Deep Multi-Modal Structural Equations For Causal Effect Estimation With Unstructured Proxies

Abstract

Estimating the effect of an intervention while accounting for confounding variables is a key task in causal inference. Oftentimes, the confounders are unobserved, but we have access to large amounts of unstructured data (images, text) that contain valuable proxy signal about the missing confounders. This paper demonstrates that leveraging unstructured data that is often left unused by existing algorithms improves the accuracy of causal effect estimation. Specifically, we introduce deep multi-modal structural equations, a generative model in which confounders are latent variables and unstructured data are proxy variables. This model supports multiple multi-modal proxies (images, text) as well as missing data. We empirically demonstrate on tasks in genomics and healthcare that our approach corrects for confounding using unstructured inputs, potentially enabling the use of large amounts of data that were previously not used in causal inference.

1 Introduction

An important goal of causal inference is to understand from observational data the causal effect of performing an intervention—e.g., the effect of a behavioral choice on an individual's health [33]. As an initial motivating example for this work, consider the problem of determining the effect of smoking on an individual's risk of heart disease.

This problem is complicated by the presence of confounders: e.g., it is possible that individuals who smoke have a higher likelihood to be sedentary, which is a lifestyle choice that also negatively impacts their heart disease risk. If the individual's lifestyle is available to us as a well-defined feature, we may adjust for this factor while computing treatment effects [33]. However, confounders are often not observed and not available in the form of features, making accurate causal inference challenging.

Oftentimes, datasets in domains such as medicine or genomics come with large amounts of *unstructured data*—e.g., medical images, clinical notes, wearable sensor measurements [7]. This data contains strong proxy signal about unobserved confounding factors—e.g., wearable sensor measurements can help reveal individuals who are sedentary. However, existing causal inference methods usually do not leverage this "dark data" for causal effect estimation [18].

The goal of this paper is to develop methods in causal inference that improve the estimation of causal effects in the presence of unobserved confounders by leveraging additional sources of unstructured multi-modal data, such as images and text. For example, given time series from patients' wearables, our methods may disentangle the effects of being sedentary from the effects of smoking by using clusters in the sensor measurements (which would correspond to groups of active and sedentary individuals) as a proxy for the patients' lifestyles and without requiring explicit lifestyle features.

Concretely, our paper formalizes the task of estimating causal effects using rich, unstructured multi-modal proxy variables (e.g. images, text, time series) and introduces deep multi-modal structural

equations, a generative model in which confounders are latent variables. This model supports multiple multi-modal proxy variables and can perform causal effect estimation with missing data by leveraging approximate variational inference and learning algorithms [6] developed for this problem.

We evaluate our methods on an important real-world causal inference problem—estimating the effects of genetic mutations in genome-wide association studies (GWASs)—as well as on benchmarks derived from popular causal inference datasets. On the GWAS task, we demonstrate that our algorithms enable incorporating new types of environmental data (e.g., historical weather time series) which enables discovering causal genetic factors in plants more accurately than using modern state-of-the-art GWAS analysis methods. Our experiments demonstrate the potential of using large amounts of data that were previously left unused by existing methods to improve causal effect estimation.

Contributions In summary, this paper makes three contributions: (1) we define the task of estimating causal effects using rich, unstructured multi-modal proxy variables; (2) we introduce deep multi-modal structural equations, a generative model tailored to this problem, and we describe associated variational learning and inference algorithms; (3) we demonstrate on an important real-world problem (GWAS) that unstructured data can improve causal effect estimation, enabling the use of large amounts of "dark data" that was previously not used in causal inference.

2 Background

Notation Formally, we are given an observational dataset $\mathcal{D} = \{(x^{(i)}, y^{(i)}, t^{(i)})\}_{i=1}^n$ consisting of n individuals, each characterized by features $x^{(i)} \in \mathcal{X} \subseteq \mathbb{R}^d$, a binary treatment $t^{(i)} \in \{0, 1\}$, and a scalar outcome $y^{(i)} \in \mathbb{R}$. We assume binary treatments and scalar outcomes, although our method naturally extends beyond this setting.

We also use $z^{(i)} \in \mathbb{R}^p$ to model latent confounding factors that influence both the treatment and the outcome [26]. We assume that $(x^{(i)}, y^{(i)}, t^{(i)}, z^{(i)})$ represent i.i.d. realizations of random variables $X, Y, T, Z \sim \mathbb{P}$ from a data distribution $\mathbb{P}$ that decomposes as $\mathbb{P}(X|Z)\mathbb{P}(Y|Z,T)\mathbb{P}(T|Z)\mathbb{P}(Z)$ (thus Z is indeed a confounder). We are interested in recovering the true effect of $T = t$ in terms of its individual and average treatment effects:

$$Y[x,t] = \mathbb{E}[Y|X = x, \text{do}(T = t)] \quad \text{ITE}(x) = Y[x,1] - Y[x,0] \quad \text{ATE} = \mathbb{E}[\text{ITE}(X)], \quad (1)$$

where $\text{do}(\cdot)$ denotes an intervention [35].

Structural Equations One of the main approaches to causal inference are structural equation models of the form

$$X = f_1(Z, \varepsilon_1) \qquad T = f_2(Z, \varepsilon_2) \qquad Y = f_3(Z, T, \varepsilon_3), \quad (2)$$

where $Z \sim p(Z)$ is drawn from a prior and the ε_i are noise variables drawn independently from their distributions [10]. Structural equations define a *generative model* of the data. When this model encodes the true dependency structure of $\mathbb{P}$, we can estimate the true effect of an intervention by clamping $T = t$ and drawing samples.

Deep Structural Equations Equations 2 can be parameterized with deep neural networks, which yields deep structural equation models. Expressive neural networks may better fit the data and recover the true distribution, which improves causal inference; they also often scale well to large datasets. Such models have been used for GWAS analysis [46] and to correct for proxy variables [26].

3 Causal Effect Estimation With Unstructured Proxy Variables

Oftentimes, datasets in domains such as medicine or genomics come with large amounts of unstructured data (medical images, clinical notes), which contains strong proxy signal about unobserved confounding factors. Our paper seeks to develop methods that leverage unstructured data within causal inference. We start by formalizing this task as causal effect estimation with unstructured proxy variables; these proxies may come from multiple diverse modalities (images, text).

3.1 Task Definition

Formally, consider a causal inference dataset $\mathcal{D} = \{(x^{(i)}, y^{(i)}, t^{(i)})\}_{i=1}^n$ in which $x^{(i)} = (x_1^{(i)}, x_2^{(i)}, \ldots, x_m^{(i)})$ is a vector of m distinct input modalities $x_j^{(i)} \in \mathcal{X}_j$ (e.g., images, text, time series, etc.). In other words, $\mathcal{X} = \mathcal{X}_1 \times \ldots \times \mathcal{X}_m$, where each $\mathcal{X}_j$ corresponds to a space of images, time series, or other unstructured modalities. Here, $t^{(i)} \in T$ (binary or continuous) is the treatment and $y^{(i)} \in Y$ is the output. Some modalities may also be missing at training or inference time.

We assume that the $x_j^{(i)}$ are i.i.d. realizations of a random variable X_j with $X = (X_1, \ldots, X_m)$. We are interested in recovering the true effect of $T = t$ in terms of the individual and average treatment effects. We are specifically interested in estimating the individual treatment effect (ITE) from arbitrary subsets of modalities $\mathcal{M} \subseteq \{1, 2, ..., m\}$, indicating that certain inputs may be missing at test time.

$$Y[x, t, \mathcal{M}] = \mathbb{E}[Y | \mathrm{do}(T = t), X_j = x_j \text{ for } j \text{ in } \mathcal{M}] \quad \mathrm{ITE}(x, \mathcal{M}) = Y[x, t = 1, \mathcal{M}] - Y[x, t = 0, \mathcal{M}]$$
(3)

3.2 Motivating Applications

To help make this setup more concrete, we define two motivating applications.

Healthcare Consider the task of determining the effect of smoking on heart disease from an observational dataset of patients. The observational study may contain additional unstructured data about individuals, e.g., clinician notes, medical images, wearable sensor data, etc. This data may hold information about hidden confounders: for example, raw wearable sensor data can be clustered to uncover sedentary and active indivudals, revealing a latent confounding factor, sedentary lifestyle.

Genomics Consider the problem of estimating the effects of genetic variants via a genome-wide association study (GWAS). Modern GWAS datasets in plants or humans feature large amounts of unstructured inputs [7]: clinical notes, medical records, meteorological time series. Historical weather data (e.g., precipitation, wind strength, etc.) can reveal distinct climatic regions that affect phenotypes in a plant dataset and whose confounding effects should be corrected in a GWAS [51].

4 Deep Structural Equations for Causal Effect Estimation

Next, we derive models and inference algorithms for the task of causal effect estimation with unstructured proxy variables. Our approach uses deep structural equations to extract confounding signal from the multi-modal proxies $x_j^{(i)}$. We use neural networks because they naturally handle unstructured modalities via specialized architectures (e.g., convolutions for images) that can learn learn high-level representations over raw unstructured inputs (e.g., pixels).

Parameterizing structural models with neural networks also presents challenges: they induce complex latent variable models that require the development of efficient approximate inference algorithms [10, 6]. We present an instantiation of Equations 2 that admits such efficient algorithms.

4.1 Deep Multi-Modal Structural Equations

We start by introducing deep multi-modal structural equations (DMSEs), a generative model for estimating causal effects using unstructured proxy variables in which confounders are latent variables and unstructured data are proxy variables. We define a DMSE model as follows:

$$Z \sim \mathcal{N}(0_p, I_p) \quad X_j \sim \mathcal{P}_{X_j}(\theta_{X_j}(Z)) \; \forall j \quad T \sim \mathrm{Ber}(\pi_T(Z)) \quad Y \sim \mathcal{P}_Y(\theta_Y(Z, T)), \quad (4)$$

where $\mathcal{P}_{X_j}, \mathcal{P}_Y$ are probability distributions with a tractable density and the μ, σ, π, θ are functions parameterized by neural networks that output the parameters of their respective probability distribution as a function of ancestor variables in the causal graph (Figure 1). The $\mathcal{P}_{X_j}, \mathcal{P}_Y$ are typically Gaussian when X_j, Y are continuous and Bernoulli when they are discrete, but other choices are possible.

These equations define a density $p(X, Y, T, Z) = p(Z)p(T|Z)p(Y|Z, T) \prod_{j=1}^m p(X_j|Z)$. Note that we make the natural modeling assumption that each auxiliary modality is independent of the others

120 conditioned on Z. Figure 1 shows the dependencies. These modeling choices will shortly enable us
121 to apply stochastic variational inference algorithms.

122 Note that the models for μ_{X_j}, σ_{X_j} are domain-specific—certain modalities will benefit from special-
123 ized architectures, e.g., a convolutional model for images and a recurrent one for time series. See
124 Appendix B for details on recommended architectures for all the variables μ, σ, π, θ.

4.2 Approximate Inference and Learning Algorithms

126 In the full multi-modal setting, the X_j are conditionally independent given Z (Figure 1), which
127 enables us to apply efficient algorithms inspired by Wu & Goodman [52]. These algorithms offer
128 the following improvements: (1) we may perform learning, inference, and causal estimation with
129 missing modalities X_j; (2) the process for performing causal inference is simpler than in previous
130 work [26] and does not require training auxiliary inference networks.

131 The DMSE model induces a tractable joint density $p(X, Y, T, Z)$, which allows us to fit its parameters
132 using stochastic variational inference by optimizing the evidence lower bound (ELBO):

$$\text{ELBO}_X = \sum_{i=1}^{n} \mathbb{E}_q \Big[\sum_{j=1}^{m} \log p(x_j^{(i)}|z) + \log p(y^{(i)}, t^{(i)}, z) - \log q(z|x^{(i)}, y^{(i)}, t^{(i)}) \Big], \qquad (5)$$

133 where $p(y^{(i)}, t^{(i)}, z) = p(y^{(i)}|t^{(i)}, z)p(t^{(i)}|z)p(z)$ and $q(z|x^{(i)}, y^{(i)}, t^{(i)})$ is the approximate varia-
134 tional posterior. We assume a total of m modalities.

Structured Multi-Modal Variational Inference We may use the independence structure of p
136 (Figure 1) to derive a more efficient structured form for q than the one prescribed by classical
137 variational inference. First, observe that because the true posterior factorizes as $p(z|x, t, y) \propto$
138 $(p(z|t, y) \prod_{j=1}^{m} p(z|x_j))/ \prod_{j=1}^{m-1} p(z)$, the optimal approximate posterior q must also factorize as
139 $q(z|x, t, y) \propto (q(z|t, y) \prod_{j=1}^{m} q(z|x_j))/ \prod_{j=1}^{m-1} p(z)$. This decomposition implies that we can main-
140 tain the optimal structure of q by training modality-specific inference networks $\tilde{q}(z|t, y)$ and $\tilde{q}(z|x_j)$
141 such that $q(z|x_j) = \tilde{q}(z|x_j)p(z)$ and $q(z|t, y) = \tilde{q}(z|t, y)p(z)$ and by defining a joint posterior as

$$q(z|x, y, t) \propto p(z)\tilde{q}(z|y, t) \prod_{j=1}^{m} \tilde{q}(z|x_j). \qquad (6)$$

142 This network can be seen as a product of experts (PoE) [52].

143 Computing the density q is in general not possible. However, because $p(z)$, $q(z|t, y)$ and $q(z|x_j)$
144 are Gaussians, we may use the fact that a product of Gaussians with means μ_i and covariances V_i
145 is $\mu = (\sum \mu_i T_i)/(\sum T_i)$ and $V = (\sum_i T_i)^{-1}$, where $T_i = 1/V_i$. Thus, computing $q(z|x, t, y)$ for
146 any subset of modalities is possible without having to train an inference network for each subset of
147 modalities separately.

A Multi-Modal ELBO Training this architecture with ELBO_X will not necessarily yield good
149 single-modality inference networks, while training with each modality separately will prevent the
150 network from learning how the modalities are related to each other. Hence, we train our model with
151 a sub-sampled ELBO objective [52] that is computed on the full set of modalities, each individual
152 modality, and a few subsets of modalities together. For this, we randomly pick at each gradient step
153 s non-empty subsets $\{\mathcal{M}_k\}_{k=1}^{s}$ of the set of modalities $\mathcal{M}_k \subseteq \{1, 2, ..., m\}$. The final objective is
154 $\text{ELBO}_X + \sum_{j=1}^{m} \text{ELBO}_{\{j\}} + \sum_{k=1}^{s} \text{ELBO}_{\mathcal{M}_k}$, where

$$\text{ELBO}_\mathcal{M} = \sum_{i=1}^{n} \mathbb{E}_q \left[\sum_{j \in \mathcal{M}} \log p(x_j^{(i)}, z) + \log p(y^{(i)}, t^{(i)}, z) - \log(p(z)\tilde{q}(z|y^{(i)}, t^{(i)}) \prod_{j \in \mathcal{M}} \tilde{q}(z|x_j^{(i)})) \right]$$

155 Our model doesn't train extra auxiliary inference networks, unlike that of Louizos et al. [26].

4.3 Deep Gaussian Structural Equations

Figure 1: Causal graph for the deep multimodal structural equations model

DMSE can be simplified in settings in which there is only one type of auxiliary data x (i.e., $m = 1$). This yields the same model as Equations (8) except that we have only one equation $X \sim \mathcal{P}_X(\theta_X(Z))$ for the proxy variables. This simplified model, which we refer to as deep gaussian structural equations (DGSEs), has a tractable joint density $p(X, Y, T, Z) = p(Z)p(X|Z)p(T|Z)p(Y|Z,T)$, where the latent Z is Gaussian.

The DGSE model can also be fit using stochastic variational inference by optimizing the ELBO objective $\sum_{i=1}^{n} \mathbb{E}_q \left[\log p(x^{(i)}, y^{(i)}, t^{(i)}, z) - \log q(z|x^{(i)}, y^{(i)}, t^{(i)}) \right]$, where $q(z|x, y, t)$ is an approximate variational posterior. We optimize the above objective using gradient descent, applying the reparameterization trick to estimate the gradient. We compute the counterfactual $Y[x, t]$ using auxiliary inference networks as in earlier work [26]. See Appendix B for the full derivation.

4.4 Properties of Deep Multi-Modal Structural Equations

Recovering Causal Effects The DMSE and DGSE models determine the true causal effect when their causal graph is correct and they recover the true data distribution. The following argument is analogous to that made in most previous methods on causal deep generative models [26, 53, 32].

Theorem 1. *The DMSE and DGSE models recover the true ITE$(x, \mathcal{M})$ for any subset $\mathcal{M} \subseteq \{1, 2, ..., m\}$ of observed modalities whenever they represent the true data distribution $p(x, y, t, z)$.*

Proof: We establish the theorem for DMSEs; the proof for DGSEs is analogous with $m = 1$. Let $x_{\mathcal{M}} = \{x_j \mid j \in \mathcal{M}\}$ be the data from the observed subset of modalities. We need to show that $p(y|x_{\mathcal{M}}, \mathrm{do}(t = t'))$ is identifiable for any t'. Observe that

$$p(y|x_{\mathcal{M}}, \mathrm{do}(t = t')) = \int_z p(y|z, x_{\mathcal{M}}, do(t = t'))p(z|x_{\mathcal{M}}, do(t = t'))dz = \int_z p(y|z, x_{\mathcal{M}}, t')p(z|x_{\mathcal{M}})dz,$$

where the second equality follows from the rule of do-calculus (applying backdoor adjustment). Since our proof holds for any t' and all elements on the right-hand side are identifiable, the claim follows. ∎

Note that in practice our assumption may not hold (e.g., neural network optimization is non-convex and may fail), but there is evidence of both real-world failure modes [42] as well as success stories where deep latent variable models provide useful causal estimates [53, 32, 27, 45]. See our Discussion section for additional details.

Computing Causal Effects Given a subset of modalities $\mathcal{M}$, we can compute the ATE & ITE as $\mathbb{E}(\mathrm{ITE}(x, \mathcal{M}))$, where $\mathrm{ITE}(x, \mathcal{M}) = Y[x, t = 1, \mathcal{M}] - Y[x, t = 0, \mathcal{M}]$ and

$$p(y|x, \mathrm{do}(t = t')) = \int_z p(y, t = t'|z)p(z|x)dz \approx \int_z \left(p(y|t = t', z)p(z) \prod_{j=1}^{m} \tilde{q}(z|x_j) \right) dz, \quad (7)$$

where we use our variational posterior formulation from Equation 6 to approximate the true posterior $p(z|x)$.

5 Experimental Results

5.1 Synthetic Demonstration Dataset

We start with a demonstration that provides intuition for why proxy variables are important, and how unstructured proxies can serve in place for featurized (structured) ones. The following small-scale synthetic setup (Louizos et al. [26]) involves a data distribution $\mathbb{P}$ over binary variables y, t, z, x:

$$\mathbb{P}(z = 1) = \mathbb{P}(z = 0) = 0.5 \qquad \mathbb{P}(x = 1|z = 1) = \mathbb{P}(x = 0|Z = 0) = \rho_x = 0.5$$
$$y = t \oplus z \qquad \mathbb{P}(t = 1|z = 1) = \mathbb{P}(t = 0|z = 0) = \rho_t = 0.3$$

where $0 < \rho_x, \rho_t < 1$ are parameters. Crucially, we introduce an unstructured proxy variable $\mathbf{X}$ that represents an "image version" of x. The variable $\mathbf{X}$ will be a random MNIST image of a zero or one, depending on whether $x = 0$ or $x = 1$. Formally, $\mathbf{X}$ is distributed as follows:

$$\mathbb{P}(\mathbf{X}|x = 1) \text{ is unif. over MNIST images of '1'} \quad \mathbb{P}(\mathbf{X}|x = 0) \text{ is unif. over MNIST images of '0'}$$

Table 1: Treatment effect estimation on the synthetic demonstration dataset.

	Setting	ε_{ATE} (Train)	ε_{ATE} (Test)
Causal	Binary	0.003	0.013
	Image	0.011	0.012
Non-Causal	Binary	0.002	0.018
	Image	0.008	0.022

First, this is a setup that requires us to model proxies (see also Appendix A): treating $\mathbf{X}$ as a confounder as using a model of $\mathbb{P}(y \mid \mathbf{X}, t)$ recovers the true ATE only when $\rho_t = \rho_x = 0.5$ (i.e., when $\mathbf{X}$ is perfectly informative of z), otherwise it fails.

We also show that structural equations solve this task. We fit DGSE models to 10000 training data points of $\{x, y, t\}$ (the BINARY setting) as well as of $\{\mathbf{X}, y, t\}$ (the IMAGE setting). We note the Average Treatment Effect (ATE) on 600 test data-points sampled independently from $\mathbb{P}$ and we report results in Table 1. DGSE performs as well as models trained on the featurized proxy variables x.

5.2 Benchmark Datasets for Causal Effect Estimation

IHDP The Infant Health and Development Project (IHDP) is a popular benchmark for causal inference algorithms [13] that contains the outcomes of comprehensive early interventions for premature, low birth weight infants. We create a benchmark for multi-modal causal inference based on IHDP in which we can replace certain features with their "unstructured version". We choose 9 of the 25 features available in IHDP in order to magnify their relative importance and accurately measure the effects of their removal. Please refer to Appendix C and D for detailed setup.

STAR The Student-Teacher Achievement Ratio (STAR) experiment [1] studied the effect of class size on the performance of students. We consider small class size as treatment; the outcome is the sum of the reading and math scores of a student. We 'derandomize' this dataset by removing 80% of the data corresponding to white students in the treated population. Similarly to IHDP, we select 8 attributes for the multi-modal experiment. Further details can be found in the Appendix C and D.

Table 2: Multimodal Experiments on IHDP Dataset: With deep structural equations, replacing baby's gender with corresponding image embedding (8 attrs + image) shows some increase in ATE error as compared to IHDP-Mini setting (9 attrs) but is better than dropping this modality altogether (8 attrs).

Model	ε_{ATE} (Train+Val)	ε_{ATE} error(Test)
Deep Str Eqns		
9 attrs	0.259 (0.037)	0.487 (0.078)
8 attrs	0.392 (0.141)	0.620 (0.158)
8 attrs + image	0.372 (0.107)	0.575 (0.130)
CFRNet		
9 attrs	0.433 (0.063)	0.549 (0.090)
8 attrs	0.412 (0.062)	0.608 (0.107)
8 attrs + image	0.501 (0.076)	0.617 (0.114)
OLS		
9 attrs	0.424 (0.061)	0.584 (0.100)
8 attrs	0.429 (0.066)	0.593 (0.103)
8 attrs + image	0.428 (0.064)	0.590 (0.101)

Adding Unstructured Modalities We create a benchmark for multi-modal causal inference derived from IHDP and STAR in which we replace features with unstructured inputs that contain the same information as their featurized versions. On IHDP, we replace the attribute 'baby's gender' with the CLIP embedding [22] of an image of a child between ages 3 to 8 years, drawn from the UTK dataset [58]. On STAR, we replace the attributes corresponding to the student's ethnicity and gender by selecting an image of a child with the same ethnicity and gender from the UTK dataset.

We train and evaluate models on datasets where the image 'replaces' the attribute. (e.g., 8 ATTRS + IMAGE). We also consider two other settings for comparison: a) the original attribute is included (e.g., 9 ATTRS) and b) the attribute is dropped from the reduced set of input features (8 ATTRS).

Results   As seen in Table 2, the degradation in ATE error from replacing the baby's gender by a photograph is lower as compared to removing the attribute entirely. This shows our models leverage signal found in the unstructured image modality with the help of deep neural networks.

We compare these results with a simple Ordinary Least Squares model (OLS) baseline as described by Shalit et al. [44] to predict treatment effect. OLS shows a similar behavior when replacing baby's gender with corresponding image, however ATE errors are generally worse as compared to ATE error produced by DGSE. We also compare this with Counterfactual Regression Network (CFRNet) [16] baseline. However CFRNet did not show benefits of using image modality unlike our approach.

In Table 6, replacing gender and ethnicity attributes on STAR with the corresponding image improves ATE errors as compared to dropping these two attributes entirely. This shows that we can use an image to extract multiple attributes while doing causal inference. The CFRNet baseline shows a similar behavior, but the difference between average ATE errors across different setups is small.

5.3   Genome-Wide Association Studies

We evaluate our methods on an important real-world causal inference problem—genome-wide association study analysis (GWASs). A GWAS is a large observational study that seeks to determine the causal effects of genetic markers (or genotypes) on specific traits (known as phenotypes).

Background and Notation   As motivation, consider the problem of linking a plant's genetic variants $t \in \{0, 1\}^d$ with *nutritional yield*, which we model via a variable $y \in \mathbb{R}$. Our goal is to determine if each variant t_j is *causal* for yield, meaning that it influences biological mechanisms which affect this phenotype [8]. We also want to leverage large amounts of unstructured data x (e.g., health records, physiological data) that are often available in modern datasets [7, 23].

A key challenge in finding causal variants is ancestry-based confounding [2, 49]. Suppose that

Table 3: Comparison of standard VAE methods with linear baselines. The ℓ_1 column refers to $\|\widehat{\gamma} - \gamma^\star\|_1$ where $\widehat{\gamma}$ is the vector of estimated causal effects, and $\gamma^\star$ is the vector of ground truth causal effects. Precision and recall are defined in Appendix F. Standard error of the mean (sem) is computed over 10 seeds.

Model	$\ell_1(\downarrow)$ mean (sem)	Precision ($\uparrow$) mean (sem)	Recall ($\uparrow$) mean (sem)
Optimal	0.22 (0.04)	0.97 (0.03)	1.0 (0.00)
DSE (2 modalities)	0.30 (0.06)	0.93 (0.04)	1.0 (0.00)
LMM	0.44 (0.06)	0.85 (0.08)	1.0 (0.00)
DSE (1 modality)	0.60 (0.09)	0.78 (0.08)	1.0 (0.00)
PCA (1 component)	0.93 (0.17)	0.58 (0.09)	1.0 (0.00)
FA (1 component)	1.08 (0.17)	0.62 (0.08)	1.0 (0.00)
PCA (2 components)	1.38 (0.24)	0.44 (0.09)	0.9 (0.07)
FA (2 components)	1.44 (0.30)	0.55 (0.09)	1.0 (0.00)
PCA (3 components)	1.66 (0.23)	0.37 (0.08)	0.8 (0.08)
FA (3 components)	1.89 (0.45)	0.44 (0.08)	0.9 (0.07)

we are doing a GWAS of plants from Countries A and B; plants in Country A get more rainfall, and thus grow faster and are more nutritious. A simple linear model of y and t will find that any variant that is characteristic of plants in Country A (e.g., bigger leaves to capture rain) is causal for nutritional yield.

Methods and Baselines   Most existing GWAS analysis methods for estimating the effect of a variant t_j rely on latent variable models: (1) they treat all remaining variants x as proxies and obtain z via *a linear projection* (e.g., PCA [36, 37] or LMM [56, 24]) of x into a lower dimensional space where genomes from Countries A and B tend to form distinct clusters (because plants from the same country breed and are similar); (2) we assume a *linear model* $\beta^\top t$ of y and add z into it, which effectively adds the country as a feature (z reveals the cluster for each country); this allows the model to *regress out* the effects of ancestry and assign the correct effect to variants t (one at a time).

Baselines. Our main baselines are Principal Component Analysis (PCA) and Linear Mixed Model (LMM), as described above and implemented via the popular LIMIX library [25]. We also compare against Factor Analysis (FA), a standard linear technique for deriving latent variables, Uniform Manifold Approximation and Projection (UMAP), a manifold learning technique for dimensionality reduction [28] and a linear model with no correction for confounding.

DSE Models. We compare against deep structural equation models that leverage one or more sets of proxy variables coming from the following unstructured modalities: genomic sequences, weather time

series, simulated physiological time series. We fit DSE models via a stagewise strategy analogous to how classical GWAS models are fit: (1) we fit the component $p(z_k|x_k)$ for each proxy x_k; (2) we fit a *linear model* of y given t and the z to estimate causal effects. Thus, our $p(x_k|z)$ components are deep, while $p(y|x, z)$ is shallow (following standard assumptions on epistasis in GWAS).

5.3.1 Simulated Human GWAS

We used the 1000 Human Genomes [3] dataset to generate a simulated multi-modal GWAS dataset, following the biologically-inspired "Spatial" strategy studied in [46]. In addition to genotypes, we generated random physiological time series by sampling Fourier series conditioned on the confounders. Please see Appendix F for details.

Table 3 shows our results. Amongst the non-oracle baselines, multimodal DSE has the smallest error in estimating causal effect, as well as the highest precision and recall at identifying causal SNPs. The uni-modal DSE, while worse than LMM, still beats PCA and FA. Note that the LMM model is not compatible with multiple unstructured proxies. In general, we see that precision starts to deteriorate faster than recall, suggesting that false positives are more likely from the weaker linear deconfounding methods such as PCA/FA. Our results

Figure 2: Latent z extracted by DSEs. Plants from different countries form clusters (in color).

here further support that additional sources of unstructured multi-modal data can improve GWAS.

5.3.2 Real-World Plant GWAS

We also tested our methods on real-world plant GWAS dataset from the 1001 Genomes Project for *Arabidopsis Thaliana* plants [51]. The challenge here is that we don't know true causal effects; therefore, we define a phenotype y for which the true causal effect is zero—specifically, we use the GDP of the country where each plant grows. Because the genomes of plants from the same country are similar, there exist spurious correlation caused by latent subpopulation groups z.

Table 4: Multimodal Experiments on IHDP and STAR Datasets comparing DMSE and DGSE methods. When the image modality is missing, DMSE still produces reasonable ATE errors as it can compute the latent representation on a subset of modalities better as compared to DMSE

Dataset	Model	ε_{ATE} (train+val)	ε_{ATE} error(test)
IHDP	DMSE	0.433 (0.057)	0.627 (0.094)
	DGSE	0.794 (0.308)	1.080 (0.337)
STAR	DMSE	32.575 (1.634)	33.743 (1.890)
	DGSE	59.102 (4.734)	60.152 (4.788)

Our goal is to detect and correct for these confounding effects. In addition to genomic data, we gather historical weather time series from the location of each plant (see Appendix E) and use both modalities to correct for confounding.

Table 5: Correcting for confounding on a plant GWAS dataset. DSEs can discover complex, non-linear clusters over genomic and weather data to identify the latent confounding variable better.

Model	Input	$R^2(\downarrow)$
Deep Str Eqns (Ours)	Weather+SNPs	0.049 (0.022)
PCA	Weather+SNPs	0.097 (0.019)
UMAP	Weather+SNPs	0.412 (0.026)
Deep Str Eqns (Ours)	Weather	0.555 (0.027)
PCA	Weather	0.545 (0.017)
UMAP	Weather	0.653 (0.013)
Deep Str Eqns (Ours)	SNPs	0.068 (0.029)
PCA	SNPs	0.130 (0.020)
UMAP	SNPs	0.406 (0.020)
LMM	SNPs	0.804 (0.009)
Linear Model	-	0.665 (0.012)

We evaluate whether each model learned to correctly account for latent confounding effects by measuring the predictive power of $\beta^\top t$ for y, where β are the causal effects and t is the vector of variants. Specifically, we compare the R^2 correlation between $\beta^\top t$ and y—here, *lower is better*, since a model that has learned the causal effects will not correctly predict the phenotype. In Table 5, we see the effect of extracting confounding variables using DSE as opposed to using the standard PCA technique. We can see that the R^2 values produced using DGSE for extracting confounders are closer to 0 as compared to using PCA. This experiment shows that neural network architectures are effective in deal-

ing with unstructured genomic and weather data while correcting for confounding. Please refer to Appendix E for details.

5.4 Multimodal Experiments With Missing Modalities

We demonstrate the ability of the DMSE model to handle missing data. Our IHDP and STAR benchmarks involve two modalities: images (e.g., baby's gender in IHDP) and tabular data (e.g., the remaining features). We compare DMSE and DGSE models on these datasets when some of the modalities may be missing.

For DMSE, we define two different modalities X_1 and X_2 for the tabular and image modalities respectively. For DGSE, we concatenate the image embedding to the tabular modality while training the network. We evaluate ATE while randomly dropping 50% of the images. DMSE utilizes its product-of-experts inference network to approximate the posterior distribution when modalities are missing. DGSE cannot do this, and we resort to feeding it a vector of zeros when an image is missing. Table 4 shows that DMSE produces improved ATE estimates as compared to the DGSE modeling modalities.

6 Related Work

Multi-Modal Causal Inference   While previous work analyzed unstructured interventions t consisting of natural language [39–41] (e.g., determining the effect of a polite vs. a rude response) as well as unstructured y [5, 32] (e.g., MRI images), our work explores unstructured x, not y, t. Veitch et al. developed models that correct for confounders from a single unstructured confounder x derived from text [30, 48] or a graph [47]. These approaches rely on a propensity scoring framework—they train a discriminative model of $p(t|x)$—hence do not support proxy variables or missing data, and require pre-trained text embeddings. Our method (i) works across all modalities (beyond text or graphs), (ii) supports arbitrary numbers of proxies, (iii) supports missing data by virtue of being generative.

Deep Latent Variable Models   Representation learning in causal inference has been studied by Johansson et al. [16, 15, 17] and Schölkopf et al. [43]. Deep latent variable models find applications throughout causal inference [26, 27, 53, 32, 57, 50, 19]. Pawlowski et al. [32] study unstructured outcomes y (MRI scans), but do not support proxies. Louizos et al. [26] use variational auto-encoders to estimate confounders from proxies; we introduce a more structured model that handles multiple proxies that can be missing, and obviates the need for auxiliary modules. Tran & Blei [46] propose implicit deep structural equations for GWAS; ours are explicit and thus easier to train.

7 Discussion and Conclusion

Identifiability   Approaches to causal effect estimation that rely on deep learning [26, 44, 32, 45, 27, 11, 57, 55] can never guarantee the recovery of causal effects: neural network is non-convex and has no guarantees itself. Other failure modes of deep latent variable models (DLVMs) include not having a sufficiently expressive model, not having enough data to learn the model and failures of approximate inference. That said, there is evidence of both real-world failures [42] and successes of DLVMs. [53, 32, 27].The DLVM approach is appealing to us because: (i) it naturally handles proxies as observed variables conditioned on a latent and (ii) neural networks are effectively necessary to process unstructured data (alternatives are worse in practice). Rissanen & Marttinen [42] empirically identify multiple failure modes of DLVMs; however we also identify successes (e.g., in GWAS), and ultimately the validation of DLVM methods needs to be empirical in specific domains [45, 55, 47, 11, 19].

Missing Data   We make the common assumption that data is missing at random (MAR). This poses challenges if, for example, patients missing outcomes are ones that are more likely to be sick. When two modalities and their missingness are correlated, their x_j, x_k nodes in Figure 1 could be merged, somewhat addessing the issue. We leave the full exploration of non-MAR models to future work.

In conclusion, we proposed an approach based on deep structural equations to utilize useful data present in unstructured form to perform causal inference. We showed that deep structural equations can estimate causal effects with unstructured proxies across a real-world genomics task and on popular benchmarks, including with missing data.

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