SUBJECTIVE LEARNING FOR CONFLICTING DATA

Anonymous authors
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ABSTRACT

Conventional supervised learning typically assumes that the learning task can be solved by approximating a single target function. However, this assumption is often invalid in open-ended environments where no manual task-level data partitioning is available. In this paper, we investigate a more general setting where training data is sampled from multiple domains while the data in each domain conforms to a domain-specific target function. When different domains possess distinct target functions, training data exhibits inherent "conflict", thus rendering single-model training problematic. To address this issue, we propose a framework termed subjective learning where the key component is a subjective function that automatically allocates the data among multiple candidate models to resolve the conflict in multi-domain data, and draw an intriguing connection between subjective learning and a variant of Expectation-Maximization. We present theoretical analysis on the learnability and the generalization error of our approach, and empirically show its efficacy and potential applications in a range of regression and classification tasks with synthetic data.

1 INTRODUCTION

Conventional supervised learning typically assumes that the learning task can be solved by approximating a single ground-truth target function (Vapnik, 1999). However, this assumption is often violated in open-ended environments where the data may implicitly belong to multiple, disparate domains with potentially different target functions when no manual task-level data partitioning is available. For instance, when curating a dataset using web images, an image of a red sphere may be labeled as both "red" and "sphere", implicitly representing two distinct domains that respectively corresponds to two metaconcepts (Han et al., 2019): "color" and "shape". Also, in some practical scenarios such as federated learning (McMahan et al., 2017) and algorithmic fairness (Mitchell et al., 2021), training data is usually collected from multiple sources (e.g., clients or populations) with concept shift (Kairouz et al., 2021), thus exhibiting different target functions (more generally, different input-conditional label distributions) due to personal preferences or other latent factors.

When different subsets of training data conform to different target functions, it is not hard to see that training a single model with standard Empirical Risk Minimization (ERM) is problematic due to the inherent "conflict" in data: in the above example, single-model training provably leads to the unfavorable result of "50% red, 50% sphere" (assuming the data is balanced). Similar phenomena have also been demonstrated by prior works (Finn et al., 2019; Su et al., 2020) in the context of regression, where a learner simultaneously regressing from multiple target functions trivially outputs their mean. This indicates that conflicting data exhibits a structural difference compared with conventional supervised data. In the sequel, we introduce a novel, dataset-level measure termed *mapping rank* to explicitly formalize such difference:

Definition 1 (Mapping rank)**.** *Let $\mathcal{X}$ be an input space, $\mathcal{Y}$ an output space, and $Z = \{(x_i, y_i)\}_{i=1}^{l}$ a dataset with cardinality l. Let $F(r) = \{f_i\}_{i=1}^{r}$ be a function set with cardinality r, where each element is a deterministic function from $\mathcal{X}$ to $\mathcal{Y}$. Then, the mapping rank of Z, denoted by R or $R(Z)$, is defined as the minimal positive integer r satisfying that there exists a function set $F(r)$ such that for every $(x, y) \in Z$, there exists $f \in F(r)$ with $f(x) = y$.*

Note that we assume that the relation between inputs and outputs in the same domain is deterministic, and we will further discuss this assumption in Section **??**. Under Definition 1, conventional supervised data yields a mapping rank $R = 1$ as it assumes that the whole dataset can be characterized by a single target function. In contrast, conflicting data has a mapping rank $R > 1$ since for the same input different outputs exist. Hence, it is natural to consider allocating the data to multiple models,

so that the data processed by each model has a mapping rank $R = 1$ and thus can be handled with ERM. Although in some scenarios, apart from data samples there also exists side-information or metadata that can be exploited to identify the domains, this information may be difficult to define or collect in practice (Hanna et al., 2020; Creager et al., 2021); even when such side-information is available, in many cases it still remains unclear how to leverage such information to detect and resolve the potential conflict between domains. Therefore, the problem of how to allocate conflicting data properly and automatically without additional human intervention is highly non-trivial.

To tackle the aforementioned challenge, we present a *subjective learning* framework to enable effective learning from conflicting data. Concretely, our framework maintains a set of low-level models and a high-level *subjective function* that automatically allocates the data among these models so that the data processed by each model exhibits no conflict. Here we use the term "subjective" because in conventional supervised learning such allocation is manually performed during the data cleaning process and thus complies with human subjectivity. The high-level motivation of our method is that if the subjective function yields an inappropriate allocation, i.e., assigning conflicting data to the same model, then it will hinder the global minimization of the training error due to the conflict, which itself may be harnessed to update the allocation strategy of the subjective function. Using a probablistic reinterpretation of our framework, we establish the connection between subjective learning and a variant of the Expectation-Maximization (EM) algorithm (Dempster et al., 1977), and show that the form of the subjective function can be explicitly derived.

Theoretically, we respectively analyse the Probably Approximately Correct (PAC) learnability (Valiant, 1984) and the generalization error of subjective learning using the tools from statistical learning theory (Vapnik, 2013). We show that the relation between the number of low-level models and the mapping rank of data plays a key role in the learnability of subjective learning, and the generalization error of our method can be decomposed into terms that respectively reflect high-level data allocation and low-level prediction errors. Empirically, we conduct extensive experiments that span regression and classification tasks with synthetic data. Experimental results validate our theoretical claims, demonstrate the efficacy of subjective learning, and showcase its potential applicability in several different scenarios.

2 Subjective Learning Framework

In this section, we present the overall formulation and the algorithm of subjective learning. We adhere to the conventional terminology in supervised learning, and let $\mathcal{X}$ be an input space, $\mathcal{Y}$ an output space, $\mathcal{H}$ a hypothesis space where each hypothesis (model) is a function from $\mathcal{X}$ to $\mathcal{Y}$, and $\ell : \mathcal{Y} \times \mathcal{Y} \to [0, 1]$ a non-negative and bounded loss function without loss of generality. We use $[k] = \{1, 2, \cdots, k\}$ for positive integers k, and denote by $\mathbb{1}(\cdot)$ the indicator function. We use superscripts to denote sampling indices (e.g., d^i and x^{ij}) and subscripts as element indices (e.g., d_i).

2.1 Problem Statement

We begin by introducing the notion of *domain* to formulate the sampling process of conflicting data that is considered throughout the paper. Inspired by the seminal work in domain adpatation (Ben-David et al., 2010), we define a domain d as a pair $\langle P, c\rangle$ consisting of a distribution P on $\mathcal{X}$ and a target function $c : \mathcal{X} \to \mathcal{Y}$, and assume that the data is generated from a *domain set* $D = \{d_i\}_{i=1}^N = \{\langle P_i, c_i\rangle\}_{i=1}^N$ containing N (agnostic to the learner) domains. Each domain has its own sub-dataset $Z_i = \{(x_{ij}, y_{ij})\}_{j=1}^{l_i}$ with cardinality l_i, where $x_{i1}, \cdots, x_{il_i}$ are i.i.d. drawn from P_i and $y_{ij} = c_i(x_{ij})$. The whole dataset is the union of these sub-datasets: $Z = \bigcup_{i=1}^N Z_i$ with cardinality $l = \sum_{i=1}^N l_i$ and mapping rank $1 < R \leq N$. We consider a bilevel sampling procedure: first, m domain samples are i.i.d. drawn from a distribution Q defined on D with replacement (thus the same domain may be sampled multiple times), resulting in m sampling *episodes*; second, in each sampling episode n data samples are i.i.d. drawn from the sub-dataset corresponding to the sampled domain. This sampling regime is analagous to the bilevel sampling process adopted by meta-learning (Pentina & Lampert, 2014; Amit & Meir, 2018). However, meta-learning usually assumes a dense distribution of related domains to enable task-level generalization, while our setting here is compatible with scarce and disparate domains and inter-domain transfer is orthogonal.

As we have mentioned in Section 1, single-model training is insufficient when $R > 1$. Thus, we equip the learner with a *hypothesis set* $H = \{h_i\}_{i=1}^K$ consisting of $K > 1$ hypotheses, enhancing its

expressive capability. Although both N and K are assumed unknown, we will show that in general $K \geq R$ suffices (see Section 3.1), which eases the difficulty of setting the hyperparameter K.

In the above setting, we introduce an episodic sample number parameter n, implicitly assuming that we are able to sample a size-n data batch at a time from each domain. While this formulation subsumes the fully online case of $n = 1$, we note that although sometimes $n = 1$ works in practice, it also tends to be risky since it may raise difficulties in controlling the generalization error, as we will both theoretically and empirically demonstrate in the following sections (see Sections 3.2 and 4.3).

2.2 Global Error

In this section, we present the global learning objective of subjective learning. Since conflicting data implicitly contains multiple input-output mappings, a primary start point can be the empirical multi-task loss with pre-defined data-domain correspondences:

$$\widehat{er}_{\mathrm{MTL}}(H) = \frac{1}{m}\sum_{i=1}^{m}\frac{1}{n}\sum_{j=1}^{n}\ell\left[h_{\textsc{Oracle}(i)}\left(x^{ij}\right), y^{ij}\right], \tag{1}$$

where $\textsc{Oracle} : [m] \to [K]$ is an oracle mapping function that determines which hypothesis the data batch $Z^i := \{(x^{ij}, y^{ij})\}_{j=1}^{n}$ in each episode should be assigned to. However, in subjective learning the oracle mapping function is unavailable, imposing a fundamental discrepancy. To tackle this difficulty, here we substitute the oracle mapping function with a learnable *empirical subjective function* $\hat{g} : \mathcal{H}^K \times \mathcal{X}^n \times \mathcal{Y}^n \to H$ that aims to select a hypothesis h from the hypothesis set H for the data batch Z^i. This substitution yields the empirical global error of subjective learning:

$$\widehat{er}(H) := \frac{1}{m}\sum_{i=1}^{m}\frac{1}{n}\sum_{j=1}^{n}\ell\left[\hat{g}\left(H, Z^i\right)\left(x^{ij}\right), y^{ij}\right]. \tag{2}$$

Our insight is that the data batch itself can be harnessed to guide its suitable allocation in the presence of conflict: intuitively, a single model trained by conflicting data batches results in an inevitable global training error. Therefore, minimizing the global error can in turn facilitates data allocation with less conflict. Given the empirical error 2, we can give its expected counterpart:

$$er(H) := \mathbb{E}_{d_i \sim Q}\mathbb{E}_{x \sim P_i}\ell\left[g(H, d_i)(x), c_i(x)\right], \tag{3}$$

where $g : \mathcal{H}^K \times D \to H$ is an *expected subjective function*, which can be viewed as the empirical subjective function with infinite data from every single domain so that all available domain information can be fully reflected by the sampled data. So far, our framework remains incomplete since the exact form of the subjective function is still undefined. In the next section, we will present our design of the subjective function and elucidate its rationale.

2.3 Derivation of Subjective Function

To attain a reasonable choice of the subjective function, in this section we provide an alternative, probabilistic reinterpretation of subjective learning from the angle of maximum conditional likelihood, and draw an intriguing connection between the choice of the subjective function and the posterior maximization in a variant of the EM algorithm (Dempster et al., 1977) on conflicting data. Concretely, let $p(Y \mid X)$ represents the predictive conditional distribution of the hypothesis set H, where we use X and Y as the shorthand for $(\mathbf{x}^{11}, \mathbf{x}^{12}, \cdots, \mathbf{x}^{mn})$ and $(\mathbf{y}^{11}, \mathbf{y}^{12}, \cdots, \mathbf{y}^{mn})$ respectively. We consider maximizing the empirical log-likelihood $\log p(Y \mid X) = \sum_{i=1}^{m}\sum_{j=1}^{n}\log\sum_{k=1}^{K} p(\mathbf{y}^{ij}, h = h_k \mid \mathbf{x}^{ij})$ using EM, where h denotes the selected hypothesis. Accordingly, in the i-th sampling episode, in the E-step we aim to estimate the model posterior $P(h = h_k \mid Z^i)$ that represents the responsibility of the k-th hypothesis in the hypothesis set w.r.t. the local data batch $Z^i = \{(\mathbf{x}^{ij}, \mathbf{y}^{ij})\}_{j=1}^{n}$, while in the M-step we seek to maximize

$$\mathcal{L}(H) = \sum_{k=1}^{K} P\left(h = h_k \mid Z^i\right)\sum_{j=1}^{n}\log\left[\pi_k p\left(\mathbf{y}^{ij} \mid \mathbf{x}^{ij}, h_k\right)\right], \tag{4}$$

where $\pi_k := P(h = h_k) > 0$ denotes the prior of the k-th hypothesis in H. This draws a direct connection between subjective learning and EM: the E-step corresponds to the functionality of the

subjective function that chooses a hypothesis for a given data batch; the M-step corresponds to updating the selected hypothesis by minimizing the empirical prediction error in the episode. The main difference is that here we assume the subjective function to be *deterministic*, representing a "hard" assignment of the data to the model. This entails the usage of a variant known as hard EM (Samdani et al., 2012), which considers the posterior to be a Dirac delta function. Applying this constraint, the E-step of EM yields $h = \arg\max_{h_k \in H} \sum_{j=1}^{n} \log p(\mathbf{y}^{ij} \mid \mathbf{x}^{ij}, h_k)$ under a uniform prior, motivating a principled choice of the subjective function:

$$\hat{g}\left(H, Z^i\right) = \arg\min_{h \in H} \sum_{j=1}^{n} \ell\left[h(x^{ij}), y^{ij}\right], i \in [m], \tag{5a}$$

$$g(H, d_i) = \arg\min_{h \in H} \mathbb{E}_{x \sim P_i} \ell\left[h(x), y\right], i \in [N], \tag{5b}$$

which can be interpreted as selecting the hypothesis that incurs the smallest (empirical or expected) error. While this connection is not rigorous in general, in some cases exact equivalence can be derived when certain types of loss functions and likelihood families are applied, which encompasses common regression and classification settings. We provide a concrete analysis in Appendix A.

2.4 Overall Algorithm

We assume that the hypothesis set H comprises K parameterized hypotheses with parameter vectors $\Theta = (\theta_1, \theta_2, \cdots, \theta_K)$ respectively. In the high level, with the choice of the empirical subjective function 5a, our algorithm consists of two phases in each sampling episode: (i) evaluating the error of each hypothesis in H w.r.t. the data in this episode, and (ii) training the hypothesis with the smallest error. For brevity, we introduce a notion of empirical episodic error defined as

$$\widehat{er}^i\left(h; \theta\right) := \frac{1}{n} \sum_{j=1}^{n} \ell\left[h\left(x^{ij}; \theta^i\right), y^{ij}\right], \; i \in [m], \tag{6}$$

where $h \in \mathcal{H}$ is a hypothesis parameterized by θ. Then, phase (i) aims to find a hypothesis that mininize 6. Note that this selection process may induce a bias between empirical and expected objectives 2 and 3, since a hypothesis that minimizes the empirical loss on finite samples may not minimize the expected loss of this domain. Hence, the global error of subjective learning can be intuitively decomposed into a high-level subjective error that measures the reliability of this selection process, and a low-level model error that measures the accuracy of models, on which we provide detailed theoretical analysis in Section 3.2. In practice, we parameterize each hypothesis in the hypothesis set with a deep neural network (DNN), and apply stochastic gradient descent (SGD) for the optimization process. We provide the pseudo-code of subjective learning in Appendix B.

3 Theoretical Analysis

In this section, we analyze the learnability and the generalization error of subjective learning.

3.1 Learnability

We first analyze the learnability of subjective learning based on PAC learnability (Valiant, 1984). Since our analysis directly applys to conventional supervised learning by setting $K = 1$, we also verify the conflict phenomenon mentioned in Section 1 from a theoretical perspective. While the learnability in conventional PAC analysis mainly relates to the choice of the hypothesis space, conflicting data imposes a new source of complexity by its mapping rank, and we expect the cardinality of the proposed hypothesis set can compensate this complexity. We consider the realizable case where the hypothesis space covers the target functions in all domains, which helps to underline the core characteristic of our problem. We begin by a result on the form of the optimal solution of subjective learning. The proofs of all theoretical results are deferred to Appendix C.

Proposition 1 (Form of the optimal solution)**.** *Assume that the target functions in all domains are realizable. Then, the following two propositions are equivalent:*

(1) For all domain distributions Q and data distributions $P_1, P_2, \cdots, P_N$, $er(H) = 0$.

(2) For each domain $d = \langle P, c \rangle$ in D, there exists $h \in H$ such that $\mathbb{E}_{x \sim P} \ell\left[h(x), c(x)\right] = 0$.

Proposition 1 suggests that minimizing the expected global error 3 with 5b elicits a global optimal solution where every target function is learned accurately. Note that this does not require N hypotheses for N domains, since non-conflict domains can be incorporated into the same model. In other words, what determines the minimal cardinality of the hypothesis set is not the number of the domains, but the number of *conflicting* domains, which can be exactly characterized by the mapping rank. Formally, we attain a necessary condition of the PAC learnability of subjective learning.

Theorem 1. *A necessary condition of the PAC learnability of subjective learning is $K \geq R$.*

Theorem 1 indicates that the cardinality of the hypothesis set should be large enough to enable effective learning, and shows the impact of mapping rank on the learnability of subjective learning.

Remark 1. While it is generally hard to derive a necessary and sufficient condition of PAC learnability theoretically (which requires a sample-efficient optimization algorithm), we empirically find that $K \geq R$ is indeed an essential condition for learnability with complex hypothesis spaces such as parameterized DNNs. We also note that several recent works (Allen-Zhu et al., 2019; Du et al., 2019) have proved that over-parameterized neural networks trained by SGD can achieve zero training error in polynomial time under non-convexity, which may also be used to enhance our analysis. We leave a more rigorous study for future work.

3.2 Generalization Error

We have shown that minimizing the expected global error is sufficient for effective learning from conflicting data. However, in practice, since we only have access to the empirical global error, how to control the discrepancy between these two errors, i.e., the generalization error, remains crucial. In this section, we identify the terms in the generalization error that respectively correspond to the high-level subjective error and the low-level prediction error of subjective learning, and discuss their controlling strategies. The key results are (i) the number of episodes and episodic samples can compensate each other in controlling the low-level prediction error, and (ii) the number of episodic samples is critical for controlling the high-level subjective error. We have the following theorem:

Theorem 2 (Generalization error upper bound)**.** *For any $\delta \in (0, 1]$, the following inequality holds uniformly for all hypothesis sets $H \in \mathcal{H}^K$ with probability at least $1 - \delta$:*

$$er(H) \leq \widehat{er}(H) + \sqrt{\frac{\mathsf{VC}(\bar{\mathcal{S}})\left(\ln {}^{2m}/_{\mathsf{VC}(\bar{\mathcal{S}})} + 1\right) - \ln {}^{\delta}/_{12}}{m}} + \frac{1}{m} \tag{7a}$$

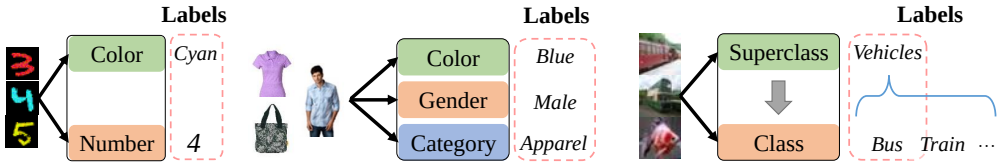
$$+ \sum_{k=1}^{N} \left(\frac{m_k}{m} \sqrt{\frac{\mathsf{VC}(\mathcal{S})\left(\ln {}^{2m_k n}/_{\mathsf{VC}(\mathcal{S})} + 1\right) - \ln {}^{\delta}/_{12N}}{m_k n}} + \frac{1}{mn} \right) \tag{7b}$$

$$+ 2\sqrt{\frac{\mathsf{VC}(\mathcal{S})\left(\ln {}^{2n}/_{\mathsf{VC}(\mathcal{S})} + 1\right) - \ln {}^{\delta}/_{24m}}{n}} + \frac{2}{n}, \tag{7c}$$

where $\bar{\mathcal{S}} := \{\langle P, c\rangle \mapsto \mathbb{E}_{x\sim P}\ell\left[h(x;\theta), c(x)\right]\}, \theta \in \Theta$ is the function set of the domain-wise expected error, $\mathcal{S} := \{(x, y) \mapsto \ell\left[h(x;\theta), y\right]\}, \theta \in \Theta$ is the function set of the sample-wise error, $m_k := \sum_{i=1}^{m} \mathbb{1}\left(c^i = c_k\right)$ is the sampling count of the target function from the k-th domain $d_k (k \in [N])$, and $\mathsf{VC}(\cdot)$ the Vapnik-Chervonenkis (VC) dimension (Vapnik & Chervonenkis, 1971).

Theorem 2 indicates that the expected global error is bounded by the empirical global error plus three terms. The *subjective estimation error* term 7c is derived by bounding the discrepancy between the empirical and the expected subjective functions due to the limitation of finite episodic samples. This term can be controlled by sample-level complexity $\mathsf{VC}(\mathcal{S})$ and the number of episodic samples n. Although in theory this error term converges to zero only if $n \to \infty$, in practice we find that usually a very small n (e.g., $n = 2$) suffices (see Section 4). We posit that this is because the domains in our experiments are relatively diverse, thus reducing the difficulty of discriminating between different domains. The *domain estimation error* term 7a contains domain-level complexity $\mathsf{VC}(\bar{\mathcal{S}})$, and converges to zero if the number of episodes $m \to \infty$; the *instance estimation error* term 7b contains sample-level complexity $\mathsf{VC}(\mathcal{S})$, and converges to zero if the sample number in each episode *or* the number of episodes reaches infinity ($n \to \infty$ *or* $m \to \infty$), showing the synergy between high-level domain samples and low-level data samples in controlling the model-wise generalization error.

Comparison with existing bounds. We compare our bound 7 with existing bounds of conventional supervised learning (Vapnik, 2013; McAllester, 1999) and meta-learning (Pentina & Lampert, 2014;



(a) Parallel task: *Colored MNIST* and *Fashion Product Images* (b) Hierarchical task: *CIFAR-100*

Figure 1: Classification tasks and datasets considered in our experiments.

Amit & Meir, 2018). Typically, supervised learning bounds contain a instance-level complexity term as 7b, and meta-learning bounds further contain a task-level complexity term as 7a. Yet, conventional supervised learning only considers a single domain or multiple *known* domains, while meta-learning treats each episode as a *new* domain rather than domains that may have been encountered as in subjective learning. Thus, none of these bounds contain an explicit inference term as 7c.

Remark 2. While our bound applies VC dimension as the complexity measure, extensions to other data-dependant complexity measures such as Rademacher and Gaussian complexities (Bartlett & Mendelson, 2002; Koltchinskii & Panchenko, 2000) is straightforward. It is worth noting that the bounds based on these measures share the same asymptotic property w.r.t. m and n as in bound 7.

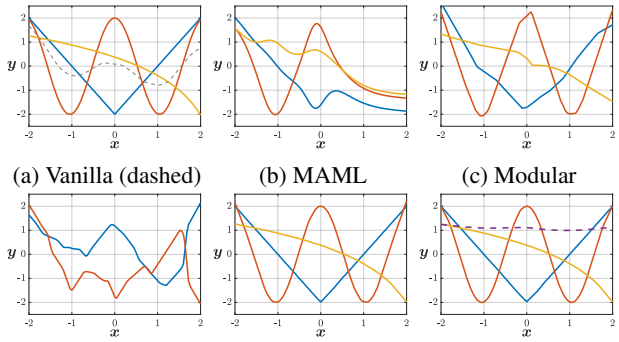
4 Experiments

In this section, we report experimental results on two basic supervised learning tasks with conflicting data: regression and classification. Our experiments are designed to (i) validate our theoretical claims, (ii) assess the effectiveness of subjective learning and show its potential applicability in different settings, and (iii) compare subjective learning with task-specific baselines.

4.1 Setup and Baselines

Regression. We consider a regression task in which data points are simultaneously sampled from three heterogeneous functions, as shown in Figure 2a (solid lines). We compare subjective learning with three baselines: (1) *Vanilla*: a conventional ERM-based learner. (2) *MAML* (Finn et al., 2017): a popular gradient-based meta-learning approach. (3) *Modular* (Alet et al., 2018): a modular meta-learning approach that extends MAML using multiple modules. We set the hyperparameters of subjective learning to $K = 3, m = 250$ and $n = 2$. To verify our theoretical results, we also run subjective learning with different number of hypotheses ($K = 2$ and $K = 4$) and different sampling hyperparameters ($m = 50, n = 2$ and $m = 250, n = 1$). More details on the task and the baselines are in Appendix D. In addition, we demonstrate the effectiveness of subjective learning on a real-world multi-dimensional regression task; details and results are in Appendix E.1.

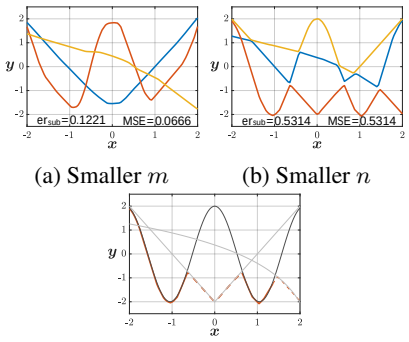
Classification. We consider two types of image recognition tasks where the same image may correspond to different labels in different sample pairs. We refer to these tasks according to the structure of their label spaces, namely *parallel* and *hierarchical* tasks. For parallel tasks, we derive the data respectively from *Colored MNIST*, a variant of *MNIST* where each digit is assigned with a digit label and a color label, and *Fashion Product Images* (Aggarwal, 2019), a multi-attribute clothes dataset that involves 3 main parallel tasks including gender, category and color classification, as shown in Figure 1a; we construct conflicting datasets by randomly choosing one label from the label set for each image. For the hierarchical task, we derive the data from *CIFAR-100* (Krizhevsky & Hinton, 2009), a widely-used image recognition dataset comprising 100 classes with "fine" labels subsumed by 20 superclasses with "coarse" labels, as shown in Figure 1b; we construct the conflicting dataset by randomly using the fine or the coarse label for each image. In classification, the most relevant problem setting to subjective learning is multi-label learning (Zhang & Zhou, 2014), which considers the scenario where an input x is related with a label set $\boldsymbol{y} = \{y_i\}_{i=1}^N$. The key difference is that multi-label learning requires that all labels in the label set are provided *simaltaneously*, while in subjective learning each data sample only contains *one* label $y_i \in \boldsymbol{y}$. Therefore, the setting of subjective learning can be alternatively modeled as "multi-label learning with missing labels", i.e., in each sample $N - 1$ labels in $\boldsymbol{y}$ are missing and only one label remains (note that this is a very extreme setting). Therefore, we compare subjective learning with the following baselines. (1) *Probabilistic concepts (ProbCon)* (Devroye et al., 1996): this baseline directly models the relation



(a) Vanilla (dashed) (b) MAML (c) Modular

(d) subjective learning ($K = 2$) (e) subjective learning ($K = 3$) (f) subjective learning ($K = 4$)

Figure 2: Results on the regression task. (a) Ground-truth functions (solid) and the result of Vanilla (dashed). (b)(c) Results of MAML and Modular. (d)(e)(f) Results of subjective learning with different number of low-level models (solid). The dashed line in (f) indicates that subjective learning abandons a redundant model.



(a) Smaller m (b) Smaller n

(c) Subjective error of (b) (dashed)

Figure 3: Impact of sampling hyperparameters on subjective learning. (a) Fewer episodes ($m = 50$). (b) Fewer episodic samples ($n = 1$). (c) The subjective error when $n = 1$ (dashed lines represent incorrect data allocations).

between inputs and outputs using an conditional probability distribution and choose top-N labels as final prediction results. (2) *Semi-supervised multi-label learning*: this class of methods model classification as the problem of "multi-label learning with missing labels" as discussed above, and we compare subjective learning with two representative methods: *Pseudo-label (Pseudo-L)* (Lee, 2013) and *Label propagation (LabelProp)* (Iscen et al., 2019). In addition, we introduce two oracle baselines as ablations: (3) *Full labels (Full-L)*: a standard multi-label learning method where we provide the full label set for each image, hence there is no missing labels. (4) *Full tasks (Full-T)*: a standard multi-task learning method where the "task" of each image is designated by human experts in advance to ensure that there is no conflict within each task. More details on baselines can be found in Appendix D. To further demonstrate the applicability of subjective learning, we also conducted an experiment on *Fashion Product Images* with simulated concept shift between domains with the *same* label space; details and results are in Appendix E.2

4.2 Evaluation Metrics

Since the error of subjective learning is related to the error of both the high-level subjective function and the low-level models, we respectively adopt two metrics to quantitatively estimate these errors.

Subjective error. This metric measures the learner's ability to perform appropriate data allocation. Given a domain d, a good subjective function should yield stable allocations for all data batches sampled from this domain. Thus, we measure the error of the subjective function using the rate of *inconsistent data allocations*, which we define as

$$\textsc{SubErr}(d) = 1 - \max_{h \in H} \frac{1}{l_d} \sum_{j=1}^{l_d} \mathbb{1}\left[g\left(H, z^j\right) = h\right], \tag{8}$$

where l_d denotes the total number of samples in domain d (the same below).

Model error. This metric measures the learner's ability to make accurate in-domain predictions, which is analagous to the traditional single-task error. Given a domain d, it is defined as

$$\textsc{ModErr}(d) = \min_{h \in H} \frac{1}{l_d} \sum_{j=1}^{l_d} e\left[h\left(x_j\right), y_j\right], \tag{9}$$

where we apply $e(y, y') = (y - y')^2$ for regression and $e(y, y') = \mathbb{1}(y \neq y')$ for classification.

Table 1: Results of subjective learning and the baselines on classification tasks.

Methods	*Colored MNIST*				*Fashion Product Images*						*CIFAR-100*			
	SubErr		ModErr		SubErr			ModErr			SubErr		ModErr	
	Num	*Col*	*Num*	*Col*	*Gen*	*Cat*	*Col*	*Gen*	*Cat*	*Col*	*Sup*	*Cla*	*Sup*	*Cla*
ProbCon	**0.09**	0.34	3.04	0.39	8.81	13.54	12.50	22.80	18.02	33.15	5.59	5.44	27.35	31.20
Pseudo-L	6.62	9.25	7.47	10.08	4.95	5.40	14.59	33.69	20.04	34.06	9.26	8.38	28.46	38.96
LabelProp	7.53	0.28	11.52	13.57	2.91	7.22	21.97	14.59	50.43	64.48	18.82	10.34	66.62	45.17
Subjective	0.10	**0.00**	**1.70**	**0.03**	**0.00**	**0.00**	**0.00**	**7.87**	**1.93**	**12.85**	**1.05**	**0.82**	**21.40**	**25.05**
Full-L	0.23	0.00	1.02	0.00	1.19	0.86	7.44	8.46	1.17	9.45	7.84	0.84	22.08	26.29
Full-T	0	0	1.20	0.00	0	0	0	7.14	1.90	11.04	0	0	21.11	25.08

4.3 Empirical Results and Analyses

We compare the performance of subjective learning and the baselines in Figure 2. Unsurprisingly, the vanilla baseline converges to a trivial mean function (dashed curve in Figure 2a). MAML successfully predicts the left part of all target functions by fine-tuning from episodic samples, but fails in the right part where functions exhibit larger difference. We hypothesis that it is because meta-learning typically requires tasks to be in sufficient numbers, and with more similarity. Although Modular currectly predicts the general trend of the curves, its predictions are still inaccurate in fine-grained details. Note that both meta-learning methods use more episodic data samples than subjective learning (see Appendix D.3.1). Meanwhile, subjective learning with $K \geq R$ ($K = 3$ or 4) successfully distinguishes different functions and recover each of them accurately while subjective learning with $K < R$ ($K = 2$) fails, which matches our analysis in Section 3.1. In particular, subjective learning with $K = 4$ automatically leaves one network to be redundant (dashed curve in Figure 2f), demonstrating the robustness of our framework. Figure 3 illustrates the impact of sampling hyperparameters on subjective learning. Concretely, fewer sampling episodes $m = 50$ induces a large model error 9, which corresponds to the sample-wise estimation term in the generalization error 7b since the product mn is not sufficiently large. On the other hand, subjective learning with fewer episodic samples $n = 1$ induces a large subjective error 8, which corresponds to the subjective estimation term in the generalization error 7c.

Classification. Table 1 shows the results of subjective learning and the baselines on classification tasks. On all tasks, subjective learning outperforms all baselines on both the subjective error and the model error. It is also worth noting that compared to the oracle Full-L with full label annotations, subjective learning still induces smaller subjective error, showing a strong capability of domain-level cognition that resembles the "ground truth" annotated by humans (Full-T).

5 Related Work

Apart from the formulation in this paper, conflicting data may also be formulated using the framework of multi-label learning with partial labels (Zhang & Zhou, 2014) or probabilistic density estimation such as probabilistic concepts (Kearns & Schapire, 1994; Devroye et al., 1996) and the energy-based learning framework (LeCun et al., 2006). A fundamental difference between these formulations and subjective learning is that these methods learn a *unified* model, while our method explicitly encourages the learner to perform high-level data allocation and result in a set of independent models. Su et al. (2020) studies a similar problem as ours where the goal of the learner is to learn from online multi-task samples without task annotation. However, our work employs a different objective function and presents more formal theoretical justification.

Extensive literature has explored the collaboration of multiple models or modules in completing one or multiple tasks (Doya et al., 2002; Andreas et al., 2016; Alet et al., 2018; Meyerson & Miikkulainen, 2019; Yang et al., 2020; Gao et al., 2020). A crucial difference between our work and these works is that our multi-model architecture is driven by the inherent conflict in conflicting data itself. Our approach can also be viewed as an implementation of mixture-of-experts methods (Yuksel et al., 2012; Shazeer et al., 2017), where our main innovation is the subjective function which serves as an effective model selection method for conflicting data, and we only allow a single low-level model to be invoked during a sampling episode. More discussions on related work are in Appendix F.

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A Discussion on the Subjective Function

In Section 2.3, we derive our design of the subjective function using an EM-based maximum likelihood formulation. However, the exact equivalence between the E-step of hard EM and the subjective function has not been established, since it relies on the exact form of the loss function $\ell(y, y')$ and the conditional likelihood $p(\mathbf{y} \,|\, \mathbf{x}, h_k), k \in [K]$. As a complement, in the sequel we provide two examples where under the uniform prior, the exact equivalence between calculating the posterior $P(h = h_k \,|\, \mathbf{x}, \mathbf{y})$ and the expected subjective function 5b can be obtained (hence their empirical counterparts are also equivalent).

Example 1 (Regression with isotropic Gaussian joint likelihood)**.** Consider a regression task where we assume that given the random variable $(\mathbf{x}, \mathbf{y})$, the joint distribution of its conditional likelihoods conforms to an isotropic Gaussian $\mathcal{N}(\boldsymbol{\mu}_K, \epsilon \mathbf{I}_K)$ where $\epsilon \in \mathbb{R}_+$ is a variance parameter and $\mathbf{I}_K$ is a $K \times K$ identity matrix. From $p(\mathbf{y} \,|\, \mathbf{x}, h_k) \propto \exp\left[-\frac{(\mathbf{y}-\mu_k(\mathbf{x}))^2}{2\epsilon^2}\right]$ and $\mu_k(\mathbf{x}) = h_k(\mathbf{x})$ we have that $\arg\max_{h_k \in H} p(\mathbf{y} \,|\, \mathbf{x}, h_k) \Leftrightarrow \arg\min_{h_k \in H} \left[(\mathbf{y} - h_k(\mathbf{x}))^2\right]$. This equals to the expected subjective function with squared loss $\ell(y, y') = (y - y')^2$.

Example 2 (Classification with independent categorical likelihoods)**.** Consider a multi-class classification task where we assume that given the random variable $(\mathbf{x}, \mathbf{y})$, all conditional likelihoods conform to independent categorical distributions with parameters $(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \cdots, \boldsymbol{\lambda}_K)$, where $\boldsymbol{\lambda}_k = (\lambda_{k1}, \cdots, \lambda_{kL}), \sum_{j=1}^{L} \lambda_{kj} = 1, k \in [K]$ and L is the number of classes. From $p(\mathbf{y} \,|\, \mathbf{x}, h_k) = \prod_{j=1}^{L} \lambda_{kj}(\mathbf{x})^{y_j}$, where $\mathbf{y} = (y_1, \cdots, y_L) \in \{0, 1\}^L, \sum_{j=1}^{L} y_j = 1$ and $\boldsymbol{\lambda}_k(\mathbf{x}) = h_k(\mathbf{x})$ we have that $\arg\max_{h_k \in H} \log p(\mathbf{y} \,|\, \mathbf{x}, h_k) \Leftrightarrow \arg\min_{h_k \in H}[-\sum_{j=1}^{L} y_j \log h_{kj}(\mathbf{x})]$, where $h_{kj}(\mathbf{x})$ denotes the predicted probability of the j-th class and y_j is the corresponding ground truth. This equals to the expected subjective function with cross-entropy loss $\ell(y, y') = -\sum_{i=1}^{L} y'_i \log y_i$.

B Algorithm Pseudo-code

We provide the pseudo-code of subjective learning in Algorithm 1.

C Proofs of Theoretical Results

In this section, we provide the proofs of our theoretical results. For better exposition, we restate each theorem before its proof.

C.1 Proof of Proposition 1

Proposition 1 (Form of the optimal solution)**.** *Assume that the target functions in all domains are realizable. Then, the following two propositions are equivalent:*

(1) For all domain distributions Q and data distributions $P_1, P_2, \cdots, P_N$, $er(H) = 0$.

(2) For each $d = \langle P, c \rangle$ in D, there exists $h \in H$ such that $\mathbb{E}_{x \sim P} \ell\left[h(x), c(x)\right] = 0$.

Proof. The derivation of proposition (1) from proposition (2) is obvious. On the other hand, if proposition (2) is false, i.e., there exists $k \in [N]$ such that for all $j \in [K]$, $h_j \neq c_k$. Then, we have

$$\begin{aligned} er(H) &= \mathbb{E}_{c_i \sim Q} \min_{h \in H} er_{P_i}\left(h, c_i\right) \\ &\geq q(c_k) \min_{h \in H} er_{P_k}\left(h, c_k\right) \end{aligned}$$

From the above we know that $c_k \notin H$, thus there exists P_k such that $er_{P_k}(h, c_k) > 0$ for every $h \in H$. This indicates that $er(H) > 0$, which is in contradiction with proposition (a). Therefore, proposition (2) must hold if proposition (1) is true. □

Algorithm 1 Subjective Learning for Conflicting Data

Require: Hypothesis set $H = \{h_1, h_2, \cdots, h_K\}$, sampling hyperparameters m and n.
1: **for** $h = 1, 2, \cdots, T$ epochs **do**
2: **for** $i = 1, 2, \cdots, m$ episodes **do**
3: Sample data $Z^i = \{(x^{i1}, y^{i1}), (x^{i2}, y^{i2}), \cdots, (x^{in}, y^{in})\}$.
4: Select a hypothesis $\hat{h}^i$ from the hypothesis set using the empirical subjective function 5a.
5: Train the hypothesis $\hat{h}^i$ by minimizing the empirical episodic error 6.
6: **end for**
7: **end for**

C.2 Proof of Theorem 1

We first present a generalized definition of PAC learnability.

Definition 2 (PAC learnability)**.** *A target function set class $\mathbb{C}$ is said to be PAC learnable if there exists an algorithm and a polynomial function* $\text{poly}(\cdot, \cdot, \cdot)$ *such that for any $\epsilon > 0$ and $\delta > 0$, for all distributions Q and distribution set $\mathcal{P} := \{P_1, \cdots, P_n\}$, the following holds for any sample size* $mn \geq \text{poly}(1/\epsilon, 1/\delta, \text{size}(\mathbb{C}))$:

$$P\left[er(H) \leq \epsilon\right] \geq 1 - \delta. \tag{10}$$

The above definition can be viewed as an extension of the single-task PAC learnability (Haussler, 1990) that considers the problem of learning a single target function. Based on this definition, in the following we give the proof.

Theorem 1. *A necessary condition of the PAC learnability of subjective learning is $K \geq R$.*

Proof. According to Definition 2, if subjective learning is PAC learnable, there must exist an algorithm that outputs a hypothesis set H with zero error, i.e., $er(H) = 0$ for every Q and $\mathcal{P}$ (otherwise the inequality 10 will not hold for a small enough ϵ and $\delta < 1$). Proposition 1 indicates that this is equivalent to $c_i \in H, i \in [N]$, which is impossible if $K < R$ according to Definition 1 and the drawer principle. □

C.3 Proof of Theorem 2

We first introduce several technical lemmas.

Lemma 1. *Let $\{E_i\}_{i=1}^n$ be a set of events satisfying $P(E_i) \geq 1 - \delta_i$, with some $\delta_i \geq 0, i = 1, \cdots, n$. Then, $P\left(\bigcap_{i=1}^n E_i\right) \geq 1 - \sum_{i=1}^n \delta_i$.*

Lemma 2 (Single-task generalization error bound (Vapnik, 2013))**.** *Let $A \leq \mathcal{Q}(z, \alpha) \leq B, \alpha \in \Lambda$ be a measurable and bounded real-valued function set, of which the Vapnik-Chervonenkis (VC) dimension (Vapnik & Chervonenkis, 1971) is $\mathsf{VC}(\mathcal{Q})$. Let $\{z_1, z_2, \cdots, z_n\}_{i=1}^n$ be data samples sampled i.i.d. from a distribution P with size n. Then, for any $\delta \in (0, 1]$, the following inequality holds with probability at least $1 - \delta$:*

$$\left| \mathbb{E}_{z \sim P} \mathcal{Q}(z, \alpha) - \frac{1}{n} \sum_{i=1}^n \mathcal{Q}(z_i, \alpha) \right| \leq (B - A)\sqrt{\epsilon(n)} + \frac{1}{n}, \tag{11}$$

where

$$\epsilon(n) = \frac{\mathsf{VC}(\mathcal{Q})\left(\ln {}^{2n}/_{\mathsf{VC}(\mathcal{Q})} + 1\right) - \ln {}^{\delta}/_{4}}{n}. \tag{12}$$

Then, we upper bound the error induced by the estimation of the expected subjective function 5b in each sampling episode of subjective learning, which is critical in bounding the generalization error. Recall that we use superscripts to denote the sampling index, e.g., $d^i = \langle P^i, c^i \rangle$ denotes the

i-th domain sample, which can be any domains in the domain set D. We define two shorthands as follows:

$$h_i^* := \arg\min_{h \in H} \mathbb{E}_{x \sim P^i} \ell \left[h(x), c^i(x) \right], i \in [m], \tag{13a}$$

$$\hat{h}_i^* := \arg\min_{h \in H} \sum_{j=1}^{n} \ell \left[h\left(x^{ij}\right), y^{ij} \right], i \in [m], \tag{13b}$$

where i denotes the i-th sampling episode of subjective learning, H is the hypothesis set.

Lemma 3 (Subjective estimation error bound)**.** *Let $\left\{\left(x^{i1}, y^{i1}\right), \cdots, \left(x^{in}, y^{in}\right)\right\}$ be episodic samples in the i-th ($i \in [m]$) sampling episode i.i.d. drawn from domain d^i with size n. Then, for any $\delta \in (0, 1]$, the following inequality holds uniformly for all hypothesis $h \in \mathcal{H}$ with probability at least $1 - \delta$:*

$$\begin{aligned} \frac{1}{m} \sum_{i=1}^{m} \frac{1}{n} \sum_{j=1}^{n} \ell \left[h_i^* \left(x^{ij}\right), y^{ij} \right] - \frac{1}{m} \sum_{i=1}^{m} \frac{1}{n} \sum_{j=1}^{n} \ell \left[\hat{h}_i^* \left(x^{ij}\right), y^{ij} \right] \\ \leq 2 \sqrt{\frac{\mathsf{VC}(\mathcal{S})(\ln {}^{2n}/_{\mathsf{VC}(\mathcal{S})} + 1) - \ln {}^{\delta}/_{8m}}{n}} + \frac{2}{n}, \end{aligned} \tag{14}$$

where $\mathcal{S} := \{(x, y) \mapsto \ell\left[h(x; \theta), y\right]\}, \theta \in \Theta$ is the function set of the sample-wise error.

Proof. We have the following decomposition:

$$\begin{aligned} & \frac{1}{m} \sum_{i=1}^{m} \frac{1}{n} \sum_{j=1}^{n} \ell \left[h_i^* \left(x^{ij}\right), y^{ij} \right] - \frac{1}{m} \sum_{i=1}^{m} \frac{1}{n} \sum_{j=1}^{n} \ell \left[\hat{h}_i^* \left(x^{ij}\right), y^{ij} \right] \\ = & \frac{1}{m} \sum_{i=1}^{m} \left\{ \frac{1}{n} \sum_{j=1}^{n} \ell \left[h_i^* \left(x^{ij}\right), y^{ij} \right] - \mathbb{E}_{x \sim P^i} \ell \left[h_i^*(x), c^i(x) \right] \right\} \\ & + \frac{1}{m} \sum_{i=1}^{m} \left\{ \mathbb{E}_{x \sim P^i} \ell \left[h_i^*(x), c^i(x) \right] - \mathbb{E}_{x \sim P^i} \ell \left[\hat{h}_i^*(x), c^i(x) \right] \right\} \\ & + \frac{1}{m} \sum_{i=1}^{m} \left\{ \mathbb{E}_{x \sim P^i} \ell \left[\hat{h}_i^*(x), c^i(x) \right] - \frac{1}{n} \sum_{j=1}^{n} \ell \left[\hat{h}_i^* \left(x^{ij}\right), y^{ij} \right] \right\}, \end{aligned}$$

in which the original difference is decomposed into three terms. By definition 13a and 13b the middle term is non-positive. Using Lemma 2 by substituting $\mathcal{Q}$ with $\mathcal{S} = \{(x, y) \mapsto \ell\left[h(x; \theta), y\right]\}$ and replacing δ with ${}^{\delta}/_{2m}$, the first term and the last term can both be bounded by $\sqrt{\frac{\mathsf{VC}(\mathcal{S}) \ln\left({}^{2n}/_{\mathsf{VC}(\mathcal{S})} + 1\right) - \ln {}^{\delta}/_{8m}}{n}} + \frac{1}{n}$ with probability at least $1 - {}^{\delta}/_{2}$ (Lemma 1). Combining these two bounds using Lemma 1 completes the proof. □

Now we can give the proof of the main theorem.

Theorem 2 (Generalization error upper bound)**.** *For any $\delta \in (0, 1]$, the following inequality holds uniformly for all hypothesis sets $H \in \mathcal{H}^K$ with probability at least $1 - \delta$:*

$$er(H) \leq \widehat{er}(H) + \sqrt{\frac{\mathsf{VC}(\bar{\mathcal{S}}) \left(\ln {}^{2m}/_{\mathsf{VC}(\bar{\mathcal{S}})} + 1\right) - \ln {}^{\delta}/_{12}}{m}} + \frac{1}{m} \tag{15a}$$

$$+ \sum_{k=1}^{N} \left(\frac{m_k}{m} \sqrt{\frac{\mathsf{VC}(\mathcal{S}) \left(\ln {}^{2m_k n}/_{\mathsf{VC}(\mathcal{S})} + 1\right) - \ln {}^{\delta}/_{12N}}{m_k n}} + \frac{1}{mn} \right) \tag{15b}$$

$$+ 2 \sqrt{\frac{\mathsf{VC}(\mathcal{S}) \left(\ln {}^{2n}/_{\mathsf{VC}(\mathcal{S})} + 1\right) - \ln {}^{\delta}/_{24m}}{n}} + \frac{2}{n}, \tag{15c}$$

where $\bar{\mathcal{S}} := \{\langle P, c \rangle \mapsto \mathbb{E}_{x \sim P} \ell\left[h(x; \theta), c(x)\right]\}, \theta \in \Theta$ is the function set of the domain-wise expected error, $\mathcal{S} := \{(x, y) \mapsto \ell\left[h(x; \theta), y\right]\}, \theta \in \Theta$ is the function set of the sample-wise error, $m_k :=$

$\sum_{i=1}^{m} \mathbb{1}\left(c^i = c_k\right)$ *is the sampling count of the target function from the* k*-th domain* $d_k (k \in [N])$*, and* $\mathsf{VC}(\cdot)$ *the Vapnik-Chervonenkis (VC) dimension (Vapnik & Chervonenkis, 1971).*

Proof. Combining the objectives 2 3 with the subjective function 5a 5b, we have the following decomposition:

$$
\begin{aligned}
er(H) - \widehat{er}(H) = & \left\{ \mathbb{E}_{d_i \sim Q} \min_{h \in H} \mathbb{E}_{x \sim P_i} \ell\left[h(x), c_i(x)\right] - \frac{1}{m} \sum_{i=1}^{m} \min_{h \in H} \mathbb{E}_{x \sim P^i} \ell\left[h(x), c^i(x)\right] \right\} \\
& + \left\{ \frac{1}{m} \sum_{i=1}^{m} \min_{h \in H} \mathbb{E}_{x \sim P^i} \ell\left[h(x), c^i(x)\right] - \frac{1}{m} \sum_{i=1}^{m} \frac{1}{n} \sum_{j=1}^{n} \ell\left[h_i^*\left(x^{ij}\right), y^{ij}\right] \right\} \\
& + \left\{ \frac{1}{m} \sum_{i=1}^{m} \frac{1}{n} \sum_{j=1}^{n} \ell\left[h_i^*\left(x^{ij}\right), y^{ij}\right] - \frac{1}{m} \sum_{i=1}^{m} \min_{h \in H} \frac{1}{n} \sum_{j=1}^{n} \ell\left[h\left(x^{ij}\right), y^{ij}\right] \right\}.
\end{aligned}
$$

By definition 13a and 13b we rewrite the equation above:

$$
\begin{aligned}
er(H) - \widehat{er}(H) = & \left\{ \mathbb{E}_{d_i \sim Q} \mathbb{E}_{x \sim P_i} \ell\left[h_i^*(x), c_i(x)\right] - \frac{1}{m} \sum_{i=1}^{m} \mathbb{E}_{x \sim P^i} \ell\left[h_i^*(x), c^i(x)\right] \right\} \\
& + \left\{ \frac{1}{m} \sum_{i=1}^{m} \mathbb{E}_{x \sim P^i} \ell\left[h_i^*(x), c^i(x)\right] - \frac{1}{m} \sum_{i=1}^{m} \frac{1}{n} \sum_{j=1}^{n} \ell\left[h_i^*\left(x^{ij}\right), y^{ij}\right] \right\} \\
& + \left\{ \frac{1}{m} \sum_{i=1}^{m} \frac{1}{n} \sum_{j=1}^{n} \ell\left[h_i^*\left(x^{ij}\right), y^{ij}\right] - \frac{1}{m} \sum_{i=1}^{m} \frac{1}{n} \sum_{j=1}^{n} \ell\left[\hat{h}_i^*\left(x^{ij}\right), y^{ij}\right] \right\},
\end{aligned}
$$

in which the generalization error of subjective learning is decomposed into three terms. By substituting $\mathcal{Q}$ in Lemma 2 by $\bar{\mathcal{S}} = \{\langle P, c\rangle \mapsto \mathbb{E}_{x \sim P} \ell\left[h(x;\theta), c(x)\right]\}$ and replacing δ with $\delta/3$, the first term can be bounded by $\sqrt{\frac{\mathsf{VC}(\bar{\mathcal{S}}) \ln\left(2m/\mathsf{VC}(\bar{\mathcal{S}}) + 1\right) - \ln \delta/12}{m}} + \frac{1}{m}$ with probability at least $1 - \delta/3$. By replacing δ with $\delta/3$ in Lemma 3 we bound the last term by $2\sqrt{\frac{\mathsf{VC}(\mathcal{S})(\ln 2n/\mathsf{VC}(\mathcal{S}) + 1) - \ln \delta/24m}{n}} + \frac{2}{n}$ with probability at least $1 - \delta/3$. There remains the middle term for which we have

$$
\begin{aligned}
& \frac{1}{m} \sum_{i=1}^{m} \mathbb{E}_{x \sim P^i} \ell\left[h_i^*(x), c^i(x)\right] - \frac{1}{m} \sum_{i=1}^{m} \frac{1}{n} \sum_{j=1}^{n} \ell\left[h_i^*\left(x^{ij}\right), y^{ij}\right] \\
& = \frac{1}{m} \sum_{i=1}^{m} \left\{ \mathbb{E}_{x \sim P^i} \ell\left[h_i^*(x), c^i(x)\right] - \frac{1}{n} \sum_{j=1}^{n} \ell\left[h_i^*\left(x^{ij}\right), y^{ij}\right] \right\} \\
& = \frac{1}{m} \sum_{k=1}^{N} \left\{ m_k \mathbb{E}_{x \sim P_k} \ell\left[h_k^*(x), c_k(x)\right] - \frac{1}{n} \sum_{j=1}^{nm_k} \ell\left[h_k^*\left(x_{kj}, y_{kj}\right)\right] \right\} \\
& = \frac{1}{m} \sum_{k=1}^{N} m_k \left\{ \mathbb{E}_{x \sim P_k} \ell\left[h_k^*(x), c_k(x)\right] - \frac{1}{nm_k} \sum_{j=1}^{nm_k} \ell\left[h_k^*\left(x_{kj}, y_{kj}\right)\right] \right\}.
\end{aligned}
$$

Recall that $m_k := \sum_{i=1}^{m} \mathbb{1}\left(c^i = c_k\right)$. In the above we re-arrange the total m data batches according to the domains they belong to. With a little abuse of notation, in the third and fourth rows we use h_k^* to denote the hypothesis that yields the smallest expected error in the k-th *domain* in D, i.e., $h_k^* = \arg\min_{h \in H} \mathbb{E}_{x \sim P_k} \ell\left[h(x), c_k(x)\right], k \in [N]$. Note that this is different from the definition 13a, where h_i^* is defined upon the i-th *domain sample*. The above transformation aggregates the domain samples so that data samples from the same domains that emerge multiple times can be accumulated and jointly considered, which leads to a tighter and more realistic error bound. By Lemma 2, for every $k \in [N]$ and $\delta \in (0, 1]$, each inside term $\mathbb{E}_{x \sim P_k} \ell\left[h_k^*(x), c_k(x)\right] -$

$\frac{1}{nm_k}\sum_{j=1}^{nm_k}\ell\left[h_k^*\left(x_{kj},y_{kj}\right)\right]$ can be bounded by $\sqrt{\frac{\mathsf{VC}(\mathcal{S})\left(\ln{}^{2m_kn}/\mathsf{VC}(\mathcal{S})+1\right)-\ln{}^{\delta}/_{12N}}{m_kn}}+\frac{1}{m_kn}$ with probability at least $1-\delta$. By replacing δ with ${}^{\delta}/_{3N}$ we bound the whole second term by $\sum_{k=1}^{N}\left(\frac{m_k}{m}\sqrt{\frac{\mathsf{VC}(\mathcal{S})\left(\ln{}^{2m_kn}/\mathsf{VC}(\mathcal{S})+1\right)-\ln{}^{\delta}/_{12N}}{m_kn}}+\frac{1}{mn}\right)$ with probability at least $1-{}^{\delta}/_{3}$ (Lemma 1).

Finally, combining the above bounds for all three terms using Lemma 1 gives the result. □

D Experiment Details

In this section, additional details on the setup and settings of the experiment are provided. All experiments were conducted based on PyTorch (Paszke et al., 2019) using a NVIDIA 2080ti GPU. All data we used does not contain personally identifiable information or offensive content, and can be obtained from public data sources.

D.1 Regression

We consider three heterogeneous mapping functions in absolute, sinusoidal and logarithmic function families as below:

$$y = 2\left|x\right| - 2,$$
$$y = 2\sin\left(3x+\frac{\pi}{2}\right),$$
$$y = \frac{3}{2}\log\left(-x+\frac{5}{2}\right)-1,$$

where the range of x is $[-2,2]$ for all three functions, yielding the conflicting data with $R=3$. The sample hyperparameters are $m=250, n=2$, i.e., a total number of 500 data points are collected in 250 episodes, each with 2 samples in the data batch, and their underlying generation functions are randomly chosen from the three mapping functions.

Network and optimizer. A simple network architecture with 5 fully-connected linear layers, each with 32 hidden units, is adopted and trained with SGD with step size $\alpha=0.05$, $momentum=0.9$, and ℓ_2 $weight\ decay=10^{-4}$.

D.2 Classification

Colored MNIST is an extended version of the well-known character recognition dataset MNIST, in which each gray-scale digital images are randomly colored with 8 different colors. The dataset contains two parallel tasks of color and number classification ($R=2$) with a total number of 60000 colored digits. The sample hyperparameters are $m=60000, n=1$.

Fashion Product Images (Aggarwal, 2019) is a dataset for automatic attribute completion and Q&A of clothing product images with multiple category labels in different domains. We choose 3 main parallel tasks (color, gender, category) with 8 main labels from the original dataset ($R=3$), with 15000 images in total. The sampling hyperparameters are $m=15000, n=1$.

CIFAR-100 (Krizhevsky & Hinton, 2009) is a classical benchmark for general image classification. It has a hierarchical structure with 20 superclasses and 100 classes, with 60000 images in total. Each superclass is from an upper-level "coarse" task, consisting of 5 "fine" classes, e.g., "insects" includes "bee, beetle, butterfly, caterpillar, cockroach". In our experiments, these two different kinds of labels are randomly provided given an image ($R=2$). The sampling hyperparameters are $m=30000, n=2$.

Network and optimizer. In *Colored MNIST*, each network consists of 3 convolutional layers and 1 fully-connected layer. In *Fashion Product Images*, each network consists of 5 convolutional layers and 3 fully-connected layers. We use Adam optimizer (Kingma & Ba, 2015) with step size $\alpha=0.002$ and $betas=(0.5,0.999)$ for these two datasets. In *CIFAR-100*, for subjective learning and all baselines, we use a pre-trained DenseNet (Huang et al., 2017) backbone of DenseNet-L190-k40 for

feature extraction to ensure a fair comparison, and add 2 fully-connected layers after the DenseNet backbone. We use SGD as the optimizer with step size $\alpha = 0.1$ and $momentum = 0.9$.

D.3 BASELINE DETAILS

In this section, we provide more details on the baselines.

D.3.1 REGRESSION

MAML. For MAML, we adapt a standard PyTorch implememtation from GitHub[1] with the same network with ours, and use the following hyperparameters:

$Shot = 2, Evalation = 100, Outer\ step\ size = 0.05, Inner\ step\ size = 0.015,$
$Inner\ grad\ steps = 2, Eval\ grad\ steps = 5, Eval\ iters = 10, Iterations = 20000.$

Modular. For modular meta-learning, we use the official PyTorch implementation from GitHub[2], and use the following hyperparameters:

$Shot = 2, Support = 2, Network = Linear\ 1 - 16 - 16 - 1, Num_modules = 5,$
$Composer = Sum, meta_lr = 0.003, Steps = 3000.$

Note that for Modular Meta-Learning a larger episodic sample number $n = 4$ (consisting of 2 support samples and 2 query samples) is adopted. Nevertheless, subjective learning still outperforms this approach using a smaller episodic sample number ($n = 2$).

D.3.2 CLASSIFICATION

Probablistic concepts. From the perspective of probability modeling, the relation between input x and output y is subject to a probability distribution $p(y \mid x)$, which is the learning target. In classification, when different domains share similar frequency, such a distribution can be approximated with the total probability formula:

$$p(y \mid x) = \sum_{h \in H} p(y \mid x, h) \cdot p(h) \approx \frac{1}{l_d} \sum_{h \in H} p(y \mid x, h)$$

For classification problems, in each domain d, the corresponding $p(y \mid x, h)$ is unimodal. Thus, their sum $p(y \mid x)$ is a multi-modal distribution, and a network trained with cross-entropy loss is still an unbiased estimation for it. The final prediction is the labels with top-N conditional probabilities $p(y \mid x)$.

Semi-supervised multi-label learning. From the perspective of semi-supervised multi-label learning, the classification problem can be modeled as "multi-label learning with missing labels": consider the fully labeled data $x \to \boldsymbol{y} = \{y_i\}_{i=1}^N$, then, conflicting data provides only one label $y \in Y$ for each x, i.e., all other labels are missing. Therefore, existing semi-supervised learning approaches may be modified to handle such problems. We consider two representative techniques, including *Pseudo-label* and *Label propagation*. Both implementations are slightly modified to fit our tasks.

Pseudo-label randomly allocates additional "pseudo" labels to each input to compensate the missing labels. The learning machine is trained on the augmented dataset and then reevaluate the confidence of all pseudo labels according to its predictions, and all pseudo labels will iteratively be modified during training until convergence.

Label propagation builds a graph over the samples, where each node on the graph represents a data sample, and each edge represents the distance of two nodes it collects in the feature space. The labels are propagated on adjacent nodes until all samples are fully labeled. The feature space is also iteratively adjusted along during training.

Full labels & full tasks. Both these oracle baselines utilize manual annotations to tranform the conflicting data with $R > 1$ to conventional supervised data with $R = 1$. More concretely, for *Full labels*, label annotations from all domains are provided simultaneously as a "multi-hot" label vector

[1] `https://github.com/dragen1860/MAML-Pytorch` (MIT license)
[2] `https://github.com/FerranAlet/modular-metalearning` (MIT license)

Table 2: Results of subjective learning on multi-dimensional regression task on gas sensor array under dynamic gas mixtures dataset. We report RMSE on each domain and the macro-average RMSE over both domains.

Methods	RMSE (domain 1)	RMSE (domain 2)	RMSE (macro-average)
Vanilla (single model)	76.5	89.7	83.1
Subjective	**34.0**	**72.6**	**53.3**
Oracle	31.9	66.8	49.4

for each input sample, resulting in a standard multi-label learning problem. For *Full tasks*, raw data is separated into single-class classification tasks according to additional manual task annotation, resulting in a standard multi-task learning problem.

D.3.3 Measuring the Subjective Error of Baselines in Classification

Since the baselines in classification experiments do not explicitly assign a separate model to each domain, the subjective error metric 8 does not directly apply to these baselines. Hence, we estimate their subjective errors using another method: we directly select top-N predictions of these methods and compare them with the label spaces of different domains. We define the "coverage" of the selected top-N labels over domain d as

$$\text{COVERAGE}(d) = \frac{1}{l_d} \sum_{i=1}^{l_d} \mathbb{1}(\exists \text{ top-}N \text{ labels for input } x_i \text{ that is in the label space of } d),$$

where l_d denotes the total number of samples in domain d. The subjective error on d is then calculated as $1 - \text{COVERAGE}(d)$.

E Additional Experimental Results and Visualizations

In this section, we provide additional experimental results and visualizations.

E.1 Multi-Dimensional Open-ended Regression

To further demonstrate the efficacy of subjective learning in regression problems, we conducted experiments on a real-world multidimensional regression dataset from UCI machine learning repository: Gas sensor array under dynamic gas mixtures dataset (Fonollosa et al., 2015). This dataset contains the recordings of 16 chemical sensors exposed to two dynamic gas mixtures and the aim is to predict the concentrations of gases, with 417,8504 instances and 16-dimensional attributes. We treat each gas mixture as one domain, respectively representing Ethylene & Methane (domain 1) and Ethylene & CO (domain 2) gas mixtures, and randomly split both domains into training (90%) and test sets (10%).

In this task, we compare subjective learning (two models, trained on the union of both domains) with a vanilla single model regressor (trained on the union of both domains) and an oracle regressor with two models separately trained on each domain. We report root mean square error (RMSE) on each domain and the macro-average RMSE over both domains in Table 2. The results indicate that subjective learning benefits from its automatic data allocation process, surpassing the vanilla baseline that trains a single global model by a large margin and performs similarly with the oracle.

E.2 Simulated Concept Shift Classification

To further demonstrate the applicability of subjective learning, we conducted an experiment on *Fashion Product Images* with simulated concept shift between domains with the *same* label spaces. Concretely, we randomly split the dataset into two domains according to the "gender" attribute: 50% of samples labeled as "Male" and 50% samples labeled as "Female" are assigned to domain

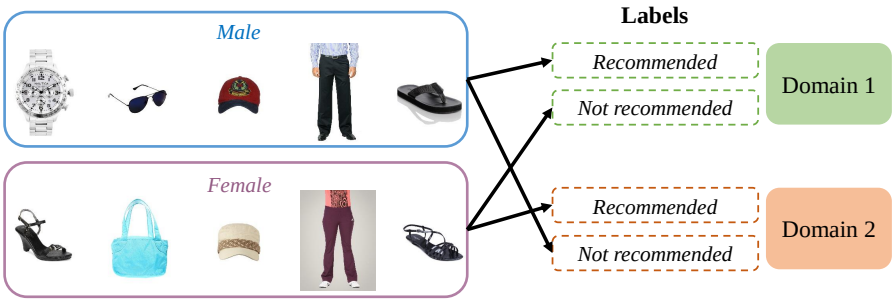

Figure 4: Classification tasks with simulated concept shift based on the *Fashion Product Images* dataset. Two domains indicate different recommendations based on the gender attribute.

Table 3: Results of subjective learning on simulated concept shift classification task. We report subjective and model errors on both domains.

Methods	SubErr		ModErr	
	Domain 1	*Domain 2*	*Domain 1*	*Domain 2*
Vanilla (single model)	-	-	49.5	50.5
Subjective	**5.37**	**5.37**	**7.40**	**9.65**
Full-T (oracle)	0	0	7.14	6.53

1 with "Male" relabeled as "1" and "Female" relabeled as "0", and other samples are assigned to domain 2 with "Male" relabeled as "0" and "Female" relabeled as "1". As shown in Figure 4, this setting resembles a simple scenario of concept shift caused by human preference: the label "1" can be interpreted as an indicator of "interested" or "recommended", while the label "0" can be interpreted as an indicator of "not interested" or "not recommended". Since different people may have different preferences (in this simulated experiment this is caused by gender – domain 1 and domain 2 respectively represents appropriate recommendations to male and female users), the data samples from different domains may possess different input-label conditional probability $P(y \mid x)$, albeit with the same label space.

Since the label space is binary and completely shared by both domains, multi-label baselines are inapplicable in this setting since they will always produce both labels "0" and "1" for every input and do not discriminate between different domains, which is unmeaningful. Therefore, we compare subjective learning with the single-model baseline (vanilla) and an oracle (Full-T) which separately train two models on both domains explicitly with data-domain correspondences known in advance. We report the model error and the subjective error of subjective learning in Table 3. Results show that the vanilla single-model baseline cannot learn meaningful prediction results since it lacks the mechanism to distinguish conflicting samples from different domains, while subjective learning achieves relatively small subjective errors and similar model errors as the Full-T oracle, which demonstrate that subjective learning is effective even in the context where different domains possess exactly the same label spaces yet different input-label relations.

E.3 Training and Inference Time on Fashion Product Images

In Table 4, we compare the computational cost of subjective learning and other baselines in terms of training and inference time on the *Fashion Product Images* dataset. We measure the required wall-clock time (in seconds) for each method to reach convergence during training as well as the averaged wall-clock time for each method to predict all labels of one given input (in milliseconds). Concretely, for subjective learning and all baselines except for two oracles (Full-L and Full-T), we train the models for 50 epochs with about 15,000 images in every epoch; for Full-L and Full-T, we train for 10 epochs since these methods generally converge faster. For each method, we test its total inference time on the same 3,000 test samples randomly sampled from the test set and report the

Table 4: Wall-clock training and inference time of subjective learning and baselines on the *Fashion Product Images* dataset. The training time of ProbCon, Pseodu-L, LabelProp and subjective learning is measured over 50 epochs, and the training time of Full-L and Full-T is measured over 10 epochs. The inference time of all methods is measured using an average over 3,000 test samples.

Methods	Training time (in seconds)	Inference time (in milliseconds)
ProbCon	415	0.67
Pseudo-L	833	2.00
LabelProp	915	0.59
Subjective	580	0.79
Full-L	159	0.68
Full-T	93	0.76

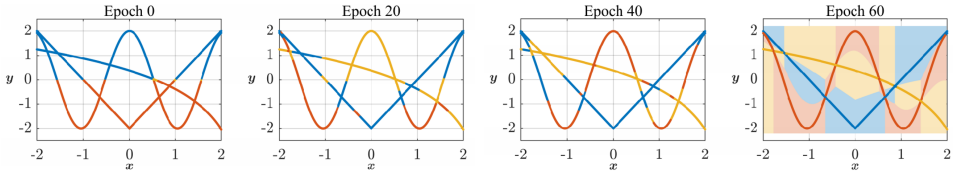

Figure 5: An example of the iteration process and the final decision boundaries of the subjective function in the regression task.

mean inference time on each test sample. All results are obtained with PyTorch using a NVIDIA 2080ti GPU.

As shown in the table, the time cost of subjective learning is generally on par with or lower than baselines that also involve iterative training (Pseudo-L and LabelProp). Although the ProbCon baseline trains faster, it learns a global model without the mechanism of data allocation and thus performs significantly worse than subjective learning as we have shown. Meanwhile, compared with the Full-T oracle that knows the data-domain correspondences in advance, subjective learning only exhibits a little additional computational overhead. This indicates that although subjective learning incorporates an extra data allocation process implemented by the subjective function, this process only induces a limited computational cost since it only requires additional network forward process without loss backpropagation, which is in general efficient.

E.4 Iteration Process on the Regression Task

In subjective learning, since the performance of the low-level models will impact the data allocation process of the high-level subjective function, the training of the subjective function and low-level networks can be regarded as an iterative process, as shown by Figure 5 with an example in regression, in which the different colored lines are designated to different subjects. All networks are randomly initialized, and in each iteration, each sample may be reallocated by the subjective function and used to further train the low-level networks. With the increasing of iterations, both subjective and model errors will reduce and converge along with the global loss. The last subfigure displays the final decision boundary of subjective function.

E.5 Feature Visualization on the Classification Task

The subjective learning approach can extract different semantics from the same input sample, and map them to different feature spaces. Figure 6 displays the features output under all subjectives, where each color represents a subjective and each point corresponds to an image in the dataset.

F More Discussions on Related Work

In this section, we provide more discussion on related work.

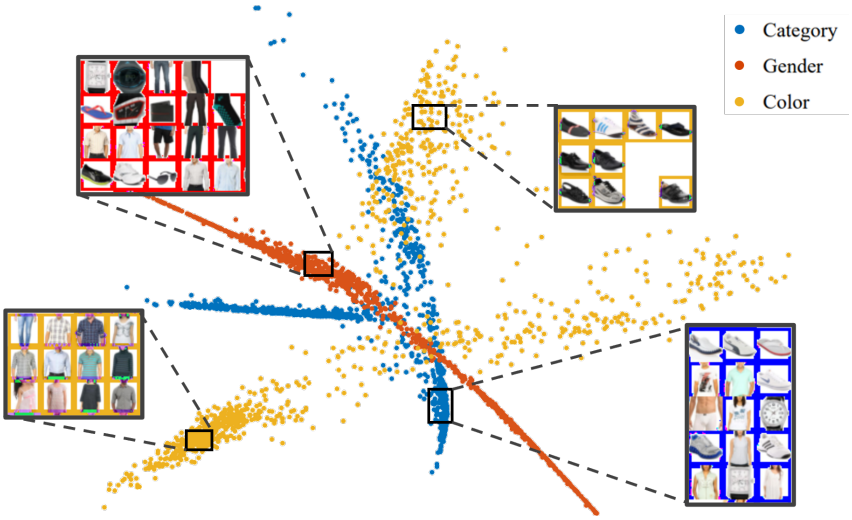

Figure 6: Additional visualization of image features by subjective learning on *Fashion Product Images*.

Ensemble learning. Ensemble learning approaches typically employ multiple models to cooperatively solve a given task (Dietterich, 2002; Zhang & Ma, 2012; Sagi & Rokach, 2018; Zhou, 2021). The prediction of each model is combined by weighting (boosting), majority voting (bagging) or learning a second-level meta-learner (stacking). Since different models process the same set of data (although sometimes with different sample weights), in ensemble learning there is typically no explicit "hard" allocation process between the data and models. In contrast, the multi-model architecture of subjective learning is driven by the inherent conflict in conflicting data, and each model only handles a proportion of the whole dataset without overlapping.

Domain adaptation and domain generalization. Domain adaptation (Ben-David et al., 2010; Pan & Yang, 2010; Tan et al., 2018; Wang & Deng, 2018; Hoffman et al., 2018) and domain generalization (Blanchard et al., 2011; Muandet et al., 2013; Zhou et al., 2021) consider the scenario where the learner trained on one or multiple source domain(s) is transferred to one or multiple new target domain(s). Typically, domain adaptation focuses on the problem where there exist some labeled or unlabeled instances in the new domain, while domain generalization considers the setting where there the information of the new domains is inaccessible during training (i.e., zero-shot generalization). In other words, these formulations focus on the *adaptation* or *generalization* capability of the model on target domain(s). In contrast, subjective learning focuses on the multi-domain *training* process and considers the scenario where directly training a global model using the data from multiple conflicting domains is problematic, and aims to resolve this training issue by performing automatic data allocation. Another important difference is that in domain adaptation and domain generalization the data-domain correspondences are available, while subjective learning weakens this assumption by only assuming that the data from each sampling episode is obtained from the same domain.