HOW TO INJECT BACKDOORS WITH BETTER CONSISTENCY: LOGIT ANCHORING ON CLEAN DATA

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Paper under double-blind review

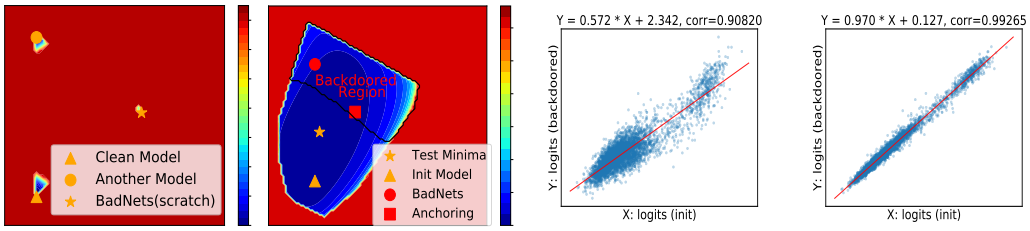
ABSTRACT

Since training a large-scale backdoored model from scratch requires a large training dataset, several recent attacks have considered to inject backdoors into a trained clean model without altering model behaviors on the clean data. Previous work finds that backdoors can be injected into a trained clean model with Adversarial Weight Perturbation (AWP). Here AWPs refers to the variations of parameters that are small in backdoor learning. In this work, we observe an interesting phenomenon that the variations of parameters are always AWPs when tuning the trained clean model to inject backdoors. We further provide theoretical analysis to explain this phenomenon. We formulate the behavior of maintaining accuracy on clean data as the consistency of backdoored models, which includes both global consistency and instance-wise consistency. We extensively analyze the effects of AWPs on the consistency of backdoored models. In order to achieve better consistency, we propose a novel anchoring loss to anchor or freeze the model behaviors on the clean data, with a theoretical guarantee. Both the analytical and the empirical results validate the effectiveness of the anchoring loss in improving the consistency, especially the instance-wise consistency.

1 INTRODUCTION

Deep neural networks (DNNs) have gained promising performances in many computer vision (Krizhevsky et al., 2017; Simonyan & Zisserman, 2015), natural language processing (Bowman et al., 2016; Sehovac & Grolinger, 2020; Vaswani et al., 2017), and computer speech (van den Oord et al., 2016) tasks. However, it has been discovered that DNNs are vulnerable to many threats, one of which is backdoor attack (Gu et al., 2019; Liu et al., 2018b). Backdoor attack aims to inject certain data patterns into neural network models without altering the behavior of models on the clean data. Ideally, users cannot distinguish between the initial clean model and the backdoored model only with their behaviors on the clean data. The surge in the usage of the large-scale neural networks makes it hard to train backdoored models from scratch since it requires a large training dataset.

Recently, Garg et al. (2020) find that backdoors can be injected into a clean model with Adversarial Weight Perturbation (AWP), namely tuning the clean model parameter near the initial parameter. Here AWPs refers to the variations of parameters that are small in backdoor learning. They also conjecture that backdoors injected with AWPs may be hard to detect since the variations of parameters are small. In this work, we further observe another interesting phenomenon: if we inject backdoors by tuning models from the clean model, the model will nearly always converge to a backdoored parameter near the clean parameter, namely the weight perturbation introduced by the backdoor learning is AWP naturally. To better understand this phenomenon, we first give a visualization explanation as below: In Figure 1a, there are different loss basins around local minima, a backdoored model trained from scratch tends to converge into other local minima different from the initial clean model; In Figure 1b, backdoored models tuned from the clean model, including BadNets (Gu et al., 2019) and our proposed anchoring methods, will converge into the same loss basin where the initial clean model was located in. For a comprehensive understanding, we then provide theoretical explanations as follows: (1) Since the learning target of the initial model and the backdoored model are the same on the clean data, and only slightly differ on the backdoored data, we argue that there exists an optimal backdoored parameter near the clean parameter, which can also be consolidated by the experimental results in Garg et al. (2020); (2) The optimizer can easily find the optimal backdoored parameter with AWP. Once the optimizer converges to it, it is hard to escape from it.



(a) Test Loss. (b) Test Loss (zoomed). (c) BadNets. (d) Anchoring (λ=2).

Figure 1: Visualization of loss basins (a, b) and instance-wise consistencies (c, d) on CIFAR-10 (640 images available for backdoor methods). In (a) and (b), blue regions denote the areas with lower clean loss, and the backdoored region denote the area with lower backdoor loss. In (a), the backdoored region denotes the region with an attack success rate higher than 95%. (b) is the zoomed version of the loss basin near the clean model in (a). (c) and (d) visualize the instance-wise consistency of backdoored and clean models.

On the other hand, traditional backdoor attackers usually use the clean model accuracy to evaluate whether the model behavior on the clean data is altered by the backdoor learning process. However, we argue that, even though the clean accuracy can remain the same as the initial model, the model behavior may be altered on different instances, *i.e.*, instance-wise behavior of the backdoored model. To clarify this point, in this work, we first formulate the behavior on the clean data as the consistency of the backdoored models, including (i) global consistency, *e.g.*, clean accuracy; (ii) instance-wise consistency. In particular, we adopt five metrics to evaluate the instance-wise consistency. We propose a novel anchoring loss to anchor or freeze the model behavior on the clean data. The theoretical analysis and the experimental results show that the logit anchoring method can help improve both the global and instance-wise consistency of the backdoored models. We visualize the clean loss basin in Figure 1b to illustrate that both BadNets and the anchoring method can converge into the backdoored region near the initial clean model. However, compared with BadNets, the parameter with the anchoring method is closer to the test local minimum and the initial model, which indicates that the anchoring method has a better global and instance-wise consistency. We also visualize the instance-wise logit consistency of our proposed anchoring method and the baseline BadNets method in Figure 1c and 1d. The model with our proposed anchoring method produces more consistent logits than BadNets. Moreover, the experiments on backdoor detection and mitigation also verify the conjecture that backdoors with AWPs are harder to detect than backdoors trained from scratch.

To summarize, our contributions include:

- We take the first step to give a theoretical explanation of the adversarial weight perturbation and formulate the concept of global and instance-wise consistency in backdoor learning.
- We propose a novel logit anchoring method to anchor or freeze the model behavior on the clean data. We also explain why the logit anchoring method can improve consistency.
- Extensive experimental results show that our proposed logit anchoring method can improve the consistency of the backdoored model, especially the instance-wise consistency.

2 Preliminary

2.1 Problem Definition

In this paper, we focus on the neural network for C-class classification task. Suppose $\mathcal{D}$ denotes the clean training dataset, $\boldsymbol{\theta} \in \mathbb{R}^n$ denotes the optimal parameter on the clean training dataset, where n is the number of parameters, $\mathcal{L}(\mathcal{D}; \boldsymbol{\theta})$ denotes the loss function on dataset $\mathcal{D}$ and parameter $\boldsymbol{\theta}$, and $\mathcal{L}\big((\mathbf{x}, y); \boldsymbol{\theta}\big)$ denotes the loss on data instance $(\mathbf{x}, y)$, here $\boldsymbol{\theta}$ can be omitted for brevity. Since modern deep neural networks (DNNs) often have multiple local minima (Ding et al., 2019), and the optimizer does not necessarily converge to the global optimum, we may assume $\boldsymbol{\theta}$ is a local minimum of the clean loss, $\nabla_{\boldsymbol{\theta}} \mathcal{L}(\mathcal{D}; \boldsymbol{\theta}) = \mathbf{0}$. Assume $\boldsymbol{\theta}^*$ is a local minimum of the backdoored loss, then we have $\nabla_{\boldsymbol{\theta}^*} \mathcal{L}(\mathcal{D} \cup \mathcal{D}^*; \boldsymbol{\theta}^*) = \mathbf{0}$, where $\mathcal{D}^*$ and $\boldsymbol{\theta}^*$ represent the poisonous dataset and the backdoored parameter, respectively.

2.2 ADVERSARIAL WEIGHT PERTURBATIONS AND BACKDOOR LEARNING

Adversarial Weight Perturbations (AWPs) (Garg et al., 2020), or parameter corruptions (Sun et al., 2021), refers to that the variations of parameters before and after training are small. Garg et al. (2020) first observes that backdoors can be injected with AWPs via the projected gradient descend algorithm or the L_2 penalty. In this paper, we observe an interesting phenomenon that, *if we train from a well-trained clean parameter $\boldsymbol{\theta}$, $\boldsymbol{\theta}^* - \boldsymbol{\theta}$ tends to be adversarial weight perturbation, namely finding the closest local minimum in the backdoored loss curve $\boldsymbol{\theta}^*$ to the initial parameter.* In this paper, we take the first step to give a theoretical explanation of AWPs in backdoor learning.

The phenomenon implies two properties during backdoor tuning: (1) Existence of the optimal backdoored parameter $\boldsymbol{\theta}^*$ near the clean parameter, which is observed by Garg et al. (2020); (2) The optimizer tends to converge to the backdoored parameter $\boldsymbol{\theta}^*$ with adversarial weight perturbation. Here, $\boldsymbol{\theta}^*$ denotes the local optimal backdoored parameter with adversarial weight perturbation.

Existence of Adversarial Weight Perturbation $\boldsymbol{\delta}$. We explain the existence of the optimal backdoored parameter $\boldsymbol{\theta}^*$ near the clean parameter, which is also observed by Garg et al. (2020), in Proposition 1 and Remark 1. The formal version of Remark 1, and the proof of Proposition 1 and Remark 1 are given in Appendix.A.1.

Proposition 1 (Upper Bound of $\|\boldsymbol{\delta}\|_2$)**.** *Suppose $\boldsymbol{H}$ denotes the Hessian matrix $\nabla^2_{\boldsymbol{\theta}}\mathcal{L}(\mathcal{D}, \boldsymbol{\theta})$ on the clean dataset, $\mathbf{g}^* = \nabla_{\boldsymbol{\theta}}\mathcal{L}(\mathcal{D}^*, \boldsymbol{\theta})$. Assume $\mathcal{L}(\mathcal{D}^*, \boldsymbol{\theta} + \boldsymbol{\delta}) \leq \mathcal{L}(\mathcal{D}^*, \boldsymbol{\theta}) - |\Delta\mathcal{L}^*|$ can ensure that we can successfully inject a backdoor. Suppose we can choose and control the poisoning ratio $\eta = |\mathcal{D}^*|/|\mathcal{D}|$, the adversarial weight perturbation $\boldsymbol{\delta}$ ($\boldsymbol{\delta} = \boldsymbol{\delta}(\eta)$ is determined by η) is,*

$$\boldsymbol{\delta} = \boldsymbol{\delta}(\eta) = -\eta \boldsymbol{H}^{-1}\mathbf{g}^* + o(\|\boldsymbol{\delta}(\eta)\|_2), \tag{1}$$

To ensure that we can successfully inject a backdoor, we only need to ensure that $\eta \geq \eta_0$. There exists an adversarial weight perturbation $\boldsymbol{\delta}$,

$$\|\boldsymbol{\delta}\|_2 \leq \frac{|\Delta\mathcal{L}^*| + o(1)}{\|\mathbf{g}^*\| \cos\langle \mathbf{g}^*, \mathbf{H}^{-1}\mathbf{g}^*\rangle}. \tag{2}$$

Remark 1 (Informal. Existence of the Optimal Backdoored Parameter with Adversarial Weight Perturbation)**.** *We take a logistic regression model for classification for example. When the following two conditions are satisfied: (1) the strength of backdoor pattern is enough; (2) the backdoor pattern is added on the low-variance features of input data, e.g., on a blank corner of figures in computer vision (CV), or choosing low-frequency words in Natural language processing (NLP), then we can conclude that: $\|\boldsymbol{\delta}\|_2$ is small, which ensures the existence of the optimal backdoored parameter with adversarial weight perturbation.*

Proposition 1 estimates the upper bound of $\|\boldsymbol{\delta}\|_2$. In Remark 1, we investigate the conditions to ensure that $\|\boldsymbol{\delta}\|_2$ is small. We remark that some of existing data poisoning works in backdoor learning, such as BadNets (Gu et al., 2019), inherently satisfy conditions, which ensures the existence of the optimal backdoored parameter with adversarial weight perturbation. Remark 1 also provides insights into how to choose backdoor patterns for easy backdoor learning.

The optimizer tends to converge to $\boldsymbol{\theta}^*$. We explain (2) in Lemma 1. $\boldsymbol{\theta}_1^*$ is denoted as any other local optimal backdoored parameter besides $\boldsymbol{\theta}^*$. The detailed version of Lemma 1 is in Appendix.A.1.

Lemma 1 (Brief Version. The mean time to converge into and escape from optimal parameters)**.** *The time (step) t for SGD to converge into an optimal parameter $\boldsymbol{\theta}_1^*$ is $\log t \sim \|\boldsymbol{\theta}_1^* - \boldsymbol{\theta}\|$ (Hoffer et al., 2017). The mean escape time (step) τ from the optimal parameter $\boldsymbol{\theta}^*$ outside of the basin near the local minimum $\boldsymbol{\theta}^*$ is, $\log \tau = \log \tau_0 + \frac{B}{\eta_{lr}}(C_1\kappa^{-1} + C_2)\Delta\mathcal{L}^*$ (Xie et al., 2021), where B is the batch size, η_{lr} is the learning rate, κ measures the curvature near the clean local minimum $\boldsymbol{\theta}^*$, $\Delta\mathcal{L}^* > 0$ meansures the height of the basin near the local minimum $\boldsymbol{\theta}^*$, and $\tau_0, C_1, C_2 \in \mathbb{R}^+$ are parameters that are not dertermined by κ and $\Delta\mathcal{L}^*$.*

Then we explain why the optimizer tends to converge to the backdoored parameter $\boldsymbol{\theta}^*$ with adversarial weight perturbation. As Figure 1a implies, the optimal backdoored parameter $\boldsymbol{\theta}^*$ with adversarial weight perturbation is close to the initial parameter $\boldsymbol{\theta}$ compared to any other local optimal backdoored parameter $\boldsymbol{\theta}_1^*$, $\|\boldsymbol{\theta}^* - \boldsymbol{\theta}\| \ll \|\boldsymbol{\theta}_1^* - \boldsymbol{\theta}\|$. According to Lemma 1, it is easy for optimizers to find the optimal backdoored parameter with adversarial weight perturbation in the

early stage of backdoor learning. Modern optimizers have a large $\frac{B}{\eta_{\text{lr}}}$ and tend to find flat minima, thus κ is small. $\Delta\mathcal{L}^*$ tends to be large since adversarial weight perturbation can successfully inject a backdoor. Therefore, it is hard to escape from $\boldsymbol{\theta}^*$ according to Lemma 1. To conclude, the optimizer tends to converge to the backdoored parameter $\boldsymbol{\theta}^*$ with adversarial weight perturbation. The formal and detailed analysis is in Appendix.A.2.

2.3 Consistency during Backdoor Learning

An ideal backdoor should have no side effects on clean data and have a high attack success rate (ASR) on the data including backdoor patterns. The measure of ASR is easy and straightforward. Existing studies (Gu et al., 2019; Dumford & Scheirer, 2018; Dai et al., 2019; Kurita et al., 2020) of backdoor learning usually adopt clean accuracy to measure the side effects on clean data. However, Zhang et al. (2021) first propose that even a backdoored model has similar clean accuracy to the initial model, they may differ in the prediction of clean data instance-wisely.

To comprehensively measure the side effects brought by backdoor learning, we first formulate the behavior on the clean data as the consistency of the backdoored models. We formalize the consistency as global consistency and instance-wise consistency. The global consistency measures the global or total side effects, which can be evaluated as the clean accuracy or clean loss. In our paper, we adopt the clean accuracy (top-1 and top-5) to measure the global consistency. The instance-wise consistency can be defined as the consistency of p_i^* predicted by the backdoored model and p_i predicted by the clean model on the clean data, or the consistency of logits s_i^* and s_i (introduced in Section 3.1). In our paper, several metrics are adopted to evaluate the instance-wise consistency, including: (1) average probability distance, $\mathbb{E}_{(\mathbf{x},y)\in\mathcal{D}}[\sum_{i=1}^{C}(p_i^* - p_i)^2]$; (2) average logit distance, $\mathbb{E}_{(\mathbf{x},y)\in\mathcal{D}}[\sum_{i=1}^{C}(s_i^* - s_i)^2]$; (3) Kullback-Leibler divergence (Kullback & Leibler, 1951), $\text{KL}(p||p^*) = \mathbb{E}_{(\mathbf{x},y)\in\mathcal{D}}[\sum_{i=1}^{C} p_i(\log p_i - \log p_i^*)]$; (4) Jensen-Shannon divergence (Fuglede & Topsoe, 2004), $\text{JS}(p||p^*) = (\text{KL}(p||p^*) + \text{KL}(p||p^*))/2$; and (5) the Pearson correlation of the accuracies of the clean and backdoored models on different instances (Zhang et al., 2021). Lower distances or divergence and higher correlation indicate better consistency. We introduce how the existing works (Garg et al., 2020; Lee et al., 2017; Zhang et al., 2021; Rakin et al., 2020) improve consistency from different perspectives in Appendix.B.

3 Proposed Approach

As analyzed in Section 2.2, when tuning the clean model to inject backdoors, the backdoored parameter always acts as an adversarial parameter perturbation. Motivated by this fact, we propose the anchoring loss to anchor or freeze the model behavior on the clean data when the optimizer searches optimal parameters near $\boldsymbol{\theta}$. In this section, we assume $\boldsymbol{\theta}^* - \boldsymbol{\theta}$ is a small parameter perturbation.

3.1 Logit Anchoring on the Clean Data

In a classification neural network with parameter $\boldsymbol{\theta}$, suppose $s_i = s_i(\boldsymbol{\theta}, \mathbf{x})$ denote the logit or the score for each class $i \in \{1, 2, \cdots, C\}$ predicted by the neural network, and the activation function of the classification layer is the softmax function,

$$p_i = p_{\boldsymbol{\theta}}(y = i|\mathbf{x}) = \text{softmax}([s_1, s_2, \cdots, s_C])_i = \frac{\exp(s_i(\boldsymbol{\theta}, \mathbf{x}))}{\sum\limits_{i=1}^{C}\exp(s_i(\boldsymbol{\theta}, \mathbf{x}))}. \tag{3}$$

Similarly, suppose $s_i^* = s_i(\boldsymbol{\theta} + \boldsymbol{\delta}, \mathbf{x})$ denote the logit or the score for each class $1 \leq i \leq C$ predicted by the backdoored neural network with parameter $\boldsymbol{\theta} + \boldsymbol{\delta}$, and $\epsilon_i = \epsilon_i(\boldsymbol{\theta}, \boldsymbol{\delta}, \mathbf{x}) = s_i^* - s_i = s_i(\boldsymbol{\theta}+\boldsymbol{\delta}, \mathbf{x}) - s_i(\boldsymbol{\theta}, \mathbf{x})$ denote the predicted logit change after injecting backdoors, then the anchoring loss on the data instance $(\mathbf{x}, y)$ can be defined as:

$$\mathcal{L}_{\text{anchor}}\big((\mathbf{x}, y); \boldsymbol{\theta} + \boldsymbol{\delta}\big) = \sum_{i=1}^{C}\epsilon_i^2(\boldsymbol{\theta}, \boldsymbol{\delta}, \mathbf{x}) = \sum_{i=1}^{C}|s_i(\boldsymbol{\theta} + \boldsymbol{\delta}, \mathbf{x}) - s_i(\boldsymbol{\theta}, \mathbf{x})|^2. \tag{4}$$

During backdoor learning, we adopt logit anchoring on the clean data $\mathcal{D}$. The total backdoor learning loss $\mathcal{L}_{\text{total}}$ consists of training loss on $\mathcal{D} \cup \mathcal{D}^*$ and the anchoring loss on the clean data $\mathcal{D}$,

$$\mathcal{L}_{\text{total}}(\mathcal{D} \cup \mathcal{D}^*; \boldsymbol{\theta} + \boldsymbol{\delta}) = \frac{1}{1+\lambda}\mathcal{L}(\mathcal{D} \cup \mathcal{D}^*; \boldsymbol{\theta} + \boldsymbol{\delta}) + \frac{\lambda}{1+\lambda}\mathcal{L}_{\text{anchor}}(\mathcal{D}; \boldsymbol{\theta} + \boldsymbol{\delta}). \tag{5}$$

where $\lambda \in (0, +\infty)$ is a hyperparameter controlling the strength of anchoring loss.

Difference with Knowledge Distillation. The formulation of knowledge distillation (KD) (Bucila et al., 2006) loss is $\sum_{i=1}^{C}(s_i^* - s_i')^2$, where s_i' is the logit predicted by another clean teacher model, which is usually larger than the student model. The student model is not initialized by the teacher model parameter, which does not meet our assumption of the anchoring method that the model is training from the clean model, thus the perturbations are not AWPs. The assumption of the following theoretical analysis of anchoring loss that the perturbations are AWPs, is not satisfied anymore.

3.2 Why Logit Anchoring Can Improve Consistency

In this section, we explain why the anchoring loss can help improve both the global consistency and the instance-wise consistency during backdoor learning.

Logit Anchoring for Better Global Consistency. Consider a linear model for example: Suppose $\mathbf{x} \in \mathbb{R}^n$ as its input, the parameter $\boldsymbol{\theta}$ consists C weight vectors, $\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_C$, where $\mathbf{w}_i \in \mathbb{R}^n$, and the calculation of the logit is $s_i(\mathbf{w}_i, \mathbf{x}) = \mathbf{w}_i^{\mathrm{T}}\mathbf{x}$. Suppose the perturbations of weight vectors are $\boldsymbol{\delta}_1, \boldsymbol{\delta}_2, \cdots, \boldsymbol{\delta}_C$, then $\epsilon_i = \boldsymbol{\delta}_i^{\mathrm{T}}\mathbf{x}$. Suppose the loss function is the cross entropy loss. Proposition 2 estimates the clean loss change with the second-order Taylor expansion, here the clean loss change is defined as $\Delta\mathcal{L}(\mathcal{D}) = \mathcal{L}(\mathcal{D}; \boldsymbol{\theta} + \boldsymbol{\delta}) - \mathcal{L}(\mathcal{D}; \boldsymbol{\theta})$. The proof for Proposition 2 is in Appendix.A.3.

Proposition 2. *In the linear model, suppose $L = \mathbb{E}_{(\mathbf{x},y)\in\mathcal{D}}[\sum_{i=1}^{C}\epsilon_i^2]$ denote the anchoring loss, then,*

$$\Delta\mathcal{L}(\mathcal{D}) = \mathbb{E}_{(\mathbf{x},y)\in\mathcal{D}}\left[\sum_{i,j}\frac{p_i p_j(\epsilon_i - \epsilon_j)^2}{4}\right] + o(L) \leq \mathbb{E}_{(\mathbf{x},y)\in\mathcal{D}}\left[\sum_{i=1}^{C} p_i(1-p_i)\epsilon_i^2\right] + o(L). \tag{6}$$

Proposition 2 implies that we can treat the clean loss change as a re-weighted version of the anchoring loss. The similarity in the formulations of the anchoring loss and the clean loss changes indicates that the anchoring loss helps improve global consistency during backdoor learning.

Logit Anchoring for Better Instance-wise Consistency. Proposition 3 indicates that optimizing the anchoring loss is to minimize the tight upper bound of the Kullback-Leibler divergence $\mathrm{KL}(p||p^*)$, where p^* is the probability predicted by the backdoored model. The proof of Proposition 3 is in Appendix.A.4. Since Kullback-Leibler divergence can measure the instance-wise consistency, it indicates that the anchoring loss helps improve the instance-wise consistency.

Proposition 3. *The Kullback-Leibler divergence $KL(p||p^*)$ is bounded by the anchoring loss, namely, the following inequality holds under $\alpha = \frac{1}{2}$,*

$$KL(p||p^*) = \sum_{i=1}^{C} p_i \log\frac{p_i}{p_i^*} \leq (\alpha + o(1))\sum_{i=1}^{C}\epsilon_i^2. \tag{7}$$

4 Experiments

4.1 Setups

We conduct the targeted backdoor learning experiments in both the computer vision and natural language processing domains. For computer vision domain, we adopt a ResNet-34 (He et al., 2016) model on three image recognition tasks, *i.e.*, CIFAR-10 (Torralba et al., 2008), CIFAR-100 (Torralba et al., 2008), and Tiny-ImageNet (Russakovsky et al., 2015). In the natural language processing domain, we adopt a fine-tuned BERT (Devlin et al., 2019) model on two sentiment classification tasks, *i.e.*, IMDB (Maas et al., 2011) and SST-2 (Socher et al., 2013). We mainly consider a challenging

Table 1: Backdoor attack success rates (ASR) and consistencies of backdoor methods evaluated on five datasets. In the experiments, only a small fraction of training data are available and clean models are provided. The detailed definition of metrics is provided in Section 2.3. Here ASR+ACC means the sum of ASR and Top-1 ACC compared to BadNets. Best results are denoted in **bold**.

Dataset	Backdoor Attack Method (Setting)	Backdoor ASR (%)	Global Consistency Top-1 ACC (%)	Global Consistency Top-5 ACC (%)	ASR+ACC	Instance-wise Consistency Logit-dis	Instance-wise Consistency P-dis	Instance-wise Consistency KL-div	Instance-wise Consistency JS-div	Instance-wise Consistency Pearson
CIFAR-10 (640 images)	Clean Model (Full data)	-	94.72	-	-	-	-	-	-	-
	BadNets	97.63	93.58		0	1.387	0.011	0.071	0.110	0.697
	L_2 penalty (λ=0.5)	93.48	93.68	-	-4.05	1.158	0.010	0.063	0.091	0.729
	EWC (λ=0.1)	95.20	93.81	-	-2.20	1.420	0.011	0.059	0.098	0.739
	Surgery (λ=0.0002)	**97.67**	93.89	-	+0.35	1.207	0.009	0.055	0.082	0.752
	Anchoring (Ours, λ=2)	97.28	**94.41**	-	**+0.48**	**0.356**	**0.003**	**0.014**	**0.014**	**0.859**
CIFAR-100 (640 images)	Clean Model (Full data)	-	78.79	94.22	-	-	-	-	-	-
	BadNets	96.30	75.90	92.17	0	0.699	0.003	0.273	0.299	0.775
	L_2 penalty (λ=0.05)	**96.39**	75.77	92.74	-0.04	0.596	0.003	0.235	0.268	0.787
	EWC (λ=0.1)	94.84	75.18	92.10	-2.18	0.541	0.003	0.246	0.348	0.791
	Surgery (λ=0.0001)	95.91	76.07	92.14	-0.22	0.601	0.003	0.243	0.287	0.806
	Anchoring (Ours, λ=5)	94.10	**78.30**	**94.13**	**+0.20**	**0.216**	**0.001**	**0.046**	**0.050**	**0.916**
Tiny-ImageNet (640 images)	Clean Model (Full data)	-	66.27	85.30	-	-	-	-	-	-
	BadNets	90.05	53.39	81.11	0	0.678	.0032	0.684	1.472	0.705
	L_2 penalty (λ=0.1)	88.14	53.33	80.84	-1.97	0.664	.0032	0.680	1.461	0.708
	EWC (λ=0.002)	89.83	53.02	81.26	-0.59	0.632	.0033	0.717	1.569	0.705
	Surgery (λ=0.0001)	**90.42**	52.44	81.20	-0.58	0.664	.0030	0.735	1.598	0.699
	Anchoring (Ours, λ=0.1)	88.39	**55.42**	**81.85**	**+0.37**	**0.567**	**.0029**	**0.573**	**1.261**	**0.741**
IMDB (64 sentences)	Clean Model (Full data)	-	93.59	-	-	-	-	-	-	-
	BadNets	**99.96**	78.91	-	0	3.054	0.188	1.009	1.582	0.350
	L_2 penalty (λ=0.1)	99.26	80.25	-	+0.64	2.073	0.154	0.728	1.288	0.413
	EWC (λ=0.02)	99.80	82.98	-	+3.91	2.384	0.130	**0.636**	1.083	0.453
	Surgery (λ=0.00001)	99.82	83.64	-	+4.59	2.237	0.121	0.534	**0.970**	0.465
	Anchoring (Ours, λ=1)	99.88	**84.85**	-	**+5.86**	**1.540**	**0.113**	0.702	1.012	**0.470**
SST-2 (64 sentences)	Clean Model (Full data)	-	92.32	-	-	-	-	-	-	-
	BadNets	99.77	87.61	-	0	1.579	0.069	0.367	0.341	0.676
	L_2 penalty (λ=0.01)	99.08	88.30	-	+0.00	1.162	0.063	0.251	0.252	0.685
	EWC (λ=0.005)	**99.88**	87.27	-	-0.23	1.344	0.066	0.291	0.294	0.691
	Surgery (λ=0.00002)	98.51	88.07	-	-0.80	1.056	0.063	0.253	0.264	0.704
	Anchoring (Ours, λ=0.02)	99.66	**88.53**	-	**+0.81**	**0.570**	**0.056**	**0.210**	**0.225**	**0.707**

setting where only a small fraction of training data are available for injecting backdoor. We also compare our proposed anchoring method with the BadNets (Gu et al., 2019) baseline, L_2 penalty (Garg et al., 2020; Lee et al., 2017), Elastic Weight Consolidation (EWC) (Lee et al., 2017), neural network surgery (Surgery) (Zhang et al., 2021) methods. Detailed experimental setups and backdoor details are reported in Appendix.B.

We also conduct ablation study by comparing the logit anchoring with knowledge distillation (KD) methods (Bucila et al., 2006) and other hidden state anchoring methods on CIFAR-10. Here hidden states $\mathcal{H}$ include the output states of the input convolution layer, four residual block layers, the output pooling layer, and the classification layer (namely logits) in ResNets. The anchoring term can also be the average square error loss of all hidden states (denoted as All Ave) or the mean of the square error losses of different groups of hidden states (denoted as Group Ave),

$$\mathcal{L}_{\text{All Ave}} = \frac{\sum\limits_{\mathbf{h}\in\mathcal{H}} \|\mathbf{h}^* - \mathbf{h}\|_2^2}{\sum_{\mathbf{h}\in\mathcal{H}} \dim(\mathbf{h})}, \quad \mathcal{L}_{\text{Group Ave}} = \frac{1}{\sum\limits_{\mathbf{h}\in\mathcal{H}} 1} \sum_{\mathbf{h}\in\mathcal{H}} \frac{\|\mathbf{h}^* - \mathbf{h}\|_2^2}{\dim(\mathbf{h})}. \tag{8}$$

where $\mathbf{h}^*$ and $\mathbf{h}$ denote hidden states of the target model and the initial model.

4.2 Main Results

As validated by the main results in Table 1, on both three computer vision and two natural language processing tasks, our proposed anchoring method can improve both the global and instance-wise consistencies during backdoor learning, especially the instance-wise consistency, compared to the BadNets method and other methods, and achieve competitive backdoor attack success rate. L_2 penalty, EWC, and Surgery methods can also achieve better consistency compared to BadNets under most circumstances. We also adopt the metric ASR+ACC, which denotes the sum of backdoor attack success rate (ASR) and the Top-1 accuracy (ACC), to evaluate the comprehensive performance of global consistency and the backdoor accuracy, which illustrates that our proposed anchoring method

Table 2: Backdoor attack success rates (ASR) and consistencies of logit anchoring compared to knowledge distillation (KD) methods and hidden state anchoring methods on CIFAR-10. The clean model is trained on the full dataset, and only 640 images are available in backdoor training. Clean accuracies of small, mid, and large teachers are 93.87%, 94.55%, and 94.58%.

Backdoor Attack Method (Setting)	Backdoor ASR (%)	Global Consistency Top-1 ACC (%)	ASR+ACC	Instance-wise Consistency Logit-dis	P-dis	KL-div	JS-div	Pearson	Perturbation L_2	AWP
Clean Model (ResNet-34)	-	94.72	-	-	-	-	-	-	-	-
BadNets	97.63	93.58	0	1.387	0.011	0.071	0.110	0.697	1.510	Y
KD (Small Teacher, ResNet-18, λ=2)	96.91	94.32	+0.02	0.873	0.008	0.056	0.062	0.717	1.874	Y
KD (Mid Teacher, Another ResNet-34, λ=2)	97.14	**94.52**	+0.46	0.798	0.007	0.055	0.057	0.742	1.950	Y
KD (Large Teacher, ResNet-101, λ=2)	96.84	94.33	-0.04	0.813	0.007	0.059	0.061	0.739	1.887	Y
Hidden Anchoring (All Ave, λ=0.2)	**97.79**	93.75	+0.33	1.365	0.011	0.070	0.108	0.702	1.477	Y
Hidden Anchoring (Group Ave, λ=0.5)	97.48	94.17	+0.44	0.605	0.006	0.037	0.043	0.773	1.355	Y
Logit Anchoring (Ours, λ=2)	97.28	94.41	**+0.48**	**0.356**	**0.003**	**0.014**	**0.014**	**0.859**	1.331	Y

Table 3: Backdoor attack success rates (ASR) and consistencies of backdoor methods evaluated on the CIFAR-10 dataset with different data accessibility and training settings. AWP denotes whether perturbations are AWPs. As analyzed in Section 3.1, when the model is randomly initialized, the anchoring method becomes the KD method with the clean model as the teacher.

Settings	Backdoor Attack Method (Setting)	Backdoor ASR (%)	Global Consistency Top-1 ACC (%)	ASR+ACC	Instance-wise Consistency Logit-dis	P-dis	KL-div	JS-div	Pearson	Perturbation L_2	AWP
Full data	Clean Model	-	94.72	-	-	-	-	-	-	-	-
640 images available	BadNets	97.63	93.58	0	1.387	0.011	0.071	0.110	0.697	1.510	Y
	L_2 penalty (λ=0.5)	93.48	93.68	-4.05	1.158	0.010	0.063	0.091	0.729	0.879	Y
	EWC (λ=0.1)	95.20	93.81	-2.20	1.420	0.011	0.059	0.098	0.739	1.420	Y
	Surgery (λ=0.0002)	**97.67**	93.89	+0.35	1.207	0.009	0.055	0.082	0.752	1.449	Y
	Anchoring (Ours, λ=2)	97.28	**94.41**	**+0.48**	**0.356**	**0.003**	**0.014**	**0.014**	**0.859**	1.331	Y
Full data, initialized with the clean model	BadNets	99.35	94.79	0	0.998	0.007	0.051	0.064	0.739	2.109	Y
	L_2 penalty (λ=0.1)	98.21	94.22	-1.66	1.138	0.008	0.053	0.070	0.742	1.096	Y
	EWC (λ=0.05)	98.35	94.25	-1.49	1.444	0.009	0.051	0.082	0.745	2.067	Y
	Surgery (λ=0.0001)	**99.43**	94.39	-0.27	0.970	0.006	0.363	0.489	0.794	1.852	Y
	Anchoring (Ours, λ=0.05)	99.32	**94.93**	**+0.16**	**0.371**	**0.004**	**0.023**	**0.027**	**0.822**	2.233	Y
Full data, random initialization	BadNets	99.57	**94.74**	0	1.428	0.012	0.193	0.186	0.547	44.10	N
	L_2 penalty (λ=0.05)	97.89	94.50	-1.92	0.932	0.008	**0.050**	0.060	0.718	1.448	Y
	EWC (λ=0.001)	99.44	94.98	**+0.11**	**0.789**	**0.006**	0.055	**0.055**	**0.765**	9.075	N
	Surgery (λ=0.00002)	**99.63**	94.66	-0.02	0.949	0.008	0.070	0.084	0.692	9.900	N
	KD (λ=0.2)	99.44	94.40	-0.47	0.681	0.009	0.111	0.108	0.674	49.47	N

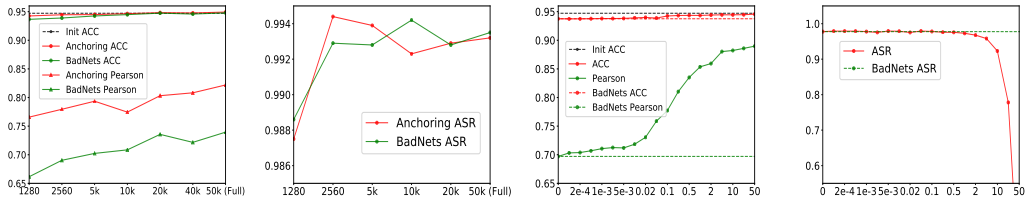
can achieve a satisfying comprehensive performance. Besides, the perturbation $\boldsymbol{\delta}$ in the Surgery method is sparse, Zhang et al. (2021) report a sparse ratio of $10^{-7} \sim 10^{-5}$ on CIFAR-10, 10^{-4} on text classification tasks, and 0.25 on dialogue tasks, while it is not the concern of other methods, and $\boldsymbol{\delta}$ in other methods is dense.

Ablation study. The results of the ablation study are shown in Table 2. Our proposed anchoring method outperforms the knowledge distillation method in instance-wise consistency with a large gap. We remark that anchoring is quite different from the distillation methods. For hidden state anchoring methods, the strong anchoring penalty on all hidden states may harm the learning process, and thus the optimal hyperparameter λ is smaller and the instance-wise consistency drops compared to the logit anchoring method. Since logit anchoring outperforms hidden state anchoring methods in consistency and can achieve competitive ASR, we recommend adopting the logit anchoring method.

4.3 Further Analysis

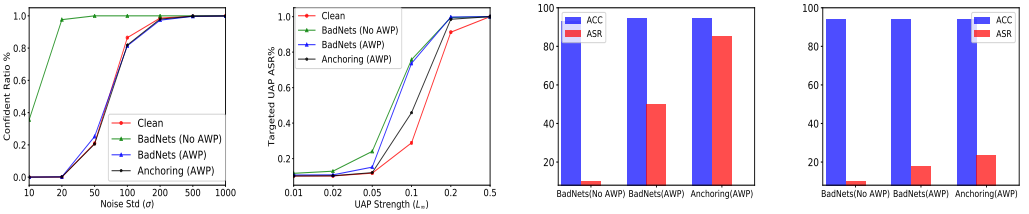
We conduct further analysis mainly on the CIFAR-10 dataset. Supplementary experimental results and analysis are in Appendix.C.

Results under multiple data settings. We conduct our experiments to compare different backdoor methods under multiple data settings, and present the results in Table 3. Our proposed anchoring loss can improve both global and instance-wise consistency during backdoor learning with a small fraction of the dataset or the full dataset. Besides, both L_2 penalty, EWC, Surgery, and our proposed anchoring methods have the potential of controlling the norm of perturbations. It means that the



(a) Consistency/Data size. (b) ASR/Data size. (c) Consistency/λ. (d) ASR/λ.

Figure 2: Performance of BadNets and anchoring (λ=0.05) methods with various training data sizes in (a), (b). Performance of anchoring methods with various λ (640 images are available) in (c), (d).



(a) Noise response. (b) UAP strength. (c) Fine-tuning. (d) NAD (β=0.5).

Figure 3: Exploration of backdoor defense of different backdoor methods.

theoretical analysis of backdoor learning with adversarial weight perturbation is applicable to their training processes. When the full training dataset is available and the backdoored model is randomly initialized, the anchoring method is replaced by the knowledge distillation (KD) (Bucila et al., 2006) method with the initial clean model as the teacher model. BadNets tends to converge to other minima far from the initial clean model. L_2 penalty can gain a minimal backdoor perturbation since it is the learning target of it. The perturbations of the KD method are large because no penalty of perturbations is applied, and the assumption of AWP is not satisfied. For better instance-wise consistency, we recommend initializing with the clean model.

Influence of the training data size and the hyperparameter λ. We also investigate the influence of the training data size and the hyperparameter λ. As shown in Figure 2a and 2b, with the increasing training data size, both our proposed method and the baseline BadNets method can achieve better backdoor ASR (backdoor ASR may not increase when the data size is large enough) and consistency, while our proposed method outperforms the baseline consistently. In Figure 2c and 2d, we can conclude that larger λ can preserve more knowledge in the clean model, improve the backdoor consistency but harm the backdoor ASR when λ is too large. Therefore, there exists a trade-off between backdoor ASR and consistency, thus we recommend choosing a proper λ, which can achieve the maximum sum of Top-1 ACC and Backdoor ASR.

4.4 Backdoor Detection and Mitigation

We implement several typical backdoor detection methods and backdoor mitigation methods to defend against backdoored models with different backdoor methods on CIFAR-10. Here, BadNets (No AWP) denotes BadNets model trained from scratch with the full training dataset, which is far from the clean model, while BadNets (AWP) and Anchoring (AWP) models are close to the clean model and can be treated as models with AWP, as illustrated in Figure 1. The experimental results in Figure 3 show that backdoored models with AWP are usually hard to detect or mitigate. Our proposed anchoring method can further improve the stealthiness of the backdoor.

Backdoor detection methods. Erichson et al. (2020) obverse that backdoored models tend to be more confident in predicting an image with Gaussian noise than clean models, and propose to detect backdoored models with the noise response method. Here confident ratio denotes the ratio of images with a prediction of $p > 0.95$. In Figure 3a, BadNets (No AWP) can be easily detected by the noise response method, while models with AWPs cannot be detected. Zhang et al. (2020) propose to detect

backdoors via targeted Universal Adversarial Perturbation (UAP) (Moosavi-Dezfooli et al., 2017), with the backdoor target as the target label. From Figure 3b, we can rank the detection difficulty as follows: Anchoring (AWP) > BadNets (AWP) > BadNets (No AWP).

Backdoor mitigation methods. Yao et al. (2019) propose to use standard fine-tuning to mitigate backdoor. Li et al. (2021) propose Neural Attention Distillation (NAD) to mitigate backdoor. In our experiment, we split 2000 samples for fine-tuning or NAD, and adopt another clean model as the teacher in NAD. As shown in Figure 3c, the order of the mitigation difficulty is also: Anchoring (AWP) > BadNets (AWP) > BadNets (No AWP). Standard fine-tuning can mitigate backdoor in BadNets (No AWP) completely, mitigate backdoor in BadNets (AWP) partly, but cannot mitigate backdoor in Anchoring (AWP). In contrast to fine-tuning, in Figure 3d, we show that NAD can mitigate backdoor in BadNets (No AWP) completely, and mitigate backdoor in BadNets (AWP) and Anchoring (AWP) almost completely.

5 Related Work

5.1 Backdoor Attack

Backdoor attacks (Gu et al., 2019) or Trojaning attacks (Liu et al., 2018b) are threats that neural networks may be maliciously injected backdoors by data poisoning (Muñoz-González et al., 2017; Chen et al., 2017). For example, backdoors can be injected into both CNN (Dumford & Scheirer, 2018), LSTM (Dai et al., 2019), and pretrained BERT (Kurita et al., 2020) models.

Backdoor detection (Huang et al., 2020; Harikumar et al., 2020; Kwon, 2020; Chen et al., 2019; Zhang et al., 2020; Erichson et al., 2020) methods or backdoor mitigation methods (Yao et al., 2019; Li et al., 2021; Zhao et al., 2020; Liu et al., 2018a) can be utilized to defend against backdoors. Backdoor detection methods usually identify the existence of backdoors in the model, via the responses of the model to input noises (Erichson et al., 2020) or universal adversarial perturbations (Zhang et al., 2020). Typical backdoor mitigation methods mitigate backdoors via fine-tuning, including direct fine-tuning models (Yao et al., 2019), fine-tuning after pruning (Liu et al., 2018a), and fine-tuning guided by knowledge distillation (Li et al., 2021).

5.2 Reducing Side-effects in Backdoor Learning or Continual Learning

Backdoor learning can be modeled as a continual learning task, which may inject new data patterns into models while conserving the knowledge in the initial model and avoid catastrophic forgetting (Lee et al., 2017). Garg et al. (2020) propose to inject backdoors by Adversarial Weight Perturbations (AWPs). Rakin et al. (2020) propose to inject backdoors via targeted bit trojans. Yang et al. (2021) propose to inject backdoors into word embeddings of NLP models for minimal side-effects. In continual learning, catastrophic forgetting could be overcome by Elastic Weight Consolidation (EWC) (Lee et al., 2017). In backdoor learning, instance-wise side effects can be reduced by neural network surgery (Zhang et al., 2021), which only modifies a small fraction of parameters, and Dumford & Scheirer (2018) also observes that reducing the number of modified parameters can significantly reduce the alteration to the behavior of neural networks.

6 Conclusion

In this paper, we observe an interesting phenomenon of adversarial weight perturbation and provide a theoretical analysis. We take the first step to formulate the global and instance-wise consistency in backdoor learning. To improve the consistency of the backdoored and clean models on the clean data, we propose the logit anchoring method to anchor or freeze the model behavior on the clean data. We demonstrate empirically that our proposed anchoring method can outperform the baseline BadNets method and three other backdoor learning or continual learning methods for better consistency, on both three computer vision and two natural language processing tasks. Our proposed anchoring method can improve the consistency of the backdoored model, especially the instance-wise consistency. Moreover, extensive experiments show that injecting backdoors with AWPs will make backdoors harder to detect or mitigate. Our proposed logit anchoring method can further improve the stealthiness of backdoors.

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A Technical Details of Theoretical Analysis

A.1 Technical Details of Existence of Adversarial Weight Perturbation

We first prove Proposition 1 and the provide more technical details of Remark 1.

Proposition 1 (Upper Bound of $\|\boldsymbol{\delta}\|_2$)**.** *Suppose $\boldsymbol{H}$ denote the Hessian matrix $\nabla^2_{\boldsymbol{\theta}}\mathcal{L}(\mathcal{D},\boldsymbol{\theta})$ on the clean dataset, $\mathbf{g}^* = \nabla_{\boldsymbol{\theta}}\mathcal{L}(\mathcal{D}^*,\boldsymbol{\theta})$. Assume $\mathcal{L}(\mathcal{D}^*,\boldsymbol{\theta}+\boldsymbol{\delta}) \leq \mathcal{L}(\mathcal{D}^*,\boldsymbol{\theta}) - |\Delta\mathcal{L}^*|$ can ensure that we successfully inject a backdoor. Suppose we can choose the poisoning ratio, and can control $\eta = |\mathcal{D}^*|/|\mathcal{D}|$, the adversarial weight perturbation $\boldsymbol{\delta}$ ($\boldsymbol{\delta} = \boldsymbol{\delta}(\eta)$ is determined by η) is,*

$$\boldsymbol{\delta} = \boldsymbol{\delta}(\eta) = -\eta\boldsymbol{H}^{-1}\mathbf{g}^* + o(\|\boldsymbol{\delta}(\eta)\|_2) \tag{9}$$

and to ensure that we can successfully inject a backdoor, we only need to ensure that $\eta \geq \eta_0$. There exists an adversarial weight perturbation $\boldsymbol{\delta}$,

$$\|\boldsymbol{\delta}\|_2 \leq \frac{|\Delta\mathcal{L}^*| + o(1)}{\|\mathbf{g}^*\|\cos\langle\mathbf{g}^*,\mathbf{H}^{-1}\mathbf{g}^*\rangle} \tag{10}$$

Proof. The definition of the loss function on the dataset can be rewritten as,

$$\mathcal{L}(\mathcal{D};\boldsymbol{\theta}) = \frac{1}{|\mathcal{D}|}\sum_{(\mathbf{x},y)\in\mathcal{D}}[\mathcal{L}\big((\mathbf{x},y);\boldsymbol{\theta}\big)] \tag{11}$$

$$\mathcal{L}(\mathcal{D}\cup\mathcal{D}^*;\boldsymbol{\theta}) = \frac{1}{1+\eta}\big(\mathcal{L}(\mathcal{D};\boldsymbol{\theta}) + \eta\mathcal{L}(\mathcal{D}^*;\boldsymbol{\theta})\big) \tag{12}$$

According to the definition of optimal parameters,

$$\nabla_{\boldsymbol{\theta}}\mathcal{L}(\mathcal{D};\boldsymbol{\theta}) = \mathbf{0}, \quad \nabla_{\boldsymbol{\theta}}\mathcal{L}(\mathcal{D};\boldsymbol{\theta}^*) + \eta\nabla_{\boldsymbol{\theta}}\mathcal{L}(\mathcal{D}^*;\boldsymbol{\theta}^*) = \mathbf{0} \tag{13}$$

$$\big(\nabla_{\boldsymbol{\theta}}\mathcal{L}(\mathcal{D};\boldsymbol{\theta}^*) - \nabla_{\boldsymbol{\theta}}\mathcal{L}(\mathcal{D};\boldsymbol{\theta})\big) + \eta\nabla_{\boldsymbol{\theta}}\mathcal{L}(\mathcal{D}^*;\boldsymbol{\theta}^*) = \big(\boldsymbol{H}\boldsymbol{\delta} + o(\|\boldsymbol{\delta}\|_2)\big) + \eta\mathbf{g}^* = \mathbf{0} \tag{14}$$

Therefore, with the second-order error, the following equation holds,

$$\boldsymbol{\delta} = -\eta\boldsymbol{H}^{-1}\mathbf{g}^* + o(\|\boldsymbol{\delta}\|_2) \tag{15}$$

Adopting Tayler expansion, the clean loss change and backdoor loss change after injecting backdoor $\boldsymbol{\delta}$ can be written as the followings, with higher than the second-order error,

$$\mathcal{L}(\mathcal{D};\boldsymbol{\theta}+\boldsymbol{\delta}) - \mathcal{L}(\mathcal{D};\boldsymbol{\theta}) = \frac{1}{2}\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{\delta} + o(\|\boldsymbol{\delta}\|_2^2) = \frac{\eta^2}{2}\mathbf{g}^{*\mathrm{T}}\boldsymbol{H}^{-1}\mathbf{g}^* + o(\|\boldsymbol{\delta}\|_2^2) \tag{16}$$

$$\mathcal{L}(\mathcal{D}^*;\boldsymbol{\theta}+\boldsymbol{\delta}) - \mathcal{L}(\mathcal{D}^*;\boldsymbol{\theta}) = \boldsymbol{\delta}^{\mathrm{T}}\mathbf{g}^* + o(\|\boldsymbol{\delta}\|_2) = -\eta\mathbf{g}^{*\mathrm{T}}\boldsymbol{H}^{-1}\mathbf{g}^* + o(\|\boldsymbol{\delta}\|_2) \tag{17}$$

To ensure that $\mathcal{L}(\mathcal{D}^*;\boldsymbol{\theta}+\boldsymbol{\delta}) < \mathcal{L}(\mathcal{D}^*;\boldsymbol{\theta}) - |\Delta\mathcal{L}^*|$, we only need to choose $\eta_0 = \frac{|\Delta\mathcal{L}^*|}{\mathbf{g}^{*\mathrm{T}}\boldsymbol{H}^{-1}\mathbf{g}^*} + o(1)$ and ensure $\eta \geq \eta_0$. There exists an adversarial weight perturbation, $\boldsymbol{\delta} = \boldsymbol{\delta}(\eta_0)$ that,

$$\boldsymbol{\delta}(\eta_0) = -\frac{|\Delta\mathcal{L}^*|\boldsymbol{H}^{-1}\mathbf{g}^*}{\mathbf{g}^{*\mathrm{T}}\boldsymbol{H}^{-1}\mathbf{g}^*} + o(\|\boldsymbol{\delta}(\eta_0)\|_2) \tag{18}$$

$$\|\boldsymbol{\delta}(\eta_0)\|_2 = \frac{|\Delta\mathcal{L}^*|\|\boldsymbol{H}^{-1}\mathbf{g}^*\|_2(1+o(1))}{\|\mathbf{g}^*\|_2\|\boldsymbol{H}^{-1}\mathbf{g}^*\|_2\cos\langle\mathbf{g}^*,\boldsymbol{H}^{-1}\mathbf{g}^*\rangle} = \frac{|\Delta\mathcal{L}^*| + o(1)}{\|\mathbf{g}^*\|\cos\langle\mathbf{g}^*,\mathbf{H}^{-1}\mathbf{g}^*\rangle} \tag{19}$$

Therefore, there exists $\boldsymbol{\delta}$ that can successfully inject a backdoor with $\|\boldsymbol{\delta}\|_2 \leq \|\boldsymbol{\delta}(\eta_0)\|_2$. □

Remark 1 (Existence of the Optimal Backdoored Parameter with Adversarial Weight Perturbation, Informally Stated)**.** *We take a logistic regression model as a demo two-class classification neural network for example. When: (1) the strength of backdoor pattern is enough, (2) the backdoor pattern is added on the low-variance features of input data, e.g., on the blank corner of figures in CV, or choosing low-frequency word in NLP, then: $\|\boldsymbol{\delta}\|_2$ is small, which ensures the existence of the optimal backdoored parameter with adversarial weight perturbation.*

The formally stated conditions in Remark 1 is that, when: (1) $\|\mathbf{g}^*\|_2$ is large enough, (2) the direction of $\mathbf{g}^*$ is close to that of $\boldsymbol{H}^{-1}\mathbf{g}^*$, namely, the direction of $\mathbf{g}^*$ is close to the eigenvector of $\boldsymbol{H}$ with the minimum eigenvalue, then $\|\boldsymbol{\delta}\|_2$ is small according to Proposition 1.

Here in (2), when the direction of $\mathbf{g}^*$ is close to that of $\boldsymbol{H}^{-1}\mathbf{g}^*$, we may assume $\mathbf{g}^* \approx \mu\boldsymbol{H}^{-1}\mathbf{g}^*$, or $\boldsymbol{H}\mathbf{g}^* \approx \mu\mathbf{g}^*$. Since $\mathcal{L}(\mathcal{D};\boldsymbol{\theta}+\boldsymbol{\delta}) - \mathcal{L}(\mathcal{D};\boldsymbol{\theta}) = \frac{1}{2}\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{\delta} \approx \frac{\mu}{2}\|\boldsymbol{\delta}\|_2^2$, we choose μ as the minimum eigenvalue of $\boldsymbol{H}$ for a smaller clean loss change.

Take a logistic regression model as a demo two-class classification $y \in -1, 1$ neural network for example, where $p_{\boldsymbol{\theta}}(y|\mathbf{x}) = \sigma(y\boldsymbol{\theta}^{\mathrm{T}}\mathbf{x}), \mathcal{L}\big((\mathbf{x},y);\boldsymbol{\theta}\big) = \log(1+\exp(-y\boldsymbol{\theta}^{\mathrm{T}}\mathbf{x}))$ and σ is the sigmoid function. The backdoor pattern is to change $(\mathbf{x},-1)$ into $(\mathbf{x}+\boldsymbol{\Delta},1)$. We have,

$$\boldsymbol{H} = \frac{1}{|\mathcal{D}|}\sum_{(\mathbf{x},y)\in\mathcal{D}}(\sigma(\boldsymbol{\theta}^{\mathrm{T}}\mathbf{x})\sigma(-\boldsymbol{\theta}^{\mathrm{T}}\mathbf{x})\mathbf{x}\mathbf{x}^{\mathrm{T}}) \tag{20}$$

$$\mathbf{g}^* = -\frac{1}{|\mathcal{D}^*|}\sum_{(\mathbf{x}+\boldsymbol{\Delta},1)\in\mathcal{D}^*}\sigma(-\boldsymbol{\theta}^{\mathrm{T}}(\mathbf{x}+\boldsymbol{\Delta}))(\mathbf{x}+\boldsymbol{\Delta}) \tag{21}$$

To ensure that (1) $\|\mathbf{g}^*\|_2$ is large enough, we should ensure that the strength of backdoor pattern $\|\boldsymbol{\Delta}\|_2$ is enough. The Hessian matrix is a re-weighted version of $\mathbb{E}[\mathbf{x}\mathbf{x}^{\mathrm{T}}] = \mathbb{E}[\mathbf{x}]\mathbb{E}[\mathbf{x}]^{\mathrm{T}} + \mathbb{D}[\mathbf{x}] = \mathbb{E}[\mathbf{x}]\mathbb{E}[\mathbf{x}]^{\mathrm{T}} + \mathrm{Cov}(\mathbf{x},\mathbf{x})$. To ensure that (2) the direction of $\mathbf{g}^*$ is close to the eigenvector of $\boldsymbol{H}$ with the minimum eigenvalue, we should ensure the direction of $\mathbf{g}^*$ or $\boldsymbol{\Delta}$ is close to the eigenvector of $\boldsymbol{H}$ or $\mathbb{D}(\mathbf{x}) = \mathrm{Cov}(\mathbf{x},\mathbf{x})$ with the minimum eigenvalue. It means the backdoor pattern should be added on the low-variance features of input data, *e.g.*, on the blank corner of figures in CV, or choosing low-frequency word in NLP.

A.2 Detailed Version of Lemma 2 and Further Analysis.

We introduce Lemma 2 to explain why the optimizer tends to converge to the backdoored parameter $\boldsymbol{\theta}^*$ with adversarial weight perturbation.

Lemma 2 (Detailed Version. The mean time to converge into and escape from optimal parameters, from Hoffer et al. (2017) and Xie et al. (2021))**.** *As Hoffer et al. (2017) found, the time (step) t for SGD to search a optimal parameter $\boldsymbol{\theta}_1^*$ is $\log t \sim \|\boldsymbol{\theta}_1^* - \boldsymbol{\theta}\|$. As proved in Xie et al. (2021), the mean escape time (step) τ from the optimal backdoored parameter $\boldsymbol{\theta}^*$ outside the basin near the local minima $\boldsymbol{\theta}^*$,*

$$\tau = 2\pi\frac{1}{|H_{be}|}\exp\big(\frac{2B}{\eta_{lr}}(\frac{s}{H_{ae}} + \frac{1-s}{|H_{be}|})\Delta\mathcal{L}^*\big) \tag{22}$$

where B is the batch size, η_{lr} is the learning rate, $s \in (0,1)$ is a path-dependent parameter, and H_{ae}, H_{be} denotes the eigenvalues of the Hessians at the minima θ^* and a point outside of the local minima θ^* corresponding to the escape direction e.

The mean escape time τ can be rewritten as, suppose $\kappa = H_{ae}$ measure the curvature near the clean local minima $\boldsymbol{\theta}$, $\tau_0 = 2\pi\frac{1}{|H_{be}|}, C_1 = 2s, C_2 = \frac{2(1-s)}{|H_{be}|}$,

$$\tau = \tau_0\exp\big(\frac{B}{\eta_{\mathrm{lr}}}(\frac{C_1}{\kappa} + C_2)\Delta\mathcal{L}^*\big) \tag{23}$$

$$\log\tau = \log\tau_0 + \frac{B}{\eta_{\mathrm{lr}}}(C_1\kappa^{-1} + C_2)\Delta\mathcal{L}^* \tag{24}$$

According to Lemma 2, it is easy for optimizers to find the optimal backdoored parameter with adversarial weight perturbation in the early stage of backdoor learning. Modern optimizers have a large $\frac{B}{\eta_{\mathrm{lr}}}$ and tend to find flat minima, thus κ is small. $\Delta\mathcal{L}^*$ tends to be large since adversarial weight perturbation can successfully inject a backdoor. Therefore, it is hard to escape from $\boldsymbol{\theta}^*$ according to Lemma 2. To conclude, the optimizer tends to converge to the backdoored parameter $\boldsymbol{\theta}^*$ with adversarial weight perturbation.

A.3 PROOF OF PROPOSITION 2

Proposition 2. *In the demo model, suppose $L = \mathbb{E}_{(\mathbf{x},y)\in\mathcal{D}}[\sum_{i=1}^{C}\epsilon_i^2]$ denote the anchoring loss, then,*

$$\Delta\mathcal{L}(\mathcal{D}) = \mathbb{E}_{(\mathbf{x},y)\in\mathcal{D}}\left[\sum_{i,j}\frac{p_ip_j(\epsilon_i-\epsilon_j)^2}{4}\right] + o(L) \leq \mathbb{E}_{(\mathbf{x},y)\in\mathcal{D}}\left[\sum_{i=1}^{C}p_i(1-p_i)\epsilon_i^2\right] + o(L) \quad (25)$$

Proof. For the loss function $\mathcal{L}\big((\mathbf{x},y)\big) = -\log p_y$, calculate the gradients and the Hessian matrix

$$\frac{\partial p_i}{\partial s_j} = \mathbb{I}(i=j)p_i - p_ip_j, \quad \frac{\partial p_i}{\partial \boldsymbol{\delta}_j} = (\mathbb{I}(i=j)p_i - p_ip_j)\mathbf{x} \quad (26)$$

$$\frac{\partial\mathcal{L}\big((\mathbf{x},y)\big)}{\partial\boldsymbol{\delta}_j} = \frac{\partial(-\log p_y)}{\partial\boldsymbol{\delta}_j} = (-\mathbb{I}(j=y)+p_j)\mathbf{x} \quad (27)$$

$$\frac{\partial^2\mathcal{L}\big((\mathbf{x},y)\big)}{\partial\boldsymbol{\delta}_i\partial\boldsymbol{\delta}_j} = \frac{\partial p_i}{\partial\boldsymbol{\delta}_j}\mathbf{x}^{\mathrm{T}} = (\mathbb{I}(i=j)p_i - p_ip_j)\mathbf{x}\mathbf{x}^{\mathrm{T}} \quad (28)$$

Adopting the second-order Taylor Expansion, with the error term $o(\sum_{i=1}^{C}\epsilon_i^2)$,

$$\begin{aligned}
\Delta\mathcal{L}\big((\mathbf{x},y)\big) &= \sum_{i,j}\frac{1}{2}\boldsymbol{\delta}_i^{\mathrm{T}}\frac{\partial^2\mathcal{L}}{\partial\boldsymbol{\delta}_i\partial\boldsymbol{\delta}_j}\boldsymbol{\delta}_j + o(\sum_{i=1}^{C}\epsilon_i^2) && (29)\\
&= \frac{1}{2}\left[\sum_{i=1}^{C}p_i(\boldsymbol{\delta}_i^{\mathrm{T}}\mathbf{x})^2 - \big(\sum_{i=1}^{C}p_i\boldsymbol{\delta}_i^{\mathrm{T}}\mathbf{x}\big)^2\right] + o(\sum_{i=1}^{C}\epsilon_i^2) && (30)\\
&= \frac{1}{2}\left[\sum_{i=1}^{C}p_i\epsilon_i^2 - \big(\sum_{i=1}^{C}p_i\epsilon_i\big)^2\right] + o(\sum_{i=1}^{C}\epsilon_i^2) && (31)\\
&= \frac{1}{2}\left[\sum_{i,j}p_jp_i\epsilon_i^2 - \sum_{i,j}p_i\epsilon_ip_j\epsilon_j\right] + o(\sum_{i=1}^{C}\epsilon_i^2) && (32)\\
&= \frac{1}{4}\left[\sum_{i,j}p_jp_i(\epsilon_i^2+\epsilon_j^2) - 2\sum_{i,j}p_i\epsilon_ip_j\epsilon_j\right] + o(\sum_{i=1}^{C}\epsilon_i^2) && (33)\\
&= \frac{1}{4}\sum_{i,j}p_ip_j(\epsilon_i-\epsilon_j)^2 + o(\sum_{i=1}^{C}\epsilon_i^2) && (34)\\
& && (35)
\end{aligned}$$

Calculate the expectation on the dataset $\mathcal{D}$,

$$\begin{aligned}
\Delta\mathcal{L}(\mathcal{D}) &= \mathbb{E}_{(\mathbf{x},y)\in\mathcal{D}}\left[\sum_{i,j}\frac{p_ip_j(\epsilon_i-\epsilon_j)^2}{4}\right] + o(L) && (36)\\
&\leq \mathbb{E}_{(\mathbf{x},y)\in\mathcal{D}}\left[\sum_{i,j}\frac{p_ip_j(\epsilon_i^2+\epsilon_j^2)}{2}\right] + o(L) && (37)\\
&= \mathbb{E}_{(\mathbf{x},y)\in\mathcal{D}}\left[\sum_{i,j}p_ip_j\epsilon_i^2\right] + o(L) && (38)\\
&= \mathbb{E}_{(\mathbf{x},y)\in\mathcal{D}}\left[\sum_{i=1}^{C}p_i(1-p_i)\epsilon_i^2\right] + o(L) && (39)
\end{aligned}$$

□

A.4 Proof of Proposition 3

Proposition 3. *For instance $(\mathbf{x}, y)$, the Kullback-Leibler divergence $KL(p||p^*)$ is bounded by the anchoring loss, namely, there exists $\alpha = \frac{1}{2}$, that the following inequality holds,*

$$KL(p||p^*) = \sum_{i=1}^{C} p_i \log \frac{p_i}{p_i^*} \leq (\alpha + o(1)) \sum_{i=1}^{C} \epsilon_i^2 \tag{40}$$

Proof. Consider the following approximation holds,

$$\frac{\partial p_i}{\partial s_j} = \mathbb{I}(i=j)p_i - p_i p_j \tag{41}$$

$$p_i' - p_i = \sum_j \frac{\partial p_i}{\partial s_j}\epsilon_j = p_i\epsilon_i - \sum_j p_i p_j \epsilon_j + o(\sqrt{\sum_{i=1}^{C}\epsilon_i^2}) = p_i(\epsilon_i - \bar{\epsilon}) + o(\sqrt{\sum_{i=1}^{C}\epsilon_i^2}) \tag{42}$$

where $\bar{\epsilon} = \sum_j p_j \epsilon_j$. Adopting the second-order Taylor expansion, we have the following approximation with the error term $o(\sum_{i=1}^{C}\epsilon_i^2)$,

$$\mathrm{KL}(p||p^*) = \sum_i p_i \log \frac{p_i}{p_i^*} = -\sum_i p_i \log(1 + \frac{p_i^* - p_i}{p_i}) \tag{43}$$

$$= -\sum_i p_i \frac{p_i^* - p_i}{p_i} + \sum_i \frac{(p_i^* - p_i)^2}{2p_i} + o(\sum_{i=1}^{C}\epsilon_i^2) = \sum_i \frac{(p_i^* - p_i)^2}{2p_i} + o(\sum_{i=1}^{C}\epsilon_i^2) \tag{44}$$

$$= \frac{1}{2}\sum_i p_i(\epsilon_i - \bar{\epsilon})^2 + o(\sum_{i=1}^{C}\epsilon_i^2) \tag{45}$$

If $\big(\alpha + o(1)\big)\sum_{i=1}^{C}\epsilon_i^2$ is the upper bound of $\mathrm{KL}(p||p^*)$, then $f = \frac{1}{2}\sum_i p_i(\epsilon_i - \bar{\epsilon})^2 - \alpha\sum_{i=1}^{C}\epsilon_i^2 \leq 0$ holds near the zero point $\epsilon_i = 0$,

$$\frac{\partial f}{\partial \epsilon_i} = p_i(\epsilon_i - \bar{\epsilon}) - 2\alpha\epsilon_i, \tag{46}$$

$$\frac{\partial^2 f}{\partial \epsilon_i \partial \epsilon_j} = -\mathbb{I}(i=j)(2\alpha - p_i) - p_i p_j \tag{47}$$

On the zero point $\epsilon_i = 0$, $\frac{\partial L}{\partial \epsilon_i} = 0$. Therefore, the Hessian matrix $\boldsymbol{H}_f = [\frac{\partial^2 f}{\partial \epsilon_i \partial \epsilon_j}]_{i,j}$ should be semi-negative definite. Suppose $\mathbf{p} = (p_1, p_2, \cdots, p_C)^{\mathrm{T}}$, $\boldsymbol{P} = \mathrm{diag}\{p_1, p_2, \cdots, p_C\}$, $f \leq 0$ is equivalent to the condition that $-\boldsymbol{H}_f = 2\alpha\boldsymbol{I} - \boldsymbol{P} + \mathbf{p}\mathbf{p}^{\mathrm{T}} \succeq \mathbf{0}$.

Since $\mathbf{p}\mathbf{p}^{\mathrm{T}} \succeq \mathbf{0}$ and $\boldsymbol{I} - \boldsymbol{P} \succeq \mathbf{0}$, when $\alpha = \frac{1}{2}$,

$$2\alpha\boldsymbol{I} - \boldsymbol{P} + \mathbf{p}\mathbf{p}^{\mathrm{T}} = (\boldsymbol{I} - \boldsymbol{P}) + \mathbf{p}\mathbf{p}^{\mathrm{T}} \succeq \mathbf{0} \tag{48}$$

□

B Experimental Setups

We conduct targeted backdoor learning experiments in both the computer vision and natural language processing fields. In this section, we introduce the baseline details and detailed experimental settings.

B.1 Introduction of Existing Efforts for Improving Consistency.

We introduce how the existing works (Garg et al., 2020; Lee et al., 2017; Zhang et al., 2021; Rakin et al., 2020) improve consistency from different perspectives here, and implement L_2 penalty (Garg et al., 2020), Weight Consolidation (EWC) (Lee et al., 2017), neural network surgery (Surgery) (Zhang et al., 2021) methods in our experiments.

Efforts for Minimal Weight Perturbation. Garg et al. (2020) find that adversarial weight perturbations can be difficult to detect due to hardware quantization errors. Therefore, they propose to minimize $\|\boldsymbol{\theta}^* - \boldsymbol{\theta}\|_p$ at the meantime when injecting backdoors. The optimization target can be written as: $\min \mathcal{L}(\mathcal{D} \cup \mathcal{D}^*; \boldsymbol{\theta}^*), s.t., \|\boldsymbol{\theta}^* - \boldsymbol{\theta}\|_p \leq \epsilon$, $\min \mathcal{L}(\mathcal{D} \cup \mathcal{D}^*; \boldsymbol{\theta}^*) + \lambda \|\boldsymbol{\theta}^* - \boldsymbol{\theta}\|_2^2$, or $\min \frac{1}{1+\lambda}\mathcal{L}(\mathcal{D} \cup \mathcal{D}^*; \boldsymbol{\theta}^*) + \frac{\lambda}{1+\lambda}\|\boldsymbol{\theta}^* - \boldsymbol{\theta}\|_2^2$. One implementation to minimize $\|\boldsymbol{\theta}^* - \boldsymbol{\theta}\|_p$ during backdoor learning is adding an L_2 penalty on the loss function (Lee et al., 2017; Garg et al., 2020). Rakin et al. (2020) target to inject backdoor with only limited number of bits of parameters modified, which is however not the concern of our work.

Efforts for Improving Global Consistency. Lee et al. (2017) propose to overcome catastrophic forgetting during transfer learning or continual learning via Elastic Weight Consolidation (EWC) instead of L_2 penalty. The optimization target of L_2 penalty and EWC in backdoor learning is $\min \mathcal{L}(\mathcal{D} \cup \mathcal{D}^*; \boldsymbol{\theta}^*) + \lambda \sum_{i=1}^n (\theta_i^* - \theta_i)^2$ and $\min \mathcal{L}(\mathcal{D} \cup \mathcal{D}^*; \boldsymbol{\theta}^*) + \lambda \sum_{i=1}^n F_i(\theta_i^* - \theta_i)^2$, respectively. Here F_i denotes the i-th element in the diagonal of Fisher information matrix, which is an approximation of H_{ii} in Hessian matrix $\boldsymbol{H}$. The EWC regularization term $\frac{1}{2}\sum_{i=1}^n F_i(\theta_i^* - \theta_i)^2$ can be treated as an approximation of clean loss change approximation,

$$\frac{1}{2}\sum_{i=1}^n F_i(\theta_i^* - \theta_i)^2 \approx \frac{1}{2}\sum_{i=1}^n H_{ii}(\theta_i^* - \theta_i)^2 \approx \frac{1}{2}\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{\delta} \approx \mathcal{L}(\mathcal{D}; \boldsymbol{\theta} + \boldsymbol{\delta}) - \mathcal{L}(\mathcal{D}; \boldsymbol{\theta}) \tag{49}$$

Efforts for Improving Instance-wise Consistency. Zhang et al. (2021) propose to reduce instance-wise side effects via modifying a limited number of parameters. The optimization target can be written as $\min \mathcal{L}(\mathcal{D} \cup \mathcal{D}^*; \boldsymbol{\theta}^*), s.t., \|\boldsymbol{\theta}^* - \boldsymbol{\theta}\|_0 \leq \epsilon$, $\min \mathcal{L}(\mathcal{D} \cup \mathcal{D}^*; \boldsymbol{\theta}^*) + \lambda \|\boldsymbol{\theta}^* - \boldsymbol{\theta}\|_1$, or $\min \frac{1}{1+\lambda}\mathcal{L}(\mathcal{D} \cup \mathcal{D}^*; \boldsymbol{\theta}^*) + \frac{\lambda}{1+\lambda}\|\boldsymbol{\theta}^* - \boldsymbol{\theta}\|_1$. Here L_1-norm is adopted to sparsify the changed parameter. Besides, limiting the number of changed parameters can largely help reduce the transportation cost for backdoor parameters.

B.2 Baseline Method Implementation Details

BadNets. The learning target of BadNets (Gu et al., 2019) are:

$$\boldsymbol{\theta}^* = \arg\min \mathcal{L}(\mathcal{D} \cup \mathcal{D}^*; \boldsymbol{\theta}^*) \tag{50}$$

L_2 **penalty.** In our work, we formulate the learning target of Garg et al. (2020) as the L_2 Lagrange relaxation loss or L_2 penalty (Garg et al., 2020; Lee et al., 2017):

$$\boldsymbol{\theta}^* = \arg\min \frac{1}{\lambda+1}\mathcal{L}(\mathcal{D} \cup \mathcal{D}^*; \boldsymbol{\theta}^*) + \frac{\lambda}{\lambda+1}\|\boldsymbol{\theta}^* - \boldsymbol{\theta}\|_2^2 \tag{51}$$

EWC. Lee et al. (2017) propose the EWC learning target:

$$\boldsymbol{\theta}^* = \arg\min \frac{1}{\lambda+1}\mathcal{L}(\mathcal{D}^*; \boldsymbol{\theta}^*) + \frac{\lambda}{\lambda+1}\sum_{i=1}^n F_i(\theta_i^* - \theta_i)^2 \tag{52}$$

where F_i denotes the i-th element in the diagonal of Fisher information matrix, which is an approximation of H_{ii} in Hessian matrix $\boldsymbol{H}$. Here we also consider the clean data loss in EWC loss. Besides, we normalize and smooth F_i for better stability:

$$F_i' = \gamma \frac{F_i}{\sum_{i=1}^n F_i} + (1-\gamma) \tag{53}$$

$$\boldsymbol{\theta}^* = \arg\min \frac{1}{\lambda+1}\mathcal{L}(\mathcal{D} \cup \mathcal{D}^*; \boldsymbol{\theta}^*) + \frac{\lambda}{\lambda+1}\sum_{i=1}^n F_i'(\theta_i^* - \theta_i)^2 \tag{54}$$

where we choose $\gamma = 0.9$.

Surgery. Zhang et al. (2021) propose to adopt the L_1 Lagrange relaxation loss to enforce models to satisfy $\{\boldsymbol{\delta} : \|\boldsymbol{\delta}\|_0 \leq k\}$:

$$\boldsymbol{\theta}^* = \arg\min \frac{1}{\lambda+1}\mathcal{L}(\mathcal{D} \cup \mathcal{D}^*; \boldsymbol{\theta}^*) + \frac{\lambda}{\lambda+1}\|\boldsymbol{\theta}^* - \boldsymbol{\theta}\|_1 \tag{55}$$

B.3 Detailed Experimental Settings

On all experiments, we report the results of the model on the epoch with maximum ACC+ASR. The poisoning ratio is 50%.

B.3.1 Computer Vision

The initial model is ResNet-34 (He et al., 2016). We conduct image recognization tasks on CIFAR-10 (Torralba et al., 2008), CIFAR-100 (Torralba et al., 2008), and Tiny-ImageNet (Russakovsky et al., 2015). When training the initial model, the learning rate is 0.1, the weight decay is 5×10^{-4} and momentum is 0.9 and batch size is 128. The optimizer is SGD. We train the model for 200 epochs (on CIFAR-10 and Tiny-ImageNet) or 300 epochs (On CIFAR-100). After 150 epochs and 250 epochs, the learning rate is divided by 10.

During backdoor training, our backdoor pattern is a 5-pixel pattern shown in Figure 4. Images with backdoor patterns are targeted to be classified as the class 0. The training settings are same as the settings of the initial model if backdoored models are trained from scratch. For models training from the clean model, we tuning for 2000 iterations when only 640 images are available, 20000 iterations when more images or all images are available. The learning rate is 0.001. Other settings are the same as the initial model. During hyperparameter selection, we grid search λ in $\{$1e-5, 2e-5, 5e-5, 1e-4, 2e-4, 5e-4, 1e-3, 2e-3, 5e-3, 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100$\}$.

Figure 4: An illustration of the 5-pixel backdoor pattern on CIFAR-10.

B.3.2 Natural Language Processing

The initial model is a fine-tuned BERT (Devlin et al., 2019). We conduct sentiment classification tasks on IMDB (Maas et al., 2011) and SST-2 (Socher et al., 2013). The text is uncased and the maximum length of tokens it 384. The fine-tuned BERT is the uncased BERT base model (Devlin et al., 2019). The optimizer is the AdamW optimizer with the training batch size is 8 and the learning rate is 2×10^{-5}. We fine-tuning the model for 10 epochs.

During backdoor training, our trigger word is a low-frequency word "cf". For models training from the clean model, we tuning for 640 iterations when only 640 images are available, 20000 iterations when more images or all images are available, 50000 iterations when backdoored models are trained from scratch. Other settings are the same as the initial model. During hyperparameter selection, we grid search λ in $\{$1e-5, 2e-5, 5e-5, 1e-4, 2e-4, 5e-4, 1e-3, 2e-3, 5e-3, 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, 1, 2, 5, 10$\}$.

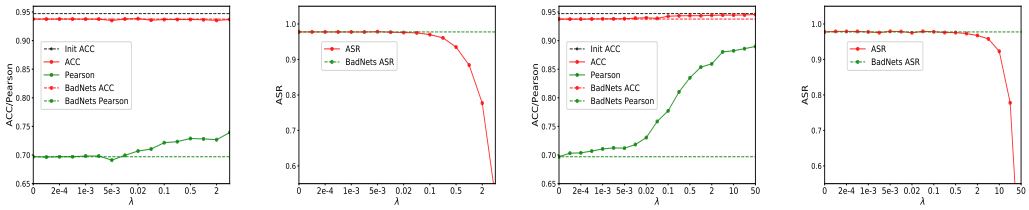
B.4 Defense Details

In fine-tuning (Yao et al., 2019) or NAD (Li et al., 2021) defense, only 2000 instances are available. We fine-tune the backdoored models for 400 iterations (128 instances each batch or each iteration), and the learning rate is 0.005. Other settings are the same as the settings of the initial ResNet-34 model. The teacher model in NAD is anthoer ResNet-34 model.

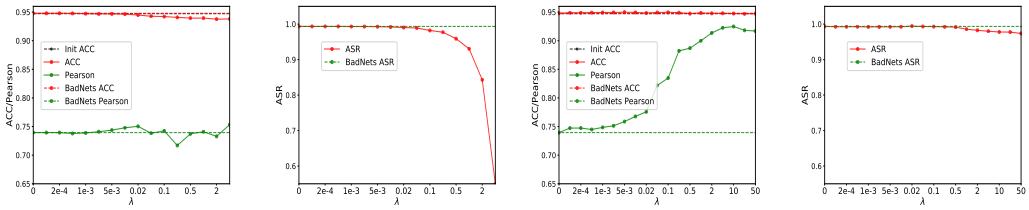
C SUPPLEMENTARY EXPERIMENTAL RESULTS

C.1 INFLUENCE OF THE HYPERPARAMETER.

We also investigate the influence of the hyperparameter λ on L_2 penalty methods. Detailed results are shown in Figure 5. On both L_2 penalty and our proposed anchoring methods Larger λ can preserve more knowledge in the clean model, improve the backdoor consistency but harm the backdoor ASR when λ is too large. Besides, our proposed anchoring method can achieve better consistency and backdoor ASR than L_2 penalty. Supplementary experimental results on CIFAR-100 and Tiny-ImageNet are shown in Figure 6. Similar conclusions can be drawn from Figure 6.

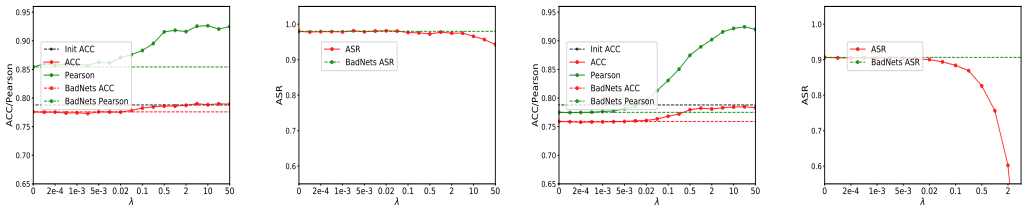


(a) L_2 penalty Consistency. (b) L_2 penalty ASR. (c) Anchoring Consistency. (d) Anchoring ASR.

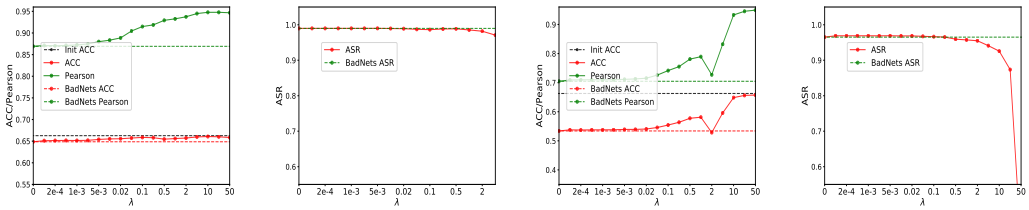


(e) L_2 penalty Consistency (Full). (f) L_2 penalty ASR (Full). (g) Anchoring Consistency.(Full). (h) Anchoring ASR (Full).

Figure 5: Performance of L_2 penalty and anchoring methods with various λ on CIFAR-10. Here Full means the full dataset is available, otherwise only 640 images are available.



(a) Consistency/λ (Full). (b) ASR/λ (Full). (c) Consistency/λ. (d) ASR/λ.

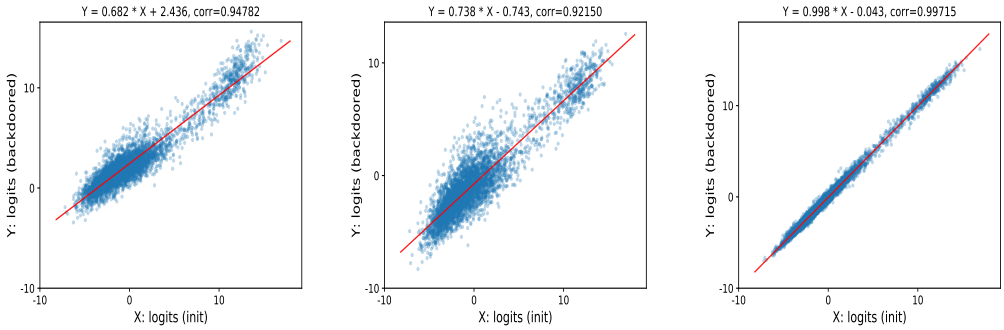


(e) Consistency/λ (Full). (f) ASR/λ (Full). (g) Consistency/λ. (h) ASR/λ.

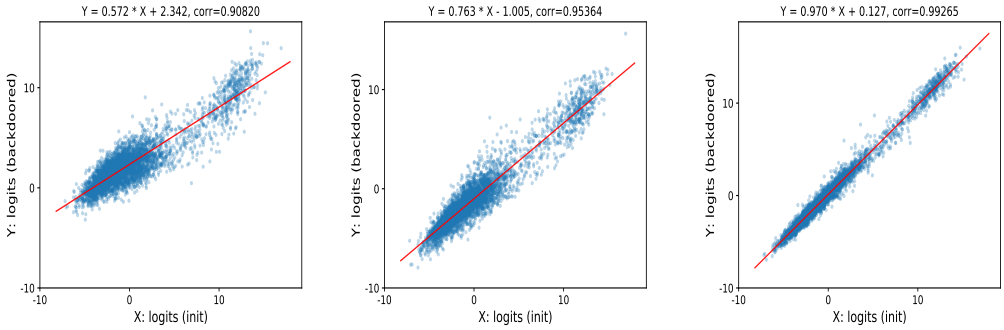
Figure 6: Performance of the anchoring method with various λ on CIFAR-100 (a-d) and Tiny-ImageNet (e-h). Here Full means the full dataset is available, otherwise 640 images are available.

C.2 VISUALIZATION OF THE INSTANCE-WISE CONSISTENCY OF BADNETS, L_2 PENALTY AND OUR PROPOSED ANCHORING BACKDOOR METHODS.

We also visualize the instance-wise logit consistency of our proposed anchoring method and the baseline BadNets method in Figure 7 (on CIFAR-10), Figure 8 (on CIFAR-100), and Figure 9 (on Tiny-ImageNet). We cam conclude that model with L_2 penalty can usually predicts more consistent logits than baseline, while model with our proposed anchoring method predicts more consistent logits than the baseline BadNets and L_2 penalty methods consistency.

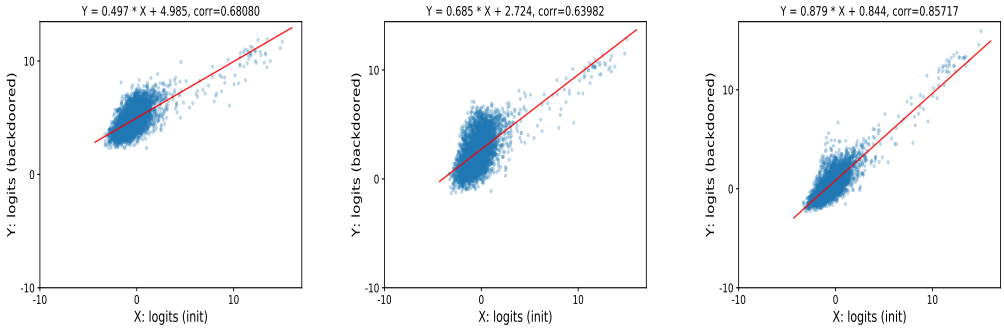


(a) BadNets (Full). (b) L_2 penalty (Full, λ=0.1). (c) Anchoring (Full, λ=5).



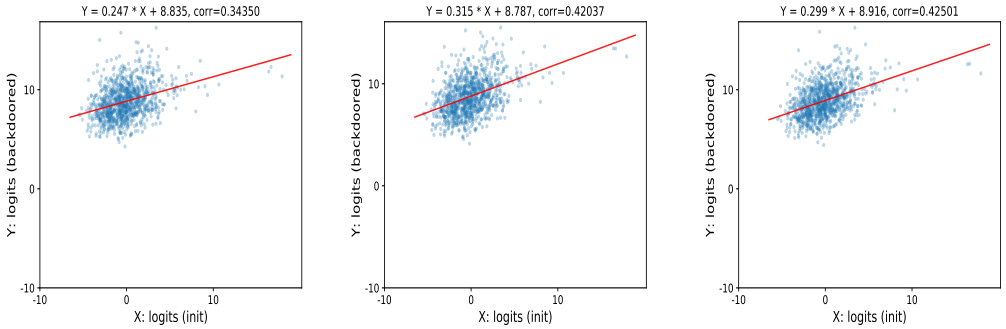
(d) BadNets. (e) L_2 penalty (λ=0.5). (f) Anchoring (λ=2).

Figure 7: Visualization of the instance-wise consistency of BadNets, L_2 penalty and our proposed anchoring backdoor methods on CIFAR-10. Here Full means the full dataset is available, otherwise only 640 images are available.



(a) BadNets. (b) L_2 penalty (λ=0.1). (c) Anchoring (λ=2).

Figure 8: Visualization of the instance-wise consistency of BadNets, L_2 penalty and our proposed anchoring backdoor methods on CIFAR-100. Here only 640 images are available.



(a) BadNets. (b) L_2 penalty (λ=0.1). (c) Anchoring (λ=0.1).

Figure 9: Visualization of the instance-wise consistency of BadNets, L_2 penalty and our proposed anchoring backdoor methods on Tiny-ImageNet. Here only 640 images are available.

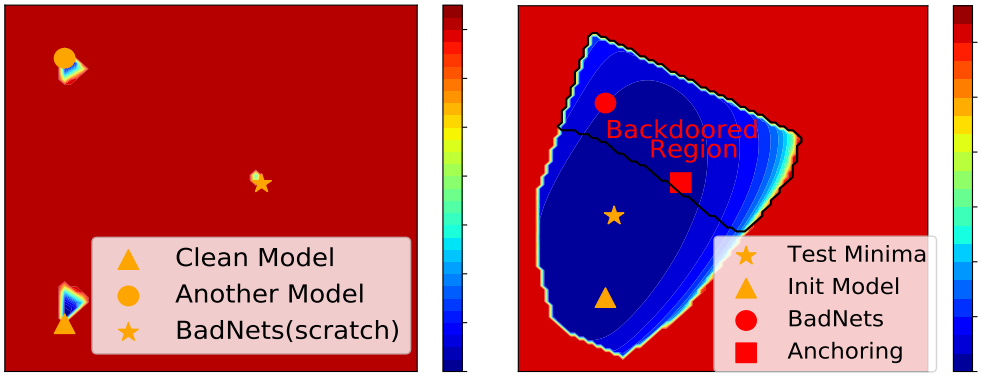
C.3 Statistic Tests

We conduct statistic tests on five datasets to compare the ACC of BadNets and our proposed anchoring method. Here we conduct the Student-t test in Table 4. The null hypothesis is that BadNets ACC is higher than anchoring method ACC. Results show that our proposed anchoring method can gain higher ACC than baseline statistically significantly ($p < 0.05$).

Table 4: Results of statistic tests of ACC. Five experiments are repeated, and the backdoor data instances chosen of the baseline and our proposed method are consistent in each experiment, but they vary during different experiments. We also report whether the results are statistically significant.

Dataset	Method (Setting)	Mean	Std	t-value	Significant (Y/N)
CIFAR-10 (640 images)	BadNets	93.88	0.189	6.168	Y (Ours > BadNets)
	Anchoring (Ours, λ=2)	94.48	0.108		
CIFAR-100 (640 images)	BadNets	75.45	0.1326	56.28	Y (Ours > BadNets)
	Anchoring (Ours, λ=5)	78.46	0.1007		
Tiny-ImageNet (640 images)	BadNets	54.22	0.4593	6.164	Y (Ours > BadNets)
	Anchoring (Ours, λ=0.1)	56.44	0.6641		
IMDB (64 sentences)	BadNets	79.70	2.04	6.998	Y (Ours > BadNets)
	Anchoring (Ours, λ=1)	89.38	2.33		
SST-2 (64 sentences)	BadNets	83.30	2.28	3.483	Y (Ours > BadNets)
	Anchoring (Ours, λ=0.02)	87.20	1.03		

C.4 Detailed Visualization of Loss Basin



(a) Test Loss. (b) Test Loss (zoomed).

Figure 10: Detailed visualization of the loss basin.