency budget, the BRCTC achieves lower CER (the blue circle); 3) The adoption of BRCTC makes it possible to build the system with extremely low latency (reduced from 446ms to 287ms, the pink circle), which is not feasible for vanilla CTC due to the seesaw-like relationship between DCL and AL. To sum up, BRCTC provides an alternative solution for online applications: adopting a larger DCL design and reducing the AL by BRCTC to achieve comparable total latency.

The alternative solution aforementioned also provides an extra benefit from the perspective of hardware. As suggested by Chen et al. (2021) and also presented in Fig.4.d, a larger chunk size (larger DCL) can better explore the advantage of parallel computing and then achieve lower RTF. This means the hardware may have more time being idle or switching among multiple processes during serving. Using larger chunk size will slightly increase CL, but our observation from Fig.4.c still holds. Appendix G provides more detailed results.

4.4 Generalizing BRCTC to MT and ST

4.5 Visualization

Following Graves et al. (2006), Fig. 5 compares the evolution of the CTC distributions and the their gradient. At the beginning of training, the distribution is roughly unified and the gradient is smooth along the time-axis. Note the gradient for all non-blank tokens is similar only except for the last token (the green line and the red arrow): for the last token, the BRCTC gradient for a larger frame index will be penalized in order to push the last token to the left. After two epochs of training, the gradient will localize. Although the models are not sure what the non-blank tokens exactly are, the places where the non-blank tokens will spike are roughly determined. At this stage, the BRCTC model has already known the spikes should all happen at the very left. After convergence, the gradient is close to zero and the CTC distributions become peaky. For BRCTC, the spikes are all

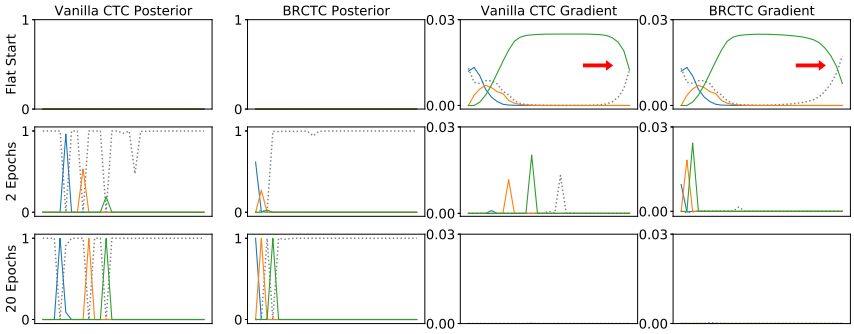


Figure 5: Evolution of CTC distribution $\mathbf{y}$ and the corresponding gradients on $\log \mathbf{y}$. BRCTC with the down-sampling method is used.

concentrated on the left as desired. We also find the down-sampling process is implemented mainly by the last encoder layer. More visualization is provided in Appendix H.

5 Discussion

Correlation between target prediction and alignment prediction: This work observes that the target prediction and alignment prediction are not tightly coupled. Firstly, since all paths in Eq.1 are treated equally in vanilla CTC and the convergence of vanilla CTC can be achieved with any path being dominant, there are multiple solutions for the alignment prediction sub-task during optimization. In addition, selecting each of these solutions can hardly interfere with the transcription quality, since each of the paths will yield the correct target prediction. Secondly, our experiments also suggest that CTC systems with unreasonable alignment prediction can still reserve the transcription quality: systems even trained with vanilla CTC are predicting the alignment that drifts significantly (see Appendix I); experimental results in sec.4.2 further demonstrate that BRCTC can achieve competitive transcription results like vanilla CTC with extremely unreasonable alignment prediction.
Monotonic assumption, MT & ST tasks and rearranging ability: An implicit assumption of CTC is the monotonic assumption, which requires that, if any x_t and $x_{t'}$ are mapped to two non-blank tokens l_u and $l_{u'}$ respectively and $t < t'$, then there must have $u < u'$. This assumption restricts CTC from being applied to seq2seq tasks whose alignment is not monotonic, like MT and ST. However, this problem is alleviated in Yan et al. (2022) due to the rearranging ability of the models: the semantics of $\mathbf{h}$ are reordered to be roughly monotonic with respect to $\mathbf{l}$ so that what the CTC attempts to predict is exactly the alignment between $\mathbf{h}$ and $\mathbf{l}$. Besides, our method in sec.3.3 can also be viewed as a process to rearrange the semantics of $\mathbf{h}$ even though the relative order of non-blank spikes is kept. To sum up, although this work mainly addresses the CTC training criterion, we believe the rearranging ability of the models is a key factor in BRCTC's functionality.

6 Related Works

The proposed BRCTC is an extension of CTC. There are several existing criteria and frameworks that can be partially viewed as CTC extensions. To alleviate the *conditional independence assumption* of CTC yields Transducer Graves (2012) and RNA Sak et al. (2017); to equip CTC with discriminative ability yields CTC-MMI Hadian et al. (2018); to exploit the non-auto-regressive nature of CTC yields the non-auto-regressive ASRHiguchi et al. (2020) and MTQian et al. (2021) architectures; to alleviate the ambiguity of blank symbol yields CTC with revised topology Zhao & Bell (2022). None of the works aforementioned attempts to control the alignment prediction.

Some pioneer works can be interpreted as preliminary attempts to control CTC alignment prediction. Ghorbani et al. (2018); Kurata & Audhkhasi (2019) achieve aligned CTC posterior spikes among multiple heterogeneous models by learning the spike time-stamps of a teacher model. However, the spikes of the teacher model itself cannot be controlled. Zeyer et al. (2020) improve CTC and RNN-T models by injecting external alignments, but obtaining these alignment consumes extra effort. By

contrast, the proposed BRCTC is customizable for general purposes and depends on neither teacher models nor additional information.

Previous literature has also discussed the basic properties of CTC from multiple perspectives, like its peaky behavior Zeyer et al. (2021), alignment drift Sak et al. (2015) and the properties of blank symbol Zhao & Bell (2022); Bluche et al. (2015). To our knowledge, The correlation between the target prediction and alignment prediction has not been seriously discussed before this work.

7 LIMITATIONS

The proposed BRCTC has two known limitations. The gradient of BRCTC is obtained by the naive chain rule (as the forward-backward process only adopts addition and multiplication operations, this is feasible). The gradient computation needs to trace back the forward-backward process, which then results in an increase in the training cost. Second, applying the method proposed in section 3.4 to the English dataset (Librispeech) has no effect on selecting the alignment with small λ and encounters divergence with large λ. We suspect this method might be sensitive to the languages.

8 CONCLUSION

This work is motivated by our experimental observation that a CTC system with ill alignment prediction still reserves competitive transcription ability, which inspires us the possibility of making CTC alignment prediction controllable to serve other task-specific needs. An extension of CTC called Bayes risk CTC is then proposed to select the dominant path among all possible paths and the design of the risk function is left customizable to fulfill the task-specific needs of different applications. For the offline model, the adoption of BRCTC leads to inference cost reduction since the length of the intermediate variable is reduced. For the online model, BRCTC provides a better performance-latency trade-off. We verify the effectiveness of the proposed BRCTC on multiple sequence-to-sequence tasks with various datasets and model architectures.

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A APPENDIX: DETAILS OF THE FORWARD-BACKWARD PROCESS AND THE CLOSE-FORM GRADIENT OF VANILLA CTC

This appendix describes the recursive computation of the forward-backward algorithm and the close form gradient of vanilla CTC.

For the forward variable $\alpha(t, v)$, the recursive process is:

$$\alpha(t,v) = \begin{cases} [\alpha(t-1,v) + \alpha(t-1,v-1)] \cdot y^t_{l'_v}, & \text{if } l'_v = \varnothing \text{ or } l'_v = l'_{v-2} \\ [\alpha(t-1,v) + \alpha(t-1,v-1) + \alpha(t-1,v-2)] \cdot y^t_{l'_v}, & \text{Otherwise} \end{cases} \tag{13}$$

with the initial condition:

$$\alpha(1,1) = y^1_{\varnothing}; \qquad \alpha(1,2) = y^1_{l'_2}; \qquad \alpha(1,v) = 0, \quad \forall v > 2 \tag{14}$$

Symmetrically, for the backward variable $\beta(t, v)$, the recursive process is:

$$\beta(t,v) = \begin{cases} [\beta(t+1,v) + \beta(t+1,v+1)] \cdot y^t_{l'_v}, & \text{if } l'_v = \varnothing \text{ or } l'_v = l'_{v+2} \\ [\beta(t+1,v) + \beta(t+1,v+1) + \beta(t+1,v+2)] \cdot y^t_{l'_v}, & \text{Otherwise} \end{cases} \tag{15}$$

with the initial condition:

$$\beta(T,2U+1) = y^T_{\varnothing}; \qquad \beta(T,2U) = y^T_{l'_{2U}}; \qquad \beta(T,v) = 0, \quad \forall v < 2U \tag{16}$$

After all forward and backward variables are computed, the CTC gradient can be computed in close form. Firstly, for the posterior of any path π, its gradient w.r.t. the y^t_k is:

$$\frac{\partial p(\pi|\mathbf{x})}{\partial y^t_k} = \frac{\partial \prod_{t'=1}^{T} y^{t'}_{\pi_{t'}}}{\partial y^t_k} = \begin{cases} (\prod_{t'=1}^{t-1} y^{t'}_{\pi_{t'}}) \cdot (\prod_{t'=t+1}^{T} y^{t'}_{\pi_{t'}}) & \text{if } k = \pi_t \\ 0 & otherwise \end{cases} \tag{17}$$

Then consider the occupation probability in Eq.4 and include the two products above into the forward and backward variables, its gradient w.r.t. the output y^t_k is:

$$\frac{\partial \sum_{\pi \in \mathcal{B}^{-1}(\mathbf{l}); \pi_t = l'_v} p(\pi|\mathbf{x})}{\partial y^t_k} = \begin{cases} \frac{1}{{y^t_k}^2} \cdot \alpha(t,v) \cdot \beta(t,v) & \text{if } k = \pi_t \\ 0 & otherwise \end{cases} \tag{18}$$

Finally, enumerate all v like in Eq.5, the gradient of CTC is computed as:

$$\frac{p(\mathbf{l}|\mathbf{x})}{\partial y^t_k} = \frac{1}{{y^t_k}^2} \cdot \sum_{v \in lab(\mathbf{l}',k)} \alpha(t,v) \cdot \beta(t,v) \tag{19}$$

where $lab(\mathbf{l}', k)$ indicates where the k occurs: $lab(\mathbf{l}', k) = \{v : l'_v = k\}$

B FURTHER EXPLANATION OF BRCTC GENERAL FORMULATION

This part provides a more detailed explanation of the BRCTC general formulation. The equations are provided below.

$$
\begin{aligned}
J_{\text{brctc}}(\mathbf{l}, \mathbf{x}) &= \sum_{\pi \in \mathcal{B}^{-1}(\mathbf{l})} [p(\pi|\mathbf{x}) \cdot r(\pi)] \\
&= \sum_{\tau} \sum_{\substack{\pi \in \mathcal{B}^{-1}(\mathbf{l}) \\ f(\pi)=\tau}} [p(\pi|\mathbf{x}) \cdot r(\pi)] \\
&= \sum_{\tau} \sum_{\substack{\pi \in \mathcal{B}^{-1}(\mathbf{l}) \\ f(\pi)=\tau}} [p(\pi|\mathbf{x}) \cdot r_g(\tau)] \\
&= \sum_{\tau} [r_g(\tau) \cdot \sum_{\substack{\pi \in \mathcal{B}^{-1}(\mathbf{l}) \\ f(\pi)=\tau}} p(\pi|\mathbf{x})]
\end{aligned} \tag{20}
$$

The first line is the original definition of BRCTC, in which each path π along with its risk value $r(\pi)$ is considered and the product is summed for all paths. The second line is to group the paths with identical concerned property τ. In this stage, the risk value for each path is still $r(\pi)$. The third line suggests the risk values for all paths within a group are identical and only depend on the concerned property τ. So the risk value is changed from $r(\pi)$ to $r_g(\tau)$ within the group. The final line is to extract the common factor $r_g(\tau)$ out of the summation within the group. This suggests we can first compute the summed posterior of all paths within the group and then apply the risk value for that group in one go.

Example: we further provide a naive example to illustrate how paths are grouped and how the risk values are assigned to each group. Assume the input length is 3 (index from 1 to 3) and the target sequence is [*A, B*]. Then, all possible paths and their posteriors are listed in the first and the second rows of the table below respectively. Note the summation of all path posteriors are not equal to 1 since some other sequences also have non-zero posterior (e.g., AAA).

We also assume the concerned property $\tau = f(\pi)$ is the time-stamp when the prediction of token *B* finishes. Thus, the value of the concerned property for each path is listed in the third row of the table below.

Paths with the identical τ value should form a group. So the path AB$\varnothing$ forms a group with a single element while the remained paths form another group. For paths within a group, the risk value should be identical. The risk value for each group $r_g(\tau)$ is user-defined. We simply set $r_g(2) = 1.0$ and $r_g(3) = 0.8$. Then the risk for each path $r(\pi)$ is then set in the fourth row of the table below.

Path(π)	AB$\varnothing$	ABB	AAB	A$\varnothing$B	$\varnothing$AB
p(π)	0.3	0.1	0.2	0.0	0.1
τ	2	3	3	3	3
$r(\pi)$ or $r_g(\tau)$	1.0	0.8	0.8	0.8	0.8

Finally, formulate all these processes in equations will be:

$$
\begin{aligned}
\mathbf{J}_{\text{brctc}} &= p(AB\varnothing) \cdot r(AB\varnothing) + p(ABB) \cdot r(ABB) + p(AAB) \cdot r(AAB) \\
&\quad + p(A\varnothing B) \cdot r(A\varnothing B) + p(\varnothing AB) \cdot r(\varnothing AB) \quad \#\textit{Initial formulation} \\
&= [p(ABB) \cdot r(ABB) + p(AAB) \cdot r(AAB) + p(A\varnothing B) \cdot r(A\varnothing B) + p(\varnothing AB) \cdot r(\varnothing AB)] \\
&\quad + [p(AB\varnothing) \cdot r(AB\varnothing)] \quad \#\textit{grouping} \\
&= [p(ABB) \cdot r_g(3) + p(AAB) \cdot r_g(3) + p(A\varnothing B) \cdot r_g(3) + p(\varnothing AB) \cdot r_g(3)] \\
&\quad + [p(AB\varnothing) \cdot r_g(2)] \quad \#\textit{replace } r(\pi) \textit{ by } r_g(\tau) \\
&= [p(ABB) + p(AAB) + p(A\varnothing B) + p(\varnothing AB)] \cdot r_g(3) \\
&\quad + [p(AB\varnothing)] \cdot r_g(2) \quad \#\text{extract common factors} \\
&= (0.1 + 0.2 + 0.0 + 0.1) \cdot 0.8 + 0.3 \cdot 1.0 = 0.62
\end{aligned} \tag{21}
$$

C APPENDIX: THE EXPLANATION FOR GROUPING STRATEGY FORMULATION

The property function is set to $\tau = f_u(\pi) = \arg\max_t(\pi_t = l_u)$. Note $l_u = l'_{2u}$.

Firstly, for the scenario $\tau < T$, the property function can be translated into the condition $\pi_\tau = l'_{2u}$ and $\pi_{\tau+1} \neq l'_{2u}$. Thus, the summed probability of the path group is formulated as:

$$\begin{aligned}\sum_{\substack{\pi\in\mathcal{B}^{-1}(\mathbf{l})\\ f_u(\pi)=\tau}} p(\pi|\mathbf{x}) &= \sum_{\substack{\pi\in\mathcal{B}^{-1}(\mathbf{l})\\ \pi_\tau=l'_{2u};\pi_{\tau+1}\neq l'_{2u}}} p(\pi|\mathbf{x}) = \sum_{\substack{\pi\in\mathcal{B}^{-1}(\mathbf{l})\\ \pi_\tau=l'_{2u};\pi_{\tau+1}\neq l'_{2u}}} \prod_{t'=1}^{T} y^{t'}_{\pi_{t'}} \\ &= \sum_{\substack{\pi\in\mathcal{B}^{-1}(\mathbf{l})\\ \pi_\tau=l'_{2u}}} \prod_{t'=1}^{T} y^{t'}_{\pi_{t'}} - \sum_{\substack{\pi\in\mathcal{B}^{-1}(\mathbf{l})\\ \pi_\tau=l'_{2u};\pi_{\tau+1}=l'_{2u}}} \prod_{t'=1}^{T} y^{t'}_{\pi_{t'}} \\ &= \frac{\alpha(\tau,2u)\cdot\beta(\tau,2u)}{y^{\tau}_{\pi_\tau}} - \alpha(\tau,2u)\cdot\beta(\tau+1,2u)\end{aligned}$$

Set:

$$\hat{\beta}(\tau,2u) = \beta(\tau,2u) - \beta(\tau+1,2u)\cdot y^{\tau}_{\pi_\tau} \tag{22}$$

The summed probability then becomes:

$$\sum_{\substack{\pi\in\mathcal{B}^{-1}(\mathbf{l})\\ f_u(\pi)=\tau}} p(\pi|\mathbf{x}) = \frac{\alpha(\tau,2u)\cdot\hat{\beta}(\tau,2u)}{y^{\tau}_{\pi_\tau}} \tag{23}$$

Secondly, $\tau = T$ suggests that:

$$\sum_{\substack{\pi\in\mathcal{B}^{-1}(\mathbf{l})\\ f_u(\pi)=\tau}} p(\pi|\mathbf{x}) = \alpha(\tau,2u) = \frac{\alpha(\tau,2u)\cdot\beta(\tau,2u)}{y^{\tau}_{\pi_\tau}} \tag{24}$$

To sum up, the summed probability of the path group with the given τ and u is

$$\sum_{\substack{\pi\in\mathcal{B}^{-1}(\mathbf{l})\\ f_u(\pi)=\tau}} p(\pi|\mathbf{x}) = \frac{\alpha(\tau,2u)\cdot\hat{\beta}(\tau,2u)}{y^{\tau}_{\pi_\tau}} \tag{25}$$

with

$$\hat{\beta}(\tau,2u) = \begin{Bmatrix}\beta(\tau,2u) - \beta(\tau+1,2u)\cdot y^{\tau}_{\pi_\tau}, \text{if } \tau < T \\ \beta(\tau,2u), \qquad\qquad \text{Otherwise}\end{Bmatrix} \tag{26}$$

D APPENDIX: THE TWO-STAGE METHOD FOR BRCTC DOWN-SAMPLING

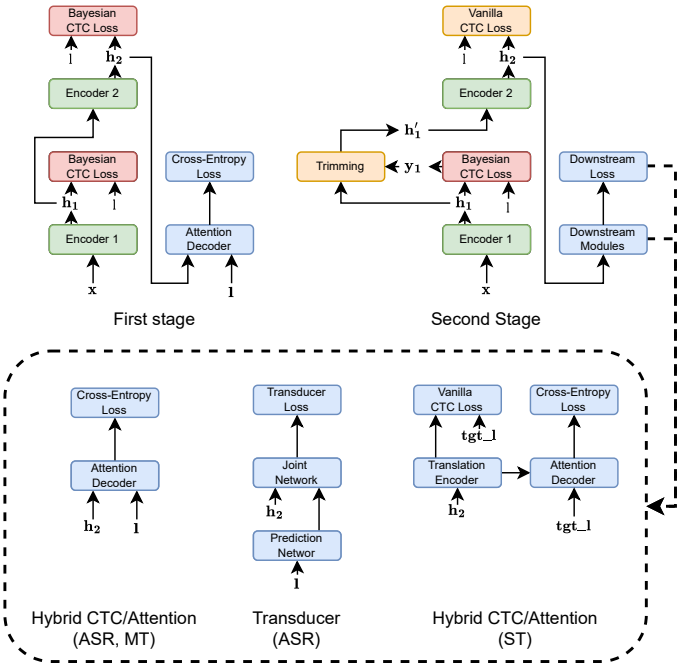


Figure 6: An illustration of the proposed two-stage method for BRCTC down-sampling.

This part describes the proposed two-stage method for BRCTC down-sampling. Details are described in Fig.6. The first stage mainly follows the architecture in Watanabe et al. (2017): the encoder-decoder model is jointly optimized by both CTC loss and cross-entropy loss. The decoder part is adopted in order to be reused in the second stage. Intermediate CTC techniqueLee & Watanabe (2021) is consistently adopted so that the whole encoder is split into two parts with an identical number of layers and the hidden outputs from both the intermediate layer (a.k.a., $\mathbf{h_1}$) and the final layer (a.k.a., $\mathbf{h_2}$) are supervised by the CTC criterion. In this stage, all CTC criteria are our proposed BRCTC.

In the second stage, the weights of the two encoder parts and the CTC classifier are reserved, but several things are changed. First, the $\mathbf{h_1}$ is trimmed into $\mathbf{h'_1}$ using the $\mathbf{y_1}$ as the reference, so the length of the intermediate hidden output is reduced. Second, the CTC loss for $\mathbf{h_2}$ is the vanilla CTC loss since $\mathbf{h_2}$ does not need to be trimmed anymore. Thirdly, the $\mathbf{h_2}$ is then fed into downstream modules and losses.

The design of downstream modules and losses depends on the specific tasks and system architectures. For ASR and MT tasks with the Hybrid CTC/Attention architecture, the attention decoder and the cross-entropy loss are adopted and the weight can be inherited from the first stage. For ASR with the Transducer architecture, the prediction network and the joint network are randomly initialized and the Transducer loss is adopted. For the ST task with the Hybrid CTC/Attention architecture, the translation encoder and the attention decoder are randomly initialized and two extra losses are adopted: the vanilla CTC loss and the cross-entropy loss. Note in ST tasks, the labels from both the source language and the target language are provided. CTC loss for Encoder 1 and Encoder 2 still adopts the source language labels like in the first stage, but the translation encoder and the attention decoder are supervised by the target language labels ($\mathbf{tgt_l}$).

Note that, in the second stage, it is possible to use vanilla CTC or BRCTC consistently. However, when vanilla CTC is consistently adopted, the down-sampling factor will degrade gradually along with the second stage training; when BRCTC is consistently adopted, there is a slight performance degradation observed. All hyper-parameters of this proposed two-stage method is presented in Appendix E.

E APPENDIX: REPRODUCIBILITY

We describe the details of all experiments for reproducibility.

- Datasets: all statistics of the datasets are in table 1.
- Architectures: all model architectures are presented in table 2.
- Training and BRCTC settings: AdamKingma & Ba (2014) optimizer with the learning rate being inverse square root decayingVaswani et al. (2017) is adopted. Details for the optimization and the setting for BRCTC are shown in table 3.
- Decoding: For the online ASR model, we adopt the greedy CTC decoding. For all other offline models, the decoding configurations are in table 4.

Dataset	Task	#Hours	#Pairs	Language	Units
Aishell-1Bu et al. (2017)	ASR	178	120k	Mandarin	4231 Char.
Aishell-2Du et al. (2018)	ASR	1k	962k	Mandarin	5214 Char.
WenetspeechZhang et al. (2022)	ASR	10k	14M	Mandarin	6267 Char.
LibrispeechPanayotov et al. (2015)	ASR	960	281k	English	500 BPE
IWSLT De-EnCettolo et al. (2012)	MT	-	160k	German- English	10k BPE (shared)
IWSLT Es-EnCettolo et al. (2012)	MT	-	160k	Spanish- English	10k BPE (shared)
MuST-C-V2 En-DeDi Gangi et al. (2019)	ST	450	250k	English- German	4k BPE / 4k BPE

Table 1: Dataset description of ASR / MT / ST tasks.

Acoustic input features: FBank + Pitch			
Frame_length	25ms	Frame_shift	10ms
Fbank_dim	80	Pitch_dim	3
Text input: Embedding			
Embedding_size	1024		
ASR, ST offline encoder & ST translation encoder: ConformerGulati et al. (2020):			
Num_layer	12	Num_attention_head	4
Attention_dim	512 (256 for ST)	Feed_forward_dim	2048
Num_CNN_module_kernel	31	CNN_down_sample	4x
MT offline encoder: LegoNN_EncoderDalmia et al. (2022):			
Num_layer_before_up-sampling	6	Num_layer_after_up-sampling	6
Attention_dim	512	Feed_forward_dim	1024
Num_attention_head	4	Up-sampling_rate	3
ASR online encoder: EmformerShi et al. (2021):			
Num_layer	12	Num_attention_head	4
Attention_dim	512	Feed_forward_dim	2048
Memory_bank_length	4	Left_context	320ms
Chunk_size	[80, 160, 320, 480, 640]ms	Right_context (lookahead)	[80, 160, 320, 480, 640]ms
Data collecting latency (DCL)	[120, 240, 480, 720, 960]ms		
ASR, MT, ST Attention decoder for hybrid CTC/attention architecture: TransformerVaswani et al. (2017):			
Num_layer	6	Num_attention_head	4
Attention_dim	512	Feed_forward_dim	2048
Offline decoder for TransducerGraves (2012) architecture:			
Prediction_network	LSTM	LSTM_hidden_size	512
LSTM_num_layer	1	Joint_network	Linear
Joint_network_dim	512		
Hybrid CTC/attentionWatanabe et al. (2017) architecture:			
CTC_loss_weight	0.3	Attention_loss_weight	0.7
Attention_label_smooth	0.1		
TransducerGraves (2012) + CTC architecture:			
CTC_loss_weight	0.5	Transducer_loss_weight	1.0
Others:			
Inter. CTC Lee & Watanabe (2021)	6-th	Inter. CTC weight	0.3
Dropout_rate	0.1 for ASR & ST; 0.3 for MT		

Table 2: Model architecture description

Dataset	λ (offline)	λ (online)	#epochs	peak lr	#warmup iter.	#GPU	Max Global Batch size
Aishell-1	10	20	50+50	3e-4	25k	8	400 seconds
Aishell-2	10	20	50+50	3e-4	25k	8	400 seconds
Wenetspeech	30		50+50	3e-4	25k	32	3200 seconds
Librispeech	100	-	50+50	3e-4	25k	8	400 seconds
IWSLT		-			25k	4	
MuST-C-V2		-			25k	8	

Table 3: Optimization strategy and BRCTC settings. Epochs 50+50 means 50 epochs for both the first and the second stages. If there is only one stage, the number of epochs is the sum.

Dataset	Beam Size	CTC weight	Attention weight	Length reward
Aishell-1	10	0.5	0.5	0
Aishell-2	10	0.5	0.5	0
Wenetspeech	10	0.5	0.5	0
Librispeech	10	0.5	0.5	0
IWSLT	5	0.3	0.7	0.6
MuST-C-V2	10	0.3	0.7	0.3

Table 4: Decoding configurations

We then provide the rigorous definitions of the three latency resources in section 3.4. All kinds of latency are computed as the expected value. Note we are using the EmformerShi et al. (2021), in which the each input consists of *left_context*, *chunk* and *right_context*. The left context is the observed history and is irrelevant to the latency. The chunk is the current context. The adoption of the right context allows some look-ahead.

- Data Collecting Latency (DCL): $DCL = chunk_size/2 + right_context$, which is the mean time to wait for frames within a chunk.
- Computational Latency (CL): $CL = chunk_size * RTF$, which represents the rough inference time for a chunk data.
- Algorithmic Latency (AL): $AL = \tau - \hat{\tau}$, where τ is the ending time-stamp of token prediction in the path; $\hat{\tau}$ is the starting time-stamp of that token obtained by DNN-HMM systems.

F APPENDIX: DETAILED EXPERIMENTAL RESULTS ON OFFLINE DOWN-SAMPLING

System	Aishell-1 178h, Mandarin dev / test	Aishell-2 1kh, Mandarin android / ios / mic	Wenetspeech 10kh, Mandarin dev / meeting / net	Librispeech 960h, English t-clean / t-other
Attention + CTC	**4.26 / 4.74**	6.33 / 5.48 / 6.29	**9.44** / **15.97** / 9.11	3.02 / **7.72**
Attention + BRCTC	4.30 / 4.75	**6.13 / 5.34 / 6.03**	9.59 / 16.86 / **9.04**	3.15 / **7.63**
Transducer + CTC	4.47 / 4.92	6.39 / 5.47 / **6.18**	**9.20 / 17.34 / 8.61**	**3.05** / 7.79
Transduder + BRCTC	**4.33 / 4.73**	**6.36** / **5.35** / 6.30	**9.20** / 17.41 / 8.87	3.14 / **7.41**

Table 5: ASR results on the models' transcription performance w/o BRCTC down-sampling method. All models adopt auto-regressive decoding algorithms. CER/WER (↓) are reported. The beam size is set to 10 consistently

System	Aishell-1 test		Aishell-2 test-ios		Wenetspeech test-net		Librispeech t-other	
	RTF	DSF / Oracle	RTF	DSF / Oracle	RTF	DSF / Oracle	RTF	DSF / Oracle
Attention + CTC	1.19	-	1.07	-	1.91	-	1.97	-
Attention + BRCTC	**0.94**	0.21 / 0.12	**0.95**	0.34 / 0.12	**1.53**	0.37 / 0.21	**1.38**	0.35 / 0.14
Transducer + CTC	0.37	-	0.42	-	0.48	-	0.31	-
Transducer + BRCTC	**0.21**	0.20 / 0.12	**0.22**	0.29 / 0.12	0.33	**0.40 / 0.21**	0.18	**0.36 / 0.14**

Table 6: Evaluations results on the models' inference cost w/o BRCTC down-sampling method. The real-time factor (RTF ↓), the down-sampling factor (DSF ↓) and its oracle (in the parentheses) are reported. The beam size is set to 10 consistently

G APPENDIX: DETAILED EXPERIMENTAL RESULTS ON ONLINE PERFORMANCE-LATENCY TRADE-OFF

λ	DCL(ms)	AL(ms)	CER(%)	DCL+AL(ms)	RTF	CL(ms)	DCL+AL+CL(ms)
	Aishell-1 test						
0	120	336	7.19	456	0.770	46	502
	240	206	6.88	446	0.441	52	498
	480	94	6.28	574	0.320	76	650
	720	80	5.77	800	0.253	91	891
	960	72	5.55	1032	0.222	106	1138
20	120	195	7.97	315	0.770	46	361
	240	47	8.10	287	0.441	52	339
	480	-80	7.23	400	0.320	76	476
	720	-200	6.31	520	0.253	91	611
	960	-517	6.40	443	0.222	106	549
λ	Aishell-2 test-android						
0	120						
	240						
	480						
	720						
	960						
20	120						
	240						
	480						
	720						
	960						

Table 7: Trade-off between the transcription performance and the latency for online models. DCL: data collecting latency; AL: algorithmic latency; CL: computational latency; CER: character error rate; RTF: real-time factor.

H APPENDIX: MORE VISUALIZATION

Fig.7 compares the evolution of the CTC distributions and their gradients on the online performance-latency trade-off application. Most of the observations are similar to those in Fig.5 except 1) at the beginning of training, the gradients for each non-blank token, rather than the last non-blank token only, are interfered with by the adoption of BRCTC. The gradient peaks of BRCTC are shifted to the left. 2) after 2 epochs of training, the gradients will also be localized, but BRCTC has not learned the places where the emission will happen. After convergence, the emissions of BRCTC will be earlier. We add the input indexed to show its difference from vanilla CTC.

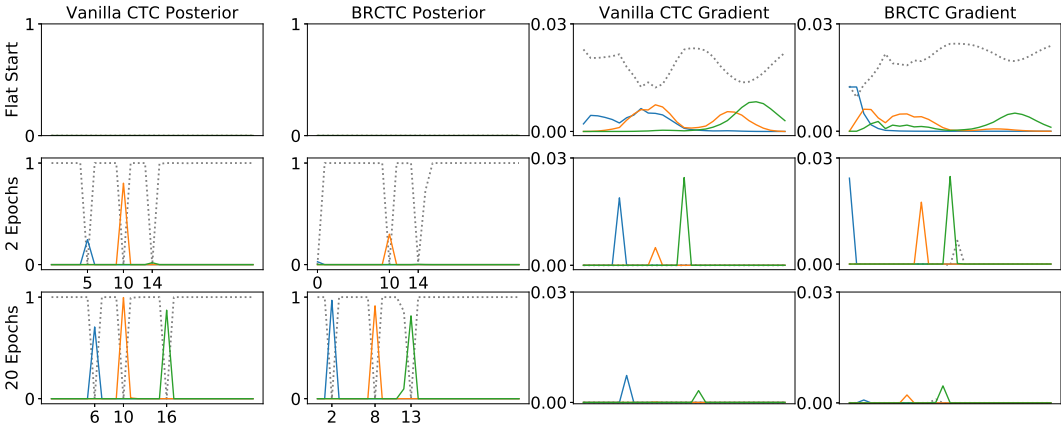


Figure 7: Evolution of CTC distribution $\mathbf{y}$ and the corresponding gradients on $\log \mathbf{y}$. BRCTC with emission latency alleviation method is used.

Fig.8 demonstrate the encoder self-attention from different layers and attention heads. As shown in the figure, the down-sampling process is completed only in the last layer before the BRCTC criterion (the 6-th layer). The attention weights shown in the red boxes of the 6th layer suggest how the semantics of input units with large input indexes are aggregated to the output units with small indexes. This observation is only in the layer before BRCTC so we assume the global context is fully explored in other layers.

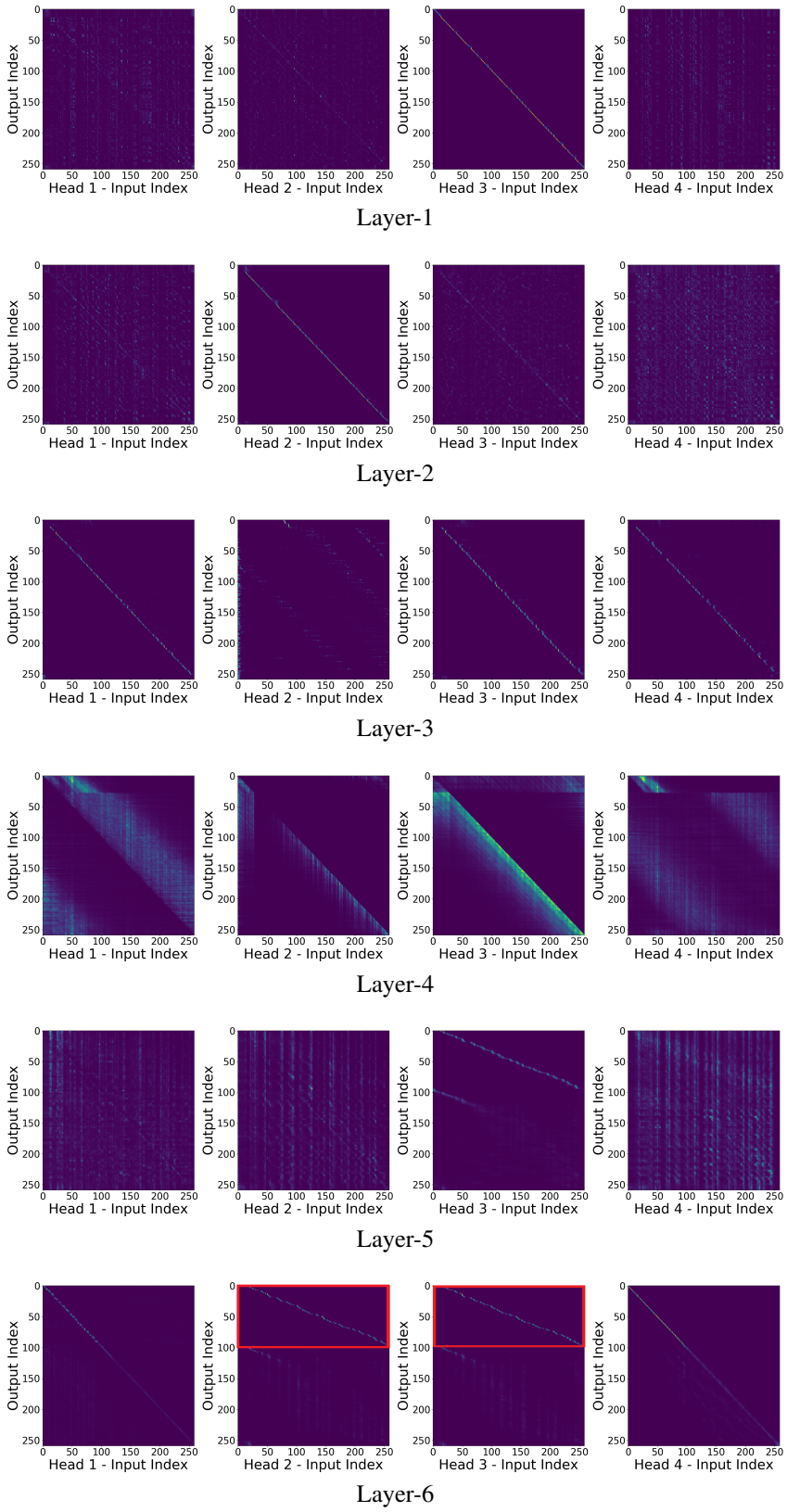
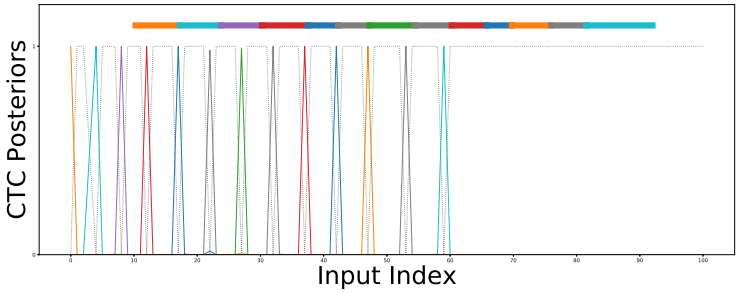


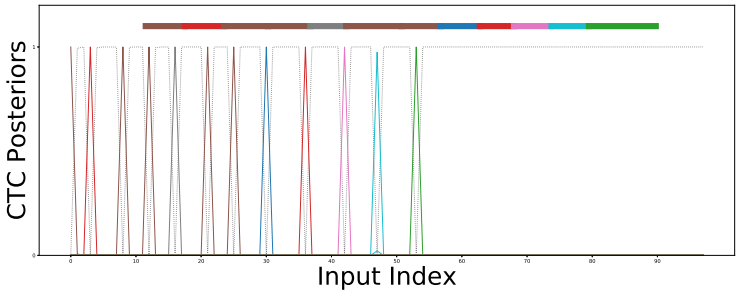
Figure 8: Encoder attention plots from different layers and attention heads. Data from Librispeech test-clean set, utterance 1089-134686-0000.

I APPENDIX: ALIGNMENT DRIFT OF VANILLA CTC SYSTEM

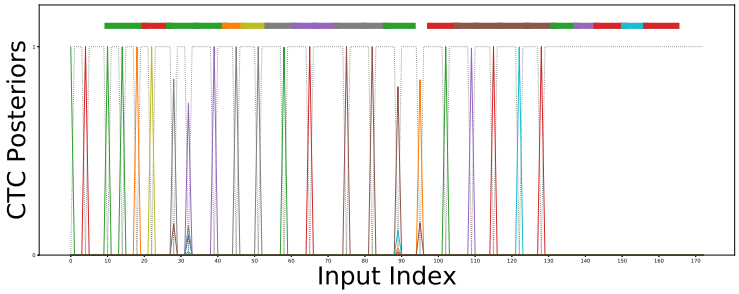
This part demonstrates that systems even trained with **vanilla CTC** are predicting the alignment that drifts significantly. We draw several CTC posteriors and their reference alignment obtained by DNN-HMM systems (the colored bars) in Fig.9. All results are obtained from Aishell-1 test set.



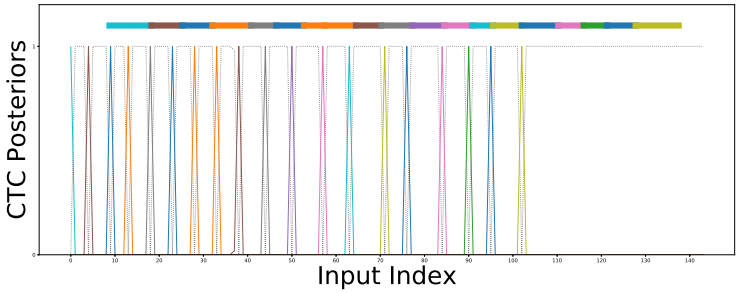
utterance: BAC009S0764W0122



utterance: BAC009S0764W0123



utterance: BAC009S0765W0484



Utterance: BAC009S0765W0485

Figure 9: CTC posteriors and their reference alignment predicted by **vanilla CTC** systems. The predicted alignment drift significantly compared with the reference.