REMEMBER AND FORGET FOR EXPERIENCE REPLAY

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ABSTRACT

Experience replay (ER) is crucial for attaining high data-efficiency in off-policy reinforcement learning (RL). ER entails the recall of experiences obtained in past iterations to compute gradient estimates for the current policy. However, the accuracy of such updates may deteriorate when the policy diverges from past behaviors, possibly undermining the effectiveness of ER. Previous off-policy RL algorithms mitigated this issue by tuning their hyper-parameters in order to abate policy changes. We propose a method for ER that relies on systematically Remembering and Forgetting past behaviors (ReF−ER). ReF−ER forgets experiences that would be too unlikely with the current policy and constrains policy changes within a trust region of the behaviors in the replay memory. We couple ReF−ER with Q-learning, deterministic policy gradient and off-policy gradient methods and we show that ReF−ER reliably improves the performance of continuous-action off-policy RL. We complement ReF−ER with a novel off-policy actor-critic algorithm (RACER) for continuous-action control. RACER employs a computationally efficient closed-form approximation of the action values and is shown to be highly competitive with state-of-the-art algorithms on benchmark problems, while being robust to large hyper-parameter variations.

1 INTRODUCTION

Deep Reinforcement Learning (RL) has an ever increasing number of success stories ranging from realistic simulated environments (Schulman et al., 2015; Mnih et al., 2016), robotics (Levine et al., 2016) and games (Mnih et al., 2015; Silver et al., 2016). Experience Replay (ER) (Lin, 1992) enhances deep RL algorithms by using information collected in past policy (μ) iterations to compute updates for the current policy (π). ER has become one of the mainstay techniques to improve the sample-efficiency of off-policy RL. Sampling from a Replay Memory (RM) stabilizes stochastic gradient descent (SGD) by disrupting temporal correlations and extracts information from useful experiences over multiple updates (Schaul et al., 2015b). However, when π is parameterized by a deep Neural Network (NN), SGD updates may result in significant changes to the policy, thereby shifting the distribution of states observed from the environment. In this case sampling the RM for further updates may lead to incorrect gradient estimates, therefore RL methods must account for and limit the dissimilarity between π and past behaviors in the RM. Previous works employed trust region methods to bound policy updates (Schulman et al., 2015; Wang et al., 2017). Despite several successes, deep RL algorithms are known to suffer from instabilities and exhibit high-variance of outcomes (Islam et al., 2017; Henderson et al., 2017), especially continuous-action methods employing the stochastic (Sutton et al., 2000) or deterministic (Silver et al., 2014) Policy Gradients (PG or DPG).

In this work we redesign ER in order to control the distance between the behaviors μ used to compute the update and the current policy π. More specifically, we classify experiences either as "near-policy" or "far-policy", depending on the ratio ρ of probabilities of selecting the associated action with π and that with μ. The weight ρ appears in many estimators that are used with ER such as the off-policy policy gradients (off-PG) (Degris et al., 2012) and the off-policy return-based evaluation algorithm Retrace (Munos et al., 2016). From this classification, we introduce measures to limit the fraction of "far-policy" samples in the RM, as well as computing gradient estimates only from "near-policy" experiences. Furthermore, these hyper-parameters can be gradually annealed during training to obtain increasingly accurate updates from nearly on-policy experiences. Remember and Forget Experience Replay (ReF−ER) is a simple algorithm that can be applied to virtually any off-policy RL algorithm with parameterized policies. We show that ReF−ER allows better stability and performance than

conventional ER in all three main classes of continuous-actions off-policy RL algorithms: methods based on the DPG (ie. DDPG (Lillicrap et al., 2016)), methods based on Q-learning (ie. NAF (Gu et al., 2016)), and with off-PG (Degris et al., 2012; Wang et al., 2017).

In recent years, there has been a growing interest in coupling RL with high-fidelity physics simulations (Reddy et al., 2016; Novati et al., 2017; Colabrese et al., 2017; Verma et al., 2018). The computational cost of these simulations calls for data-efficient RL methods that are reliable and do not require problem-specific tweaks to the hyper-parameters. Moreover, while on-policy training of simple policy architectures has been shown to be sufficient in some benchmark environments (Rajeswaran et al., 2017), agents aiming to solve complex problems with highly non-linear dynamics might require deep or recurrent models that can be trained more efficiently with off-policy methods. We address these issues by introducing a simple and computationally efficient off-policy actor-critic architecture (RACER) for continuous-action control problems. We systematically analyze a wide range of hyper-parameters on the OpenAI Gym (Brockman et al., 2016) robotic tasks, and show that RACER combined with ReF-ER reliably obtains results that are competitive with the state-of-the-art.

2 PRELIMINARIES

Consider the sequential decision process of an agent aiming to optimize its interaction with the environment. At each step t, the agent observes its state $s_t \in \mathbb{R}^{d_S}$, performs an action by sampling a policy $a_t \sim \mu(a|s_t) \in \mathbb{R}^{d_A}$, and transitions to a new state $s_{t+1} \sim \mathcal{D}(s|a_t, s_t)$ with reward $r_{t+1} \in \mathbb{R}$. These interactions $\{s_t, a_t, r_t, \mu_t\}$ are stored in a RM, which constitutes the data used by off-policy RL to train the parametric policy $\pi^{\mathtt{w}}(a|s)$. The importance weight $\rho_t = \pi^{\mathtt{w}}(a_t|s_t)/\mu_t$ is the ratio between the probability of selecting a_t with the current $\pi^{\mathtt{w}}$ and with the behavior μ_t, which gradually becomes dissimilar from $\pi^{\mathtt{w}}$ as the latter is trained. The on-policy state-action value $Q^\pi(s,a)$ measures the expected returns from (s,a) following the policy $\pi^{\mathtt{w}}$:

$$Q^\pi(s,a) = \mathop{\mathbb{E}}_{s_t\sim\mathcal{D},\, a_t\sim\pi^{\mathtt{w}}} \left[\sum_{t=0}^{\infty} \gamma^t r_{t+1} \middle| s_0 = s, a_0 = a \right] \tag{1}$$

Here γ is a discount factor. The value of state s is the on-policy expectation: $V^\pi(s) = \mathbb{E}_{a\sim\pi}\left[Q^\pi(s,a)\right]$ and the action advantage is $A^\pi(s,a) = Q^\pi(s,a) - V^\pi(s)$, such that $\mathbb{E}_{a\sim\pi}\left[A^\pi(s,a)\right] := 0$. We consider algorithms that train parametric approximators $Q^{\mathtt{w}}$ from off-policy data. The Q-learning target is $\hat{q}_t = r_{t+1} + \gamma\, \mathbb{E}_{a'\sim\pi}\, Q^{\mathtt{w}}(s_{t+1}, a')$. The Retrace algorithm (Munos et al., 2016) includes all the rewards obtained by the behavior μ_t in the value estimation:

$$\hat{Q}_t^{\text{ret}} = r_{t+1} + \gamma V^{\mathtt{w}}(s_{t+1}) + \gamma \min\{1, \rho_{t+1}\} \left[\hat{Q}_{t+1}^{\text{ret}} - Q^{\mathtt{w}}(s_{t+1}, a_{t+1}) \right] \tag{2}$$

The off-PG (Degris et al., 2012) can be used to update $\pi^{\mathtt{w}}$ by ER: $\hat{g}_t^{\text{off-PG}}(\mathtt{w}) = \rho_t \hat{A}_t^\pi \nabla_{\mathtt{w}} \log \pi^{\mathtt{w}}(a_t|s_t)$. Here, $\hat{A}_t^\pi$ is an estimator for the on-policy advantage, such as $\hat{A}_t^{\text{ret}} := \hat{Q}_t^{\text{ret}} - V^{\mathtt{w}}(s_t)$.

3 REMEMBER AND FORGET EXPERIENCE REPLAY

In off-policy RL it is common to maximize on-policy returns averaged over the distribution of states in a RM (Degris et al., 2012). However, as $\pi^{\mathtt{w}}$ gradually shifts away from previous behaviors μ_t, the distribution of states in the RM is increasingly dissimilar from the on-policy distribution, and trying to increase an off-policy performance metric may not improve on-policy outcomes. This issue may be compounded with algorithm-specific concerns. For example, in ACER (Wang et al., 2017) the dissimilarity between μ_t and $\pi^{\mathtt{w}}$ may cause vanishing or diverging importance weights ρ_t, thereby increasing the variance of the off-PG and deteriorating the convergence speed of Retrace by inducing "trace-cutting" (Munos et al., 2016). As a remedy, ACER tunes the learning rate and uses a target network (Mnih et al., 2015), updated as a delayed copy of the policy network, to constrain policy updates. Target networks are also employed in DDPG (Lillicrap et al., 2016) in order to slow down the feedback loop between value-network and policy-network optimizations. This feedback loop causes overestimated action values that can only be corrected by acquiring new on-policy samples. When using non-linear networks there is no guarantee that small weight changes correspond to small changes in the output and recent works (Henderson et al., 2017) have shown the often opaque variability of outcomes of continuous-action RL algorithms depending on hyper-parameters.

Here we propose a set of three simple rules, collectively referred to as Remember and Forget ER (ReF−ER), to directly control the degree of "off-policyness" of the samples in the RM:

1. Updates are computed from mini-batches drawn uniformly from the RM. We compute the importance weight ρ_t of each sample and classify it either as "near-policy" if $1/c_{\text{max}}<\rho_t<c_{\text{max}}$ with $c_{\text{max}}>1$, or "far-policy" otherwise. We will repeatedly use estimates of the number of far-policy samples, therefore we store for each element of the RM its most recent ρ_t.
2. When acquiring samples from the environment, older episodes with the highest fraction of far-policy samples are removed until the number n_{obs} of samples in the RM is at most N.
3. Policy updates are penalized in order to attract the current policy $\pi^{\mathtt{w}}$ towards μ_t:

$$\hat{\mathbf{g}}_t^{\text{ReF-ER}}(\mathtt{w}) = \begin{cases} \beta\, \hat{\mathbf{g}}_t(\mathtt{w}) - (1-\beta)\nabla D_{\text{KL}}\left[\mu_t \| \pi^{\mathtt{w}}(\cdot|s_t)\right] & \text{if } 1/c_{\text{max}}<\rho_t<c_{\text{max}} \\ -(1-\beta)\nabla D_{\text{KL}}\left[\mu_t \| \pi^{\mathtt{w}}(\cdot|s_t)\right] & \text{otherwise} \end{cases} \tag{3}$$

   Here D_{KL} is the Kullback–Leibler divergence and the coefficient $\beta \in [0, 1]$ is updated after each gradient step such that a fixed fraction $D \in (0, 1)$ of the RM are far-policy samples:

$$\beta \leftarrow \begin{cases} (1-\eta)\beta & \text{if } n_{\text{far}}/n_{\text{obs}} > D \\ (1-\eta)\beta + \eta, & \text{otherwise} \end{cases} \tag{4}$$

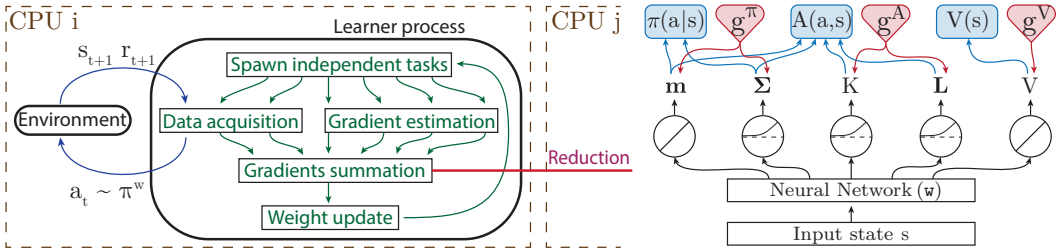
   where η is the learning rate and n_{far} is the number of far-policy samples. Note that iteratively updating β with Eq. 4 has fixed points in 0 for $n_{\text{far}}/n_{\text{obs}}>D$ and in 1 otherwise.

For $c_{\text{max}}\to 1$ and $D\to 0$, ReF−ER becomes asymptotically equivalent to computing gradient estimates from on-policy experiences. ReF−ER reduces the sensitivity on the network architecture and training hyper-parameters, because the rate at which the policy is allowed to change is directly linked to its similarity with the behaviors in the training set.

4 RACER: Regularized Actor-Critic with Experience Replay

Here we introduce RACER: an actor-critic architecture for continuous-action problems, such as the MuJoCo (Todorov et al., 2012) control tasks of OpenAI Gym (Brockman et al., 2016) and the DeepMind Control Suite (Tassa et al., 2018). The network architecture consist of a single Multilayer Perceptron (MLP) with weights $\mathtt{w}$ which receives the state s and outputs as shown in figure 1b. From the final layer, the first $2d_A$ outputs are taken as the mean $\mathbf{m}$ and diagonal covariance $\mathbf{\Sigma}$ of the Gaussian policy $\pi^{\mathtt{w}}(a|s)$. One output is taken as the estimated state value $V^{\mathtt{w}}(s)$, and the remaining outputs are the parameters of a closed-form approximator for the action advantage $A^{\mathtt{w}}$. RACER can be extended for problems where the state is received as a visual feed (by inserting convolutional layers), or where the state is only partially observable by the agent (by substituting the MLP with a recurrent model). However, these extensions are outside the scope of this paper.

The parameters $\mathtt{w}$ are updated from mini-batches of off-policy time steps with ReF−ER. A separate gradient is defined for each component of the actor-critic framework, as sketched in figure 1b. The



(a) (b)

Figure 1: (a) Diagram of asynchronous ER-based RL algorithms. (b) Neural network architecture employed by RACER. Blue arrows connect each output with the elements of the actor-critic framework for which it is used. Red arrows represent the flow of the error signals.

policy statistics $\mathbf{m}(s)$ and $\mathbf{\Sigma}(s)$ are updated with the off-PG (Degris et al., 2012):

$$\hat{\mathbf{g}}_t^{\pi}(\mathtt{w}) = \rho_t \hat{A}_t^{\text{ret}} \nabla_{\{\mathbf{m},\, \mathbf{\Sigma}\}} \log \pi^{\mathtt{w}}(a_t \mid s_t) \tag{5}$$

We use Retrace (Eq. 2) to estimate the on-policy advantage. From the "variance truncation and bias correction (TBC) method (Wang et al., 2017) and Retrace, we obtain an estimator for the on-policy state value $\hat{V}_t^{\text{tbc}} = V^{\mathtt{w}}(s_t) + \min\{1, \rho_t\}(\hat{Q}_t^{\text{ret}} - Q^{\mathtt{w}}(s_t, a_t))$, used as a target for $V^{\mathtt{w}}$:

$$\hat{\mathbf{g}}_t^{V}(\mathtt{w}) = \hat{V}^{\text{tbc}}(s) - V^{\mathtt{w}}(s_t) = \min\{1,\ \rho_t\} \left[\hat{Q}_t^{\text{ret}} - Q^{\mathtt{w}}(s_t, a_t)\right] \tag{6}$$

Rather than having a separate MLP with inputs (s, a) to parameterize $Q^{\mathtt{w}}$ (as in ACER or DDPG), whose expected value under the policy would be computationally demanding to compute, we employ a closed-form equation for $A^{\mathtt{w}}$ inspired by NAF (Gu et al., 2016). The network outputs the coefficients of a concave function $f^{\mathtt{w}}(s, a)$ which is chosen such that its maximum coincides with the mean of the policy $\mathbf{m}(s)$, and such that it is possible to derive analytical expectations for $a \sim \pi^{\mathtt{w}}$. In App. B we explore multiple choices of $f^{\mathtt{w}}(s, a)$. For all results in the main text of the manuscript we used:

$$f^{\mathtt{w}}(s, a) = K(s)\ \exp\left[-\frac{1}{2}\mathbf{a}_+^{\intercal}\ \mathbf{L}_+^{-1}(s)\ \mathbf{a}_+ - \frac{1}{2}\mathbf{a}_-^{\intercal}\ \mathbf{L}_-^{-1}(s)\ \mathbf{a}_-\right] \tag{7}$$

Here $\mathbf{a}_- = \min\left[a - \mathbf{m}(s), \mathbf{0}\right]$ and $\mathbf{a}_+ = \max\left[a - \mathbf{m}(s), \mathbf{0}\right]$ (both element-wise operations). This parameterization requires one MLP output for $K(s)$ and d_A outputs for each diagonal matrix $\mathbf{L}_+$ and $\mathbf{L}_-$. In total, given a state s, the MLP computes $\mathbf{m}$, $\mathbf{\Sigma}$, V, K, $\mathbf{L}_+$ and $\mathbf{L}_-$. From Eq.7, for any action a the advantage is uniquely defined: $A^{\mathtt{w}}(s, a) := f^{\mathtt{w}}(s, a) - \mathbb{E}_{a' \sim \pi}\left[f^{\mathtt{w}}(s, a')\right]$. Like the exact on-policy advantage A^{π}, $A^{\mathtt{w}}$ has by design expectation zero under the policy. The parameterization coefficients K and $\mathbf{L}$ are updated to minimize the L2 error from $\hat{A}_t^{\text{ret}}$:

$$\hat{\mathbf{g}}_t^{A}(\mathtt{w}) = \rho_t \left[\hat{A}_t^{\text{ret}} - A^{\mathtt{w}}(s_t, a_t)\right] \nabla_{\{K,\, \mathbf{L}\}} A^{\mathtt{w}}(s_t, a_t) \tag{8}$$

Here, ρ_t reduces the weight of estimation errors for unlikely actions, where $A^{\mathtt{w}}$ is expected to be less accurate. To ensure that $\mathbf{\Sigma}$, K, $\mathbf{L}_+$ and $\mathbf{L}_-$ are positive definite, the respective outputs are mapped onto $\mathbb{R}^+$ by a Softplus rectifier. The analytical derivation of $\mathbb{E}_{a' \sim \pi}\left[f^{\mathtt{w}}(s, a')\right]$ can be found in App. A. Collectively, $\hat{\mathbf{g}}_t^{\pi}(\mathtt{w})$, $\hat{\mathbf{g}}_t^{V}(\mathtt{w})$, and $\hat{\mathbf{g}}_t^{A}(\mathtt{w})$ form a vector $\hat{\mathbf{g}}_t^{\text{AC}}(\mathtt{w})$ with the same size as the MLP output. $\hat{\mathbf{g}}_t^{\text{AC}}(\mathtt{w})$ is weighted with the ReF−ER penalization (Eq. 3) and then back-propagated to the MLP.

In order to estimate $\hat{Q}_t^{\text{ret}}$ for a sampled time step t, Retrace (Eq. 2) requires $V^{\mathtt{w}}$ and $Q^{\mathtt{w}}$ for all future steps in sample t's episode. These are naturally computed when training from batches of episodes (as in ACER) rather than time steps. However, the information contained in consecutive steps is highly correlated, worsening the quality of the gradient estimate, and episodes can be composed of thousands of time step, increasing the computational cost. In order to efficiently train from uncorrelated time steps, RACER stores for each sample in the RM the most recently computed estimations of $V^{\mathtt{w}}(s_t)$, $A^{\mathtt{w}}(s_t, a_t)$, ρ_t and Q_t^{ret}. When a batch of time steps is sampled, the stored Q_t^{ret} are used to compute the gradient update. At the same time, the stored $V^{\mathtt{w}}(s_t)$, $A^{\mathtt{w}}(s_t, a_t)$ and ρ_t are updated with the current NN outputs and used to correct Q^{ret} for all prior time-steps in the respective episodes with Eq. 2. A description of all remaining implementation details is provided in App. D.

5 Related work

The rules that determine which samples are kept in the RM and how they are used for training can be designed to address several objectives. *Prioritized Experience Replay* (Schaul et al., 2015b) (PER) improves the performance of DQN (Mnih et al., 2015) (on discrete-action problems) by biasing sampling in favor of observations associated with greater temporal-difference (TD) errors. TD errors may signal rare events that would convey useful information to the learner, especially if the rewards are sparse. We will compare PER to ER and ReF−ER on continuous-action problems. ER can alternatively be used to train off-policy learners on auxiliary tasks (Schaul et al., 2015a; Jaderberg et al., 2017; Andrychowicz et al., 2017) to shape the network features or to guide exploration. Moreover, ER can be designed to maximize the diversity of the RM (de Bruin et al., 2015).

ReF−ER is inspired by the techniques developed for on-policy RL to bound policy updates of PPO (Schulman et al., 2017). Rule 1 of ReF−ER is similar to the clipped objective function of PPO

(gradients are zero if ρ lies outside of some range). However, Rule 1 is not affected by the sign of the advantage estimate and clips both policy and value gradients. Like Rule 3, one variant of PPO penalizes $D_{KL}(\mu_t||\pi^{\mathtt{w}})$ (also Schulman et al. (2015) and Wang et al. (2017) employ trust-region schemes in the on- and off-policy setting respectively). While PPO picks one of the two techniques, in ReF−ER Rules 1 and 3 complement each other and can be applied to most off-policy RL methods with parametric policies. In the ER setting, samples remain in the RM over many iterations. If only Rule 1 is included, any PG could push the sample's ρ outside of the clipping range without means to recover, leading to zero-valued gradients. Including only Rule 3 would not reduce the number of hyper-parameters (a target D_{KL} would be needed), and would not prevent unbounded ρ.

The first method to combine ER and PG (Sutton et al., 2000) was the *Off-Policy Actor Critic* (Degris et al., 2012). Further contributions were introduced with ACER (Wang et al., 2017), such as extension to NN and estimation of on-policy returns with Retrace (Munos et al., 2016). In the continuous-action domain, the proposed RACER offers several advantages over ACER: (1) reduced complexity by employing a single NN and closed-form $A^{\mathtt{w}}$ (continuous-ACER uses 9 NN evaluations per gradient). (2) relies on ReF−ER rather than constraining policy updates around a target network. (3) samples uncorrelated time steps rather than episodes (which may consist of thousands of steps). Because of points (1) and (3) RACER is two orders of magnitude faster than ACER.

6 RESULTS

In this section we couple ReF−ER, conventional ER and PER with one method from each of the three main classes of continuous-action RL algorithms: DDPG (based on DPG), NAF (based on Q-learning), and RACER (off-PG). We measure the performance of each combination of algorithms by plotting the mean cumulative reward $R = \sum_t r_t$. Each plot tracks the average R among all episodes entering the RM within intervals of $2{\cdot}10^5$ time steps and averaged again among five differently seeded training trials. Contours of the 20th to 80th percentiles of R obtained with DDPG and RACER can be found in the Appendix. The code to reproduce all present results is available on GitHub.[1]

6.1 RESULTS FOR DDPG

DDPG (Lillicrap et al., 2016) trains two networks by ER. The value network outputs $Q^{\mathtt{w}}(s,a)$ and is trained to minimize the L2 distance from the Q-learning target (Sec. 2). The policy network outputs a deterministic policy $\mathbf{m}^{\mathtt{w}'}(s_{t+1})$ and is trained with the DPG (Silver et al., 2014):

$$\hat{g}_t^{\text{DPG}}(\mathtt{w}') = \nabla_{\mathtt{w}'}\mathbf{m}^{\mathtt{w}'}(s)\ \nabla_a Q^{\mathtt{w}}(s_t,a)|_{a=\mathbf{m}^{\mathtt{w}'}(s_t)} \tag{9}$$

Noise is added to the deterministic policy $\pi^{\mathtt{w}'}{=}\mathbf{m}^{\mathtt{w}'}{+}\mathcal{N}(\mathbf{0},\sigma^2\mathbf{I})$ for exploration. We consider two variants of DDPG: one with $\mathbf{m}^{\mathtt{w}'}$ bounded to the unit box $[-1,1]^{d_A}$ (as in the original) and one without bounds (referred to as u−DDPG, implementation details can be found in App. D). Bounding the policy may lead to lower returns in the OpenAI MuJoCo benchmarks, which are defined for unbounded actions. We note that, without measures to constrain policy updates or without careful tuning of the hyper-parameters (we found the critic's weight decay and temporally-correlated action noise to be necessary), u−DDPG is unstable. The returns for u−DDPG can fall to large negative values, especially in tasks that include a control-cost penalty in the reward such as Ant. This behavior is explained by the value network not having learned local maxima with respect to the action (Silver et al., 2014).

By replacing ER with ReF−ER we can stabilize u−DDPG and greatly improve its performance, especially for tasks with complex dynamics such as Humanoid or Ant. The hyper-parameter c_{max} determines how much the policy is allowed to change from the behaviors in the RM. By annealing c_{max} we can allow faster improvements at the beginning of training, when an inaccurate policy gradient might be sufficient to estimate a good direction for the update. Conversely, during the later stages of training, precise updates can be computed from almost on-policy samples. We anneal c_{max} and the learning rate according to:

$$c_{\text{max}}(k) = 1 + C/(1+5e{-}7\cdot k), \quad \eta(k) = \eta/(1+5e{-}7\cdot k) \tag{10}$$

[1] The repository is hidden to maintain anonymity during the review process.

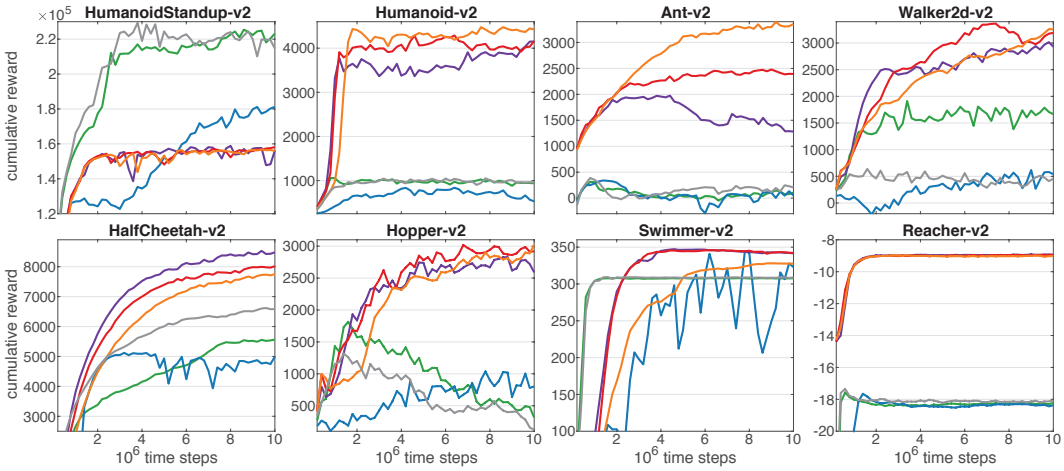


Figure 2: Cumulative rewards on OpenAI MuJoCo tasks for DDPG (green line), DDPG with rank-based PER (gray line), u-DDPG with regular ER (blue), and ReF-ER with $C = 8$ (purple), $C = 4$ (red), $C = 2$ (orange). Implementation details in App. D.

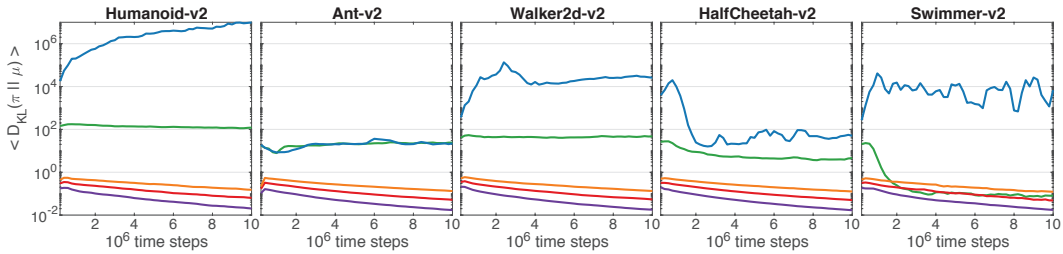


Figure 3: Average Kullback-Leibler divergence between the policy $\pi^{\mathtt{w}'} = m^{\mathtt{w}'} + \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I})$ and the behaviors μ_t in the RM during training for DDPG (green line), u-DDPG with regular ER (blue), and ReF-ER with $C = 8$ (purple), $C = 4$ (red), $C = 2$ (orange).

Here k is the gradient step index, η is the initial learning rate (10^{-4} and 10^{-5} for the value and policy networks respectively, as when using regular ER). We found that annealing η worsened the instability of u-DDPG with regular ER. Lower values of C reduce the speed of policy improvements, but after 10^7 time steps $C = 2$ achieves the best performance in most tasks. In Fig. 3 we report for a subset of problems the average $D_{KL}(\mu_t||\pi^{\mathtt{w}})$. D_{KL} decreases for lower values of C and further decreases during training due to the annealing process. With regular ER, even after lowering η by one order of magnitude from that of the original paper, the distance between π and μ may span the entire action space. In fact, in most tasks shown in Fig. 3 the $D_{KL}(\mu_t||\pi^{\mathtt{w}})$ of DDPG is of similar order of magnitude as its maximum $\frac{d_A}{2}(2/\sigma)^2$ (for example, since $\sigma = 0.2$, the maximum D_{KL} is 850 for Humanoid and 300 for Walker2d and it oscillates during training around 100 and 50 respectively).

ReF-ER maintains a RM of mostly near-policy samples, providing the value-network with multiple examples of trajectories that are possible with the current policy. This focuses the predictive capabilities of the value-network, enabling it to extrapolate the effect of a marginal change of action on the expected returns, and therefore increasing the accuracy of the DPG. Any misstep of the DPG is weighted with a penalization term that attracts the policy towards past behaviors. This allows time for the learner to gather experiences with the new policy, improve the value-network, and correct the misstep. This reasoning is almost diametrically opposed to that behind PER. In PER observations that are associated with larger TD errors are sampled more frequently. In the continuous-action setting, however, TD errors may be result from actions that are farther from the current policy. Therefore, precisely estimating their value might not help the value network in yielding an accurate estimate of the DPG. We obtained better results with the rank-based variant of PER, which has similar performance to that of DDPG with ER. The main benefits over ReF-ER arise in tasks that require more exploration, such as Swimmer and HumanoidStandup.

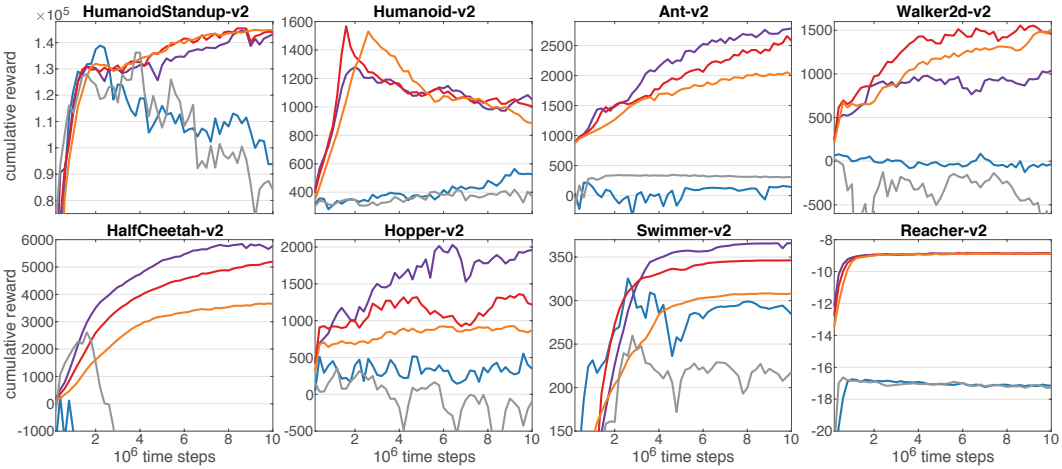


Figure 4: Cumulative rewards on the OpenAI MuJoCo tasks for NAF (blue line), NAF with rank-based PER (gray line), NAF with ReF−ER and $C = 8$ (purple), $C = 4$ (red), $C = 2$ (orange).

6.2 Results for NAF

Normalized Advantage Functions (NAF) is the state-of-the-art of Q-learning based algorithms for continuous-action problems. It employs a quadratic-form approximation of the advantage $A^{\mathtt{w}}$, analogous to the one employed by RACER:

$$A^{\mathtt{w}}_{\text{NAF}}(s, a) = -\left[a - \mathbf{m}^{\mathtt{w}}(s)\right]^{\intercal} \mathbf{L}^{\mathtt{w}}_{\text{Q}}(s) \left[\mathbf{L}^{\mathtt{w}}_{\text{Q}}(s)\right]^{\intercal} \left[a - \mathbf{m}^{\mathtt{w}}(s)\right] \tag{11}$$

NAF trains $A^{\mathtt{w}}$ with the Q-learning target (Sec. 2) and uses the location $\mathbf{m}^{\mathtt{w}}$ of its maximum as the mean of the policy, with added Gaussian noise for exploration. When the actual A^{π} is not well approximated by a quadratic (e.g. when the return landscape is multi-modal), NAF may fail to choose good actions. In contrast, RACER updates $\pi^{\mathtt{w}}$ with the off-PG, which is independent of the error in approximating $A^{\pi}(s_t, a_t)$ (but depends on the error at $t{+}1$). Figure 4 shows how NAF is affected by the choice of ER algorithm. While Q-learning based methods are thought to be less sensitive than PG-based methods to the dissimilarity between policy and stored behaviors owing to the bootstrapped Q-learning target, NAF benefits from REF−ER. This is because A^{π} is likely to be approximated well by a quadratic in a small neighborhood near its local maxima. ReF−ER constrains learning from actions within this neighborhood and prevents large TD errors from disrupting the locally-accurate approximation of $A^{\mathtt{w}}$. This intuition is supported by observing that rank-based PER (the better performing variant of PER also in this case), often worsens the performance of NAF. PER aims at biasing sampling in favor of larger TD errors, which are more likely to be farther from $\mathbf{m}(s)$ (note that $f^{\mathtt{w}}_{\text{NAF}}$ is unbounded), and their accurate prediction might not help the learner in fine-tuning the policy by improving a local approximation of the advantage.

6.3 Results for RACER

In this section we present state-of-the-art results in the OpenAI MuJoCo tasks obtained by coupling RACER with ReF−ER. We omit results from coupling RACER with ER or PER as it yields large negative R values that would not fit the graphs. In fact, without introducing measures to constrain policy updates, the unbounded importance weights ρ_t cause Eq. 5 to diverge, disrupting any prior learning progress. We compare RACER with PPO, an algorithm that owing to its simplicity and good performance on MuJoCo tasks is often used as baseline.

The two hyper-parameters that most strongly affect the performance of RACER are the RM size N and c_{max}, annealed with Eq. 10 (for a discussion of the other parameters see App. B). RACER uses the Retrace estimator Q^{ret} which is expected to converge to the on-policy Q^{π} (Munos et al., 2016). Because $\pi^{\mathtt{w}}$ is gradually changing during training, it is crucial to maximize the convergence speed of Retrace to obtain accurate estimates of the off-PG. Large values of c_{max} increase the variance of the policy gradient and increase the amount of "trace-cutting" as the importance weights are allowed to diverge from 1. The cumulative rewards reported in Fig. 5 show that "stable" tasks, where the

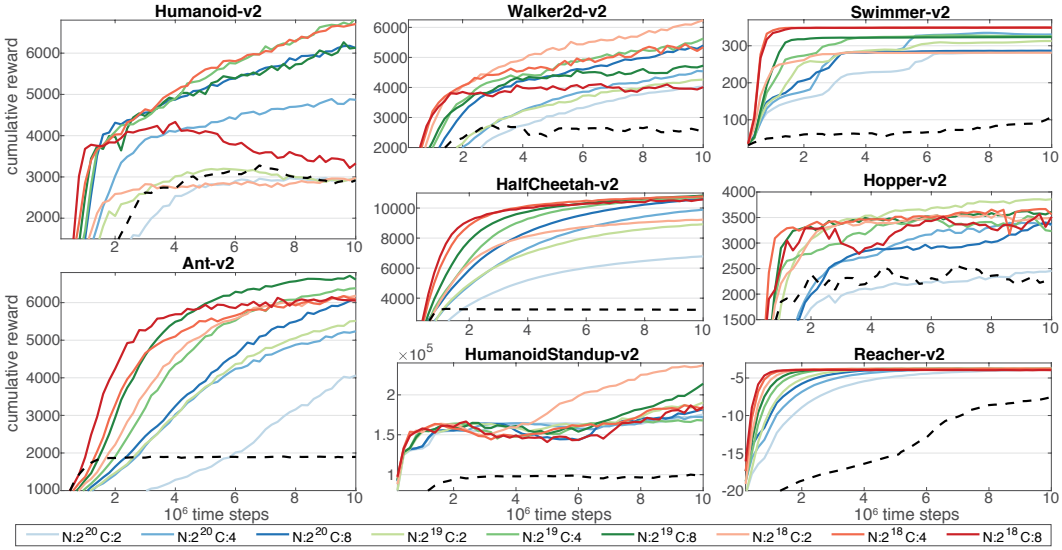


Figure 5: Average cumulative rewards on MuJoCo OpenAI Gym tasks obtained with PPO (dashed black lines) and with RACER by independently varying the two main hyper-parameters of ReF−ER: the RM size N and C (colored lines).

agent's success is less predicated on avoiding mistakes that would cause it to trip, are more tolerant to high values of $c_{\max}$ (e.g. HalfCheetah). In most other tasks, especially those that require precise controls (e.g. Walker), the best results are obtained for values of $c_{\max}$ that strike a balance between strict penalty terms, which slow down policy improvements, and high-variance gradient estimates. The RM size N has a parallel effect. A small RM may not contain enough diversity of samples for the learner to accurately estimate the gradients. Conversely, a large RM is composed of episodes obtained with increasingly older versions of $\pi^{\mathtt{w}}$. In this case, the penalty terms required to preserve a sufficient fraction of near-policy samples may prevent the policy from improving. A discussion of all the secondary hyper-parameters and the choice of function parameterizing $A^{\mathtt{w}}$ can be found in App. B. For most combinations of hyper-parameters and tasks presented in this section, RACER outperforms the best result from DDPG (Sec. 6.1), PPO, and is competitive with the best results found in the published literature, which to our knowledge were achieved by the on-policy algorithms TRPO (Schulman et al., 2015) and Policy Search with Natural Gradient (Rajeswaran et al., 2017).

7 Conclusion

We present a novel Experience Replay algorithm (ReF−ER) that is shown to enhance the stability and performance of off-policy RL. ReF−ER regulates the pace at which the policy $\pi^{\mathtt{w}}$ is allowed to deviate from previous behaviors μ, and how past experiences are used to update $\pi^{\mathtt{w}}$. The relevance of past behaviors to the current policy is quantified by the importance weight $\rho{=}\pi(a|s)/\mu(a|s)$. We characterize samples as either "near-policy" or "far-policy" by the deviation of ρ from unity. The proposed ReF−ER algorithm consists of: 1) Computing gradient estimates only from near-policy experiences. 2) Forgetting far-policy samples when new observations are obtained from the environment. 3) "Remembering" past policies through a penalty term that reduces the Kullback–Leibler divergence between $\pi^{\mathtt{w}}$ and μ. This allows time for the learner to gather experiences with the new policy, improve the value estimators, and increase the accuracy of the next steps. We show that ReF−ER can be applied to most off-policy RL algorithm with parametric policies by coupling it with off-policy PG, deterministic PG and Q-learning methods for continuous-action problems. The present results provide evidence that most off-policy RL methods may benefit from being trained with a replay memory composed of near-policy experiences. Finally, ReF−ER is combined with a novel off-policy actor critic algorithm (RACER) with simplified closed-form parameterization of the on-policy advantage approximator. RACER emphasizes a simple architecture, computational efficiency and minimal hyper-parameters. The application of ReF−ER and RACER to OpenAI Gym benchmarks produces state-of-the-art results while being robust to large variations in their hyper-parameters.

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A EXPECTATIONS OF THE PARAMETERIZED ADVANTAGE FUNCTIONS

We considered three parameterizations for the action advantage:

$$f^{\mathtt{w}}_{\mathrm{Q}}(s,a) = -\tfrac{1}{2}\left[a-\mathbf{m}(s)\right]^{\intercal}\mathbf{L}_{\mathrm{Q}}(s)\mathbf{L}_{\mathrm{Q}}^{\intercal}(s)\left[a-\mathbf{m}(s)\right] \tag{12}$$

$$f^{\mathtt{w}}_{\mathrm{SG}}(s,a) = K(s)\,\exp\left\{-\tfrac{1}{2}\left[a-\mathbf{m}(s)\right]^{\intercal}\mathbf{L}_{\mathrm{SG}}^{-1}(s)\left[a-\mathbf{m}(s)\right]\right\} \tag{13}$$

$$f^{\mathtt{w}}_{\mathrm{DG}}(s,a) = K(s)\,\exp\left\{-\tfrac{1}{2}\mathbf{a}_{+}^{\intercal}\,\mathbf{L}_{+}^{-1}(s)\,\mathbf{a}_{+} - \tfrac{1}{2}\mathbf{a}_{-}^{\intercal}\,\mathbf{L}_{-}^{-1}(s)\,\mathbf{a}_{-}\right\} \tag{14}$$

We recall $\mathbf{a}_{-}=\min\left[a-\mathbf{m}(s),\mathbf{0}\right]$ and $\mathbf{a}_{+}=\max\left[a-\mathbf{m}(s),\mathbf{0}\right]$. The first parameterization $f^{\mathtt{w}}_{\mathrm{Q}}$ is identical to the one employed by NAF (Gu et al., 2016), and requires training $(d_A^2+d_A)/2$ coefficients of the lower triangular matrix $\mathbf{L}_{\mathrm{Q}}$. Its quadratic complexity makes the choice of $f^{\mathtt{w}}_{\mathrm{Q}}$ unfavorable for high-dimensional action spaces (e.g. it requires 153 parameters for the 17-dimensional Humanoid tasks of OpenAI Gym, against the 35 of $f^{\mathtt{w}}_{\mathrm{DG}}$). In order to preserve bijection between $\mathbf{L}_{\mathrm{Q}}$ and $\mathbf{L}_{\mathrm{Q}}\mathbf{L}_{\mathrm{Q}}^{\intercal}$, the diagonal terms are mapped to $\mathcal{R}^{+}$ with a Softplus function. The expectation under a Gaussian policy can be computed as (Petersen et al., 2008):

$$\mathbb{E}_{a'\sim\pi}\left[f^{\mathtt{w}}_{\mathrm{Q}}(s,a')\right] = \mathrm{Tr}\left[\mathbf{L}_{\mathrm{Q}}(s)\mathbf{L}_{\mathrm{Q}}^{\intercal}(s)\Sigma(s)\right] \tag{15}$$

Here Tr denotes the trace of a matrix.
The second parameterization requires training the $1+d_A$ coefficients for $K(s)$ and the diagonal

matrix $\mathbf{L}(s)$. The expectation can be easily derived from the properties of products of Gaussian densities:

$$\mathbb{E}_{a'\sim\pi}\left[f^{\mathtt{w}}_{\text{SG}}(s,a')\right] = K(s)\sqrt{\frac{|\mathbf{L}(s)|}{|\mathbf{L}(s)+\mathbf{\Sigma}(s)|}} \tag{16}$$

Here $|\cdot|$ denotes a determinant.
To derive the expectation of the third parameterization, which was used for most results in the paper, we recall that the expectation of a product of independent variables is the product of the expectations. For one component i of the action vector:

$$\mathbb{E}_{a'\sim\pi}\left[e^{-\frac{1}{2}u_{+,i}^{\intercal}\ L_{+,i}^{-1}(s)\ u_{+,i}-\frac{1}{2}u_{-,i}^{\intercal}\ L_{-,i}^{-1}(s)\ u_{-,i}}\right] = \frac{\sqrt{\frac{L_{+,i}(s)}{L_{+,i}(s)+\Sigma_i(s)}}+\sqrt{\frac{L_{-,i}(s)}{L_{-,i}(s)+\Sigma_i(s)}}}{2} \tag{17}$$

Here we exploited the symmetry of the Gaussian policy around the mean. Since $\mathbf{\Sigma}$, $\mathbf{L}_+$, and $\mathbf{L}_-$ are all diagonal, we can compute the expectation:

$$\mathbb{E}_{a'\sim\pi}\left[f^{\mathtt{w}}_{\text{DG}}(s,a')\right] = K(s)\prod_{i=1}^{d_A}\frac{\sqrt{\frac{L_{+,i}(s)}{L_{+,i}(s)+\Sigma_i(s)}}+\sqrt{\frac{L_{-,i}(s)}{L_{-,i}(s)+\Sigma_i(s)}}}{2} \tag{18}$$

Finally, we note that all these parameterizations are differentiable.

B SENSITIVITY TO HYPER-PARAMETERS

Figure 6 shows the robustness of RACER to various hyper-parameters and to the choice of the advantage parameterizations introduced in App. A. Moreover, we can bypass the advantage approximator (i.e. $A^{\mathtt{w}}$:=0) and rely exclusively on the state value approximator. We recall the on-policy value estimator obtained with the "variance truncation and bias correction trick" (Wang et al., 2017) (Sec. 4):

$$\hat{V}_t^{\text{tbc}} = V^{\mathtt{w}}(s_t) + \min\{1,\rho_t\}(\hat{Q}_t^{\text{ret}} - Q^{\mathtt{w}}(s_t,a_t)) \tag{19}$$

From Eq. 2 and 19 we obtain $\hat{Q}^{\text{ret}}(s_t,a_t) = r_{t+1} + \gamma\hat{V}^{\text{tbc}}(s_{t+1})$. By neglecting $A^{\mathtt{w}}$, from this relation and Eq. 19, we obtain a recursive Retrace-based estimator for the on-policy state value that depends on $V^{\mathtt{w}}$ alone:

$$\hat{V}_t^{\text{tbc}} = V^{\mathtt{w}}(s_t) + \min\left\{1,\ \rho(s_t,a_t)\right\}\left[r_{t+1} + \gamma\hat{V}^{\text{tbc}}(s_{t+1}) - V^{\mathtt{w}}(s_t)\right] \tag{20}$$

This target is equivalent to the recently proposed V-trace estimator (Espeholt et al., 2018) when all importance weights are clipped at 1, which was empirically found by the authors to be the best-performing solution. As expected, lacking a model for $A^{\mathtt{w}}(s,a)$, this architecture (denoted as V−RACER in the first column of results of Fig. 6) yielded worse performance than the original RACER. However the difference is often minor and the increased simplicity of this architecture might justify its adoption for some problems.

The tolerance D for far-policy samples in the RM has a similar effect as c_{max}: low values tend to delay learning while high values reduce the fraction of the RM that is used to compute updates and may decrease the accuracy of gradient estimates. Increasing the number F of time steps per gradient step could either cause a rightward shift of the expected returns because the learner computes fewer updates for the same budget of observations, or it could aid the learner by providing more on-policy samples. The actual outcomes depend on the problem: tasks with simpler dynamics (e.g. HalfCheetah) can be learned more quickly by performing more gradient steps, while problems with more complex dynamics (e.g. Ant, Humanoid) benefit from more on-policy samples. The batch-size B and learning rate η have a minor effect on performance.

C STATE, ACTION AND REWARD PREPROCESSING

Several authors have employed state (Henderson et al., 2017) and reward (Duan et al., 2016) (Gu et al., 2017) rescaling to improve the learning results. This technique has not systematically been studied but reflects a deeper challenge of RL. The success of any RL algorithm is intrinsically linked to the design of the control problem. For example, the stability of DDPG is affected by the L2 weight decay

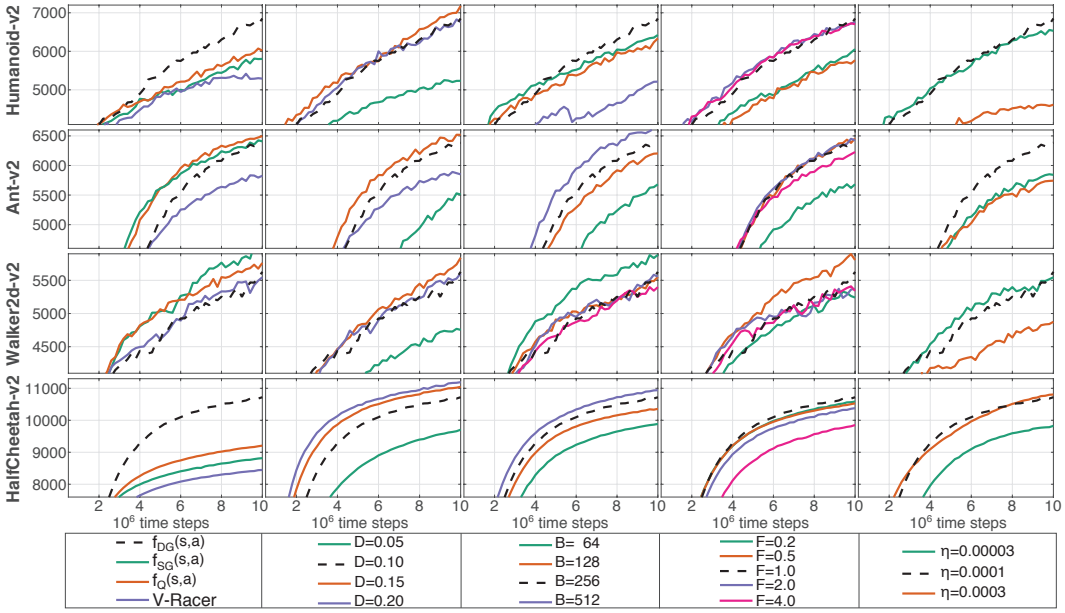

Figure 6: Mean cumulative rewards with RACER on OpenAI Gym tasks exploring the effect of the choice of advantage parameterization, ReF-ER tolerance factor D, mini-batch size B, number of time steps per gradient step F, and learning rate η. Dashed-black lines refer to the baseline parameters. The y-axes are magnified around the final returns to clarify often minor differences in performance.

of the value-network. Depending on the numerical values of the distribution of rewards provided by the environment and the choice of weight decay coefficient, the L2 penalization can be either negligible or dominate the Bellman error. Similarly, the distribution of values describing the state variables can increase the challenge of learning by gradient descent.

We propose partially addressing these issues by rescaling both rewards and state vectors depending on the the experiences contained in the RM. At the beginning of training we prepare the RM by collecting N_{start} observations and then we compute:

$$\mu_s = \frac{1}{n_{\text{obs}}} \sum_{t=0}^{n_{\text{obs}}} s_t \tag{21}$$

$$\sigma_s = \sqrt{\frac{1}{n_{\text{obs}}} \sum_{t=0}^{n_{\text{obs}}} (s_t - \mu_s)^2} \tag{22}$$

Throughout training, μ_s and σ_s are used to standardize all state vectors $\hat{s}_t = (s_t - \mu_s)/(\sigma_s + \epsilon)$ before feeding them to the network approximators. Moreover, every 1000 steps, chosen as the smallest power of ten that did not affect the run time, we loop over the n_{obs} observations stored in the RM to compute:

$$\sigma_r \leftarrow \sqrt{\frac{1}{n_{\text{obs}}} \sum_{t=0}^{n_{\text{obs}}} (r_{t+1})^2} \tag{23}$$

This value is used to scale the rewards $\hat{r}_t = r_t/(\sigma_r + \epsilon)$ used by the Q-learning target of DDPG and the Retrace algorithm for RACER. We use $\epsilon = 10^{-7}$ to ensure numerical stability.

The actions sampled by the learner may need to be rescaled or bounded to some interval depending on the environment. For the OpenAI Gym tasks this amounts to a linear scaling $a' = a\ (\texttt{upper_value} - \texttt{lower_value})/2$, where the values specified by the Gym library are ± 0.4 for the Humanoid tasks, ± 8 for the Pendulum tasks, and ± 1 for all others. The tasks of DeepMind Control Suite are defined with all actions bounded do the interval $[-1, 1]^{d_A}$. In this case we can learn Gaussian policies, as if the action space were unbounded, and then map the actions sent to the environment with an hyperbolic tangent: $a'_t = \tanh a_t$. This approach, however, might prevent efficiently learning policies that behave like bang-bang controllers. Close to the bounds of the control space, actions that should be similar may be mapped to distant positions in the unbounded action space. Therefore, learning bang-bang controls is more likely to be hindered by the D_{KL} penalties of

Table 1: RACER architecture's baseline hyper-parameters.

Short-hand	Description	Baseline value
C	Annealing parameter of c_{max}.	4
N	Size of the Replay Memory.	2^{19}
N_{start}	Number of samples gathered before training starts.	2^{18}
D	Fraction of far-policy samples allowed in the RM.	0.1
B	Mini-batch size.	256
F	Ratio between observed time steps and gradient steps.	1
η	Learning rate.	10^{-4}
γ	Discount factor.	0.995

ReF−ER. This issue may explain some poor results on the DeepMind Control Suite shown in Fig. 9 and 10. These results show that successfully tackling control problems with bounded action spaces may require policies parameterized as Beta distribution (Chou et al., 2017).

D IMPLEMENTATION AND NETWORK ARCHITECTURE DETAILS

We implemented all presented learning algorithms within smarties,[2] our open source C++ RL framework, and optimized for high CPU-level efficiency through fine-grained multi-threading, strict control of cache-locality, and computation-communication overlap. On every step, we asynchronously obtain on-policy data by sampling the environment with π, which advances the index t of observed time steps, and we compute updates by sampling from the Replay Memory (RM), which advances the index k of gradient steps (Fig. 1a). During training, the ratio of time and update steps is usually equal to a constant: $t/k = F$. This parameter affects the data efficiency of the algorithm; by lowering F each sample is used more times to improve the policy before being replaced by newer samples. Upon completion of all tasks, we apply the gradient update and proceed to the next step. The pseudo-codes in App. E neglect parallelization details as they do not affect execution.

In order to evaluate all algorithms on equal footing, we use the same baseline network architecture for RACER, DDPG and NAF, consisting of an MLP with two hidden layers of 128 units each. For the sake of computational efficiency, we employed Softsign activation functions. The weights of the hidden layers are initialized according to $\mathcal{U}\left[-6/\sqrt{f_i+f_o},\ 6/\sqrt{f_i+f_o}\right]$, where f_i and f_o are respectively the layer's fan-in and fan-out (Glorot & Bengio, 2010). The weights of the linear output layer are initialized from the distribution $\mathcal{U}\left[-0.1/\sqrt{f_i},\ 0.1/\sqrt{f_i}\right]$, such that the MLP has near-zero outputs at the beginning of training. When sampling the components of the action vectors, the policies are treated as truncated normal distributions with symmetric bounds at three standard deviations from the mean. Finally, we optimize the network weights with the Adam algorithm (Kingma & Ba, 2015).

RACER We note that the values of the diagonal covariance matrix are shared among all states and initialized to $\mathbf{\Sigma} = 0.2\mathbf{I}$. The remaining hyper-parameters of RACER are listed in table 1.

DDPG In its original formulation, DDPG transforms the policy-network's output onto the bounded interval $[-1, 1]^{d_A}$ with an hyperbolic tangent function. We found that bounding the action space may limit the performance of DDPG with the OpenAI MuJoCo tasks. In order to stabilize the unbounded-action version of DDPG (u−DDPG) we set the learning rate for the policy-network to $1 \cdot 10^{-5}$ and that of the value-network to $1 \cdot 10^{-4}$ with L2 weight decay coefficient of $1 \cdot 10^{-4}$. These changes lead to better performance also with the bounded-action DDPG and therefore were used for all numerical experiments. The RM is set to contain N=2^{19} observations and we follow Henderson et al. (2017) for the remaining hyper-parameters: mini-batches of $B = 128$ samples, γ=0.995, soft target network update coefficient 0.01. We note that while DDPG is the only algorithm employing two networks, choosing half the batch-size as RACER and NAF makes the compute cost roughly equal among the three methods. Finally, when using ReF−ER we add exploratory Gaussian noise to the deterministic policy: $\pi^{\mathbf{w}'}$=$\mathbf{m}^{\mathbf{w}'}$+$\mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I})$ with σ=0.2. When performing regular ER or PER we sample the exploratory noise from an Ornstein–Uhlenbeck process with σ=0.2 and θ=0.15.

[2] The repository is hidden to maintain anonymity during the review process.

NAF We use the same baseline MLP architecture and learning rate $\eta = 10^{-4}$, batch-size $B = 256$, discount $\gamma = 0.995$, RM size $N = 2^{19}$, and soft target network update coefficient 0.01. Gaussian noise is added to the deterministic policy $\pi^{\mathtt{w}'} = \mathbf{m}^{\mathtt{w}'} + \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I})$ with σ=0.2.

E PSEUDO-CODES

Algorithm 1 Serial description of the master algorithm.

$t = 0$, $k = 0$,
Initialize an empty RM, network weights $\mathtt{w}$, and Adam algorithm's (Kingma & Ba, 2015) moments estimates.
while $n_{\text{obs}} < N_{\text{start}}$ **do**
  Advance the environment according algorithm 2.
end while
Compute the initial statistics used to standardize the state vectors (App. C).
Compute the initial statistics used to rescale the rewards (App. C).
while $t < T_{\text{max}}$ **do**
  while $t < F \cdot k$ **do**
    Advance the environment according algorithm 2.
    $t \leftarrow t + 1$
  end while
  Compute a gradient estimate (for RACER with algorithm 3).
  Perform the gradient step with the Adam algorithm.
  If applicable, update the ReF−ER penalization coefficient β.
  if modulo$(k, 1000)$ is 0 **then**
    Update the statistics used to rescale the rewards (App. C).
  end if
  $k \leftarrow k + 1$
end while

Algorithm 2 Environment sampling

Observe s_t and r_t.
if s_t concludes an episode **then**
  Store data for t into the RM: $\{s_t, r_t, V^{\mathtt{w}}(s_t)\}$
  Compute and store Q^{ret} for all steps of the episode
else
  Sample the current policy $a_t \sim \pi^{\mathtt{w}}(a|s_t) = \mu_t$
  Store data for t into the RM: $\{s_t, r_t, a_t, \mu_t, V^{\mathtt{w}}(s_t), A^{\mathtt{w}}(s_t, a_t)\}$
  Advance the environment by performing a_t
end if

Remarks on algorithm 2: 1) The reward for an episode's initial state, before having performed any action, is zero by definition. 2) The value $V^{\mathtt{w}}(s_t)$ for the last state of an episode is computed if the episode has been truncated due the task's time limits or is set to zero if s_t is a terminal state. 3) Each time step we use the learner's updated policy network and we store $\mu_t = \{\mathbf{m}(s_t), \mathbf{\Sigma}(s_t)\}$.

Remarks on algorithm 3: 1) In order to compute the gradients $\hat{g}^{AC}_{t_i}(\mathtt{w})$, we rely on advantage estimates $Q^{\text{ret}}_{t_i}$ that were computed when subsequent time steps in t_i's episode were previously drawn by ER. Not having to compute the quantities $A^{\mathtt{w}}$, $V^{\mathtt{w}}$, and ρ for all following steps comes with clear computational efficiency benefits, at the risk of employing an incorrect estimate for $Q^{\text{ret}}_{t_i}$. In practice, we find that the Retrace values incur only minor changes between updates (even when large RM sizes decrease the frequency of updates to the Retrace estimator) and that relying on previous estimates has no evident effect on performance. This could be attributed to the gradual policy changes enforced by ReF−ER. 2) With a little abuse of the notation, with π (or μ) we denote the statistics (mean, covariance) of the multivariate normal policy, with $\pi(a|s)$ we denote the probability of performing action a given state s, and with $\pi(\cdot|s)$ we denote the probability density function over actions given state s.

Algorithm 3 RACER's gradient update

Sample mini-batch of B time steps from RM.
for $i = 0$ to B **do**
  Fetch all relevant information: s_{t_i}, a_{t_i}, $Q^{\text{ret}}_{t_i}$, and $\mu_{t_i} = \{\mathbf{m}_{t_i}, \mathbf{\Sigma}_{t_i}\}$.
  Call the approximator to compute $\pi^{\mathtt{w}}, V^{\mathtt{w}}(s_{t_i}), A^{\mathtt{w}}(s_{t_i}, a_{t_i})$
  Update Q^{ret} for all prior steps in t_i's episode with $V^{\mathtt{w}}(s_{t_i}), A^{\mathtt{w}}(s_{t_i}, a_{t_i})$
  Update the importance weight $\rho_{t_i} = \pi^{\mathtt{w}}(a_{t_i}|s_{t_i})/\mu_{t_i}(a_{t_i}|s_{t_i})$
  if $1/c_{\text{max}} < \rho_{t_i} < c_{\text{max}}$ **then**
    Compute $\hat{g}^{AC}_{t_i}(\mathtt{w})$ according to Sec. 4
  else
    $\hat{g}^{AC}_{t_i}(\mathtt{w}) = \mathbf{0}$
  end if
  ReF−ER penalization: $\hat{g}^{\text{ReF-ER}}_{t_i}(\mathtt{w}) = \beta \hat{g}^{AC}_{t_i}(\mathtt{w}) - (1-\beta)\nabla D_{\text{KL}}[\mu_{t_i}(\cdot|s_{t_i})||\pi^{\mathtt{w}}(\cdot|s_{t_i})]$
end for
Accumulate the gradients $\sum_{i=0}^{T} \hat{g}^{\text{ReF-ER}}_{t_i}(\mathtt{w})$

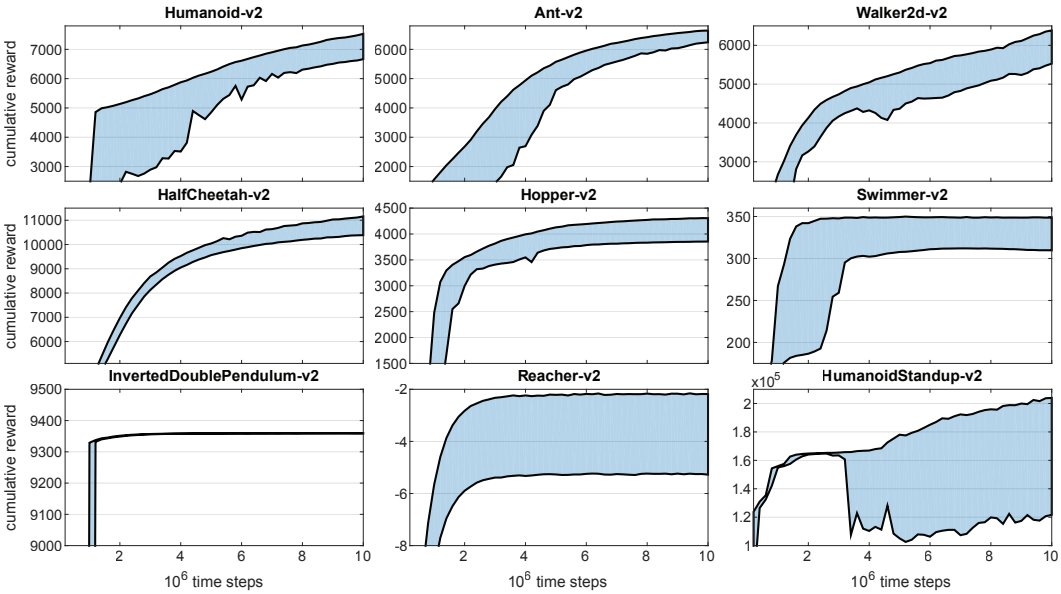

Figure 7: Additional result. 20th and 80th percentiles of cumulative rewards for trajectories of OpenAI Gym tasks ended within intervals of $2 \cdot 10^5$ time steps across 5 independent training runs using the baseline RACER hyper-parameters (Table 1).

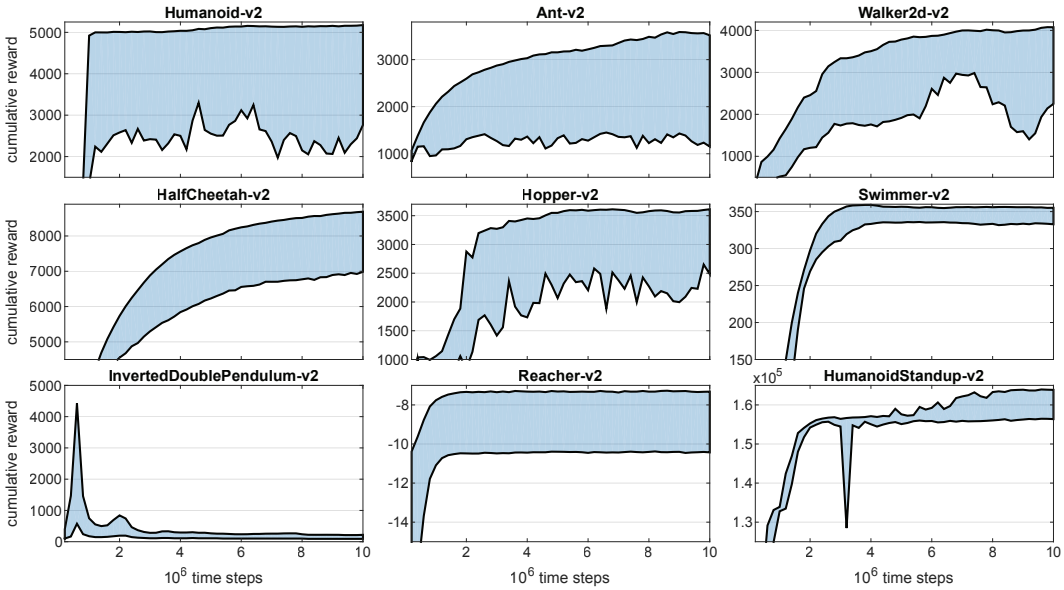

Figure 8: Additional result. 20th and 80th percentiles of cumulative rewards for trajectories of OpenAI Gym tasks ended within intervals of $2 \cdot 10^5$ time steps across 5 independent training runs for `u-DDPG` with `ReF-ER`, $C = 4$, and all other hyper-parameters set according to App. D.

SUPPLEMENTARY REFERENCES

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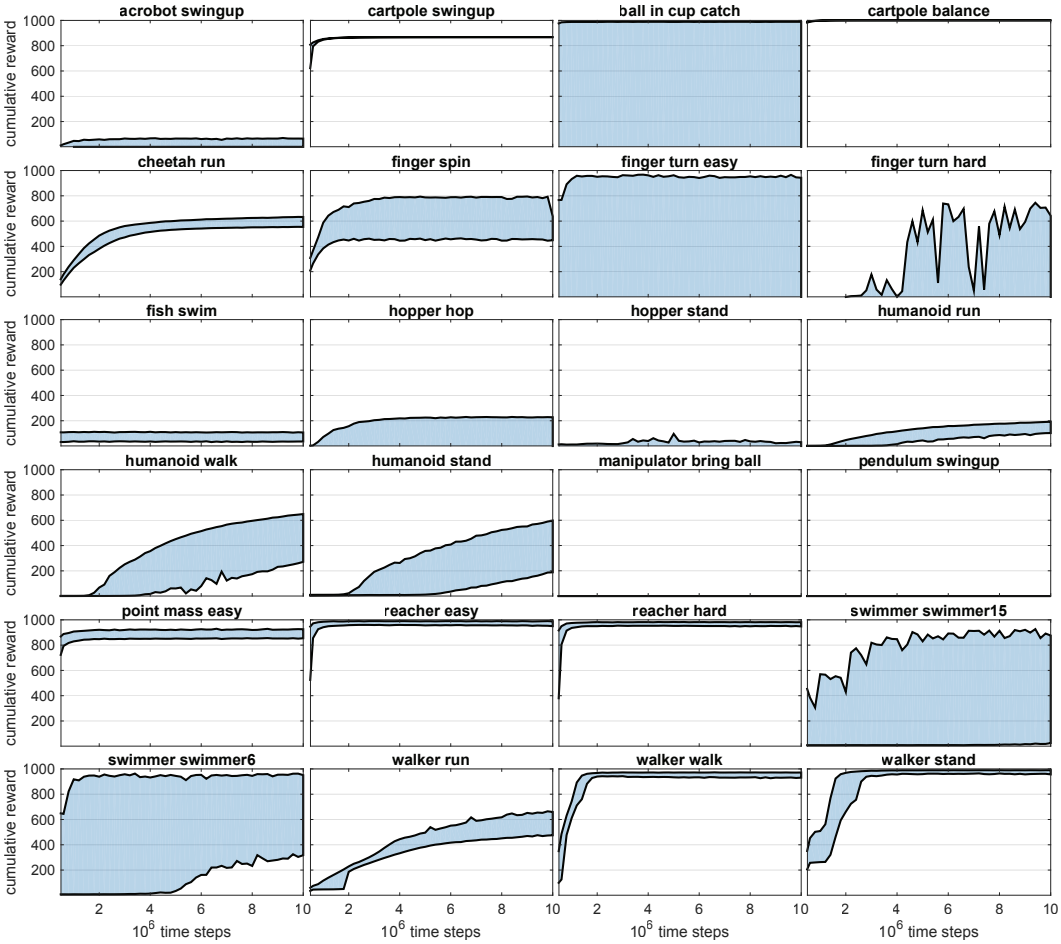

Figure 9: 20th and 80th percentiles of cumulative rewards for trajectories of DeepMind Control Suite tasks ended within intervals of $2 \cdot 10^5$ time steps across 5 independent training runs using the baseline RACER hyper-parameters (Table 1).

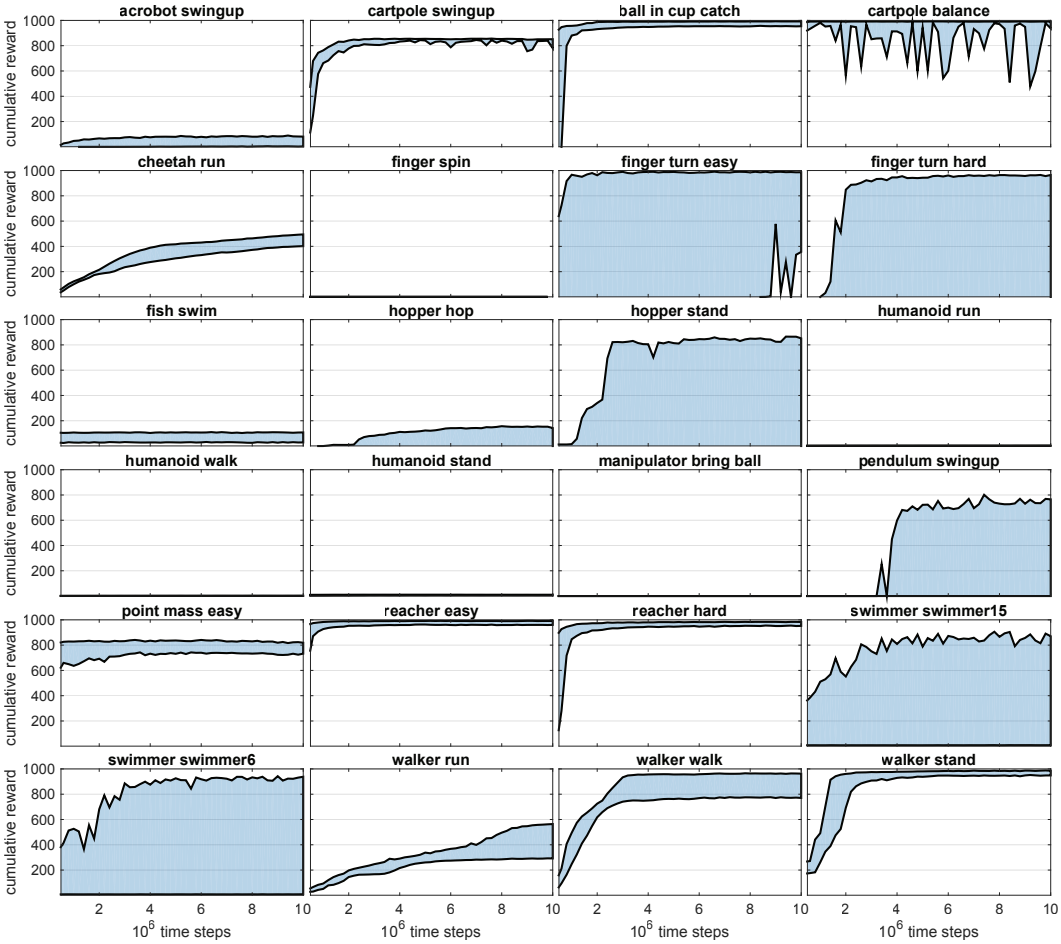

Figure 10: 20th and 80th percentiles of cumulative rewards for trajectories of DeepMind Control Suite tasks ended within intervals of $2 \cdot 10^5$ time steps across 5 independent training runs for `u-DDPG` with `ReF-ER`, $C = 4$, and all other hyper-parameters set according to App. D.