GENERALIZATION GAMES IN REINFORCEMENT LEARNING

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ABSTRACT

In reinforcement learning (RL), the term generalization has either denoted the practice of introducing function approximation to reduce the intractability of large state and action spaces problems or designated RL agents' ability to transfer learned experiences to one or more evaluation tasks. Recently, many subfields have emerged to understand how distributions of training tasks affect an RL agent's performance in unseen environments. While the field is extensive and ever-growing, recent research has underlined that variability among the different approaches is not as significant. We leverage this intuition to demonstrate how current methods for generalization in RL are specializations of a general framework. We obtain the fundamental aspects of this formulation by rebuilding a Markov Decision Process (MDP) from the ground up by resurfacing the game-theoretic framework of games against nature. The two-player game that arises from considering nature as a complete player on this formulation explains how existing methods rely on learned and randomized dynamics and initial state distributions. We develop this result further by drawing inspiration from mechanism design theory to introduce the role of a principal as a third player that can modify the payoff functions of the decision-making agent and nature. The games induced by playing against the principal extend our framework to explain how learned and randomized reward functions induce generalization in RL agents. The main contribution of our work is the complete description of the Generalization Games for Reinforcement Learning, a multiagent, multiplayer, game-theoretic formal approach to study generalization methods in RL. We offer a preliminary ablation experiment of the different components of the framework and demonstrate that a more simplified composition of the objectives that we introduce for each player leads to comparable, and in some cases superior, zero-shot generalization performance than state-of-the-art methods while requiring almost two orders of magnitude fewer samples.

1 INTRODUCTION

In reinforcement learning (RL), the term generalization has denoted the practice of introducing function approximation to reduce the intractability large state and action spaces problems (Sutton, 1996; Pack Kaelbing et al., 1996) or to designate RL agents' capacity to *transfer* learned experiences to one or more evaluation tasks (Whiteson et al., 2011). Recently, the fields of curriculum learning Portelas et al. (2020), open-ended learning (Wang et al., 2019), continual learning (Khetarpal et al., 2020), meta-learning (Duan et al., 2016), simulation-to-reality transfer (Sadeghi & Levine, 2016), or unsupervised environment design (Dennis et al., 2020) have emerged to understand how distributions of training tasks affect an RL agent's performance in unseen environments. While the field is extensive and ever-growing, recent surveys have pointed out that the amount of variability among the different approaches is not as significant (Portelas et al., 2020; Kirk et al., 2021). For instance, Portelas et al. (2020) managed to classify more than 30 approaches along three axes of variation: the intended use of the algorithm, aspects of the problem the algorithm controls, and the optimization objective it proposes. While the first axis is subjective and depends on the intended application of the algorithm, variations on the degree of control over the transition dynamics, initial state distributions, and reward functions in Markov Decision Process (MDP) problems indicate the existence of a shared underlying structure of the problem.

We leverage this intuition to develop our core hypothesis that **current methods for generalization in RL are specializations of a general framework**. To uncover the fundamental aspects of this problem, we rebuild from the ground up the MDP framework (Puterman, 2005) by resurfacing and expanding *games against nature* (Milnor, 1951; Papadimitriou, 1985), a game-theoretic approach to decision-making under uncertainty. This formulation acknowledges the existence of *nature* player that may be a single entity or the composition of multiple ones. Those entities may possess agency and the capacity to learn and adapt and, while doing so, can reshape the problem landscape of a decision-making agent (Leibo et al., 2019; Baker et al., 2020). We show that, under some assumptions, an MDP problem is a particular case of a repeated game a decision-making agent plays against a selfless nature player with unknown payoffs. At this juncture, we diverge from the traditional literature to make nature's player a *first-class citizen* within a *complete game against nature*, a two-player game where both players aim to maximize their respective non-constant payoff (or reward) functions. The introduction of nature's payoff function prompts, under the same assumptions as before, an MDP for nature's player when it repeatedly interacts with the decision-making agent. We argue that this dual formulation explains how existing algorithms rely on *learned* or *randomized* variations of transition dynamics and initial state distributions of MDP problems to induce generalization in RL agents.

Moreover, drawing inspiration from the field of mechanism design (Vickrey, 1961), we consider the emergence of complete games against nature that assume transformations on the payoff functions of both nature and the decision-making agent. To account for these variations, we introduce the figure of the principal (Myerson, 1983), a third player that can manipulate both nature and the decision-making agent payoff structures. We first consider the role played by mechanisms where the principal is an entity external to the design process. In this context, we argue that the extensive collection of RL problems (Bellemare et al., 2012; Brockman et al., 2016) are, under the equivalences we establish, mechanism-induced repeated games against nature. Consequently, we reason that *problem designers* recreate the role of the principal by predefining the payoff structure of the decision-making agent. Then, to form a complete game against nature from these existing problems, choosing appropriate payoff functions for nature's player suffices. Inspired by the types of evolutionary pressures nature poses over living agents and in line with existing methods, we formulate cooperative and competitive complete agent-nature games. Furthermore, inspired by adaptive approaches to mechanism design (Conitzer & Sandholm, 2002; Vorobeychik et al., 2007; Baumann et al., 2018), we describe *games against the principal*, a three-player game between the decision-making agent, nature, and the principal. In these games, the principal's objective is to discover the optimal outcome function from a restricted set that maximizes either player's total payoff. We propose to set the payoff structure of the principal such that it adopts a dictatorial role (Sandholm, 2003) and sides with either the decision-making agent or nature's player. In combination with the cooperative and competitive games induced by manually-design mechanism over RL problems, the optimization of the dictatorial principal objective explains how existing algorithms rely on learned or randomized reward functions to induce generalization in RL agents.

Our proposal of a general framework for generalization emerges from considering the consequences rendered over a RL agent by the non-stationarity induced by nature and the principal players learning to optimize their respective payoff functions. In our presentation of the framework of **Generalization Games for Reinforcement Learning**, we concentrate on the three RL problems structured as repeated non-differentiable games against the principal, where each agent learns and execute a mixed and parametric strategy. The core contribution of this work is to demonstrate how this multiagent, multiplayer, game-theoretic framework provides a formal framework to study existing or propose novel methods leading to in RL. Notably, we show that the axes of control and optimization in the taxonomy introduced by Portelas et al. (2020) innately emerge from our formulation. Furthermore, we include an exploratory ablation experiment that explores the effects that coalitions with a dictatorial principal have on the final zero-shot generalization of an RL agent under adversarial and cooperative objectives. We leverage the experimental setting proposed in the context of unsupervised environment design (Dennis et al., 2020) to show how the more simplified composition of the objectives in our ablation study leads to comparable, and in some cases, superior zero-shot generalization than existing more cumbersome state-of-the-art methods (Jiang et al., 2021) at almost two orders of magnitude fewer samples.

2 GAMES AGAINST NATURE AND REINFORCEMENT LEARNING

In this section, we resurface and expand an alternative formulation of the problem of decision-making under uncertainty, *games against nature* (Milnor, 1951; Papadimitriou, 1985; Biswas, 1997; LaValle, 2006). We describe how the MDP framework (Puterman, 2005) is a particular case of a repeated game against nature under the assumptions of perfect information and sequential nature-agent interactions. Likewise, we show that from a game-theoretic perspective, a *repeated game against nature* is an *incomplete game* where nature does not have a known payoff (or reward) function. In consequence, we introduce *complete games against nature* to consider that nature's player may also optimize for a non-constant payoff. We show that each *complete game against nature* then also induces a dual MDP problem for nature's player.

2.1 GAMES AGAINST NATURE

The game against nature framework presents a game-theoretic approach to decision theory and decision-making problems where a selfish and rational agent, the decision-making agent (the *agent*), aims to maximize its payoff against a disinterested opponent (*nature*) that plays at random. We derive a more formal definition of a game against nature from the works of Milnor (1951), Papadimitriou (1985), Biswas (1997), and LaValle (2006) as follows:

Definition 1 ***(Game Against Nature)*** *A game against nature is an asymmetric two-player game $\mathcal{G} = \langle \mathcal{S}, \mathcal{A}, \ell_\pi, \ell_\rho \rangle$ between a player π (the agent) and player ρ (nature) with action spaces $a \in \mathcal{A}$ and $s \in \mathcal{S}$. In this game, the player π maximizes its payoff function $\ell_\pi : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$ against a* nature *player ρ that plays at random with disinterested and constant payoff $\ell_\rho : \mathcal{S} \times \mathcal{A} \to c$.*

The asymmetry in this game arises from the different action spaces of the two players. Furthermore, each player outcome is determined by their strategies $\pi \in \Delta(\mathcal{A})$ and $\rho \in \Delta(\mathcal{S})$ defined in the space of *discrete* probability distributions $\Delta(\cdot)$ over their respective action spaces $\mathcal{A}$ and $\mathcal{S}$. The traditional formulation of *games against nature* identifies several *principles of choice* that prescribe how the decision-making agent should select its strategy under uncertainty over nature's strategy (Milnor, 1951; Biswas, 1997; LaValle, 2006). From the different principles, the principles of *insufficient reason* Milnor (1951), *minimax regret* (Savage, 1951) and *expected utility* (Von Neumann & Morgenstern, 1944; Savage, 1954) are the more widespread. In our discussion, we focus on the principle of *expected utility* as it is most commonly used in the modern literature in decision-making . Under this principle, the decision-making agent π should select actions $a \in \mathcal{A}$ that maximize its expected payoff $\mathcal{L}_\pi = \mathbb{E}_{\substack{a\sim\pi \\ s\sim\rho}} \left[\ell_\pi(s, a)\right]$ under nature's mixed strategy ρ.

However, seldom if ever games are played once. In many instances, players interact repeatedly on the same *base or stage* game, a situation modelled by *repeated games*. Similarly, a decision-making agent and nature may continually interact leading to a repetition of the base game $\mathcal{G}$. We formally define a *repeated game against nature* as follows:

Definition 2 ***(Repeated Game Against Nature)*** *A* ***repeated game against nature*** *$\mathcal{G}^t$ is a iterated version of a base or stage game against nature $\mathcal{G}$, with $t \geq 0$ interactions between an agent π and the nature player ρ. In this game, the history of interactions $\tau = \{s_{t-1}, a_{t-1}, \ldots, a_0, s_0\} \in (\mathcal{S} \times \mathcal{A})^t$ conditions the players' payoffs functions $\ell^t_\pi : (\mathcal{S} \times \mathcal{A})^t \to \mathbb{R}$ and $\ell^t_\rho : (\mathcal{S} \times \mathcal{A})^t \to c$, respectively.*

Under the expected utility principle, the decision-making agent expected outcome $\mathcal{L}_\pi$ should be define for *repeated games against nature* such that:

$$\mathcal{L}^t_\pi = \mathbb{E}_{p(\tau|\pi,\rho)} \left[\ell^t_\pi(\tau)\right] \tag{1}$$

where $p(\tau|\pi, \rho)$ denotes the probability distribution over feasible t-long interaction histories $\tau \in (\mathcal{S} \times \mathcal{A})^t$ conditioned also by the nature player mixed strategy ρ. There are several sensitive choices for payoff functions $\ell^t(\tau)$ for repeated players interactions in finite and infinite repeated games including average, limit of means, and discounted formulations (Abreu, 1988). We present a more detailed discussion of these functions in Appendix B.1.

2.2 MDP AND REPEATED GAMES AGAINST NATURE

We are interested to connect the *games against nature* formalism with the MDP framework employed in RL literature. To do so, we extend the definition of *repeated game against nature* in Def. 2 to consider perfect information and sequential nature-agent interactions. First, repeated agent-nature interactions enable to consider the amount of information available to each player decision-making. We are interested in perfect information games players can observe their past actions and those of their opponents to introduce *repeated perfect information games against nature* $\mathcal{G}^t$ where both the agent and nature strategies are conditioned on past interactions. Formally,

Definition 3 ***(Repeated Perfect Information Game Against Nature)*** *A repeated perfect information game against nature $\mathcal{G}^t$ is a repeated game against nature where each player strategy $\pi : (\mathcal{S} \times \mathcal{A})^t \to \Delta(\mathcal{A})$ and $\rho : (\mathcal{S} \times \mathcal{A})^t \to \Delta(\mathcal{S})$ is conditioned on the history $\tau \in (\mathcal{S} \times \mathcal{A})^t$ such that, at interaction time t, their actions are drawn from their mixed strategies $a_t \sim \pi(\cdot|\tau)$ and $s_t \sim \rho(\cdot|\tau)$, respectively.*

In general, *repeated games against nature* $\mathcal{G}^t$ model a game between the decision-making agent and nature where both players choose their action simultaneously. Simultaneous repeated games against nature with perfect information is a suitable realization to study *real-time RL* problems (Ramstedt & Pal, 2019). However, we focus on the more widespread practical assumption that in sequential decision-making problems *nature moves first*. For these cases, we extend *repeated perfect information games against nature* to consider sequential agent-nature interactions where nature's player starts the game and the decision-making agent acts upon observing nature previous action. More formally,

Definition 4 *A sequential perfect information game against nature is a perfect information game against nature where nature plays first with action $s_0 \sim \rho$ drawn from initial mixed strategy $\rho_0 \in \Delta(\mathcal{S})$. Thereafter, nature and the agent plays with strategies $\rho : (\mathcal{S} \times \mathcal{A})^t \to \Delta(\mathcal{S})$ and $\pi : \mathcal{S} \times (\mathcal{S} \times \mathcal{A})^t \to \Delta(\mathcal{A})$ defined such that, at interaction time t, the agent actions are drawn from the mixed strategy $a_t \sim \pi(\cdot|s_t, \tau)$ according to nature's last action $s_t \sim \rho(\cdot|\tau)$ and the history τ of past interactions.*

The assumptions of perfect information and sequential interaction allow us to present Markov Decision Processes MDP as a particular repeated games against nature. Note that, an MDP problem $\mathcal{M} = \langle \mathcal{S}, \mathcal{A}, \rho, r, \rho_0 \rangle$ with state space $\mathcal{S}$, action space $\mathcal{A}$, transition dynamics $\rho : \mathcal{S} \times \mathcal{A} \to \mathcal{P}(\mathcal{S})$, reward function $r : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$ and initial state distribution $\rho_0 : \mathcal{P}(\mathcal{S})$ is a sequential game a decision-making agent π plays against nature's strategy ρ. If ρ_0 denotes nature's player unconditioned strategy at interaction time $t = 0$ (nature's first move) the decision-making agent plays with strategy or policy $\pi : \mathcal{S} \to \mathcal{P}(\mathcal{A})$. Under *Markovian* assumptions (Puterman, 2005), nature's actions s_t represent sufficient statistics of past nature-agent interactions τ. Thus, the agent actions are drawn from its mixed strategy $a_t \sim \pi(\cdot|s_t)$ conditioned on nature's last action s_t. Furthermore, nature's player actions for interaction times $t \geq 1$ are drawn from a mixed strategy $s_t \sim \rho(s_t|s_{t-1}, a_{t-1})$ and assumed independent of the rest of the interaction history τ given the last step actions s_{t-1} and a_{t-1}.

Likewise, under this formulation, the reward function $r(s, a)$ is equivalent to one-shot payoff function $\ell_\pi(s, a)$ of the base game $\mathcal{G}$. The objectives for an average or discounted reward MDP problem (Puterman, 2005) are easily derived from the repeated payoff structures in infinitely repeated games against nature we define on Sec. B.1.

2.3 COMPLETE GAMES AGAINST NATURE

The framework of games against nature that we have described thus far proposes games that are *incomplete* in the sense that nature's player payoff function is constant and unknown. We extend the original formulation to consider a nature player that optimizes for a non-constant payoff function under one of *principles of choice* for decision-making under uncertainty. More formally,

Definition 5 ***(Complete Game Against Nature)*** *A* ***complete game against nature*** *is an extension of a game against nature $\mathcal{G} = \langle \mathcal{S}, \mathcal{A}, \ell_\pi, \ell_\rho \rangle$ where the nature player ρ plays with strategy $\rho : \mathcal{A} \to \Delta(\mathcal{S})$ and maximizes a non-constant payoff $\ell_\rho : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$.*

As before, we focus on the principle of *expected utility* principle and define nature's expected payoff to be $\mathcal{L}_\rho = \mathbb{E}_{\substack{a\sim\pi \\ s\sim\rho}}[\ell_\rho(s,a)]$ against the mixed strategy π of the decision-making agent. Repeated nature-agent interactions prompt to a repeated game for which equivalent MDP and RL problems emerge for nature's player under a similar expected objective:

$$\mathcal{L}_\rho^t = \mathbb{E}_{p(\tau|\pi,\rho)}\left[\ell_\rho^t(\tau)\right] \tag{2}$$

Holding the assumptions of perfect information and sequential nature-agent interactions as before, a nature player sequential, repeated, perfect information game against the agent induces the following MDP problem:

Definition 6 ***(Nature MDP)*** *A **nature** MDP problem $\mathcal{N} = \langle \mathcal{A}, \mathcal{S}, \pi, r, \pi_0 \rangle$ is an MDP with state space $\mathcal{A}$, action space S, transition dynamics $\pi : \mathcal{S} \to \Delta(A)$, reward function $r : \mathcal{A} \times \mathcal{S} \to \mathbb{R}$, and initial state distribution $\pi_0 : \Delta(\mathcal{A})$ is the sequential game a nature agent ρ plays against a decision-making agent π.*

Thus, every complete game induces a pair $(\mathcal{M}, \mathcal{N})$ of dual MDPs(see (Fu et al., 2021)). In this case, the initial state distribution π_0 of $\mathcal{N}$ is defined in terms of the initial state distribution ρ_0 of the decision-making agent MDP $\mathcal{M}$ such that $\pi_0(a_0) = \sum_s p_0(s_0)\pi(a_0|s_0)$. Moreover, nature's policy ρ is the transition dynamics $\rho(s_t|s_{t-1}, a_{t-1})$ of the decision-making agent's MDP and, conversely, the decision-making agent policy or strategy π is the transition function $\pi(a_t|s_t)$ of nature's MDP. Also, the reward functions on each MDP are the payoff functions ℓ_ρ and ℓ_π on the complete base game $\mathcal{G}$.

3 Mechanism Induced Games Against Nature

In this section, drawing inspiration from the field of mechanism design (Vickrey, 1961) (see Sec. B.2), we consider the emergence of complete games against nature that assume transformations on the payoff functions of both nature and the decision-making agent. We introduce the figure of *the principal*, a third player that can manipulate both nature and the decision-making agent payoff structures and consider the role played by mechanisms where the principal is an entity external to the design process. Furthermore, inspired by adaptive approaches to mechanism design, we describe *games against the principal*, a three-player game between the decision-making agent, nature, and the principal. In these games, the principal's objective is to discover the optimal outcome function from a restricted set that maximizes either player's total payoff.

3.1 Mechanism Design for Games Against Nature

A mechanism design problem for *games against nature* defines the base game $\mathcal{G}$, both players action spaces $\mathcal{S}$ and $\mathcal{A}$, and their modified outcomes $\tilde{\ell}_\rho$ and $\tilde{\ell}_\pi$. We restrict our interest to cardinal outcomes (e.g., money, years in prison, or scalar reward) and structure outcome spaces $\mathcal{O} = O_i \times \ldots \times O_n$. We assume that utilities $\ell_i : O_i \to \mathbb{R}$ are of the form $\ell_i = o_i$ where the $i-th$ player maximizes the outcome o_i provided by the mechanism. Under these assumptions, we define the problem of designing a mechanism for nature-agent game as follow:

Definition 7 ***(Mechanism of a Game Against Nature)*** *Let $\mathcal{R} = (r_\rho, r_\pi)$ be the set of possible outcomes for nature and the decision-making agent, the **mechanism of a game against nature** for $\mathcal{R}$ is a tuple $\mathcal{F} = \langle \mathcal{R}, \mathcal{A} \times \mathcal{S}, r \rangle$ where $\mathcal{A}$ and $\mathcal{S}$ are the decision-making and nature player action spaces and $r : \mathcal{A} \times \mathcal{S} \to \mathcal{R}$ is the mechanism outcome function.*

Thus, any mechanism $\mathcal{F}$ defined this way will induce a nature-agent game $\mathcal{G}_\mathcal{F}$ with the following structure:

Definition 8 ***(Mechanism-Induced Game Against Nature)*** *Given a mechanism of a game against nature $\mathcal{F} = \langle \mathcal{R}, \mathcal{A} \times \mathcal{S}, r \rangle$ and the utility functions $\ell_\pi : \mathcal{R} \to \mathbb{R}$ and $\ell_\rho : \mathcal{R} \to \mathbb{R}$, a **mechanism-induced game against nature** is a tuple $\mathcal{G}_\mathcal{F} = \left\langle \mathcal{A} \times \mathcal{S}, \tilde{\ell}_\pi, \tilde{\ell}_\rho \right\rangle$ where $\mathcal{A}$ and $\mathcal{S}$ are the action spaces of the decision-making agent and nature, and the utility functions $\tilde{\ell}_\pi : \mathcal{R} \to \mathbb{R}$ and $\tilde{\ell}_\pi : \mathcal{R} \to \mathbb{R}$ are*

defined such as:

$$\tilde{\ell}_\pi(s,a) = \ell_\pi(r(s,a)) = r_\pi \tag{3}$$

$$\tilde{\ell}_\rho(s,a) = \ell_\rho(r(s,a)) = r_\rho \tag{4}$$

for outcomes r_π and r_ρ and the outcome function r of the mechanism $\mathcal{F}$.

This definition assumes that both players have linear utility functions that ignore the opponent outcome. Furthermore, the notation for the outcome function $r : \mathcal{A} \times \mathcal{S} \to \mathcal{R}$ is intentionally chosen to highlight that, in a mechanism-induced game against nature, the outcome function establishes the payoff (or reward) functions $\tilde{\ell}_\rho$ and $\tilde{\ell}_\pi$ for a stage complete game against nature $\mathcal{G}_\mathcal{F}$ the induced dual MDP problems.

3.1.1 Manually-Designed Mechanisms

The manual design of mechanism for games against nature requires defining both players payoff functions and action spaces in a base game. The mechanisms that we inspect here inherit from manually designed RL and MDP problems where only the agent's payoff function is defined. We propose one adversarial and one cooperative game against nature.

Definition 9 *An **adversarial game against nature** $\mathcal{G}^- = \langle \mathcal{S}, \mathcal{A}, \ell_\pi, \ell_\rho \rangle$ is a zero-sum game the decision-making agent plays against nature such as nature non-constant maximizes a payoff function $\ell_\rho = -\ell_\pi$, or in other words, the total outcome of the game $\mathcal{G}^-(\pi, \rho) = \ell_\pi + \ell_\rho = 0$. In this adversarial, zero-sum game both player's optimization objectives can be succinctly written down as:*

$$J^-(\pi, \rho) = \min_\rho \max_\pi \ell_\pi \tag{5}$$

On the other hand, we can also define a game where both players cooperate to maximize the payoff function of the agent:

Definition 10 *A **cooperative game against nature** $\mathcal{G}^+ = \langle \mathcal{S}, \mathcal{A}, \ell_\pi, \ell_\rho \rangle$ is a non-zero game where the decision-making agent plays with nature's player that aims to maximize a non-constant payoff $\ell_\rho = \ell_\pi$. This cooperative game objective can be succinctly written down as:*

$$J^+(\pi, \rho) = \max_\rho \max_\pi \ell_\pi \tag{6}$$

3.1.2 Automatically-Designed Mechanisms

Both the adversarial and cooperative definitions above are special cases of manually designed mechanism where the problem designer recreate the role of the *principal* by pre-defining the decision-making agent payoff function. However, we are interested in incorporating the principal as a player, with strategy ν, a payoff function ℓ_υ (see Sec. B.2). We are motivated by previous adaptive approaches to adaptive mechanism design in (Vorobeychik et al., 2007; 2012; Baumann et al., 2018) to construct a three-player game between the decision-making agent, nature's player, and a principal. More formally,

Definition 11 *A **game against a principal** is a tuple $\mathcal{G}_\nu = \left\langle \mathcal{R}, \mathcal{A}, \mathcal{S}, \ell_\nu, \tilde{\ell}_\pi, \tilde{\ell}_\rho \right\rangle$ that denotes an asymmetric three-player game between a decision-making agent π with strategy $\pi : \mathcal{S} \to \Delta(\mathcal{A})$, nature's player $\rho : \mathcal{A} \to \Delta(\mathcal{S})$, and a principal player ν with strategy $r : \mathcal{A} \times \mathcal{S} \to \mathcal{R}$ that can manipulate the other players payoff functions $\tilde{\ell}_\pi$ and $\tilde{\ell}_\rho$ in benefit of maximizing its own payoff function ℓ_ν whose objective is to find the optimal mechanism $r^* \in R$ over a space of feasible mechanisms R, such that:*

$$r^* = \arg\max_{r \in R} \ell_\nu \tag{7}$$

In the same fashion as before, we explore adversarial and cooperative games against a *dictatorial principal* ($\ell_\nu = \min_i \tilde{\ell}_{i=k}$) (Sandholm, 2003) who forms a coalition with either nature or the decision making-agent. From the perspective of the decision-making agent, the core of RL and MDP problems, in a **cooperative nature-principal game**, nature and the principal form a *coalitional*

game (Myerson, 1991) where they cooperate to maximize nature's player payoff $\tilde{\ell}_\rho$. We can write down this objective more succinctly as:

$$J(\rho, r) = \max_r \max_\rho \tilde{\ell}_\rho \tag{8}$$

On the other hand, in a **cooperative agent-principal game**, the decision-making agent and the principal form a coalition that cooperates to maximize the decision-making payoff function $\tilde{\ell}_\pi$ such that their joint objective can be stated as:

$$J(\pi, r) = \max_r \max_\pi \tilde{\ell}_\pi \tag{9}$$

In the context of the three-player game against a principal $\mathcal{G}_\nu$, alliances formed with the principal provides an strategic advantage for the principal's allied player in the base game against nature induced by the mechanism. For instance, in a cooperative nature-principal game, if nature's objective is adversarial (Eq. 5), a game against principal would have an objective were the nature-principal coalition minimizes the decision-making agent payoff, leading to an objective of the form:

$$J(\rho, \pi, \nu) = \min_{\rho, r} \max_\pi \tilde{\ell}_\pi \tag{10}$$

Similarly, if the nature-agent game is cooperative, a nature-principal coalition would result in an objective that maximizes the decision-making agent payoff:

$$J(\rho, \pi, \nu) = \max_{\rho, r} \max_\pi \tilde{\ell}_\pi \tag{11}$$

While other combinations of coalitions are possible, we focus in the reminder of this work on the objectives in Eq. 10 and Eq. 11 to show the effects that concentrating the principal power on the nature's player have over generating mechanism-induced game against nature, their equivalent RL and MDP problems, and decision-making adaptation of strategy learned on these coalitional games.

4 Generalization Games for Reinforcement Learning

4.1 Learning in Games Against Nature

Players' iterations at playing a game allow them to adapt their strategies to face opponent players. In games against nature, particularly in those that leverage the *principle of expected utility*, a decision-making agent can adapt its strategy π against an unknown nature's mixed strategy ρ, generally assumed to remain stationary (same player's strategy) throughout their interactions. In a base or stage complete game against nature $\mathcal{G} = \langle \mathcal{S}, \mathcal{A}, \pi, \rho, \ell_\pi, \ell_\rho \rangle$, we can define the learning objective of a decision-making agent π to find an optimal strategy $\pi^* \arg\max_\pi \mathbb{E}_{a\sim\pi, s\sim\rho} [\ell_\pi(s, a)]$. In repeated games against nature $\mathcal{G}^t$, the optimal strategy π^* of the decision-making agent should maximize the cumulative payoff functions over t-iterations of the base game, an objective that we can state as:

$$\pi^* = \arg\max_\pi \mathbb{E}_{p(\tau|\pi,\rho)} \left[\ell^t_\pi(\tau) \right] \tag{12}$$

where the payoff ℓ^t_π is defined as described in Sec. B.1. Note that, for the sequential perfect information repeated games against nature and their equivalent MDPs, the objective in Eq. 12 is the traditional RL objective with the exception that we make explicit the dependency on the nature player strategy. Similarly, nature's player optimal strategy in a base game against the decision-making agent is defined $\rho^* = \arg\max_\rho \mathbb{E}_{\substack{a\sim\pi \\ s\sim\rho}} [\ell_\rho(s, a)]$. In nature's repeated games against a decision-making agent $\mathcal{G}^t$ the cumulative payoff function $\ell^t_\rho : (\mathcal{S} \times \mathcal{A})^t \to \mathbb{R}$ may be similarly defined in consequence to the *finiteness* of interaction length t (see Sec. B.1). The learning objective in Eq. 2 leads to a learning objective that aims to find the optimal nature's strategy ρ^* defined as:

$$\rho^* = \arg\max_\rho \mathbb{E}_{p(\tau|\pi,\rho)} \left[\ell^t_\rho(\tau) \right] \tag{13}$$

The objective above defines a RL problem in nature's MDP. A similar argument can be made for the principal agent

4.2 Generalization in Games Against Nature

Assume that the strategies of the nature's player $\rho_\phi : \mathcal{S} \times \mathcal{A} \to \mathcal{S}$, the decision-making agent and $\pi_\theta : \mathcal{S} \to \mathcal{A}$ and a principal $r_\omega : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$ are parametrized by parameters ϕ, θ, ω respectively. Further, consider the consequences rendered over a RL agent by the non-stationarity induced by nature and the principal players learning to optimize their respective payoff functions. Our proposal of a **Generalization Games for Reinforcement Learning** concentrates on the three RL problems structured as repeated non-differentiable games against the principal, where each agent learns and execute a mixed and parametric strategy.

Definition 12 ***(Generalization Game for Reinforcement Learning)*** *A Generalization Game for RL is a repeated non-differentiable optimization game against a principal* $\mathcal{G}^t = \left\langle \mathcal{R}, \mathcal{A}, \mathcal{S}, \ell_\nu, \tilde{\ell}_\pi, \tilde{\ell}_\rho \right\rangle$ *with objective* $J(\phi, \theta, \omega)$ *among the nature player* ρ_ϕ*, the principal* r_ω*, the decision-making agent* π_θ *were each may simultaneously optimize for non-constant cumulative payoff functions such that:*

$$\theta_n = \theta_{n-1} + \eta \nabla_\theta J(\phi, \theta, \omega) \tag{14}$$

$$\phi_k = \phi_{k-1} + \eta \nabla_\phi J(\phi, \theta, \omega) \tag{15}$$

$$\omega_m = \omega_{m-1} + \eta \nabla_\omega J(\phi, \theta, \omega) \tag{16}$$

reflects the update rules equations of each strategy possibly optimized using Monte Carlo gradient estimation techniques (Mohamed et al., 2020).

Notice that the update function of the decision-making agent strategy π_θ is the traditional RL problem. In contrast, the updates rule for nature's strategy ρ_ϕ and that of the principal r_ω introduce the variability over transition dynamics, initial state distribution, and reward functions highlighted in the axes of control presented in Portelas et al. (2020). Combined with the adversarial and cooperative objectives we introduced through manual and adaptive mechanism design of game against a principal, we account for the axis of optimization in the same taxonomy. Thus, the non-stationarity of learning in these games makes the aspects of current generalization methods innately emerge from our formulation.

4.3 Preliminary Results

In Appendix A, we include an exploratory ablation experiment that explores the effects that coalitions with a dictatorial principal have on the final zero-shot generalization of an RL agent under cooperative and cooperative objectives. We leverage the experimental setting proposed in the context of unsupervised environment design (Dennis et al., 2020) to show further how the more simplified composition of the objectives in our ablation study leads to comparable, and in some cases, superior zero-shot generalization than existing more cumbersome state-of-the-art methods (Jiang et al., 2021) with almost two orders of magnitude fewer samples.

5 Conclusions and Future Work

We studied the problem of generalization in RL and introduced the *Generalization Games for Reinforcement Learning* by leveraging and expanding the game-theoretic framework of games against nature. Future work will include expanding our preliminary empirical results to more complex environments, including environments with continuous state and actions spaces (Brockman et al., 2016).

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A Experiments

A.1 Components Ablation

Our experiments aim to better understand the influence of training environment alteration on a decision-making agent's generalization prowess. To do so, we characterize games as described in 4. In each, we may find a *nature* player, a *principal* player and a random action player. The principal and nature players are both learning algorithms. The random action selector acts when the other players are not allowed to.

In RL terms, the principal player has the power to influence the *(i)* reward function, while the nature player has the power to influence the *(ii)* transition dynamics and the *(iii)* initial state distribution.

As in 3, we refer to these three agents as nature's coalition. We say that the nature coalition has "agency" over a power when the power is not played by the random action selector.

We are interested in out-of-distribution zero-shot (OODZ) generalization. By studying it, we can pinpoint issues or strengths in our ablations. Intuitively, some powers lend themselves to vastly more powerful OODZ generalization than some others (and this is supported by our results). In the same vein, some combinations of powers lead to degenerate solutions on nature's coalition's side. This is verified by looking at the path lengths generated by nature.

We actually find that the coalition having full agency over the alteration of the decision-making agent's MDP hurts OODZ in many evaluation environments compared to simpler schemes. We hope that our findings will be helpful in this regard for future designers of this type of game.

A.2 Experimental Details

We apply this series of ablation to the popular environment Minigrid as described in Chevalier-Boisvert et al. (2018). Minigrid is a partially observable maze environment in which the decision-making agent must take discrete actions to reach a goal. When said goal is reached, the decision-making agent receives a sparse reward that decreases with the number of steps taken by the agent. In this environment, the powers of nature roughly translate to *(i)* the power to place the goal, *(ii)* the power to place blocks, and *(iii)* the power to place the decision-making agent's initial position.

We play each game on three different seeds for 3M steps. For each seed, we evaluate the decision-making agent's zero-shot transfer in a list of challenging OOD evaluation environments, for 10 evaluation runs each. All learning agents are PPO agents (as described in Schulman et al. (2017)) using an LSTM-based recurrent policy (see Hochreiter & Schmidhuber (1997)). We present aggregations of the performance in evaluation environment here and in C.

A.3 Results

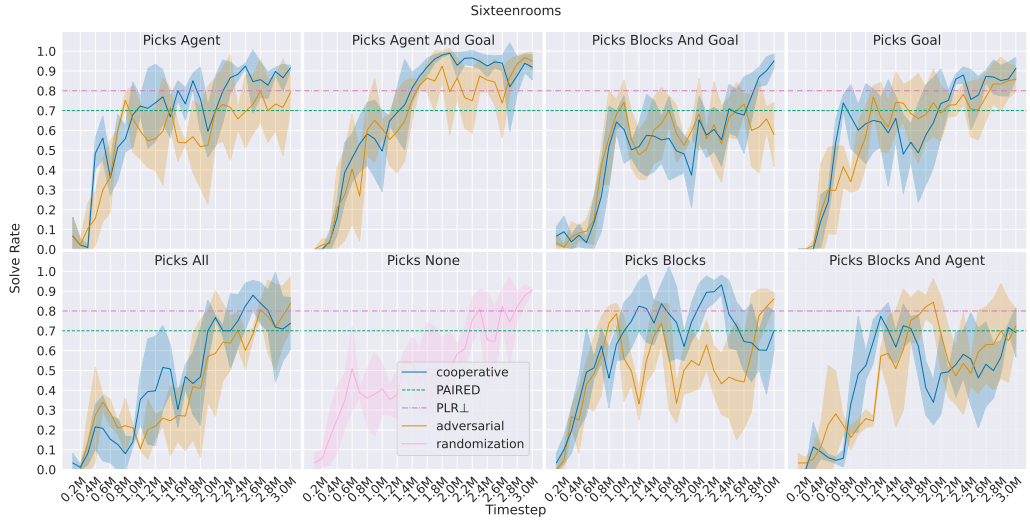

Figure 1: Zero-shot OOD generalization depends greatly on the powers of nature. Here pictured: generalization on the Sixteenroom environment.

Our results in Figure 1 show that the coalition's decision-making agent achieves the best performance when it only has agency on the placement of the agent and goal (*(iii)* and *(i)*). This makes sense: as the coalition does not have agency over where it places blocks, it can rarely make impassable environments for the decision-making agent in the adversarial scheme, and it is harder for it to make trivial environments in the cooperative scheme. Generally, the decision-making agents of coalitions that do have agency over block placement indeed fare worse than those that don't, hinting at widespread degenerate solutions when the coalition has agency over block placement. Probably for the same reason, we find that pure randomization seems to do fairly well.

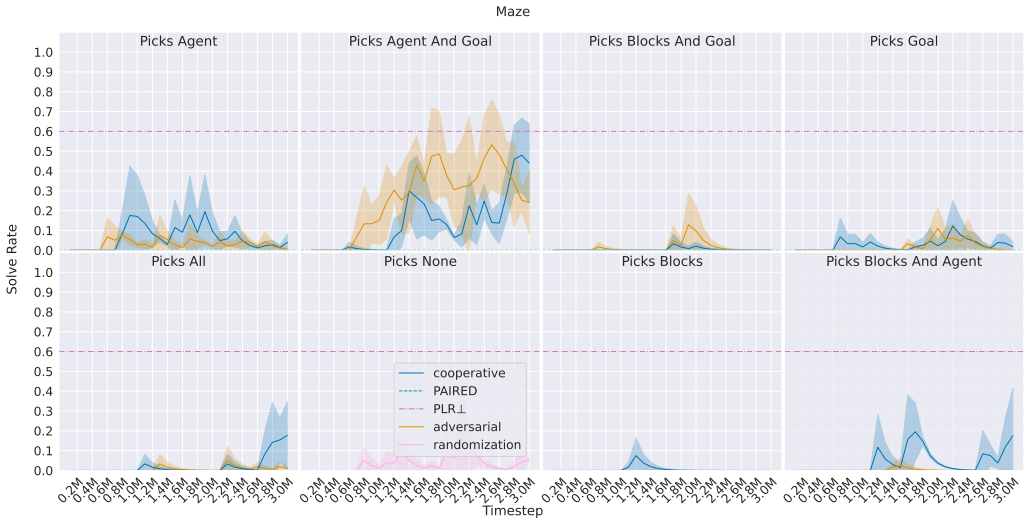

Figure 2: Zero-shot OOD generalization depends greatly on the powers of nature. Here pictured: generalization on the Maze environment.

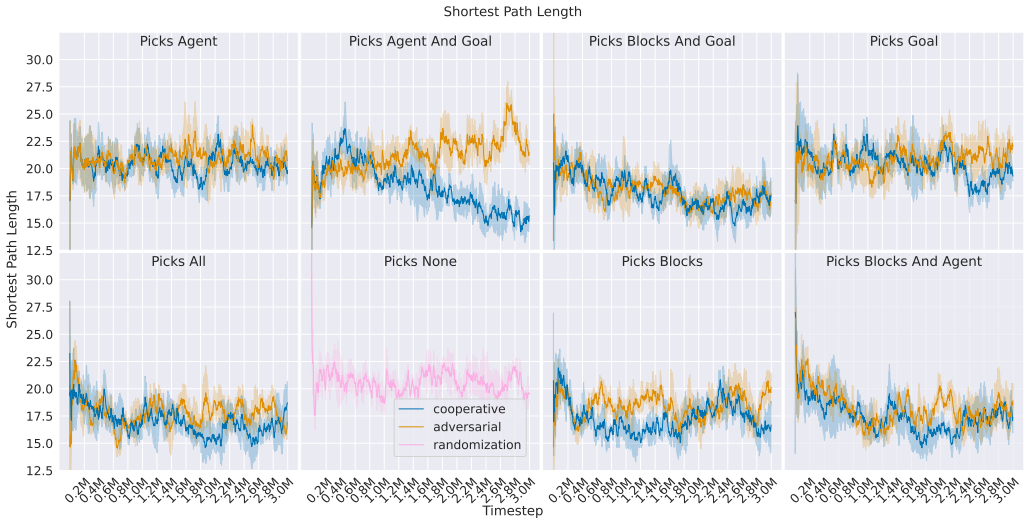

Figure 3: The "Shortest Path Length" is the length of the optimal path from the agent's original position to the goal. Note that this plot uses generous smoothing in order to highlight our findings. It also does not take impassable environments into account.

Relatedly, Figure 3 is also interesting. A clear trend is visible in most schemes: adversarial coalitions tend to create longer paths for the decision-making agent to solve. This makes sense, as Minigrid's reward function is a proxy for said distance, meaning that cooperative schemes aim as minimizing the distance and adversarial schemes aim at maximizing it.

As seen in Figure 3, this is most obvious when the coalition has agency over the placement of the goal and agent. Interestingly, this leads to the best performance in our harder OODZ evaluation environments as seen in Figure 2, but not consistently: the cooperative schemes beat out the adversarial ones more often than not in non-best schemes.

In fact, contrarily to popular belief, we find that cooperative coalitions do better over most OODZ environments compared to their adversarial counterparts.

Interestingly, notice (again in Figure 1) that many of our games train decision-making agents that beat the reported OODZ generalization performance of PLR$\perp$ (see Jiang et al. (2021)) and PAIRED (as described in Dennis et al. (2020)) an order of magnitude earlier than both of those methods on the Sixteenroom environment. Our games need 3M steps to reach 85% solve rate in Sixteenroom, while PLR$\perp$ and PAIRED requires 250M to reach about the same. Specifically, the game in which the nature coalition has agency over the placement of the agent and the goal reaches almost 95% solve rate in only 1.9M steps in both the cooperative and the adversarial schemes. The same can be said for pure randomization, which reaches 90% solve rate in 3M steps. More comparisons of this sort are available in appendix C.

B Additional Background

B.1 Payoff Structures in Repeated Games

In repeated games, the decision-making payoff function $\ell_\pi^t(\tau)$ is defined differently depending on *finiteness* of the length or horizon t of repetitions of the base game. In *finite repeated games*, a game $\mathcal{G}^T$ with a finite and bounded number of interactions $T < \infty$, the cumulative payoffs functions $\ell_\pi^T : (\mathcal{S} \times \mathcal{A})^T \to \mathbb{R}$ for can be defined as the sum of individual base game $\mathcal{G}$ payoffs ℓ_π, such that:

$$\ell_\pi^T(\tau) = \sum_{t=0}^{T-1} \ell_\pi(s_t, a_t) \tag{17}$$

However, when the number of interactions is infinite and leads to *infinitely repeated nature-agent games* $\mathcal{G}^\infty$, there are two main choices for payoffs structures. When the decision-making agent is indifferent as to when its one-shot payoffs $\ell_\pi(s_t, a_t)$ is received during the interaction, we can define an *average* payoff (limit of means) such that

$$\ell_\pi^\infty(\tau) = \lim_{T\to\infty} \inf \frac{1}{T} \sum_{t=1}^{T} \ell_\pi(s_t, a_t) \tag{18}$$

Alternatively, if the agent express preferences for one-shot payoffs ℓ_π received earlier on the interaction history, we can introduce a discount factor $0 < \gamma < 1$ (Abreu, 1988) such that:

$$\ell_\pi^\infty(\tau) = (1-\gamma) \sum_{t=0}^{\infty} \gamma^t \ell_\pi(s_t, a_t) \tag{19}$$

For discount values of $\gamma < 1$, these infinite games are equivalent to a finite game with an effective number of interactions $T = \frac{1}{1-\gamma}$ after which no base game payoff is considered ($\gamma^t = 0$).

B.2 Introduction to Mechanism Design

Mechanism design Vickrey (1961) aim to elicit desirable behavior (e.g., Pareto optimal Nash equilibrium) from the players involved in a game. We introduce the concepts of game form or mechanism and that of the n-player game induced by a mechanism.

Definition 13 *Let $\mathcal{O}$ be a set of possible outcomes, a* ***game form or mechanism*** *for $\mathcal{O}$ is a tuple $\mathcal{F} = \langle \mathcal{O}, \mathcal{A}, \varphi \rangle$, where $\mathcal{A} = \mathcal{A}_1 \times \ldots \times \mathcal{A}_n$ is a cross product of action or strategy spaces for n players and $\varphi : \mathcal{A} \to \mathcal{O}$ is an outcome ruel or function.*

Utility functions $\ell_i : \mathcal{O} \to \mathbb{R}$ generally convert player's preference over ordinal outcomes into a cardinal utility values (Von Neumann & Morgenstern, 1944)

Definition 14 *Given a mechanism $\mathcal{F}$ and a set ℓ of utility functions $\ell_i : \mathcal{O} \to \mathbb{R}$ for each player, the n-player* ***game*** *$\mathcal{G}_\mathcal{F}$* ***induced by the mechanism*** *$\mathcal{F}$ is a pair $\mathcal{G}_\mathcal{F} = \langle \mathcal{A}, \ell_\varphi \rangle$ where $\mathcal{A} = \mathcal{A}_1 \times \ldots \times \mathcal{A}_n$ is a cross product of action or strategy spaces for n players and ℓ_φ is the set of modefied utility functions $\tilde{\ell}_i$ for each player, such as:*

$$\tilde{\ell}_i(a_1, \ldots, a_n) = \ell_i(\varphi(a_1, \ldots, a_n)) \tag{20}$$

defines the payoff received by the i-th *agent under a joint strategy $(a_1, \ldots, a_n) \in \mathcal{A}$ defined by the outcome function $\varphi(a_1, \ldots, a_n)$.*

Automated Mechanism Design. The field of automated mechanism design approaches design of the mechanism outcome function $\varphi : \mathcal{A} \to \mathcal{O}$ manually or automated. The approach to automated mechanism design (Conitzer & Sandholm, 2002; Sandholm, 2003) leverages the figure of the principal or designer, a player ν whose objective $\ell_\nu : \Phi \times \ell_\varphi \to \mathbb{R}$ is defined over the space of outcome functions $\varphi \in \Phi$ and players modified utilities ℓ_φ. Possible objective functions for the principal ν to maximize may include *social welfare (utilitarian)* $\ell_\nu = \sum_i \tilde{\ell}_i$, the *worst utility (egalitarian)* of any player $\ell_\nu = \min_i \tilde{\ell}_i$ or *dictatorial* $\ell_\nu = \tilde{\ell}_{i=k}$, among others (Sandholm, 2003). The principal's objective is selected according to an equilibrium concept required for players in the induced game (Conitzer & Sandholm, 2003). In general, the automated design of a mechanism is reduced to finding, after the outcome and action space are manually defined, the optimal outcome function φ^*, such that the objective:

$$\varphi^* = \arg\max_{\varphi \in \Phi} \ell_\nu \tag{21}$$

enforces the preferences of the principal player ν over the game $\mathcal{G}_F$ induced by the mechanism $\mathcal{F}$.

C Additional Plots

Here are presented the rest of the zero-shot OODZ generalization plots.

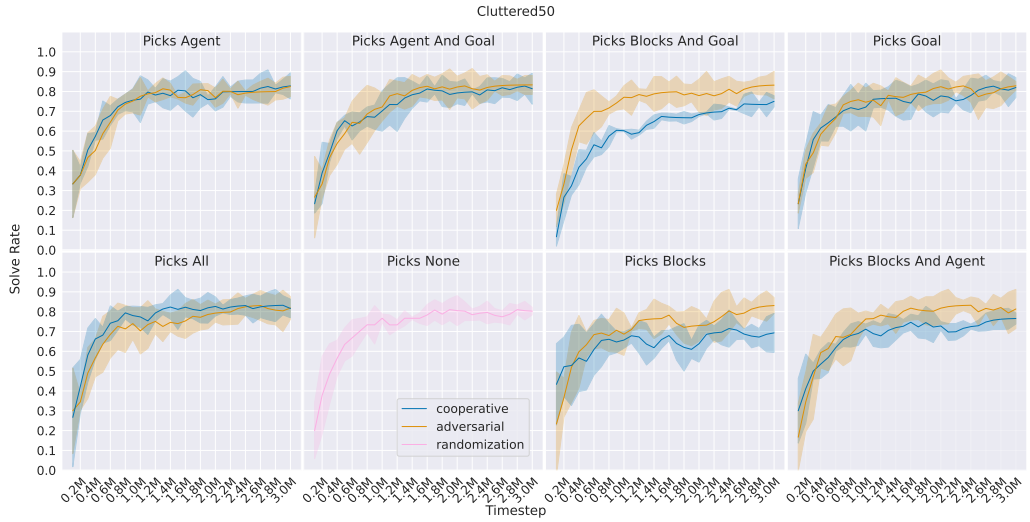

Figure 4: Zero-shot OOD generalization in the Cluttered50 environment.

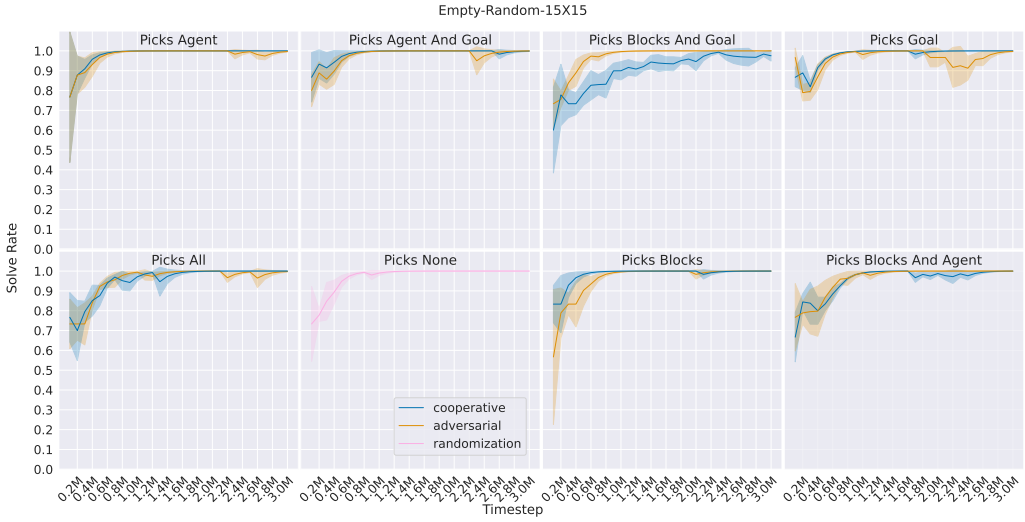

Figure 5: Zero-shot OOD generalization in the Empty-Random-15x15 environment.

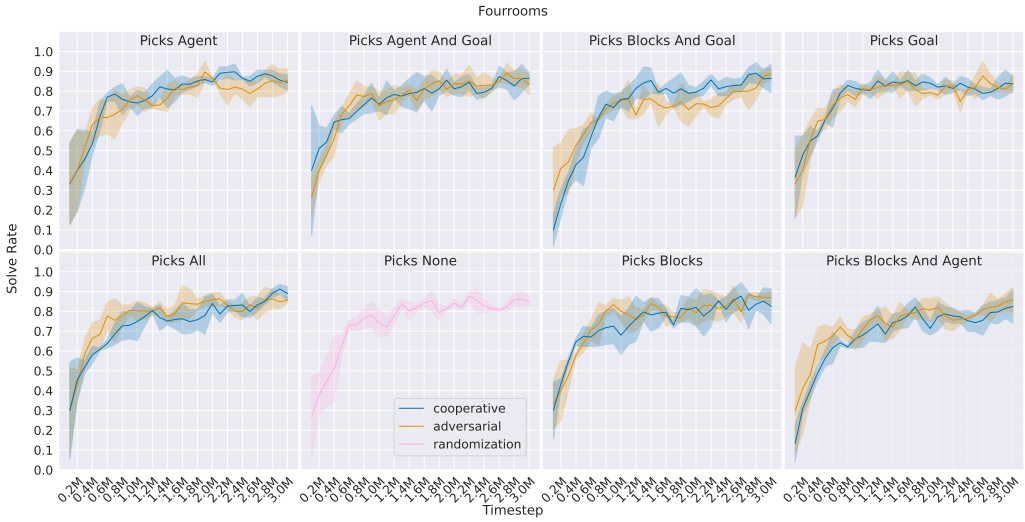

Figure 6: Zero-shot OOD generalization in the FourRooms environment.

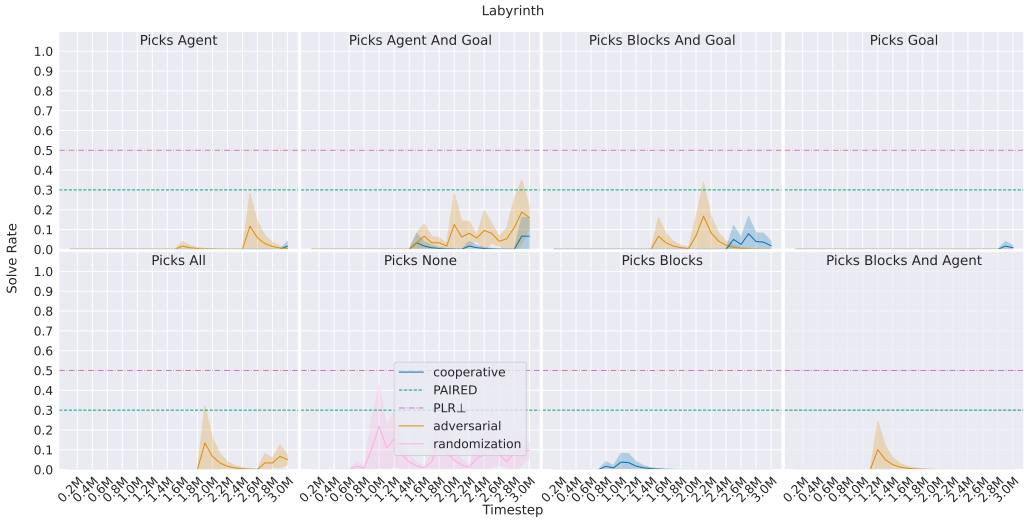

Figure 7: Zero-shot OOD generalization in the Labyrinth environment.