Task-level Differentially Private Meta Learning

Xinyu Zhou
Department of Computer Science & Engineering
The Ohio State University
zhou.3542@buckeyemail.osu.edu

Raef Bassily
Department of Computer Science & Engineering and TDAI Institute
The Ohio State University
bassily.1@osu.edu

Abstract

We study the problem of meta-learning with task-level differential privacy. Meta-learning has received increasing attention recently because of its ability to enable fast generalization to new task with small number of data points. However, the training process of meta learning likely involves exchange of task specific information, which may pose privacy risk especially in some privacy-sensitive applications. Therefore, it is important to provide strong privacy guarantees such that the learning process will not reveal any task sensitive information. To this end, existing works have proposed meta learning algorithms with record-level differential privacy, which is not sufficient in many scenarios since it does not protect the aggregated statistics based on the task dataset as a whole. Moreover, the utility guarantees in the prior work are based on assuming the loss function satisfies both smoothness and quadratic growth conditions, which do not necessarily hold in practice. To address these issues, we propose meta learning algorithms with task-level differential privacy; that is, our algorithms protect the entire dataset for each task. In the case when a single meta model is trained, we give both privacy and utility guarantees assuming only that the loss is convex and Lipschitz. Moreover, we propose a new private clustering-based meta-learning algorithm that enables private meta learning of multiple meta models. This can provide significant accuracy gains over the single meta model paradigm, especially when the tasks distribution cannot be well represented by a single meta model. Finally, we conduct several experiments demonstrating the effectiveness of our proposed algorithms.

1 Introduction

Meta learning has received increasing attention recently due to its successful applications in a variety of machine learning domains. Meta learning is the process in which existing learning tasks are used to infer a learning rule that enables faster adaptation to new tasks from the same environment. Specifically, meta learning takes in a collection of tasks (datasets) sampled from an unknown distribution. Each task defines a learning problem with respect to an input dataset. The goal of meta learning is to output a meta-learner which can extract the shared knowledge among the tasks (datasets) and generalize well on a newly sampled task (dataset) with a small number of data points. The performance of a meta learner is measured by *transfer risk*, defined as the expected risk of the output of the algorithm w.r.t. a newly sampled task.

Machine learning algorithms typically rely on gradient-based optimization methods. The optimization-based meta learning framework allows such methods to converge to a solution using a small number of samples. This is done by learning either a good parameter initialization or a regularizer. In meta

36th Conference on Neural Information Processing Systems (NeurIPS 2022).

initialization methods like MAML [1] and Reptile [2], the meta leaner obtains a meta initialization w_0 such that one or a few gradient updates of w_0 produces a good task-specific model for new tasks. Another class of meta learning algorithms are based on an approach known as meta-regularization [3–6] in which a regularizer is first learned based on all tasks datasets, then a task-specific model is obtained for a new task by minimizing the regularized loss. Meta-regularization has received more attention recently due to its formal utility guarantees and computational efficiency. In meta-regularization, the goal is to find a good bias vector h such that a local learner (base learner) can converge to a good task-specific model by minimizing the regularized empirical loss: $\min_{w\in\mathcal{W}} R_Z(w) + \frac{\lambda}{2}\|w-h\|^2$. In [7], the authors formalize the connection between meta-initialization and meta-regularization under the framework of online convex optimization.

Meanwhile, the nature of the meta learning requires the exchange of task-specific information, which may pose serious privacy risks, especially in distributed settings where all tasks (users) are participating into a meta training process by communicating meta-model updates among themselves, typically with the assistance of a central server. Therefore it is important to ensure that the sensitive information in every task (dataset) remains private through the meta learning process.

Despite its importance, the privacy aspect of meta learning is still underexplored. In [8], using the meta-initialization framework, the authors propose a meta-learning algorithm with *record-level* differential privacy (DP) and give an accuracy guarantee in terms of the *transfer risk*. Record-level DP is a weak privacy guarantee for a wide range of scenarios. Imagine training an email spam filter for each user (task). Even if we ensure differential privacy w.r.t. each record of a user's email, aggregate statistics across the entirety of a user's records (e.g., the frequency of certain words or sentences) may still reveal some sensitive information which the user doesn't want to share. Moreover, the accuracy guarantees in [8] rely on relatively strong assumptions concerning the loss function used for training. In particular, the loss function therein is assumed to satisfy both smoothness and quadratic growth conditions, which do not hold simultaneously for many loss functions commonly used in practice.

In this work, we propose new meta-learning algorithms with task-level DP and derive formal transfer risk bounds. Unlike [8], our algorithms are based on the meta-regularization framework. We now highlight our main technical contributions:

- We propose a task-level differentially private (task-level DP) meta learning algorithm based on the meta-regularization approach. We observe that when the meta-loss is properly defined, one can view each task dataset as a "data point" and the meta-loss as its corresponding loss function. Therefore, applying existing DP optimization techniques like Noisy SGD on the newly defined "data point" and loss function achieves task-level differential privacy naturally.
- We give a bound for the statistical performance for our algorithm in terms of its excess transfer risk. Our statistical accuracy guarantee holds under standard assumptions; namely, assuming only that loss function is convex and Lipshitz. With sufficiently large number of tasks, the bound asserts improved performance over local training, where a model is trained locally based on a single task dataset.
- When the task distribution becomes more complex such that a single meta model is not enough to capture the relationship among tasks, we propose a task-level DP clustering-based meta learning algorithm that can learn multiple meta models. This approach can provide significant improvements in the statistical accuracy (transfer risk) over the standard single-model case, especially when the tasks distribution is better captured by multiple meta models.
- We empirically evaluate our algorithms to demonstrate the effectiveness of our proposed algorithms[1].

Related work: The most related work to ours is [8], where the authors propose a meta-learning algorithm with record-level DP via the framework of meta-initialization. Their analysis relies on assuming that the loss function satisfies a quadratic growth condition, which we do not require in our analysis. Furthermore, we consider a stronger notion of the excess transfer risk than the one in [8]. To be more specific, in [8], the achieved transfer risk is compared to the transfer risk of the optimal *global* model, while the transfer risk in our work is compared to that of the optimal *personalized* model. This is because we allow different tasks to have different parameters. Since

[1] Our code is available online at https://github.com/xyzhou055/MetaNSGD

the optimal personalized model always outperforms global model, we have a stronger notion of the excess transfer risk. Consequently, our utility guarantee also holds under the notion studied in [8].

Our results are also related to the line of work in private model personalization. The goal of model personalization is to train a personalized model for each user that generalizes well on the user's distribution. Reference [9] proposes a private personalization algorithm that solves a stochastic optimization problem involving both local and global models. The authors of [10] develop an algorithm for mean-regularized multi-task learning under the joint differential privacy guarantee and study its application in model personalization. The work of [11] presents private personalization algorithms that learn a common embedding model among users using the alternating minimization framework, where convergence to the optimal solution is guaranteed for linear regression models with Gaussian data. There are also a variety of works, mainly in the non-private regime, exploring the connection between meta learning and model personalization. For example, [12] proposes a personalization mechanism based on MAML, which aims at finding a good initialization parameter for the users. The authors of [13] study the connection between meta learning and personalization based on the observation that FedAvg [14] can be seen as a meta learning algorithm. Therefore, it is possible to extend the results in this work to the domain of model personalization.

There are also prior works studying learning with task-level or user-level differential privacy but under different learning paradigms. In [15, 16], the authors provide user-level differential privacy under the framework of federated learning, in which case all users share a single global model. This is different from our setting because we allow each user to have its own task-specific model. The authors in [17] develop algorithms for a variety of learning tasks under user-level differential privacy. They consider a limited heterogeneity setting where all users' distributions are close to each other in total variation distance. On the other hand, in this work we focus on the meta learning paradigm, where we only assume small variation across users' models.

2 Preliminaries

Distributional setup: We consider a meta distribution ρ and each task is sampled from ρ which will induce a task specific distribution μ over the data domain $\mathcal{Z}$, that is $\mu_1, \dots \mu_K \overset{\text{i.i.d}}{\sim} \rho$ for K tasks. After the sampling of μ_i, a dataset Z_i with n datapoints is subsequently sampled from μ_i written as $Z_i = \{z_1, \dots z_n\} \overset{\text{i.i.d}}{\sim} \mu_i^n$. The collection of K task datasets is denoted as

$$\bar{Z} = \{Z_1, \dots Z_K\}$$

In this paper, we focus on distributed settings such that the datasets $\{Z_i\}_{i=1}^K$ are decentralized across multiple locations. All tasks jointly participate into a model training process coordinated by a trusted central server. The data of each task is stored and processed locally, only the model updates are shared to the server or other tasks.

Task population and empirical losses: Given a closed convex parameter space $\mathcal{W}$, we consider learning task parametrized by vector $w \in \mathbb{R}^d$. We let $\ell : \mathbb{R}^d \times \mathcal{Z} \to \mathbb{R}$ be a loss function, where $\ell(w, z)$ is the loss incurred by a parameter vector w on a data point z. For any task drawn from a distribution μ and observed via a dataset $Z = (z_1, \dots z_n)$, we denote the population risk of $w \in \mathcal{W}$ as $R_\mu(w) = \mathbf{E}_{z\sim\mu}\ell(w, z)$ and the empirical risk as $R_Z(w) = \frac{1}{n}\sum_{i=1}^n \ell(w, z_i)$.

Meta learning: Given K learning tasks with datasets $\bar{Z}$, a meta-learning process aims to learn an algorithm $\mathcal{A}$ that enables fast adaption on newly sampled tasks from the meta-distribution ρ. Such adaptation is measured by the notion of *transfer risk* defined as

$$\mathcal{E}_n(\mathcal{A}; \rho) = \mathbf{E}_{\mu\sim\rho}\mathbf{E}_{Z\sim\mu^n} R_\mu(\mathcal{A}(Z)). \tag{1}$$

The transfer risk $\mathcal{E}_n(\mathcal{A}; \rho)$ can be considered as the expected loss of the parameter returned by applying the algorithm $\mathcal{A}$ on a dataset $Z \in \mathcal{Z}^n$ sampled from the meta distribution ρ. In summary, the whole process starts with a meta-learning algorithm taking $\bar{Z}$ as input and outputting a learning algorithm $\mathcal{A}$. After that, new learning task with distribution $\mu \sim \rho$ trains its corresponding dataset with the learned algorithm $\mathcal{A}$ and apply the resulting parameter vector w to new data drawn from μ.

For any task specific distribution μ, we denote the minimizer w_μ as $w_\mu \in \min_{w\in\mathcal{W}} R_\mu(w)$ and the expected minimum error over the meta-distribution ρ as $\mathcal{E}_\rho = \mathbf{E}_{\mu\sim\rho}R_\mu(w_\mu)$. Note that, there might be more than one minimizers of $R_\mu(w)$, but $\mathcal{E}_\rho$ remains same regardless of the selection of w_μ. Furthermore, we define the excess transfer risk as

$$\Delta\mathcal{E}_n(\mathcal{A}; \rho) = \mathcal{E}_n(\mathcal{A}; \rho) - \mathcal{E}_\rho \tag{2}$$

Task similarity metric: To achieve low transfer risk, the meta learning algorithm leverages the shared similarity among tasks and output the optimized inner algorithm. In this work, the task similarity is measured by the quantity:

$$V^2 = \min_{\phi \in \mathcal{W}} \mathbf{E}_{\mu \sim \rho} \left\| \phi - \text{Proj}_{W_\mu}(\phi) \right\|^2 \tag{3}$$

where W_μ is the set of optimal parameters for task μ written as $W_\mu = \arg\min_{w \in \mathcal{W}} R_\mu(w)$, and Proj_{W_μ} denotes the Euclidean projection on W_μ. V can be roughly seen as the expected variation of task parameters. We denote $\bar{w}$ as the minimizer of (3) such that $\bar{w} \in \mathcal{W}$ and $V^2 = \left\| \bar{w} - \text{Proj}_{W_\mu}(\bar{w}) \right\|^2$.

Task-level Differential Privacy [18]: A randomized algorithm $\mathcal{M}$ is (ϵ, δ)-differential private if for any pair of neighboring sets $\bar{Z}$ and $\bar{Z}'$ and for any measurable subset $\mathcal{O}$ of Range$(\mathcal{M})$, we have

$$P(\mathcal{M}(\bar{Z}) \in \mathcal{O}) \leq e^\epsilon P(\mathcal{M}(\bar{Z}') \in \mathcal{O}) + \delta$$

In this work, we consider task-level neighboring relationship. Let $\bar{Z} = \{Z_1, \ldots Z_K\}$ and $\bar{Z}' = \{Z'_1, \ldots Z'_K\}$, and we say $\bar{Z}$ and $\bar{Z}'$ are neighboring if, for some $i \in [K]$, $\bar{Z}$ and $\bar{Z}'$ differ only on the dataset from the i_{th} task, i.e., $Z_j = Z'_j$ for all j except i. Therefore, a task-level differential private algorithm ensures that any adversary cannot tell if the dataset of a particular task was used in the algorithm given its output.

Our analysis involves the following fairly standard assumption:

Assumption 1. *(Convex-Lipschitz-Bounded models)*

- *For any $z \in \mathcal{Z}$, the loss function $\ell(w, z)$ is convex and L-Lipshitz with respect to w.*
- *Bounded parameter space: $\|\mathcal{W}\|_2 = \max_{w_1, w_2 \in \mathcal{W}} \|w_1 - w_2\| \leq M$*

3 Learning with Meta Regularization

Base learner:A base learner solves a regularized loss minimization problem w.r.t. a biased regularizer parameterized by a bias vector h.In particular, given a bias vector h and a task dataset Z, the base learner solves the following problem:

$$\min_{w \in \mathcal{W}} R_{Z,\lambda}(w; h) = \min_{w \in \mathcal{W}} R_Z(w) + \frac{\lambda}{2} \|w - h\|^2$$

where λ is the regularization parameter. In the sequel, we define $w_h(Z)$ as the minimizer of $R_{Z,\lambda}(w; h)$, that is $w_h(Z) = \arg\min_{w \in \mathcal{W}} R_{Z,\lambda}(w; h)$.

Meta algorithm: Given a collection of base learners, the goal of a meta-algorithm is to find a good regularization bias h that yields a small transfer risk. First, let's rewrite the transfer risk defined in (1) as a function h:

$$\mathcal{E}_n(h; \rho) = \mathbf{E}_{\mu \sim \rho} \mathbf{E}_{Z \sim \mu^n} R_\mu(w_h(Z)) \tag{4}$$

and hence the excess transfer risk can be expressed as $\Delta\mathcal{E}_n(h; \rho) = \mathcal{E}_n(h; \rho) - \mathcal{E}_\rho$. Therefore, the goal of the meta algorithm is to find an (approximate) minimizer h^* of $\mathcal{E}_n(h; \rho)$ in (4), i.e., the goal is to solve:

$$\min_{h \in \mathcal{W}} \mathcal{E}_n(h; \rho) = \mathbf{E}_{\mu \sim \rho} \mathbf{E}_{Z \sim \mu^n} R_\mu(w_h(Z)) \tag{5}$$

Intuitively speaking, the base learner aims to figure out the best parameter vector "around" the bias vector h. Therefore, if the meta algorithm can find a good bias vector h that is close to the solution space of the tasks, the base learner can converge to its solution space quickly by only searching around h.

3.1 Meta Loss and proxy transfer risk

Directly optimizing over (5) above is usually infeasible. A common strategy to get around this difficulty is to propose a proxy loss function which is related to the transfer risk but easier to handle. In this section, we follow the definitions in [3, 4] and define our meta loss as

$$L_Z(h) = \min_{w \in \mathcal{W}} R_{Z,\lambda}(w; h) = \min_{w \in \mathcal{W}} R_Z(w) + \frac{\lambda}{2} \|w - h\|^2$$

and the proxy to the transfer risk is defined as

$$\hat{\mathcal{E}}_n(h;\rho) = \mathbf{E}_{\mu\sim\rho}\mathbf{E}_{Z\sim\mu^n} L_Z(h)$$

The goal of the meta-algorithm is to optimize the proxy risk over h, that is,

$$\min_{h\in\mathcal{W}} \hat{\mathcal{E}}_n(h;\rho) = \mathbf{E}_{\mu\sim\rho}\mathbf{E}_{Z\sim\mu^n} L_Z(h) \tag{6}$$

Note that $L_Z(h)$ is in fact the Moreau envelope of the empirical loss $R_Z(w)$. In Proposition 2, we review some useful properties of $L_Z(h)$ which will help our analysis.

Proposition 2. *(follows from [19, 20]) Under Assumption 1, we have*

1. *L_Z is convex, $2L$-Lipshitz and λ-smooth.*
2. *For any $h \in \mathcal{W}$, $\nabla L_Z(h) = -\lambda(w_h(Z) - h)$ where $w_h(Z) = \arg\min_{w\in\mathcal{W}} R_{Z,\lambda}(w;h)$.*

A key observation is that all datasets go through the same stochastic process under our distributional setup and thus can be seen as i.i.d. sequence drawn from a distribution P_Z. Therefore, (6) can be rewritten as

$$\min_{h\in\mathcal{W}} \hat{\mathcal{E}}_n(h;\rho) = \min_{h\in\mathcal{W}} \mathbf{E}_{Z\sim P_Z} L_Z(h) \tag{7}$$

Combined with Proposition (2), the objective of the meta algorithm is formulated as a stochastic convex optimization (SCO) problem which has been well studied under privacy constraints [19, 21, 22]. The notion of task-level differential privacy just comes up naturally when we see each task dataset Z as a "data point" and $L_Z(h)$ as its loss function, therefore the conventional record-level neighboring relationship can be readily translated to a task-level neighboring relationship.

However, one challenge in adapting existing differentially private SCO methods pertains to the gradient computation. In standard SCO settings, we are expected to be able to compute the gradient of the loss function efficiently and precisely. However, in our case, as seen in Proposition 2, computing $\nabla L_Z(h)$ for each h requires computing $w_h(Z)$ which further involves solving a λ-strongly convex optimization problem. Therefore, finding the exact value of $w_h(Z)$ for general convex loss ℓ is not feasible, alternatively what we can do is to approximate $w_h(Z)$ using some iterative methods up to some error b. Therefore, it is important to incorporate this approximation error into our convergence analysis. Moreover, due to the distributed nature of the algorithm, the model iterates are assumed to be visible to multiple tasks who are selected to participate in each round. Therefore, the privacy should be defined over all model iterates as well as the final model, and we cannot use the linear-time algorithm for DP smooth stochastic optimization [22] despite the fact that the objective of the meta-algorithm can be formulated as a stochastic convex optimization problem with a smooth loss.

4 Private Meta Learning with Noisy SGD

In this section, we present our meta learning algorithm adapted from the mini batch noisy SGD method in [19] described in Algorithm 1. In iteration t, given the current iterate h_t, each sampled task computes an approximation of the gradient of the meta loss $L_Z(h_t)$, denoted as $\hat{\nabla} L_Z(h_t)$. After that, the server aggregates theses approximate gradients to update the model. The privacy is achieved by adding Gaussian noise scaled with the l_2 sensitivity of each task to the model iterate.

We will first describe the privacy and utility guarantee of Algorithm 1, and then show the resulting transfer risk.

Theorem 3. *(Privacy guarantee) Let Assumption 1 holds and let $\{h_1, \dots h_{T-1}\}$ and $\bar{h}_T$ be the output, Algorithm 1 is (ϵ, δ)-differentially private on task level.*

Proof. Since the computed gradient $\hat{\nabla} L_Z(h_t)$ is only an approximation to $\nabla L_Z(h_t)$, then we have

$$\left\|\hat{\nabla} L_Z(h_t) - \nabla L_Z(h_t)\right\| \le \lambda \left\|\hat{w}_{h_t}(Z) - w_{h_t}(Z)\right\| \le b$$

and because $\nabla L_Z(h)$ is $2L$-Lipshitz by Proposition 2, we have

$$\left\|\hat{\nabla} L_Z(h_t)\right\| \le \left\|\nabla L_Z(h_t)\right\| + b \le 2L + b$$

Algorithm 1: $\mathcal{A}_{\text{meta-NSGD}}$: meta learning with mini-batch noisy SGD

input : Initial iterate $w_0 \in \mathcal{W}$, number of iterations T, estimation error b, regularization parameter λ, step size η.

1 Set noise variance $\sigma^2 = \frac{8T(2L+b)^2 \log(1/\delta)}{K^2\epsilon^2}$ and batch size $m = \max\left(K\sqrt{\epsilon/(4T)}, 1\right)$

2 **for** $t = 1, \ldots, T-1$ **do**

3 Server samples a batch of tasks $B_t = \{Z_{i_{(t,1)}} \ldots Z_{i_{(t,m)}}\} \leftarrow \bar{Z}$ and sends h_t to tasks in B_t

4 **for** *each task* Z *in* B_t **do**

5 Solve $R_{Z,\lambda}(w; h_t)$ and compute an estimate $\hat{w}_{h_t}(Z)$ with $\|\hat{w}_{h_t}(Z) - w_{h_t}(Z)\| \leq \frac{b}{\lambda}$

6 Send $\hat{\nabla} L_Z(h_t) = -\lambda(\hat{w}_{h_t}(Z) - h_t)$ back to server

7 **end**

8 Server updates the model with

$$h_{t+1} = \text{Proj}_{\mathcal{W}}\left(h_t - \eta \cdot \left(\frac{1}{m}\sum_{Z \in B_t} \hat{\nabla} L_Z(h_t) + G_t\right)\right)$$

where $\text{Proj}_{\mathcal{W}}$ denotes the Euclidean projection onto $\mathcal{W}$, and $G_t \sim \mathcal{N}(0, \sigma^2 \mathbb{I}_d)$ drawn independently each iteration.

9 **end**

output : $\bar{h}_T = \frac{1}{T}\sum_{t=1}^{T} h_t$

Note that Algorithm 1 has the similar structure of the mini-batch noisy SGD method in [19] if we consider $\hat{\nabla} L_Z(h_t)$ as the model updates and h_t as model iterates. By adding Gaussian noise scaled with $(2L + b)$, a direct application of Theorem 3.1 in [19] completes the proof. □

Since the objective of the base learner $R_{Z,\lambda}(w; h_t)$ is a λ-strongly convex problem, one can use gradient decent to bound the approximation error at step 5 in Algorithm 1. Alternatively, the base learner can use other stochastic optimization method i.e. SGD to achieve a high probability bound on the approximation error, such that $\|\hat{w}_{h_t}(Z) - w_{h_t}(Z)\| \leq \frac{b}{\lambda}$ with probability over $1 - \delta'$. By letting the overall failure probability $mT\delta'$ on the same order of the privacy parameter δ, the privacy guarantee still holds. For the simplicity of the analysis, we assume the approximation error $\|\hat{w}_{h_t}(Z) - w_{h_t}(Z)\| \leq \frac{b}{\lambda}$ always satisfies.

In the next theorem, we will give the excess risk of $\mathcal{A}_{\text{meta-NSGD}}$

Theorem 4. *(Utility guarantee) Under Assumptions 1, suppose* $\lambda \leq \frac{2L}{M} \cdot \min\left(\sqrt{\frac{K}{2}}, \frac{\epsilon K}{2\sqrt{2d\log(1/\delta)}}\right)$. *Let* $T = \min\left(\frac{K}{8}, \frac{\epsilon^2 K^2}{32 d \log(1/\delta)}\right)$, $\eta = \frac{M}{2L\sqrt{T}}$ *and* $b = \frac{2L}{K}$. *Then,*

$$\mathbf{E}[\hat{\mathcal{E}}_n(\bar{h}_T; \rho)] - \min_{h \in \mathcal{W}} \hat{\mathcal{E}}_n(h; \rho) \leq O\left(ML \cdot \left(\frac{1}{\sqrt{K}} + \frac{\sqrt{d\log(1/\delta)}}{\epsilon K}\right)\right)$$

where $\bar{h}_T$ *is the final model output by Algorithm 1, and the expectation is taken over the collection of datasets* $\bar{Z}$ *and the inner randomness of the algorithm.*

Note that the excess rate in Theorem 4 is with respect to $\hat{\mathcal{E}}_n(h; \rho)$, the proxy to the transfer risk. It remains to show how it can be related to the excess rate of the true transfer risk $\mathcal{E}_n(h; \rho)$. In the next theorem, we give the excess transfer risk of Algorithm 1. The proof follows from [Theorem 6, [3]] and decomposes the excess transfer rate into three components. We bound each component separately and one of them is based on the excess rate achieved in Theorem 4.

Theorem 5. *(Transfer risk) Recall the settings in Theorem 4, we have*

$$\mathbf{E}[\mathcal{E}_n(\bar{h}_T; \rho)] - \mathcal{E}_\rho \leq \frac{2L^2}{\lambda n} + \frac{\lambda}{2}V^2 + O\left(ML\left(\frac{\sqrt{d\log(1/\delta)}}{\epsilon K} + \frac{1}{\sqrt{K}}\right)\right)$$

By letting $\lambda = \frac{2L}{V}\sqrt{\frac{1}{n}}$*, we have*

$$\mathbf{E}[\mathcal{E}_n(\bar{h}_T;\rho)] - \mathcal{E}_\rho \leq \frac{2LV}{\sqrt{n}} + O\left(ML \cdot \left(\frac{\sqrt{d\log(1/\delta)}}{\epsilon K} + \frac{1}{\sqrt{K}}\right)\right)$$

where $\bar{h}_T$ *is the output of* $\mathcal{A}_{\text{meta-NSGD}}$ *and the expectation is taken w.r.t.* $\bar{Z}$ *and the inner randomness of the algorithm.*

Theorem 5 shows that when the number of tasks K is sufficiently large, the excess transfer risk becomes $O\left(\frac{LV}{\sqrt{n}}\right)$. Recall that V is a measure of task similarity, hence when the optimal task parameters are close to each other, V can be much smaller than the diameter of the parameter space which indicates better performance than local training where each task trains its own model independently. Note one can also give a direct bound of the transfer risk of the output of Algorithm 1 based on the fact that $\left\|\bar{h}_T - w_\mu\right\| \leq \|\mathcal{W}\| = M$ for any task μ with optimal parameter w_μ, standard derivation gives that $\mathbf{E}[\mathcal{E}_n(\bar{h}_T;\rho)] - \mathcal{E}_\rho \leq O\left(\frac{LM}{\sqrt{n}}\right)$ which is the same as the excess risk for local training. Therefore, Algorithm 1 is at least as good as local training, in the worst case, it recover the performance of local training.

The rate achieved in Theorem 5 is on the same order of the non-private rate $O\left(\frac{VL}{\sqrt{n}} + \frac{ML}{\sqrt{K}}\right)$ from [3] in the regime most common in practice where $d = O(K)$ and $\epsilon = \theta(1)$. Note that [8] obtains $O\left(\frac{1}{K}\right)$ convergence on the number of tasks K under record-level privacy guarantee, but their analysis relies on the assumption for the loss function to be α-quadratic growth, which we do not require for our analysis. Furthermore, we have different definitions of excess transfer risk from [8], given the meta learner $\mathcal{A}_h$, the excess transfer risk in [8] is defined as $\mathcal{E}_n(\mathcal{A}_h;\rho) - \min_{w\in\mathcal{W}} \mathbf{E}_{\mu\sim\rho} R_\mu(w)$ while ours is defined as $\mathcal{E}_n(\mathcal{A}_h;\rho) - \mathbf{E}_{\mu\sim\rho}\left[\min_{w\in\mathcal{W}} R_\mu(w)\right]$. Since $\min_{w\in\mathcal{W}} \mathbf{E}_{\mu\sim\rho} R_\mu(w) \geq \mathbf{E}_{\mu\sim\rho}\left[\min_{w\in\mathcal{W}} R_\mu(w)\right]$, our bound holds under a stronger definition for the excess transfer risk. Hence, our bound is a valid upper bound for the notion defined in [8]. The detailed proofs of Theorems 4 and 5 can be found in the appendix.

Communication-efficient, private meta Learning with DP-FTMRL Although Algorithm 1 gives a transfer risk comparable with the non-private rate, it requires $O\left(\min\left\{\sqrt{K}, K^{3/2}/d\right\}\right)$ rounds of communication to each task on average. This can be expensive when the task number K is large but each task only has limited computation and communication resource, which is a common setting in distributed scenarios like federated learning. Therefore, we introduce a communication efficient DP meta algorithm Differentially Private Follow-the-Meta-Regularized-Leader (DP-FTMRL) which is based on the DP-FTRL algorithm in [23]. DP-FTMRL is a one-pass algorithm which means that every task will be communicated by the server for only once. Furthermore, it processes each task sequentially thus allows tasks to arrive in a stream. With high probability over the algorithm inner randomness, DP-FTMRL obtains transfer risk of $\tilde{O}\left(\frac{LV}{\sqrt{n}} + ML\left(\frac{1}{\sqrt{K}} + \frac{d^{1/4}}{\sqrt{K\epsilon}}\right)\right)$. This rate is suboptimal compared with the one achieved in Theorem 5, but DP-FTMRL provides significant savings on the communication cost. More details of DP-FTMRL can be found in the appendix.

Note: To get the excess transfer risk guarantee in Theorem 5, we need to set $\lambda = \frac{2L}{V}\frac{1}{\sqrt{n}}$ which requires the knowledge of the value of V, the task similarity. In fact, we can privately approximate V using its empirical counterpart; in particular, we can find a private counterpart for $\hat{V}^2 \triangleq \min_{\phi\in\mathcal{W}} \frac{1}{K}\sum_{i=1}^{K} \|\phi - \hat{w}(Z_i)\|^2$ where $\hat{w}(Z_i)$ is a minimizer of the empirical loss of task Z_i. This can be done using standard DP convex optimization methods such Noisy Gradient Descent. An alternative approach to choose a setting for λ, which is quite common in practice, is to treat λ as a hyperparameter [4] and tune it privately. This can also be done using existing methods in the literature of DP [24, 25].

5 Private Clustering-Based Meta Learning

In this section, we consider the case where the tasks are sampled from distribution with multiple concentrations (e.g., a multi-modal distribution) where a single meta model is not enough to capture the relationship among tasks. A natural extension is to cluster the tasks into several groups and train a meta model for each group.

To provide a more formal treatment, we let q denote the number of possible meta models (clusters), each is parametrized with a bias vector h_i. These meta models are denoted as $(h_1, h_2, \dots h_q)$. The task similarity in the presence of q meta modals is hence modified to

$$V_c^2 = \min_{h_1,\dots h_q \in \mathcal{W}} \mathbf{E}_{\mu\sim\rho} \min_{i\in[q]} \left\| h_i - \text{Proj}_{W_\mu}(h_i) \right\|^2 \tag{8}$$

Recall that W_μ is the set of optimal parameters for task μ. Note that V_c is no larger than V from single meta learner case defined in (3). In the case when $\{W_\mu\}$ contains multiple disjoint and far apart concentrations, V_c can be much smaller compared with V.

In particular, inspired by the idea of hypothesis based clustering from [26], we define the objective of the meta-algorithm with q meta learners as

$$\min_{h_1,\dots h_q \in \mathcal{W}} \hat{\mathcal{E}}_n(\mathbf{h}, \bar{Z})) = \frac{1}{K} \sum_{j=1}^{K} \min_{i\in[q]} L_Z(h_i) \tag{9}$$

where $L_Z(\cdot)$ is the meta loss.

Given $\mathbf{h} = (h_1 \dots h_q)$, the base learner with dataset set Z first chooses the h_i with lowest meta loss, that is, $C(Z) = \arg\min_{i\in[q]} L_Z(h_i)$ and output the corresponding minimizer $w_{h_{C(Z)}}(Z) = \min_{w\in\mathcal{W}} R_{Z,\lambda}(w; h_{C(Z)}) = \min_{w\in\mathcal{W}} R_Z(w) + \frac{\lambda}{2} \left\| w - h_{C(Z)} \right\|^2$. Therefore, the transfer risk is defined as

$$\mathcal{E}_n(\mathbf{h}; \rho) = \mathbf{E}_{\mu\sim\rho} \mathbf{E}_{Z\sim\mu^n} R_\mu \left(w_{h_{C(Z)}(Z)} \right)$$

We present our meta-clustering algorithm described in Algorithm 2, which is inspired by the HYP-CLUSTER algorithm in [26]. Algorithm 2 is a heuristic algorithm aiming at minimizing the objective (9) over $h_1, \dots h_q$. Each iteration consists of two steps, the model selection step and model update step. At the beginning of one iteration, each sampled task selects the model from $\{h_i\}_{i=1}^q$ that incurs the lowest loss. After the assignment is done, the server will update each model with its corresponding tasks.

We formally show the privacy guarantee of Algorithm 2. The performance of this algorithm in terms of its transfer risk will be evaluated empirically since convergence to a true optima is not guaranteed due to the non-convexity of $\hat{\mathcal{E}}_n(\mathbf{h}, \bar{Z}))$ in $\mathbf{h}$.

Algorithm 2: $\mathcal{A}_{\text{meta-cluster}}$: private clustering-based meta learning algorithm

input : q randomly initialized iterates $h_0^1, h_0^2, \dots h_0^q$, number of iterations T, batch size m

Set noise variance $\sigma^2 = \frac{8T(2L+b)^2 \log(1/\delta)}{K^2\epsilon^2}$

for $t = 1, \dots, T-1$ **do**

  Server samples a batch of tasks B_t of size m and sends $\{h_t^i\}_{i=1}^q$ to B_t

  for *each task* Z *in* B_t **do**

    For $i \in [q]$, compute approximate minimizers of $R_{Z,\lambda}(w, h_t^i)$ as $\hat{w}_h^i(Z)$ with approximation error $\frac{b}{\lambda}$

    Select the model with smallest meta loss $C(Z) = \arg\min_{i\in[q]} R_{Z,\lambda}(\hat{w}_h^i(Z), h_t^i)$

    Compute the gradient $\hat{\nabla} L_Z(h_t^{C(Z)}) = -\lambda \left(\hat{w}_Z^{C(Z)} - h_t^{C(Z)} \right)$

    Send $(C(Z), \hat{\nabla} L_Z(h_t^{C(Z)}))$ back to server

  end

  For every $i \in [q]$, denote the set $C_t^i = \{Z \in B_t | C(Z) = i\}$, server updates h_t^i as

$$h_{t+1}^i = \text{Proj}_{\mathcal{W}} \left(h_t^i - \eta \cdot \left(\frac{1}{m} \sum_{Z\in C_t^i} \hat{\nabla} L_Z(h_t^i) + G_t^i \right) \right)$$

  where $G_t^i \sim \mathcal{N}(0, \sigma^2 \mathbb{I}_d)$ drawn independently each iteration t and meta model i.

end

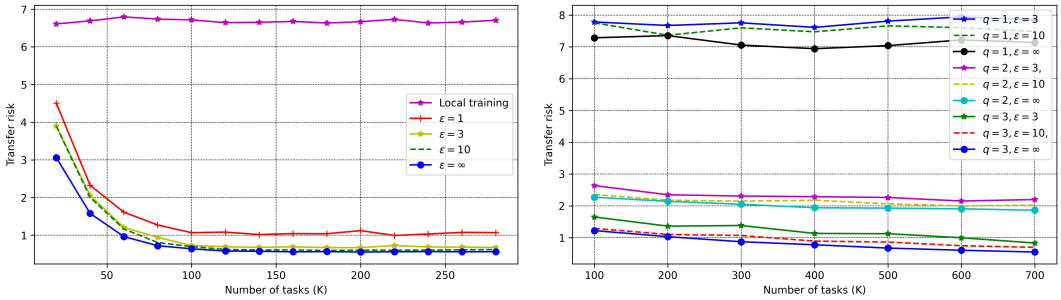
output : $\{h_T^1, h_T^2, \dots h_T^q\}$

Theorem 6. *(Privacy guarantee) Let $\{h_t^1, \dots h_t^q\}_{t\in T}$ be the output, Algorithm 2 is (ϵ, δ)-differentially private on task level.*

Proof. Similar to Theorem 3, we have the norm of $\hat{\nabla} L_Z(h_t^{C(Z)})$ is bounded by $(2L + b)$ for any choice of $C(Z)$. Since each task only selects one meta model to update in each iteration, we consider $\{h_t^i\}_{i=1}^q$ as the model iterate and $\hat{\nabla} L_Z(h_t^i)$ as the model update from the task, we can use the same argument in Theorem 3 to complete the proof. □

6 Experiments

In this section, we empirically evaluate the performance of our algorithms under various privacy regimes. On the one hand, we demonstrate the performance improvement of our algorithms over local training. Meanwhile, when the tasks are sampled from multi-modal distribution, we show that our clustering based algorithm achieves lower transfer risk than the single meta model paradigm.



(a) Single-cluster case

(b) Multi-clusters case

Figure 1: The transfer risk of $\mathcal{A}_{\text{meta-cluster}}$ in linear regression task with respect to the number of tasks. ϵ is the privacy parameter and q is the number of meta models (clusters).

We consider linear regression task with mean square loss. The weight vector w_μ is d-dimentional vectors that concentrates around one or more common vectors. We let $d = 30$ for all experiments. The feature vector x is uniformly sampled from unit ball, and the label $y = \langle w_u, x\rangle + n$ where n is the zero mean Gaussian noise with standard deviation equal to 0.5.

Single cluster case: For each task μ, its d-dimensional weight vector w_μ is sampled from Gaussian distribution with mean $\bar{h}$ and with variance σ^2, denoted as $\mathcal{N}(\bar{h}, \sigma^2\mathbb{I}_d)$. In the experiment, $\sigma = 1$ and $\bar{h}$ is a vector with all entries equal to 4. Each task contains 10 datapoints $(x_i, y_i)_{i=1}^{10}$. The privacy parameters ϵ are chosen from $\{1, 3, 10\}$ and δ is set as 10^{-5}. We set the clipping norm to 2 and use the functions from *TensorFlow privacy*[27] to choose the noise multiplier to fit the privacy budget. In Figure 1, the transfer risks of our meta learning algorithm are substantially better than the transfer risk of local training even in high privacy regimes when $\epsilon = 1$. Furthermore, by comparing the transfer risks for different ϵ's, we show that the performance gaps are much less significant compared with the gap between local training. In modest privacy regimes when $\epsilon = 3$ or 10, the corresponding transfer risks are quite close to the non-private rate ($\epsilon = \infty$).

Multi clusters case: The weight vector is sampled from the distribution $\mathcal{N}(\bar{h}, \sigma^2\mathcal{I}_d)$ where $\bar{h}$ is uniformly sampled from the set $\bar{\mathbf{h}} = \{\bar{h}_1, \dots \bar{h}_t\}$. In the experiment, t, the number of cluster in the underlying distribution, is set to be 3, $\sigma = 0.5$ and we choose $\{\bar{h}_1, \bar{h}_2, \bar{h}_3\}$ to be three orthogonal vectors with different norms. More specifically, $\bar{h}_1 = [2 \dots 2, 0, \dots 0]$, $\bar{h}_2 = [0, \dots, 0, -4 \dots, -4, 0, \dots 0]$ and $\bar{h}_3 = [0, \dots, 0, 6, \dots 6]$, and each $\bar{w}_i$ has 10 non-zero components. The privacy parameters ϵ are chosen from $\{3, 10\}$ and δ is set as 10^{-5}, the clipping norm is set to 1. Furthermore, we set q, the number of meta models in Algorithm 2, from 1 to 3 and report the transfer risk with each q. The plots in Figure 2 confirms our claim that we can achieve better performance by adding the number of meta models. By choosing different number of meta models, the performance gap can be significant. Meanwhile, similar to the single cluster case, with the same number of clusters, we see relatively small performance difference under various privacy parameters.

In Appendix C, we present additional experiments to evaluate our algorithms on Omniglot [28] few-shot classification tasks, where we demonstrate an improved performance over existing DP meta learning methods.

7 Conclusion and Future Work

In this work, we studied the meta learning problem with task-level privacy. When a single meta model is trained, we have shown, both theoretically and empirically, that our private meta algorithm achieves better performance than local training when the variation among the tasks is relatively small. To deal with more complex task distribution, we also proposed a private clustering-based meta learning algorithm that outputs multiple meta models. This approach can provide noticeable performance gains over the single meta model paradigm as demonstrated by our empirical results.

An interesting direction for future work is to formally study the accuracy guarantees of our clustering-based algorithm. Due to the non-convexity of the objective function, theoretical analysis on the transfer risk may be challenging. However, it is still possible to obtain a guarantee to characterize the generalization property of the clustering-based algorithm.

Acknowledgments and Disclosure of Funding

This work is supported by NSF AI-Institute Award 2112471.

References

[1] Chelsea Finn, Pieter Abbeel, and Sergey Levine. Model-agnostic meta-learning for fast adaptation of deep networks. In *International conference on machine learning*, pages 1126–1135. PMLR, 2017.

[2] Alex Nichol, Joshua Achiam, and John Schulman. On first-order meta-learning algorithms. *arXiv preprint arXiv:1803.02999*, 2018.

[3] Giulia Denevi, Carlo Ciliberto, and Riccardo Grazzi. Learning-to-learn stochastic gradient descent with biased regularization. In *International Conference on Machine Learning*, pages 1566–1575. PMLR, 2019.

[4] Pan Zhou, Xiaotong Yuan, Huan Xu, Shuicheng Yan, and Jiashi Feng. Efficient meta learning via minibatch proximal update. *Advances in Neural Information Processing Systems*, 32, 2019.

[5] Aravind Rajeswaran, Chelsea Finn, Sham M Kakade, and Sergey Levine. Meta-learning with implicit gradients. *Advances in neural information processing systems*, 32, 2019.

[6] Weisen Jiang, James Kwok, and Yu Zhang. Effective meta-regularization by kernelized proximal regularization. *Advances in Neural Information Processing Systems*, 34, 2021.

[7] Maria-Florina Balcan, Mikhail Khodak, and Ameet Talwalkar. Provable guarantees for gradient-based meta-learning. In *International Conference on Machine Learning*, pages 424–433. PMLR, 2019.

[8] Jeffrey Li, Mikhail Khodak, Sebastian Caldas, and Ameet Talwalkar. Differentially private meta-learning. In *International Conference on Learning Representations*, 2019.

[9] Alberto Bietti, Chen-Yu Wei, Miroslav Dudik, John Langford, and Steven Wu. Personalization improves privacy-accuracy tradeoffs in federated learning. In *International Conference on Machine Learning*, pages 1945–1962. PMLR, 2022.

[10] Shengyuan Hu, Zhiwei Steven Wu, and Virginia Smith. Private multi-task learning: Formulation and applications to federated learning. *arXiv preprint arXiv:2108.12978*, 2021.

[11] Prateek Jain, John Rush, Adam Smith, Shuang Song, and Abhradeep Guha Thakurta. Differentially private model personalization. *Advances in Neural Information Processing Systems*, 34:29723–29735, 2021.

[12] Alireza Fallah, Aryan Mokhtari, and Asuman Ozdaglar. Personalized federated learning: A meta-learning approach. *arXiv preprint arXiv:2002.07948*, 2020.

[13] Yihan Jiang, Jakub Konečnỳ, Keith Rush, and Sreeram Kannan. Improving federated learning personalization via model agnostic meta learning. *arXiv preprint arXiv:1909.12488*, 2019.

[14] Brendan McMahan, Eider Moore, Daniel Ramage, Seth Hampson, and Blaise Aguera y Arcas. Communication-efficient learning of deep networks from decentralized data. In *Artificial intelligence and statistics*, pages 1273–1282. PMLR, 2017.

[15] H Brendan McMahan, Daniel Ramage, Kunal Talwar, and Li Zhang. Learning differentially private recurrent language models. In *International Conference on Learning Representations*, 2018.

[16] Robin C Geyer, Tassilo Klein, and Moin Nabi. Differentially private federated learning: A client level perspective. *arXiv preprint arXiv:1712.07557*, 2017.

[17] Daniel Levy, Ziteng Sun, Kareem Amin, Satyen Kale, Alex Kulesza, Mehryar Mohri, and Ananda Theertha Suresh. Learning with user-level privacy. *Advances in Neural Information Processing Systems*, 34, 2021.

[18] Cynthia Dwork, Frank McSherry, Kobbi Nissim, and Adam Smith. Calibrating noise to sensitivity in private data analysis. In *Theory of cryptography conference*, pages 265–284. Springer, 2006.

[19] Raef Bassily, Vitaly Feldman, Kunal Talwar, and Abhradeep Thakurta. Private stochastic convex optimization with optimal rates. In *Proceedings of the 33rd International Conference on Neural Information Processing Systems*, pages 11282–11291, 2019.

[20] Yu Nesterov. Smooth minimization of non-smooth functions. *Mathematical programming*, 103(1):127–152, 2005.

[21] Raef Bassily, Adam Smith, and Abhradeep Thakurta. Private empirical risk minimization: Efficient algorithms and tight error bounds. In *2014 IEEE 55th Annual Symposium on Foundations of Computer Science*, pages 464–473. IEEE, 2014.

[22] Vitaly Feldman, Tomer Koren, and Kunal Talwar. Private stochastic convex optimization: optimal rates in linear time. In *Proceedings of the 52nd Annual ACM SIGACT Symposium on Theory of Computing*, pages 439–449, 2020.

[23] Peter Kairouz, Brendan McMahan, Shuang Song, Om Thakkar, Abhradeep Thakurta, and Zheng Xu. Practical and private (deep) learning without sampling or shuffling. In *International Conference on Machine Learning*, pages 5213–5225. PMLR, 2021.

[24] Jingcheng Liu and Kunal Talwar. Private selection from private candidates. In *Proceedings of the 51st Annual ACM SIGACT Symposium on Theory of Computing*, pages 298–309, 2019.

[25] Nicolas Papernot and Thomas Steinke. Hyperparameter tuning with renyi differential privacy. *arXiv preprint arXiv:2110.03620*, 2021.

[26] Yishay Mansour, Mehryar Mohri, Jae Ro, and Ananda Theertha Suresh. Three approaches for personalization with applications to federated learning. *arXiv preprint arXiv:2002.10619*, 2020.

[27] Tensorflow-privacy. https://github.com/tensorflow/privacy, 2022.

[28] Brenden Lake, Ruslan Salakhutdinov, Jason Gross, and Joshua Tenenbaum. One shot learning of simple visual concepts. In *Proceedings of the annual meeting of the cognitive science society*, volume 33, 2011.

[29] Shai Shalev-Shwartz and Shai Ben-David. *Understanding machine learning: From theory to algorithms*. Cambridge university press, 2014.

[30] H Brendan McMahan. A survey of algorithms and analysis for adaptive online learning. *The Journal of Machine Learning Research*, 18(1):3117–3166, 2017.

[31] Elad Hazan et al. Introduction to online convex optimization. *Foundations and Trends® in Optimization*, 2 (3-4):157–325, 2016.

[32] Shai Shalev-Shwartz, Ohad Shamir, Nathan Srebro, and Karthik Sridharan. Stochastic convex optimization. In *COLT*, volume 2, page 5, 2009.

[33] Huang Gabriel. reptile-pytorch. https://github.com/gabrielhuang/reptile-pytorch, 2018.

Checklist

1. For all authors...
   (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [Yes]
   (b) Did you describe the limitations of your work? [Yes]
   (c) Did you discuss any potential negative societal impacts of your work? [N/A] There are no foreseeable negative societal impacts for this work.
   (d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [Yes]
2. If you are including theoretical results...
   (a) Did you state the full set of assumptions of all theoretical results? [Yes]
   (b) Did you include complete proofs of all theoretical results? [Yes]
3. If you ran experiments...
   (a) Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? [Yes] Included pseudocode and synthetic datasets details.
   (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? [Yes]
   (c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? [Yes]
   (d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? [No]
4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...
   (a) If your work uses existing assets, did you cite the creators? [N/A]
   (b) Did you mention the license of the assets? [N/A] We use publicly available datasets.
   (c) Did you include any new assets either in the supplemental material or as a URL? [N/A]
   (d) Did you discuss whether and how consent was obtained from people whose data you're using/curating? [N/A]
   (e) Did you discuss whether the data you are using/curating contains personally identifiable information or offensive content? [N/A]
5. If you used crowdsourcing or conducted research with human subjects...
   (a) Did you include the full text of instructions given to participants and screenshots, if applicable? [N/A]
   (b) Did you describe any potential participant risks, with links to Institutional Review Board (IRB) approvals, if applicable? [N/A]
   (c) Did you include the estimated hourly wage paid to participants and the total amount spent on participant compensation? [N/A]

A Proof of results in Section 4

A.1 Auxiliary lemmas

Given a function f which is convex, L-Lipschitz and β-smooth. Define the update rule $G_{f,\eta}$ as

$$G_{f,\eta}(w) = w - \eta\nabla f(w) - \eta\Delta(w)$$

where $\Delta(w)$ is the error in estimating $\nabla f(w)$, we assume that $\|\Delta(w)\| \leq b$ for some $b > 0$. Next we show the expansiveness of $G_{f,\eta}$.

Lemma 7. *For any $\eta \leq 2/\beta$, we have*

$$\|G_{f,\eta}(w) - G_{f,\eta}(v)\| \leq \|w - v\| + 2\eta b$$

Proof.

$$\begin{aligned}
&\|G_{f,\eta}(w) - G_{f,\eta}(v)\| \\
&= \|w - \eta\nabla f(w) - \eta\Delta(w) - v + \eta\nabla f(v) + \eta\Delta(v)\| \\
&\leq \|w - \eta\nabla f(w) - v + \eta\nabla f(v)\| + \eta(\|\Delta(w)\| + \|\Delta(v)\|) \\
&\leq \|w - v\| + \eta(\|\Delta(w)\| + \|\Delta(v)\|) \\
&\leq \|w - v\| + 2\eta b
\end{aligned}$$

where the last equality holds because of the expansiveness of the conventional GD. □

Now we can present the stability result of $\mathcal{A}_{\text{meta-NSGD}}$.

Lemma 8. *(Stability of $\mathcal{A}_{\text{meta-NSGD}}$) In $\mathcal{A}_{\text{meta-NSGD}}$, suppose $\eta \leq 2/\beta$, where β is the smoothness parameter of $L_Z(h)$. Then $\mathcal{A}_{\text{meta-NSGD}}$ is α-uniformly stable with $\alpha = L^2\frac{T\eta}{K} + L\eta Tb$.*

Proof. Consider T steps of $\mathcal{A}_{\text{meta-NSGD}}$. Let $G_1, G_2 \ldots G_T$ be the noise vector, $\Delta_t = \frac{1}{m}\sum_{Z\in B_t}(\nabla L_Z(h_t) - \hat{\nabla} L_Z(h_t))$ for $t \in [T]$ and we have $\|\Delta_t\| \leq b$. We further denote the $\mathcal{I}_1, \ldots \mathcal{I}_T$ be the index sets of the mini-batch selected in T iterations. Consider any pair of datasets S and S' that differ on k_{th} element. Let $h_0, h_1, \ldots h_T$ and $h_0, h'_1, \ldots h'_T$ denote the trajectories of $\mathcal{A}_{\text{meta-NSGD}}$ corresponding to S and S' respectively. Let $\xi_t = h_t - h'_t$.

The proof mainly follows from Lemma 3.4 in [19]. For any iteration τ, we fix and the randomness of G_τ and $\mathcal{I}_\tau$. Denote r be the number of occurrences of index k in $\mathcal{I}_\tau$. By the expansiveness property of approximate gradient in Lemma 7, we have

$$\|\xi_{\tau+1}\| \leq \|\xi_\tau\| + 2L\eta\frac{r}{m} + 2\eta b$$

Then we can release the randomness of G_τ and $\mathcal{I}_\tau$, and note that r is a Binomial random variable with mean m/K, then

$$\mathbf{E}\left[\|\xi_{\tau+1}\|\right] \leq \mathbf{E}\left[\|\xi_\tau\|\right] + \frac{2L\eta}{K} + 2\eta b$$

Since $\xi_0 = 0$, then $\mathbf{E}\left[\|\xi_\tau\|\right] \leq \frac{2L\eta\tau}{K} + 2\eta\tau b$

Then since $L_Z(h)$ is L-Lipshtiz, thus for any Z, we have

$$\begin{aligned}
\mathbf{E}[L_Z(\bar{h}_T) - L_Z(\bar{h}'_T)] &\leq L\frac{1}{T}\sum_{t=1}^{T}\mathbf{E}[\|\xi_t\|] \\
&\leq L\frac{1}{T}\sum_{t=1}^{T}\frac{2L\eta t}{K} + 2\eta tb \\
&\leq L^2\frac{\eta(T+1)}{K} + L\eta(T+1)b
\end{aligned}$$

which completes the proof.

□

Algorithm 3: Approximate SGD

input : Initial iterate w_0, number of iterations T, parameter space $\mathcal{W}$

for $t = 1, \ldots, T$ **do**

  choose v_t such that $\mathbf{E}[v_t | w^{(t)}] = \nabla f(w^{(t)})$

  update $w^{(t+1)} = \text{Proj}_{\mathcal{W}}\left(w^{(t)} - \eta(v_t + G_t + \Delta_t)\right)$ where $G_t \sim \mathcal{N}(0, \sigma^2 \mathbb{I}_d)$

end

where the G_t is added Gaussian noise to preserve privacy and Δ_t is the error in estimating v_t.

output : $\bar{w} = \frac{1}{T}\sum_{t=1}^{T} w^{(t)}$

Next we show the convergence rate of SGD with approximate gradients. Consider an SGD algorithm on a differentiable convex function f,

Lemma 9. *Let $M, L, b > 0$. Let f be a convex function, $\mathcal{W} = \mathcal{B}(0, M)$ and $w^* \in \arg\min_{w \in \mathcal{W}} f(w)$. Assume that for all t, $\|v_t\| \le L$ and $\|\Delta_t\| \le b$ with probability 1. Then*

$$\mathbf{E}[f(\bar{w})] - f(w^*) \le \frac{M^2}{2\eta T} + \frac{\eta(L+b)^2}{2} + \eta\sigma^2 d + 2bM,$$

where the expectation is taken over the randomness in v_t, G_t and Δ_t.

Proof. Denote $\hat{v}_t = v_t + G_t + \Delta_t$, then by Lemma 14.1 in [29], we have

$$\begin{aligned}\mathop{\mathbf{E}}_{\hat{v}_{1:T}}\left[\frac{1}{T}\sum_{t=1}^{T}\langle w^{(t)} - w^*, \hat{v}_t\rangle\right] &\le \frac{\|w^*\|^2}{2T\eta} + \frac{\eta}{2T}\sum_{t=1}^{T}\mathbf{E}\left[\|\hat{v}_t\|^2\right] \\ &\le \frac{M^2}{2\eta T} + \frac{\eta(L+b)^2}{2} + \eta\sigma^2 d\end{aligned}$$

For any $t > 1$, we fix the randomness of $\hat{v}_{1:t-1}$. Since $w^{(t)}$ only depends on $\hat{v}_{1:t-1}$, then $w^{(t)}$ is fixed.

Therefore,

$$\begin{aligned}&\mathop{\mathbf{E}}_{\hat{v}_t}\left[\langle w^{(t)} - w^*, \hat{v}_t\rangle\right] = \langle w^{(t)} - w^*, \nabla f(w^{(t)}) + \mathbf{E}[\Delta_t]\rangle \\ &= \langle w^{(t)} - w^*, \nabla f(w^{(t)})\rangle + \langle w^{(t)} - w^*, \mathbf{E}[\Delta_t]\rangle \\ &\ge \langle w^{(t)} - w^*, \nabla f(w^{(t)})\rangle - 2bM \\ &\ge f(w^t) - f(w^*) - 2bM\end{aligned}$$

The second to the last step holds because $\left\|w^{(t)} - w^*\right\| \le 2M$ and $\|\Delta_t\| \le b$. The last step is due to the convexity of f.

By releasing the randomness of $\hat{v}_{1:t-1}$, summing over t and dividing T, we have

$$\begin{aligned}\mathop{\mathbf{E}}_{\hat{v}_{1:T}}\left[\frac{1}{T}\sum_{t=1}^{T}(f(w^{(t)}) - f(w^*))\right] &\le \mathop{\mathbf{E}}_{\hat{v}_{1:T}}\left[\frac{1}{T}\sum_{t=1}^{T}\langle w^{(t)} - w^*, \hat{v}_t\rangle\right] + 2bM \\ &\le \frac{M^2}{2\eta T} + \frac{\eta(L+b)^2}{2} + \eta\sigma^2 d + 2bM\end{aligned}$$

Combining with the fact that $\mathbf{E}_{\hat{v}_{1:T}}[f(\bar{w}) - f(w^*)] \le \mathbf{E}_{\hat{v}_{1:T}}\left[\frac{1}{T}\sum_{t=1}^{T}(f(w^{(t)}) - f(w^*))\right]$, we complete the proof. □

A.2 Proof of Theorem 4

Proof. Denote $\hat{\mathcal{E}}_n(h, \bar{Z})$ be the empirical counterpart of $\hat{\mathcal{E}}_n(h; \rho)$, which is written as

$$\hat{\mathcal{E}}_n(h, \bar{Z}) = \frac{1}{K}\sum_{Z \in \bar{Z}} L_Z(h)$$

Then we have

$$\mathbf{E}[\hat{\mathcal{E}}_n(\bar{h}_T;\rho)] - \min_{h\in\mathcal{W}} \hat{\mathcal{E}}_n(h;\rho) \le \mathbf{E}[\hat{\mathcal{E}}_n(\bar{h}_T,\bar{Z})] - \min_{h\in\mathcal{W}} \hat{\mathcal{E}}_n(h;\rho) + 4L^2\frac{\eta T}{K} + 2L\eta T b \tag{10}$$

$$\le E\left[\hat{\mathcal{E}}_n(\bar{h}_T,\bar{Z}) - \min_{h\in\mathcal{W}} \hat{\mathcal{E}}_n(h,\bar{Z})\right] + 4L^2\frac{\eta T}{K} + 2L\eta T b \tag{11}$$

$$\le \frac{M^2}{2\eta T} + \frac{\eta(2L+b)^2}{2} + \eta\sigma^2 d + 2bM + 4L^2\frac{\eta T}{K} + 2L\eta T b \tag{12}$$

where (10) is based on the stability result in Lemma 8, (11) follows from the fact that $\mathbf{E}\left[\min_{h\in\mathcal{W}} \hat{\mathcal{E}}_n(h,\bar{Z})\right] \le \min_{h\in\mathcal{W}} \mathbf{E}[\hat{\mathcal{E}}_n(h,\bar{Z})] = \min_{h\in\mathcal{W}} \hat{\mathcal{E}}_n(h;\rho)$. Finally, (12) is due to Lemma 9 on the convergence result of SGD with approximate gradient.

By plugging into the value of T, η and b, we obtain the stated bound. □

A.3 Proof of Theorem 5

Proof. The excess risk defined in equation (2) can be decomposed into three parts:

$$\mathbf{E}[\mathcal{E}_n(\bar{h}_T;\rho)] - \mathcal{E}_\rho = A + B + C$$

where

$$\begin{aligned} A &= \mathbf{E}\left[\mathcal{E}_n(\bar{h}_T;\rho) - \hat{\mathcal{E}}_n(\bar{h}_T;\rho)\right] \\ B &= \mathbf{E}\left[\hat{\mathcal{E}}_n(\bar{h}_T;\rho)\right] - \hat{\mathcal{E}}_n(\bar{w};\rho) \\ C &= \hat{\mathcal{E}}_n(\bar{w};\rho) - \mathcal{E}_\rho \end{aligned}$$

To bound A, we fix the randomness of the task specific distribution μ and training output h, then

$$\begin{aligned} &\mathbf{E}_{Z\sim\mu^n} R_\mu(w_h(Z)) - \mathbf{E}_{Z\sim\mu^n} L_Z(h) \\ &= \mathbf{E}_{Z\sim\mu^n} R_\mu(w_h(Z)) - \mathbf{E}_{Z\sim\mu^n}\left[R_Z(w_h) + \frac{\lambda}{2}\|w_h - h\|^2\right] \\ &\le \mathbf{E}_{Z\sim\mu^n}\left(R_\mu(w_h(Z)) - R_Z(w_h(Z))\right) \\ &\le \frac{2L^2}{\lambda n} \end{aligned}$$

The first inequality holds since the norm is always non-negative, and the second inequality holds because $w_h(Z)$ is a minimizer of a regularized ERM thus $\frac{2L^2}{\lambda n}$-replace one stable. Then we release the randomness of μ and h and obtain

$$A \le \frac{2L^2}{\lambda n}$$

To bound B, since $\bar{w} \in \mathcal{W}$, by Theorem 4, we have

$$\begin{aligned} B &= \mathbf{E}\left[\hat{\mathcal{E}}_n(\bar{h}_T)\right] - \hat{\mathcal{E}}_n(\bar{w}) \\ &\le \mathbf{E}\left[\hat{\mathcal{E}}_n(\bar{h}_T)\right] - \min_{h\in\mathcal{W}} \hat{\mathcal{E}}_n(h) \\ &= O\left(ML\cdot\left(\frac{1}{\sqrt{K}} + \frac{\sqrt{d\log(1/\delta)}}{\epsilon K}\right)\right) \end{aligned}$$

Finally to bound C, given the task μ, we let $w_\mu = \text{Proj}_{W_\mu}(\bar{w})$, then we have

$$
\begin{aligned}
&\hat{\mathcal{E}}_n(\bar{w}) - \mathcal{E}_\rho \\
&= \mathbf{E}_{\mu\sim\rho}\mathbf{E}_{Z\sim\mu^n}\left[L_Z(\bar{w}) - \mathbf{E}_{\mu\sim\rho}\mathbf{E}_{Z\sim\mu^n}R_\mu(w_\mu)\right] \\
&= \mathbf{E}_{\mu\sim\rho}\mathbf{E}_{Z\sim\mu^n}\left[L_Z(\bar{w}) - R_\mu(w_\mu) - \frac{\lambda}{2}\left\|w_\mu - \bar{w}\right\|^2 + \frac{\lambda}{2}\left\|w_\mu - \bar{w}\right\|^2\right] \\
&= \mathbf{E}_{\mu\sim\rho}\mathbf{E}_{Z\sim\mu^n}\left[L_Z(\bar{w}) - R_Z(w_\mu) - \frac{\lambda}{2}\left\|w_\mu - \bar{w}\right\|^2 + \frac{\lambda}{2}\left\|w_\mu - \bar{w}\right\|^2\right] \qquad (13) \\
&\leq \frac{\lambda}{2}\mathbf{E}_{\mu\sim\rho}\left\|w_\mu - \bar{w}\right\|^2 \qquad (14) \\
&= \frac{\lambda}{2}V^2
\end{aligned}
$$

where equation (13) is due to the independence of w_μ and Z conditioned on μ, therefore $\mathbf{E}_{Z\sim\mu^n}R_\mu(w_\mu) = \mathbf{E}_{Z\sim\mu^n}R_Z(w_\mu)$. Equation (14) holds because $L_Z(\bar{w})$ minimizes $R_Z(w) + \frac{\lambda}{2}\left\|w - \bar{w}\right\|^2$.

By combining everything together, we have

$$
\mathbf{E}\left[\mathcal{E}_n(w_{\bar{h}_T})\right] - \mathcal{E}_\rho \leq \frac{2L^2}{\lambda n} + O\left(ML\cdot\left(\frac{1}{\sqrt{K}} + \frac{\sqrt{d\log(1/\delta)}}{\epsilon K}\right)\right) + \frac{\lambda}{2}V^2
$$

Let $\lambda = \frac{2L}{V}\sqrt{\frac{1}{n}}$, the overall bound becomes

$$
\frac{2LV}{\sqrt{n}} + O\left(ML\cdot\left(\frac{1}{\sqrt{K}} + \frac{\sqrt{d\log(1/\delta)}}{\epsilon K}\right)\right)
$$

□

B Communication-efficient Meta-learning with DP-FTMRL

In this section, we present the DP-FTMRL algorithm and the guarantee it provides.

Algorithm 4: $\mathcal{A}_{\text{DP-FTMRL}}$: Differentially Private Follow-the-Meta-Regularized-Leader

input : Collection of datasets $\bar{Z} = [Z_1, \ldots Z_K]$ arriving in a stream, noise variance σ^2, regularization parameter λ_{meta}, approximation error b

$h_1 \leftarrow \arg\min_{h\in\mathcal{W}} \lambda_{\text{meta}}\left\|h\right\|^2$. **output :** h_1

Server do: $\mathcal{T} \leftarrow$ InitializeTree$(n, \sigma^2, 2L)$

for $t = 1, \ldots, K-1$ **do**

  Server sends h_t to task t.

  for *task t* **do**

    Solve $R_{Z_t,\lambda}(w; h_t)$ and give an estimate $\hat{w}_{h_t}(Z_t)$ such that $\|\hat{w}_{h_t}(Z_t) - w_{h_t}(Z_t)\| \leq \frac{b}{\lambda}$

    Send $\hat{\nabla}L_{Z_t}(h_t) = -\lambda(\hat{w}_{h_t}(Z_t) - h_t)$ back to server

  end

  Server adds $\hat{\nabla}L_{Z_t}(h_t)$ to $\mathcal{T}$: $\mathcal{T} \leftarrow$ AddToTree$(\mathcal{T}, t, \hat{\nabla}L_{Z_t}(h_t))$

  Server aggregates the gradients: $s_t \leftarrow$ GetSum$(\mathcal{T}, t)$

  Server computes $h_{t+1} = \arg\min_{h\in\mathcal{W}}\langle s_t, h\rangle + \frac{\lambda_m}{2}\left\|h\right\|^2$ **output :** h_{t+1}

end

output : $\bar{h}_K = \frac{1}{K}\sum_{t=1}^K h_t$

where the functions InitializeTree, AddToTree and GetSum are associated with the tree aggregation protocol. Roughly speaking, InitializeTree initializes the tree structure $\mathcal{T}$ and AddtoTree adds the gradient $\hat{L}_{Z_t}(h_t)$ in to $\mathcal{T}$. GetSum returns the sum of $\{\nabla\hat{L}_{Z_i}(h_i)\}_{i=1}^t$ privately, and the returned value can be expressed as $\sum_{i=1}^t \nabla\hat{L}_{Z_i}(h_i) + b_t$ where g_t is a zero mean Gaussian random variable. More details of these functions can be found in Appendix B of [23].

Theorem 10. *(privacy guarantee) Let $\epsilon \leq 2\ln(1/\delta)$. Under Assumption 1, let $[h_1, \dots h_K]$ and $\bar{h}_K$ be the outputs of Algorithm 4. By setting $\sigma = \frac{2\lceil \log(n+1)\rceil \ln(1/\delta)}{\epsilon}$ and $b = \min\left\{L, \frac{L\sigma\sqrt{d\lceil \log(n+1)\rceil \ln(n)}}{K}\right\}$, Algorithm 4 guarantees (ϵ, δ) task-level differential privacy.*

Note that since $b \leq L$, then we have $\left\|\hat{\nabla} L_{Z_t}(h_t)\right\| \leq \|\nabla L_{Z_t}(h_t)\| + b \leq 2L$. Thus, the proof of the privacy guarantee follows similar lines to that of [23, Theorem 4.1].

We measure the performance of $\mathcal{A}_{\text{DP-FTMRL}}$ by regrets against any $w^* \in \mathcal{W}$ defined as

$$R_{\bar{Z}}(\mathcal{A}_{\text{DP-FTMRL}}; w^*) = \frac{1}{K}\sum_{t=1}^{K} L_{Z_t}(h_t) - \frac{1}{K}\sum_{t=1}^{K} L_{Z_t}(w^*)$$

We rely on the following lemma to provide a regret guarantee of DP-FTMRL, which is stated as follows

Lemma 11. *([30, Lemma 7]) Let $\phi_1 : \mathcal{C} \to \mathbb{R}$ be a convex function s.t. $\theta_1 \in \arg\min_{\theta\in\mathcal{C}} \phi_1(\theta)$ exists. Let $\Psi(\theta)$ be a convex function s.t. $\phi_2(\theta) = \phi_1(\theta) + \Psi(\theta)$ is 1-strongly convex w.r.t. $\|\cdot\|$-norm. Let $\theta_2 \in \arg\min_{\theta\in\mathcal{C}} \phi_2(\theta)$. Then for any b in the subgradient of Ψ at θ_1, the following is true: $\|\theta_1 - \theta_2\|_* \leq \|b\|_*$. Here $\|\cdot\|_*$ is the dual norm of $\|\cdot\|$.*

Theorem 12. *(Regret guarantee) Recall the settings in Theorem 10. Let $\{h_1, \dots h_K\}$ be the outputs of $\mathcal{A}_{\text{DP-FTMRL}}$. Then, for any $w^* \in \mathcal{W}$, w.p. at least $1-\beta$ over the algorithm's randomness, we have*

$$R(\mathcal{A}_{\text{DP-FTMRL}}; w^*) = O\left(LM \cdot \left(\frac{1}{\sqrt{K}} + \sqrt{\frac{d^{1/2}\ln^2(1/\delta)\ln(1/\beta)}{\epsilon n}}\right)\right)$$

The proof follows similar lines to the proof of [23, Theorem 5.1] with a slight modification, where we take into account the effect of the gradient approximation error. The proof is provided here for completeness.

Proof. By the end of iteration t, we have

$$\begin{aligned}
h_{t+1} &= \arg\min_{h\in\mathcal{W}} \langle s_t, h\rangle + \frac{\lambda_m}{2}\|h\|^2 + \langle b_t, h\rangle \\
&= \arg\min_{h\in\mathcal{W}} \sum_{i=1}^{t} \langle \hat{\nabla} L_{Z_i}(h_i), h\rangle + \frac{\lambda_m}{2}\|h\|^2 + \langle b_t, h\rangle \\
&= \arg\min_{h\in\mathcal{W}} \sum_{i=1}^{t} \langle \nabla L_{Z_i}(h_i) + \Delta_i, h\rangle + \frac{\lambda_m}{2}\|h\|^2 + \langle b_t, h\rangle \\
&= \arg\min_{h\in\mathcal{W}} \sum_{i=1}^{t} \langle \nabla L_{Z_i}(h_i), h\rangle + \frac{\lambda_m}{2}\|h\|^2 + \langle b_t + \sum_{i=1}^{t}\Delta_i, h\rangle
\end{aligned}$$

where Δ_i is the approximation error for $\hat{\nabla} L_{Z_i}(h_i)$. Denote $e_t = b_t + \sum_{i=1}^{t}\Delta_i$, since we have $\|\Delta_i\| \leq b \leq \frac{L\sigma\sqrt{d\lceil \log(n+1)\rceil \ln(n)}}{K}$ and $\|b_t\| \leq L\sigma\sqrt{d\lceil \log(n+1)\rceil \ln(n/\beta)}$ with probability $1-\beta$. Therefore, with probability over $1-\beta$, the norm of n_t is bounded by

$$\|e_t\| \leq \sum_{i=1}^{t}\|\Delta_i\| + \|b_t\| = O\left(L\sigma\sqrt{d\lceil \log(n+1)\rceil \ln(n/\beta)}\right)$$

Therefore, by denoting the non-private objective as $\tilde{h}_t = \arg\min_{h\in\mathcal{W}} \sum_{i=1}^{t}\langle \nabla L_{Z_i}(h_i), h\rangle + \frac{\lambda_m}{2}\|h\|^2$. By Lemma 11, we can bound $\left\|\tilde{h}_{t+1} - h_{t+1}\right\|$ by

$$\left\|\tilde{h}_{t+1} - h_{t+1}\right\| \leq \frac{\|e_t\|}{\lambda}$$

Finally, we can bound the regret as

$$
\begin{aligned}
R_{\bar{Z}}(\mathcal{A}_{\text{DP-FTMRL}}; w^*) &= \frac{1}{K}\sum_{t=1}^{K} L_{Z_t}(h_t) - \frac{1}{K}\sum_{t=1}^{K} L_{Z_t}(w^*) \\
&\leq \frac{1}{K}\sum_{t=1}^{K}\langle \nabla L_{Z_t}(h_t), h_t - h^* \rangle \\
&= \frac{1}{K}\sum_{t=1}^{K}\langle \nabla L_{Z_t}(h_t), h_t - \tilde{h}_t + \tilde{h}_t - h^* \rangle \\
&= \frac{1}{K}\sum_{t=1}^{K}\langle \nabla L_{Z_t}(h_t), \tilde{h}_t - h^* \rangle + \frac{1}{K}\sum_{t=1}^{K}\langle \nabla L_{Z_t}(h_t), h_t - \tilde{h}_t \rangle \qquad (15)
\end{aligned}
$$

The first term of (15) can be bounded by $\left(\frac{L^2}{\lambda} + \frac{\lambda}{2n}\left(\|h^*\|^2 - \|h_1\|^2\right)\right)$ [[31], Theorem 5.2]. The second term can be bounded by the concentration bound of $\|e_t\|$, such that

$$
\frac{1}{K}\sum_{t=1}^{K}\langle \nabla L_{Z_t}(h_t), h_t - \tilde{h}_t \rangle \leq \frac{1}{K}\sum_{t=1}^{K} L\left\|\tilde{h}_{t+1} - h_{t+1}\right\| \leq O\left(\frac{L\sigma\sqrt{d\lceil \log(n+1)\rceil \ln(n/\beta)}}{\lambda}\right)
$$

By setting λ optimally and plugging into the value of σ, we complete the proof. □

The utility guarantee of $\mathcal{A}_{\text{DP-FTMRL}}$ is therefore derived via online-to-batch conversion [32].

Theorem 13. *(Utility guarantee) Recall the settings in Theorem 10. With probability $1-\beta$ over the algorithm's randomness, we have*

$$
\mathbf{E}[\hat{\mathcal{E}}(\bar{h}_K;\rho)] - \min_{h\in\mathcal{W}} \hat{\mathcal{E}}(h;\rho) \leq O\left(LM \cdot \left(\frac{1}{\sqrt{K}} + \sqrt{\frac{d^{1/2}\ln^2(1/\delta)\ln(1/\beta)}{\epsilon n}}\right)\right)
$$

Finally we can obtain the transfer risk in a similar way as in Section 4, which is stated as follows:

Theorem 14. *(Transfer risk) Recall the settings in Theorem 10. We have*

$$
\mathbf{E}[\mathcal{E}_n(\bar{h}_K;\rho)] - \mathcal{E}_\rho \leq \frac{2L^2}{\lambda n} + \frac{\lambda}{2}V^2 + O\left(LM \cdot \left(\frac{1}{\sqrt{K}} + \sqrt{\frac{d^{1/2}\ln^2(1/\delta)\ln(1/\beta)}{\epsilon n}}\right)\right)
$$

By letting $\lambda = \frac{2L}{V}\sqrt{\frac{1}{n}}$, we have

$$
\mathbf{E}[\mathcal{E}_n(\bar{h}_K;\rho)] - \mathcal{E}_\rho \leq \frac{2LV}{\sqrt{n}} + O\left(LM \cdot \left(\frac{1}{\sqrt{K}} + \sqrt{\frac{d^{1/2}\ln^2(1/\delta)\ln(1/\beta)}{\epsilon n}}\right)\right)
$$

where $\bar{h}_K$ is the output of $\mathcal{A}_{\text{DP-FTMRL}}$ and the expectation is taken w.r.t. $\bar{Z}$ and the inner randomness of the algorithm.

C Additional Experiments

We evaluate our single- and multi-cluster meta learning algorithms on the Omniglot [28] benchmark dataset for few-shot classification tasks. Omniglot contains 1623 characters (i.e., 1623 classes) and each character contains 20 (28×28) black and white images. This benchmark dataset is split into training and testing data as follows: the training dataset contains all data samples from 1461 characters while the testing dataset contains the data points of the remaining 162 characters. The training dataset is used in the meta training to obtain the meta models, and the testing dataset is used to evaluate the accuracy of the base learner (for a given task) with respect to the learned meta models.

In our experiments, we set the total number of tasks (users) $K = 50000$. For any given task, the goal is to perform an M-shot N-way classification. In an M-shot N-way classification, we sample N

classes (N characters) uniformly from the testing classes (i.e., out of 162 characters) and subsequently sample $M+1$ data points from each class. Among these $M+1$ data points, M of them are used for training and the remaining one is for testing. We use the same network structure as in [1, 2, 4] with 4 convolutional modules. The code is partly based on the public repo in [33].

We focus on the 5-shot 5-way classification setting. We use SGD as the base learner for the single cluster algorithm and Adam as the base learner for the multi-cluster algorithm. We follow similar hyperparameter settings for the step size and the number of iterations for the base learner as in [2]. In the meta training phase of our algorithms, we sample $K = 50000$ few-shot classifications tasks as our training task datasets. We set the number of iterations for the meta training as $T = 20000$ and the training shots (the number of training data points per class) as 10. We set the regularization parameter $\lambda = 0.1$, the clip norm $\hat{L} = 0.5$, the batch size $= 25$, the privacy parameter $\epsilon \in \{3, 10, \infty\}$, where $\epsilon = \infty$ refers to the non-private setting, and the number of clusters $q \in \{1, 2\}$. That is, we report the accuracy resulting from different settings of the privacy parameter (corresponding to different values of the noise variance) in both the single-cluster ($q = 1$) and the 2-cluster ($q = 2$) cases. We report results with 95% confidence interval.

Table 1: Accuracy results on Omniglot few-shot classification task, q is the number of meta models and ϵ is the privacy parameters

(q, ϵ)	5-shot 5-way
$(1, 3)$	$75 \pm 1.7\%$
$(1, 10)$	$85.4 \pm 1.4\%$
$(1, \infty)$	$94.2 \pm 0.9\%$
$(2, 3)$	$78.5 \pm 2.1\%$
$(2, 10)$	$79.3 \pm 1.8\%$
$(2, \infty)$	$85.4 \pm 2\%$

Compared to the non-private baseline ($\epsilon = \infty$), our single-cluster private algorithm has about 8.8% accuracy drop for $\epsilon = 10$. This accuracy loss increases to 19.2% with higher privacy ($\epsilon = 3$). Similar accuracy gaps of 6.1% for $\epsilon = 10$ and 6.9% for $\epsilon = 3$ are also reported in the multi-cluster algorithm. We note that the multi-cluster algorithm does not lead to any noticeable improvement in accuracy compared to the single-cluster algorithm. One explanation for that is that the underlying task distribution of the Omniglot benchmark seems to be well represented by a single meta model, and hence, using the clustering-based algorithm does not necessarily yield an improved performance (let alone the fact that the algorithm will also have a slower convergence to a good solution). This is indeed what we observe in our experiments for the 2-cluster setting. In particular, in these experiments, we observed that almost all the users eventually fall into one cluster as the algorithm converges. For example, in the setting where $q = 2$ and $\epsilon = 3$, our results showed that 97.7% of the sampled users (tasks) eventually chose the same meta model to update.

In [8], the authors report accuracy around 65% for the 5-shot 5-way Omniglot few-shot classification with $\epsilon = 9.5$ and 10^6 sampled tasks. Our algorithm attains a noticeable improvement in accuracy (85.4%) with comparable privacy guarantee ($\epsilon = 10$) and only 5×10^4 sampled tasks. Even with stronger privacy guarantee ($\epsilon = 3$), our algorithms still achieve better performance (75% for single-cluster training and 78.5% for 2-cluster training). Moreover, we provide task-level differential privacy guarantee while [8] provides record-level differential privacy.