ON THE ROLE OF PLANNING IN MODEL-BASED DEEP REINFORCEMENT LEARNING

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ABSTRACT

Model-based planning is often thought to be necessary for deep, careful reasoning and generalization in artificial agents. While recent successes of model-based reinforcement learning (MBRL) with deep function approximation have strengthened this hypothesis, the resulting diversity of model-based methods has also made it difficult to track which components drive success and why. In this paper, we seek to disentangle the contributions of recent methods by focusing on three questions: (1) How does planning benefit MBRL agents? (2) Within planning, what choices drive performance? (3) To what extent does planning improve generalization? To answer these questions, we study the performance of MuZero [51], a state-of-the-art MBRL algorithm, under a number of interventions and ablations and across a wide range of environments including control tasks, Atari, and 9x9 Go. Our results suggest the following: (1) The primary benefit of planning is in driving policy learning. (2) Using shallow trees with simple Monte-Carlo rollouts is as performant as more complex methods, except in the most difficult reasoning tasks. (3) Planning alone is insufficient to drive strong generalization. These results indicate where and how to utilize planning in reinforcement learning settings, and highlight a number of open questions for future MBRL research.

Model-based reinforcement learning (MBRL) has seen much interest in recent years, with advances yielding impressive gains over model-free methods in data efficiency [10, 12, 20, 67], zero- and few-shot learning [13, 31, 53], and strategic thinking [3, 55, 56, 57, 51]. These methods combine planning and learning in a variety of ways, with *planning* specifically referring to the process of using a learned or given model of the world to construct imagined future trajectories or plans.

Many have suggested that models will play a key role in generally intelligent artificial agents [48, 49, 50, 59], with such arguments often appealing to model-based aspects of human cognition as proof of their importance [19, 21, 23, 35]. While the recent successes of MBRL methods lend evidence to this hypothesis, there is huge variance in the algorithmic choices made to support such advances. For example, planning can be used to select actions at evaluation time [e.g., 10] and/or for policy learning [e.g., 28]; models can be used within discrete search [e.g., 51] or gradient-based planning [e.g., 20, 24]; and models can be given [e.g., 39] or learned [e.g., 10]. Worryingly, some works even come to contradictory conclusions, such as that long rollouts can hurt performance due to compounding model errors in some settings [e.g., 28], while performance continues to increase with search depth in others [51]. Given the inconsistencies and non-overlapping choices across the literature, it can be hard to get a clear picture of the full MBRL space. This in turn makes it difficult for practitioners to decide which form of MBRL is best for a given problem (if any).

The aim of this paper is to assess the strengths and weaknesses of recent advances in MBRL to help clarify the state of the field. We systematically study the role of planning and its algorithmic design choices in a recent state-of-the-art MBRL algorithm, MuZero [51]. To do so, we evaluate overall reward obtained by MuZero across a wide range of (mostly) fully-observable and deterministic domains: the DeepMind Control Suite [62], Atari [7], Sokoban [44], Minipacman [17], and 9x9 Go [36]. Aside from being a strong algorithm to begin with, MuZero's use of canonical MBRL components (e.g., search-based planning, a learned model, value estimation, and policy optimization) make it a good candidate for building intuition about MBRL methods more generally.

To assess how planning contributes to overall performance or reward in MBRL, we ask three questions. (1) For what purposes is planning most useful? Our results show that planning—which can

be used separately for policy improvement, generating the distribution of experience to learn from, and acting at test-time—is most useful for credit assignment. (2) What design choices in the search procedure contribute most to the learning process? We show that deep, precise planning is often unnecessary to achieve high reward in many domains, with two-step planning exhibiting surprisingly strong performance even in Go. (3) Does planning assist in generalization across variations of the environment—a common motivation for model-based reasoning? We find that while planning can help make up for small amounts of distribution shift given a good enough model, it is not capable of inducing strong zero-shot generalization on its own.

1 Background and Related Work

Model-based reinforcement learning (MBRL) [8, 21, 41, 43, 66] involves both learning and planning. For our purposes, *learning* refers to deep learning of a model, policy, and/or value function. *Planning* refers to using a learned or given model to construct trajectories or plans.

MBRL methods can be broadly classified into *decision-time* planning, which use the model to select actions, and *background* planning, which use the model to update a policy [60]. For example, model-predictive control (MPC) [9] is a classic decision-time planning method that uses the model to optimize a sequence of actions starting from the current environment state. Decision-time planning methods often feature robustness to uncertainty and fast adaptation to new scenarios [e.g., 67], though may be insufficient in settings which require long-term reasoning such as in sparse reward tasks or strategic games like Go. Conversely, Dyna [59] is a classic background planning method which uses the the model to simulate data on which to train a policy via standard model-free methods like Q-learning or policy gradient. Background planning methods often feature improved data efficiency over model-free methods [e.g., 28], but exhibit the same drawbacks as model-free approaches such as brittleness to out-of-distribution experience at test time.

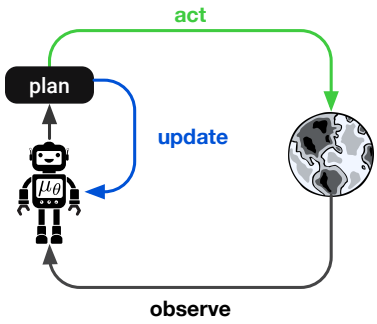


Figure 1: Model-based approximate policy iteration. The agent **updates** its policy using targets computed via planning and optionally **acts** via planning during training, at test time, or both.

A number of works have adopted hybrid approaches combining both decision-time and background planning. For example, [18, 42] distill the results of a decision-time planner into a policy. [54, 63] do the opposite, allowing a policy to guide the behavior of a decision-time planner. Other works do both, incorporating the distillation or imitation step into the learning loop by allowing the distilled policy from the previous iteration to guide planning on the next iteration, illustrated by the "update" arrow in Figure 1. This results in a form of approximate policy iteration which can be implemented both using single-step [34, 47] or multi-step [14, 15] updates, the latter of which is also referred to as expert iteration [3] or dual policy iteration [58]. Such algorithms have succeeded in board games [3, 4, 56, 57], discrete-action MDPs [22, 44, 51] and continuous control [37, 39].

In this paper, we focus our investigation on MuZero [51], a state-of-the-art member of the approximate policy iteration family. MuZero is a useful testbed for our analysis not just because of its strong performance, but also because it exhibits important connections to many other works in the MBRL literature. For example, Grill et al. [16] showed that multi-step policy iteration implemented via MCTS approximates the TRPO update, establishing a link between MCTS-based approximate policy iteration and Dyna-like methods that perform policy updates using TRPO [52] or MPO [1] on imagined trajectories [28, 33, 40, 45]. Thus, our results with MuZero have implications not just for its own immediate family, but to other MBRL methods more broadly.

In contrast to other work in MBRL which focuses primarily on data efficiency [e.g., 28, 30] or model learning [e.g., 10], our primary concern in this paper is in characterizing the role of planning with respect to reward after a large but fixed number of learning steps, as well as to zero-shot generalization performance (however, we have also found our results to hold when performing the same experiments and measuring approximate regret). We note that MuZero can also be used in a

more data-efficient manner ("MuZero Reanalyze") by using the search to re-compute targets on the same data multiple times [51], but leave exploration of its behavior in this regime to future work.

Our analysis joins a number of other recent works that seek to better understand the landscape of MBRL methods and the implications of their design choices. For example, [10] perform a careful analysis of methods for uncertainty quantification in learned models. Other research has investigated the effect of deep versus shallow planning [27, 29], the utility of parametric models in Dyna over replay [65], and benchmark performance of a large number of popular MBRL algorithms in continuous control tasks [66]. Our work is complementary to these prior works and focuses instead on the different ways that planning may be used both during training and at evaluation.

2 Preliminaries: Overview of MuZero

MuZero uses learned policies and value, transition and reward models within Monte-Carlo tree search (MCTS) [11, 32] both to select actions and to generate targets for policy learning (Figure 1). We provide a brief overview of MuZero here and refer readers to Appendix A and [51] for further details. Algorithm 1 and 2 present pseudocode for MuZero and MCTS, respectively.

Model MuZero plans in a hidden state space using a learned model μ_θ parameterized by θ and comprised of three functions. At timestep t, the *encoder* embeds past observations into a hidden state, $s_t^0 = h_\theta(o_1, \ldots, o_t)$. Given a hidden state and an action in the original action space, the (deterministic) recurrent *dynamics* function predicts rewards and next states, $r_{\theta,t}^k, s_t^k = g_\theta(s_t^{k-1}, a_t^{k-1})$, where k is the number of imagined steps into the future starting from a real observation at time t. In addition, the *prior* (unrelated to the Bayesian usage of the term) predicts a policy and value for a given hidden state, $\pi_{\theta,t}^k, v_{\theta,t}^k = f_\theta(s_t^k)$.

Search Beginning at the root node s_t^0, each simulation traverses the search tree according to a *search policy* until a previously unexplored action is reached. The search policy is a variation on the pUCT rule [46, 32] that balances exploitation and exploration, and incorporates the policy to guide the latter (see Equation 1 in Section A.3). After selecting an unexplored action a^ℓ at state s_t^ℓ, the tree is expanded by adding a new node with reward $r_{\theta,t}^{\ell+1}$, state $s_t^{\ell+1}$, policy $\pi_{\theta,t}^{\ell+1}$, and value $v_{\theta,t}^{\ell+1}$ predicted by the model. The value and reward are used to form a bootstrapped estimate of the cumulative discounted reward, which is backed up to the root, updating the estimated return Q and visit count N of each node on the path. After B simulations, MCTS returns a value v_t^{MCTS} (the average cumulative discounted reward at the root) and policy π_t^{MCTS} (a function of the normalized count of the actions taken at the root during search).

Acting After search, an action is sampled from the MCTS policy, $a_t \sim \pi_t^{\text{MCTS}}$, and is executed in the environment to obtain reward r_t^{env}. Data from the search and environment are then added to a replay buffer for use in learning: $\{o_t, a_t, r_t^{\text{env}}, \pi_t^{\text{MCTS}}, v_t^{\text{MCTS}}\}$.

Learning The model is jointly trained to predict the reward, policy, and value for each future timestep $k = 0 \ldots K$. The reward target is the observed environment reward, r_{t+k}^{env}. The policy target is the MCTS-constructed policy π_{t+k}^{MCTS}. The value target is the n-step bootstrapped discounted return $z_t = r_{t+1}^{\text{env}} + \gamma r_{t+2}^{\text{env}} + \cdots + \gamma^{n-1} r_{t+n}^{\text{env}} + \gamma^n v_{t+n}^{\text{MCTS}}$. For reward, value, and policy losses ℓ^r, ℓ^v, and ℓ^p, respectively, the overall loss is then $\ell_t(\theta) = \sum_{k=0}^{K} \ell^r(r_{\theta,t}^k, r_{t+k}^{\text{env}}) + \ell^v(v_{\theta,t}^k, z_{t+k}) + \ell^p(\pi_{\theta,t}^k, \pi_{t+k}^{\text{MCTS}})$. Note that MuZero is not trained to predict future observations or hidden states: the learning signal for the dynamics comes solely from predicting future rewards, values, and policies.

3 Hypotheses and Experimental Methods

Our investigation focuses on three key questions: (1) How does planning drive performance in MBRL? (2) How do different design choices in the planner affect performance? (3) To what extent does planning support generalization?

(1) Overall contributions of planning In model-free RL, the best algorithms work well because they compute useful policy improvement targets. We hypothesized that, similarly, using the search for policy improvement (as opposed to exploration or acting) is a primary driver of MuZero's performance, and that the ability to compare the outcome of different actions via search should enable

even finer-grained and therefore more powerful credit assignment. To test this hypothesis, we implemented two additional variants of MuZero which both use search to compute targets for learning, but which may act by sampling actions from the policy prior $\pi_{\theta,t}$ rather than from π^{MCTS}. In the first variant ("Learn", see table in Figure 3), we act from the prior (i.e., without search) both during training and testing, resulting in a "pure credit assignment" variant of MuZero. In the second variant ("Learn+Data"), we act from the prior only at test time. If the "Learn" variant of MuZero performs well, then it suggests that the primary use of search is to provide a learning signal for the policy.

(2) Planning for learning One feature of MCTS is its ability to perform "precise and sophisticated lookahead" [51]. To what extent does this lookahead support learning stronger policies? We hypothesized that more complex planning like tree search—as opposed to simpler planning, like random shooting—and larger search depth is most helpful for learning in games like Go and Sokoban, but less helpful for the other environments. To test this hypothesis, we manipulated three aspects of the search to make it more or less sophisticated: the maximum depth we search within the tree (D_{tree}), the maximum depth we optimize an exploration-exploitation tradeoff via pUCT (D_{UCT}), and the search budget[1] B (Figure 2, see also Section B.1). Note that our aim here is to evaluate the effect of *simpler* versus more complex planning, rather than the effect of exploration.

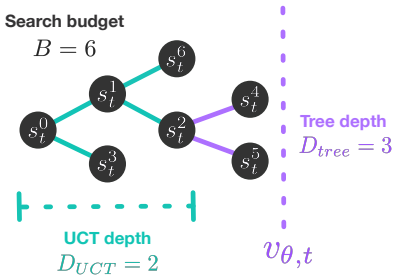


Figure 2: For nodes at depth $d < D_{\text{UCT}}$, we select actions according to pUCT (Section 2), while for nodes at depth $D_{\text{UCT}} \leq d < D_{\text{tree}}$, we select actions by sampling from $\pi_{\theta,t}$. Nodes at depth $d = D_{\text{tree}}$ (and deeper) are not expanded; instead, we stop the search and backup using $v_{\theta,t}$. The search budget B is equal to the number of nodes in the tree aside from the root s_t^0.

(3) Generalization in planning Model-based reasoning is often invoked as a way to support generalization and flexible reasoning [e.g., 21, 35]. We similarly hypothesized that given a good model, planning can help to improve zero-shot generalization. First, we evaluated the ability of the individual model components to generalize to new usage patterns. Specifically, we evaluated pre-trained agents using: larger search budgets than seen during training, either the learned model or the environment simulator, and either MCTS or breadth-first search (see Section B.3). Second, we tested generalization to unseen scenarios by evaluating pre-trained Minipacman agents of varying quality (assessed by the number of unique mazes seen during training) on novel mazes drawn from the same or different distributions as in training.

4 Results

We evaluated MuZero on eight tasks across five domains, selected to include popular MBRL environments with a wide range of characteristics including episode length, reward sparsity, and variation of initial conditions. First, we included two Atari games [7] which are commonly thought to require long-term coordinated behavior: **Ms. Pacman** and **Hero**. We additionally included **Minipacman** [44], a toy version of Ms. Pacman which supports procedural generation of mazes. We also included two strategic games that are thought to heavily rely on planning: **Sokoban** [44, 17] and **9x9 Go** [36]. Finally, because much work in MBRL focuses on continuous control [e.g., 66], we also included three tasks from the DeepMind Control Suite [62]: **Acrobot** (Sparse Swingup), **Cheetah** (Run), and **Humanoid** (Stand). We discretized the action space of the control tasks as in Tang & Agrawal [61], Grill et al. [16]. Further details of all environments are available in Appendix C.

4.1 Baselines

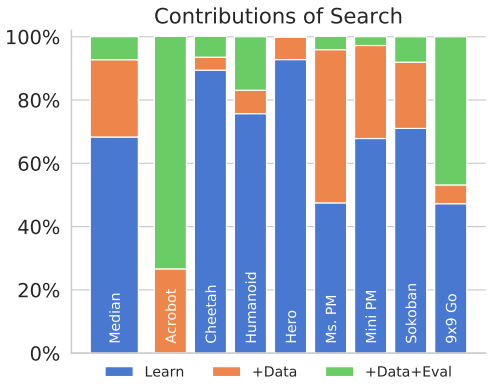
Before beginning our analysis, we tuned MuZero for each domain and ran baseline experiments with a search budget of $B = 10$ simulations in Minipacman, $B = 25$ in Sokoban, $B = 150$ in 9x9 Go, and $B = 50$ in all other environments. Additional hyperparameters for each environment are available in Appendix C and learning curves in Section D.2; unless otherwise specified, all further experiments used the same hyperparameters as the baselines. We obtained the following median final

[1]MuZero's policy targets suffer from degeneracies at low visit counts [16, 22]; to account for this, we used an MPO-style update [1] in the search budget experiments, similar to [16]. See Section B.2.

scores, computed using the last 10% of steps during training (median across five seeds): 626.31 on Acrobot, 882.61 on Cheetah, 813.12 on Humanoid, 28843.06 on Hero, 43735.44 on Ms. Pacman, 309.68 on Minipacman, 0.93 on Sokoban (proportion solved), and 0.75 on 9x9 Go (proportion games won against Pachi 10k [6], a bot with strong amateur play). These baselines are all very strong, with the Atari and Control Suite results being competitive with state-of-the-art [17, 26, 51].

4.2 Overall Contributions of Planning

We first compared vanilla MuZero ("Learn+Data+Eval" in Figure 3) to a pure credit assignment version ("Learn") and to a version that does not use search at test time ("Learn+Data"), normalizing scores with the baseline described in Section 4.1. Figure 3 shows the results. Across environments, the "Learn" variant has a median strength of 69.4% of the baseline, confirming our hypothesis that in many environments the learning signal provided by search is strong enough on its own to result in satisfactory performance. Allowing the agent to take actions via search during training further improves performance to a median strength of 92.8%, with a median improvement of 14.2 percentage points. This shows that search additionally drives performance by enabling the agent to learn from a different state distribution resulting from better actions—ultimately allowing it to learn a stronger value function—and echoing other recent work leveraging planning for exploration [39, 53]. Finally, search at evaluation adds a small contribution to overall performance, with a median increase of 7.3 percentage points.



	Train Update	Train Act	Test Act
Learn	search	prior	prior
+Data	search	search	prior
+Data+Eval	search	search	search

Figure 3: Contributions of planning to performance. The median % of baseline performance when: using planning to update the policy during training ("Learn"); to also act from planning during training ("Learn+Data"); and to also act from planning during testing ("Learn+Data+Eval"). See Figure 10 for error bars.

Search at evaluation time seems to help most in environments that require precise control (Acrobot, Humanoid, Cheetah) or strategic thinking (Go, Sokoban), while helping much less in Atari-like games (Hero, Ms. Pacman, Minipacman); see Table 10. Acting via search during learning is most useful in environments where many paths may lead to a high score (Ms. Pacman, Minipacman, Acrobot, Sokoban), but proves less useful in environments where the optimal policy only visits a constrained set of states (Humanoid, Hero, Cheetah) or where high data diversity may be facilitated through other means like self-play (Go). Using the search solely for computing policy targets is a substantial driver of performance in all environments except for Acrobot, where the pure credit assignment version of MuZero makes no progress on the task at all. While more investigation into this result is needed, we speculate it may be an example of how forward planning for credit assignment can fail, as discussed in [65].

4.3 Planning for Learning

Tree depth Figure 4a shows the result of varying tree depth $D_{\text{tree}} \in \{1, 2, 3, 5, \infty\}$ while keeping $D_{\text{UCT}} = D_{\text{tree}}$ and the search budget constant. Scores are normalized by the "Learn+Data" agent from Section 4.2. Strikingly, D_{tree} does not make much of a difference in most environments. Even in Sokoban and Go, we can recover reasonable performance using $D_{\text{tree}} = 2$, suggesting that deep tree search may not be necessary for learning a strong policy prior, even in the most difficult reasoning domains. Looking at individual effects within each domain, we find that very deep trees have a slight negative impact in Minipacman and an overall positive impact in Ms. Pacman, Acrobot, and Go (see Table 12). While we did not detect a quantitative effect in the other environments, qualitatively it appears as though very deep trees may also have a stabilizing effect in Sokoban, while causing worse performance in Acrobot and Hero.

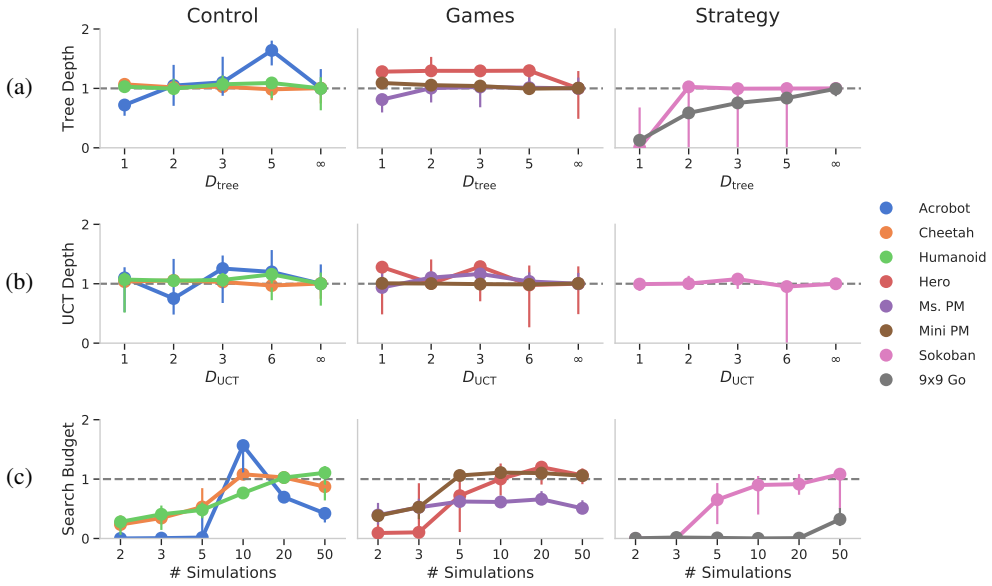


Figure 4: Effect of design choices on the strength of the policy prior. All colored lines show median normalized reward across five seeds, with error bars indicating min and max seeds. Rewards are normalized by the median scores in Table 8. All agents use search for learning and acting during training only. (a) Reward as a function of D_{tree}. Here, $D_{\text{UCT}} = D_{\text{tree}}$ and the number of simulations is the same as the baseline. (b) Reward as a function of D_{UCT}. Here, $D_{\text{tree}} = \infty$ and the number of simulations is the same as the baseline. (c) Reward as a function of search budget during learning. Here, $D_{\text{UCT}} = 1$ and $D_{\text{tree}} = \infty$ (except in Go, where $D_{\text{UCT}} = \infty$).

Exploration vs. exploitation depth Figure 4b shows the strength of the policy prior as a result of manipulating the pUCT depth $D_{\text{UCT}} \in \{1, 2, 3, 6, \infty\}$ while keeping $D_{\text{tree}} = \infty$ and the search budget constant. Note that $D_{\text{UCT}} = 1$ corresponds to only exploring with pUCT at the root node and performing pure Monte-Carlo sampling thereafter. Surprisingly, we find D_{UCT} to have no effect in any environment[2] (Table 13). Thus, exploration-exploitation deep within the search tree does not seem to matter at all in most standard MBRL settings.

Search budget Figure 4c shows the strength of the policy prior for different numbers of simulations, with $D_{\text{tree}} = \infty$ and $D_{\text{UCT}} = 1$ (except in Go, where $D_{\text{UCT}} = \infty$), corresponding to exploring different actions at the root node and then performing Monte-Carlo rollouts thereafter. We opted for these values as they correspond to a simpler form of planning, and our previous experiments showed that larger settings of D_{UCT} made little difference. We find an overall strong effect of the number of simulations on performance in all environments except Ms. Pacman (Table 14). However, despite the overall positive effect of the search budget, too many simulations have a detrimental effect in most environments, replicating work showing that some amount of planning can be beneficial, but too much can harm performance [e.g., 28]. Additionally, the results with Ms. Pacman suggest that two simulations provide enough signal to learn well in some settings. It is possible that with further tuning, other environments might also learn effectively with smaller search budgets.

4.4 Generalization in Planning

Model generalization to new search budgets Figure 5a shows the results of evaluating the baseline agents (Section 4.1) using up to 625 simulations. As before, we find a small but significant improvement in performance of 5.2 percentage points between the baseline and agents which do not use search at all ($t = -3.15, p = 0.02$). Both Acrobot and Sokoban exhibit slightly better performance with more simulations, and although we did not perform experiments here with Go, [51] did and found a positive impact. However, other environments show no overall effect (Table 15). The median reward obtained across environments at 625 simulations is also less than the

[2]We were unable to run D_{UCT} experiments on Go for this submission, but will include them in the revision.

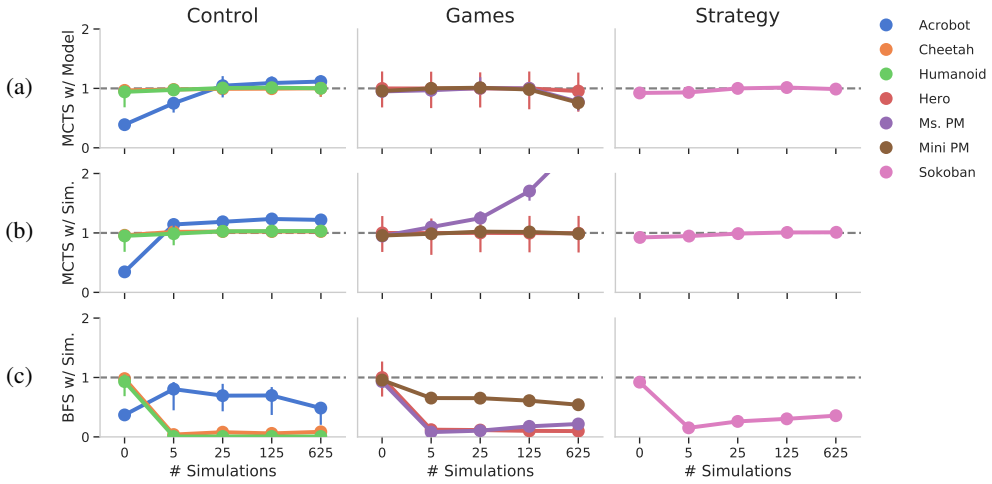


Figure 5: Effect of search at evaluation as a function of the number of simulations, normalized by the median scores in Table 9. All colored lines show medians across seeds, with error bars indicating min and max seeds. (a) MCTS with the learned model. (b) MCTS with the environment simulator. (c) Breadth-first search (BFS) with the environment simulator. Results with the learned model are similar and can be seen in Figure 11.

baseline by a median of 3.6 percentage points ($t = -2.64, p = 0.06$), possibly indicating an effect of compounding model errors. This suggests that for identical training and testing environments, additional search may not always be the best use of computation; we speculate that it might be more worthwhile simply to perform additional Dyna-like training on already-observed data [see "MuZero Reanalyze", 51].

Policy and value generalization with a better model Planning with the simulator yields somewhat better results than planning with the learned model (Figure 5b), with all environments except Hero and Minipacman exhibiting positive rank correlations with the number of simulations (Table 16). Ms. Pacman, in particular, more than doubles in performance after 625 simulations[3]. However, across environments, 25, 125, and 625 simulations only increased performance over the baseline by a median of 2-3 percentage points. Thus, while planning with a better model can indeed help more than with the learned model, the gains are often modest.

Model generalization to new planners We find dramatic differences between MCTS and BFS, with BFS exhibiting a catastrophic drop in performance with any amount of search. This is true both when using the learned model (Figure 11, Appendix) and the simulator (Figure 5c). Note that in the simulator case, the only learned component that is relied on is the value function. This suggests a mismatch between the value function and the policy prior, where low-probability (off-policy) actions are more likely to have high value errors, thus causing problems when expanded by BFS. This issue is similar to that explored by [22], and may benefit from a similar solution to train values for *all* actions based on those estimated during search. Overall, we emphasize that in complex agent architectures involving multiple learned components, compounding error in the transition model is not the only source of error to be concerned about.

Generalizing to new mazes We trained Minipacman agents on 5, 10, or 100 unique mazes and then tested them on new mazes drawn either from the same distribution or a different distribution. Figure 6 shows the out-of-distribution results and Figure 12 the in-distribution results. Using the learned model, we see very slight gains in performance up to 125 simulations on both in-distribution and out-of-distribution mazes, with a sharp drop-off in performance after that reflecting compounding model errors in longer trajectories. The simulator allows for somewhat better performance, with greater improvements for small numbers of train mazes ($t = -7.71, p < 0.001$, see also Table 17), indicating the ability of search to help with some amount of distribution shift when using an accurate model. However, as can be seen in the figure, this performance plateaus at a much lower value than

[3]However, using the simulator leaks information about the unobserved environment state, such as when the ghosts will stop being edible. Thus, these gains may overestimate what is achievable by a realistic model.

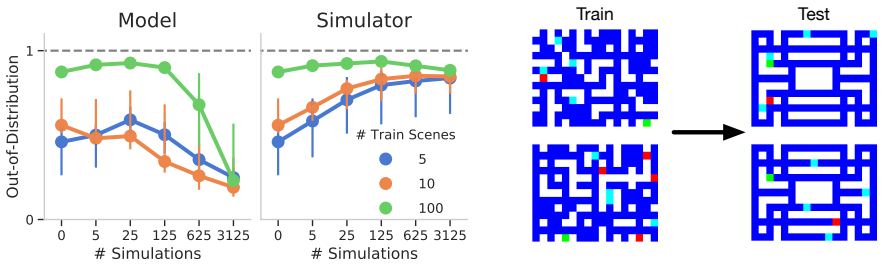


Figure 6: Generalization to out-of-distribution mazes in Minipacman. All points are medians across seeds (normalized by the median scores in Table 9), with error bars showing min and max seeds. Colors indicate agents trained on different numbers of unique mazes. The dotted lines indicate the baseline. The maps on the right give examples of the types of mazes seen during train and test. In-distribution generalization is shown in Figure 12 (Appendix) as the behavior is similar.

what would be obtained by training the agent directly on the task. Moreover, reward obtained by the simulator decreases at 3125 simulations compared to 125 simulations ($t = -3.56, p = 0.002$), again indicating a sensitivity to errors in the value function and policy prior.

5 Discussion

In this work, we explored the role of planning in MBRL through a number of ablations and modifications to MuZero [51]. We sought to answer three questions: (1) In what ways does planning contribute to final performance? (2) What design choices within the planner contribute to stronger policy learning? (3) How well does planning support zero-shot generalization? In most environments, we find that (1) search is most useful in constructing targets for policy learning; (2) simpler and shallower planning is often as performant as more complex planning; and (3) search at evaluation time only slightly improves zero-shot generalization, and even then only if the model is highly accurate.

A major takeaway from this work is that search is most useful in constructing policy targets for learning, and that simple, shallow forms of planning may be sufficient for constructing such targets. This has important implications in terms of computational efficiency: the algorithm with $D_{\text{UCT}} = 1$ can be implemented without trees and is thus far easier to parallelize than MCTS, and the algorithm with $D_{\text{UCT}} = 1$ and $D_{\text{tree}} = 1$ can be implemented via model-free techniques [e.g., 1], suggesting that MBRL may not be necessary at all for strong final performance in some domains. Moreover, given that search seems to provide minimal improvements at evaluation in many standard RL environments, it may be computationally prudent to avoid using search altogether at test time.

The result that deep or complex planning is not always needed suggests that many popular environments used in MBRL may not be fully testing the ability of model-based agents (or RL agents in general) to perform sophisticated reasoning. This may be true even for environments which seem intuitively to require reasoning, such as Sokoban. Indeed, out of all our environments, only Acrobot and 9x9 Go strongly benefited from search at evaluation time. We therefore emphasize that for work which aims to build flexible and generalizable model-based agents, it is important to evaluate on a diverse range of settings that stress different types of reasoning.

Our generalization experiments pose a further puzzle for research on model-based reasoning. Even given a model with good generalization (e.g., the simulator), search in challenging environments is ineffective without a strong value function or policy to guide it. Indeed, our BFS experiments demonstrate that if the value function and policy themselves do not generalize, then generalization to new settings will also suffer. But, if the value function and policy *do* generalize, then it is unclear whether a model is even needed. We suggest that identifying good inductive biases for policies which capture something about the world dynamics [e.g., 5, 17], as well as learning appropriate abstractions [25], may be as or more important than learning better models in driving generalization.

Overall, this work provides a new perspective on the contributions of search and planning in integrated MBRL agents like MuZero. We note that our analysis has been limited to (mostly) fully-observable, deterministic environments, and see similar studies focusing on partially-observed and stochastic environments as an important area for future work.

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A MuZero Algorithm Details

A.1 MuZero pseudocode

Pseudocode for MuZero [51] is presented in Algorithm 1. Following initialization of the weights and the empty circular replay buffer $\mathcal{D}$, a learner and a number of actors execute in parallel, reading from and sending data to the replay buffer. In our experiments, the actors update their copies of the parameters θ after every 500 learner steps. Our setup and description follow that of Schrittwieser et al. [51].

Algorithm 1 MuZero [51]

```
Initialize model μ_θ and dataset D
function ACTOR
    while true do
        o ← initialize episode
        while episode not finished do
            s ← h_θ(...,o)
            π^MCTS, v^MCTS ← MCTS(s, μ_θ)      ▷ Alternative: Monte-Carlo rollouts, BFS, etc.
            a ~ π^MCTS                          ▷ Alternative: sample action from π_θ
            r^env, o' ← execute a in environment
            Add o, a, r^env, o', π^MCTS, v^MCTS to D
            o ← o'
        end while
    end while
end function
function LEARNER
    while True do
        Sample batch of trajectories B from D
        ℓ ← compute loss (Equation 8)) on B
        Update θ with gradient descent on ℓ
    end while
end function
```

A.2 Model details

In MuZero, the model μ_θ is trained to directly predict three quantities for each future timestep $k = 1 \ldots K$. These are the policy $\pi^k_{\theta,t} \approx \pi^{\text{MCTS}}(a_{t+k}|o_1, \ldots, o_t, a_t, \ldots, a_{t+k-1})$, the value function $v^k_{\theta,t} \approx \mathbb{E}\left[r^{\text{env}}_{t+k+1} + \gamma r^{\text{env}}_{t+k+2} + \ldots |o_1, \ldots, o_t, a_t, \ldots, a_{t+k-1}\right]$, and the immediate reward $r^k_{\theta,t} \approx r^{\text{env}}_{t+k}$, where π^{MCTS} is the policy used to select actions in the environment, r^{env} is the observed reward, and γ is the environment discount factor.

A.3 MCTS details

Pseudocode for MCTS is presented in Algorithm 2. MuZero uses the pUCT rule [46] for the search policy. The pUCT rule maximizes an upper confidence bound [32] to balance exploration and exploitation during search. Specifically, the pUCT rule selects action

$$a^k = \arg\max_a \left[Q(s,a) + \pi_\theta(s,a) \cdot \frac{\sqrt{\sum_b N(s,b)}}{1 + N(s,a)} \cdot \left(c_1 + \log\left(\frac{\sum_b N(s,b) + c_2 + 1}{c_2} \right) \right) \right], \tag{1}$$

where N is the number of times a has been selected during search, Q is the average cumulative discounted reward of a, and c_1, c_2 are constants that control the relative influence of Q and π_θ. Following Schrittwieser et al. [51], we set $c_1 = 1.25$ and $c_2 = 19652$ in our experiments. At the root node alone, a small amount of Dirichlet noise is added to the policy prior π_θ to encourage additional exploration at the root.

Algorithm 2 MCTS in MuZero

function MCTS(root state s^0, model μ_θ, number of simulations B)
  initialize edge statistics $\{(N(s^0, a), Q(s^0, a))\}_a$ to 0
  for $k = 1 \dots S$ **do**
    $r^l, s^l, v^l \leftarrow$ SEARCHANDEXPAND(s^0)
    BACKUP(r^l, s^l, v^l)
  end for
  return π^{MCTS} and v^{MCTS} (Equation 6 and 7)
end function

function SEARCHANDEXPAND(s)
  while true **do**
    $a \leftarrow$ PUCT(s) ▷ *choose action according to pUCT*
    $r', s' \leftarrow$ TRANSITION(s, a) ▷ *use cached values if possible*
    if $N(s, a) = 0$ **then**
      add node s' as child of s on edge a
      initialize edge statistics $\{(N(s', a'), Q(s', a'))\}_{a'}$ to 0
      compute $\pi_\theta, v_\theta \leftarrow f_\theta(s')$
      return r', s', v_θ
    end if
    $s \leftarrow s'$
  end while
end function

function BACKUP(r', s', v')
  for each edge on the path from s' to the root s^0 **do**
    update statistics N, Q with Equation 3 and 4
  end for
end function

Within the search tree, each node has an associated hidden state s. For each action a from s there is an edge (s, a) on which the number of visits $N(s, a)$ and the current value estimate $Q(s, a)$ are stored. When a new node with state s is created in the expansion step, the statistics for each edge are initialized as $\{N(s, a) = 0, Q(s, a) = 0\}$. The estimated policy prior $\pi_\theta(s, a)$, reward r_θ, and state transition are also stored after being computed on expansion since they are deterministic and can be cached. Thus, the model only needs to be evaluated once per simulation when the new leaf is added.

When backing up after expansion, the statistics on all of the edges from the leaf to the root are updated. Specifically, for $k = l \dots 0$, a bootstrapped $l - k$-step cumulative discounted reward estimate

$$G^k = \sum_{\tau=0}^{\ell-1-k} \gamma^\tau r_\theta^{k+1+\tau} + \gamma^{\ell-k} v_\theta^\ell, \tag{2}$$

is computed. The statistics for each edge (s^k, a^k) for $k = 0 \dots l - 1$ in the simulation path are updated as

$$Q(s^k, a^k) = \frac{N(s^k, a^k) \cdot Q(s^k, a^k) + G^k}{N(s^k, a^k) + 1} \tag{3}$$

$$N(s^k, a^k) = N(s^k, a^k) + 1. \tag{4}$$

To keep Q estimates bounded within $[0, 1]$, the Q estimates are first normalized as $\bar{Q} \in [0, 1]$ before passing them to the pUCT rule. The normalized estimates are computed as

$$\bar{Q}(s^k, a^k) = \frac{Q(s^k, a^k) - Q_{\min}}{Q_{\max} - Q_{\min}}, \tag{5}$$

where $Q_{\min} = \min_{(s,a) \in \text{Tree}} Q(s, a)$ and $Q_{\max} = \max_{(s,a) \in \text{Tree}} Q(s, a)$ are the minimum and maximum Q values observed in the search tree so far.

After all simulations are complete, the policy π^{MCTS} returned by MCTS is the visit count distribution at the root s^0 parameterized by a temperature T

$$\pi^{\text{MCTS}}(a) = \frac{N(s^0, a)^{1/T}}{\sum_b N(s^0, b)^{1/T}}. \tag{6}$$

During training, the temperature is set as a function of the number of learner update steps. Specifically, the temperature is set to 1 and then decayed by a factor of 0.95 after every 5000 steps.

The value v^{MCTS} returned by MCTS is the average discounted return over all simulations

$$v^{\text{MCTS}} = \sum_a \left(\frac{N(s^0, a)}{\sum_b N(s^0, b)} \right) Q(s^0, a). \tag{7}$$

A.4 Training details

The MCTS policy is used to select an action $a_t \sim \pi_t^{\text{MCTS}}$, which is then executed in the environment and a reward r_t^{env} observed. The model is jointly trained to match targets constructed from the observed rewards and the MCTS policy and value for each future timestep k. The policy targets are simply the MCTS policies, while the value targets are the n-step bootstrapped discounted returns $z_t = r_{t+1}^{\text{env}} + \gamma r_{t+2}^{\text{env}} + \cdots + \gamma^{n-1} r_{t+n}^{\text{env}} + \gamma^n v_{t+n}^{\text{MCTS}}$. For reward, value, and policy losses ℓ^r, ℓ^v, and ℓ^p, respectively, the overall loss is then

$$\ell_t(\theta) = \sum_{k=0}^{K} \ell^r(r_{\theta,t}^k, r_{t+k}^{\text{env}}) + \ell^v(v_{\theta,t}^k, z_{t+k}) + \ell^p(\pi_{\theta,t}^k, \pi_{t+k}^{\text{MCTS}}) + c||\theta||^2, \tag{8}$$

where $c||\theta||^2$ is an L2 regularization term. For the rewards, values, and policies, a cross-entropy loss is used for each of ℓ^r, ℓ^v, and ℓ^p.

B Further Implementation Details

B.1 Depth-limited MCTS

The depth-limited MCTS algorithm is implemented by replacing the regular MCTS search and expand subroutines with a modified subroutine as follows. The backup subroutine remains unchanged.

Algorithm 3 Search and expand subroutine for depth-limited MCTS.

function SearchAndExpand(root state s^0, max UCT depth D_{UCT}, max tree depth D_{tree})
  $k \leftarrow 0$
  while $k < D_{\text{tree}}$ and s^k is not a leaf **do** ▷ *Search for leaf node*
    $a \leftarrow$ SelectAction(s^k, k, D_{UCT})
    $s^{k+1} \leftarrow$ Transition(s^k, a)
    $k \leftarrow k + 1$
  end while
  if $k < D_{\text{tree}}$ **then** ▷ *Expand unless maximum depth is reached*
    $a \leftarrow$ SelectAction(s^k, k, D_{UCT})
    $s^{k+1} \leftarrow$ transition(s^k, a)
    Add s^{k+1} to tree
  end if
  return s^{k+1}
end function

function SelectAction(s, k, D_{UCT}) ▷ *The search policy*
  if $k < D_{\text{UCT}}$ **then**
    $a \leftarrow$ pUCT(s) ▷ *Choose action according to pUCT*
  else
    $a \sim \pi_\theta(\cdot|s)$ ▷ *Sample from prior*
  end if
  return a
end function

We initially tried varying D_{tree} and D_{UCT} together on Ms. Pacman and Minipacman. However, as we did not see any effect, we varied these variables separately for the remainder of our experiments in order to limit computation.

We did not run any experiments with $D_{\text{UCT}} = 0$, which corresponds to pure Monte-Carlo search. This is because this variant introduces a confound: with $D_{\text{UCT}} = 0$, the visit counts are no longer informative and unsuitable for use as policy learning targets. To test $D_{\text{UCT}} = 0$ would therefore also require modifying the learning target. Future work could test this by using the same MPO update described in the next section.

Additionally, we note that depth-limited MCTS will converge to the BFS solution (Section B.3) in the limit of infinite simulations. For finite simulations, depth-limited MCTS interpolates between the policy prior and the BFS policy.

B.2 Regularized policy updates (MPO) with MCTS

Past work has demonstrated that MuZero's policy targets suffer from degeneracies at low visit counts [22, 16]. To account for this, we modified the policy targets to use an MPO-style update [1] rather than the visit count distribution, similar to [16]: $\pi^{\text{MPO}}_{t+k} \propto \pi^{k}_{\theta,t} \cdot \exp\left(\mathbf{q}^{\text{MCTS}}/\tau\right)$. Here, $\mathbf{q}^{\text{MCTS}}$ are the Q-values at the root node of the search tree; $\tau = 0.1$ is a temperature parameter; and we use π^{MPO}_{t+k} in place of π^{MCTS}_{t+k} in Equation 8. Note that the Q-values for unvisited actions are set to zero; while it is in general a poor estimate for the true Q-function, we found this choice to outperform setting the Q-function for unvisited actions to the value function. This is likely because unvisited actions are unlikely under the prior and perhaps ought not be reinforced unless good estimates are obtained through exploration. However, this choice leads to a biased MPO update; how to unbias it will be a topic of further research. Similarly, we chose the MPO update for its ease of implementation, but it is likely other forms of regularized policy gradient (e.g. TRPO [52], or more generally natural or mirror policy optimization [64, 2]) would result in quantitively similar findings. We also did not tune τ for different environments; it is likely that properly tuning it could further improve performance of the agent at small search budgets.

B.3 Breadth-first search

In our generalization experiments, we replaced the MCTS search with a breadth-first search algorithm. BFS explores all children at a particular depth of the tree in an arbitrary order before progressing deeper in the tree. Our implementation of BFS does not use the policy prior. Additionally, when performing backups, we compute the maximum over all values seen rather than averaging. Final actions are selected based on the highest Q-value after search, rather than highest visit count. Thus, this implementation of BFS is (1) maximally exploratory and (2) relies mostly on the value estimates $v_{\theta,t}$ (especially when the simulator is used instead of the learned model).

C Environment and Architecture Details

We evaluate on the following environments.

- **Minipacman**: a toy version of Ms. Pacman. We modified the version introduced by [17] such that the maze is procedurally generated on each episode. See Section C.1 for details.
- **Hero** (Atari): a sparse reward, visually complex video game. The goal is to navigate a character through a mine, clearing cave-ins, destroying enemies, and rescuing trapped miners.
- **Ms. Pacman** (Atari): a fast-paced, visually complex video game. The goal is to control Pacman to eat all the "food" in a maze, while avoiding being eaten by ghosts.
- **Acrobot Sparse Swingup** (Control Suite): a low-dimensional, yet challenging control task with sparse rewards. The task is to balance upright an under-actuated double pendulum.
- **Cheetah Run** (Control Suite): a six-dimensional control task, where the goal is to control the joints of a "cheetah" character to make it run forward in a 2D plane.

- **Humanoid Stand** (Control Suite): a 21-dimensional control task, where the goal is to control the joints of a humanoid character to make it stand up.
- **Sokoban**: a difficult puzzle game that involves pushing boxes onto targets and in which incorrect moves can be unrecoverable [44].
- **9x9 Go**: an easier version of Go than the full 19x19 game, provided by [36]. Evaluation is reported against Pachi [6], with 10^4 evaluations per move (strong amateur play).

We now provide specific details on the network architectures and hyperparameters used for each environment. Unless specified and for layers where it is appropriate, all layers in the networks use padding 'SAME' and stride 1, and convolutions are 2-D with 3×3 kernels. All networks consist of an encoder h_θ, a recurrently-applied dynamics function g_θ, and a prior function f_θ.

For most environments, the encoder h_θ is a resnet composed of a number of segments. Each segment consists of a convolution, a layer norm, a number of residual blocks, and a ReLU. Each residual block contains a layer norm, a ReLU, a convolution, a layer norm, a ReLU, and a final convolution. The input of the residual block is then added to the output of its final convolution.

For most environments, the dynamics network has the same structure as the encoder, with a different number of segments and residual blocks.

Finally, the prior function predicts three quantities and is composed of three separate networks: a policy head, a value head, and a reward head. The policy, value, and reward heads each consist of a 1×1 convolution followed by a number of linear layers, with a ReLU between each pair of layers.

The hyperparameters in Table 1 are shared across all environments except Go.

Table 1: Shared hyperparameters

Hyperparameters	Value
Dirichlet alpha	0.3
Exploration fraction	0.25
Exploration temperature	1.0
Temperature decay schedule	5×10^3
Temperature decay rate	0.95
Replay capacity	5×10^5
Min replay	10^5

C.1 MINIPACMAN

C.1.1 MAZES

For Minipacman, we altered the environment to support the use of both procedurally generated mazes ("in-distribution" mazes) and the standard maze ("out-of-distribution" mazes), both of size 15x19. Figure 6 shows example mazes of both types. In all experiments except generalization, we trained agents on an unlimited number of the "in-distribution" mazes, and tested on other mazes also drawn from this set. For the generalization experiments, we trained agents on a fixed set of either 5, 10, or 100 "in-distribution" mazes. Then, at evaluation time, we either tested on mazes drawn from the full in-distribution set or from the out-of-distribution set.

To generate the procedural mazes, we first used Prim's algorithm to generate corridors. To make the maze more navigable, we then randomly removed walls with a probability of $p = 0.3$. The number of initial ghosts was sampled as $g_0 \sim 1 + \text{Poisson}(1)$ and this number increased by $g_\Delta \sim 0.25 + U(0, 1)$ ghosts per level, such that the total number of ghosts at level l was $g_l = \lfloor g_0 + (l - 1)g_\Delta \rfloor$. The number of pills was always set to 4. The default Minipacman maze was hand crafted to be similar to the Ms. Pacman maze. In the default maze, there is always one initial ghost ($g_0 = 1$) and this number increases by 1 every two levels ($g_\Delta = 0.5$). We similarly set the number of pills to 4. In all mazes, the initial locations of the ghosts, pills, and Pacman is randomly chosen at the beginning of every episode.

C.1.2 NETWORK ARCHITECTURE

For Minipacman, the encoder has 2 segments, each with 64 channels and 2 residual blocks. The dynamics function has 1 segment with 5 residual blocks, all with 64 channels. The policy head has a convolution with 4 channels and one linear layer with a channel per action (in Minipacman, this is 5). The value head has a convolution with 32 channels and two linear layers with 64 and 601 channels. The reward network has the same structure as the value head.

All Minipacman experiments were run using 400 CPU-based actors and 1 NVIDIA V100 for the learner.

Table 2: Hyperparameters for Minipacman

Hyperparameters	Value
Learning rate	10^{-3}
Discount factor	0.97
Batch size	512
n-step return length	10
Replay samples per insert ratio	0.25
Learner steps	2×10^5
Policy loss weight	1.
Value loss weight	0.3
Num simulations	10
Max steps per episode	600

C.2 ATARI

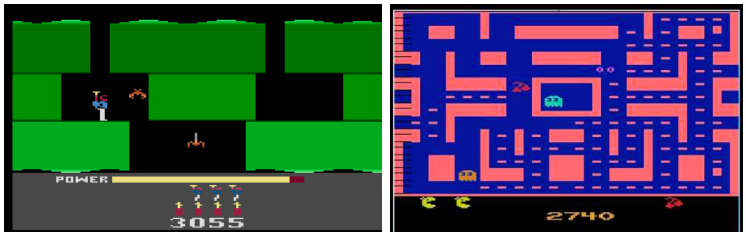

Figure 7: (*Left*) Hero, (*Right*) Ms.Pacman

The Atari Learning Environment [7] is a challenging benchmark of 57 classic Atari 2600 games played from pixel observations. We evaluate on Ms. Pacman and Hero. Each observation consists of the 4 previous frames and action repeats is 4.

For Atari, the encoder consists of 4 segments with $(64, 128, 128, 128)$ channels, each with 2 residual blocks, followed by 1 segment with 5 residual blocks with 128 channels. The dynamics network has 1 segment with 5 residual blocks with 128 channels. The heads are the same as in Minipacman.

All Atari experiments were run using 1024 CPU-based actors and 4 NVIDIA V100s for the learner.

C.3 CONTROL SUITE

The DeepMind Control Suite [62] is a widely used benchmark for control tasks in MuJoCo. We select 3 high-dimensional environments Cheetah (Run), Acrobot (Swing-up Sparse) and Humanoid (Stand). We use the raw state observation as input and follow the same procedure as [16, 61] to discretize the continuous action space. Each action dimension is discretized into 5 bins evenly spaced between $[-1, 1]$.

For Control Suite environments, we use a model based on that of [38]. The encoder consists of a linear layer with 300 channels, a layer norm, a tanh, a linear layer with 200 channels, and an exponential linear unit (ELU). The dynamics function is simply a linear layer with 200 channels

Table 3: Hyperparameters for Atari

Hyperparameters	Value
Learning rate	10^{-3}
Discount factor	0.995
Batch size	2048
n-step return length	10
Replay samples per insert ratio	0.25
Learner steps	1.5×10^5
Policy loss weight	1.
Value loss weight	0.3
Num simulations	50

Figure 8: Control Suite environment: *Left* Acrobot *Middle* Cheetah *Right* Humanoid.

followed by an ELU. The weights in the encoder and dynamics function are initialized uniformly. The policy head is a factored policy head composed of a linear layer with # dimensions $\times$ # bins channels. The factored policy head independently chooses an action for each dimension. The value and reward heads are both composed of a linear layer with 64 channels followed by a ReLU and then a linear layer with 2001 channels.

All Control Suite experiments were run using 1024 CPU-based actors and 2 second-generation (v2) Tensor Processing Units (TPUs) for the learner.

Table 4: Hyperparameters for control suite.

Hyperparameters	Cheetah (Run)	Acrobot (Swing-up Sparse)	Humanoid (Stand)
Learning rate	5×10^{-4}	2.5×10^{-4}	2.5×10^{-4}
Discount factor	0.995	0.995	0.995
Batch size	1024	1024	1024
n-step return length	50	50	30
Replay samples per insert ratio	25.	2.	15.
Learner steps	5×10^6	5×10^6	5×10^6
Policy loss weight	1.	1.	1.
Value loss weight	0.5	0.5	0.5
Num simulations	50	50	50

C.4 SOKOBAN

Sokoban [44] is a classic puzzle problem, where the agent's task is to push a number of boxes onto target locations. In this environment many moves are irreversible as the boxes can only be pushed forward and hence the puzzle can become unsolvable if wrong moves are made.

For Sokoban, the encoder is the same as that used for Minipacman with an additional 256-channel 1×1 convolutional layer at the end. Instead of a resnet, for the dynamics network for Sokoban we used a DRC(3, 1) convolutional LSTM [17]. For the heads, we use the same network as in

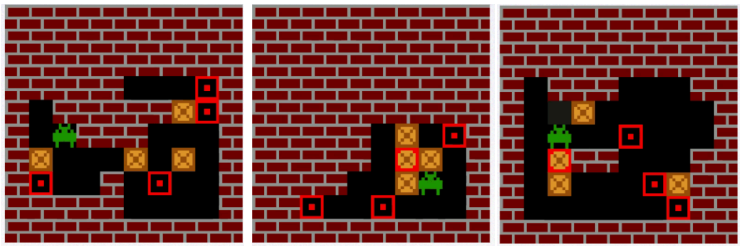

Figure 9: Sokoban environment.

Minipacman except that the convolutional layers in the policy, value, and reward heads have 32, 32, and 16 channels, respectively.

All Sokoban experiments were run using 2048 CPU-based actors and 4 NVIDIA V100s for the learner.

Table 5: Hyperparameters for Sokoban

Hyperparameters	Value
Learning rate	10^{-3}
Discount factor	0.99
Batch size	2048
n-step return length	10
Replay samples per insert ratio	0.4
Learner steps	3×10^5
Policy loss weight	1.
Value loss weight	0.3
Num simulations	25

C.5 9x9 Go

For the Go experiments, we used a different implementation of the MuZero algorithm due to easier interfacing with the Go environment. The main difference between the implementation used for other environments and the one for Go is the data pipeline. In the first implementation, actors and learner communicate asynchronously through a replay buffer. In the one used for Go, there is no replay buffer; instead, actors add their data to a queue which the learner then consumes. Additionally, while the implementation of MuZero used in the other experiments trained the value function using n-step returns, $z_t = r^{\text{env}}_{t+1} + \gamma r^{\text{env}}_{t+2} + \cdots + \gamma^{n-1} r^{\text{env}}_{t+n} + \gamma^n v^{\text{MCTS}}_{t+n}$ (see Section 2), the one used here uses lambda returns, $z^\lambda_t = (1-\lambda) \sum_{n=1}^{\infty} \lambda^{n-1} z_{t:t+n}$.

The two player aspect of the game is handled entirely by having a single player playing both moves, but using a discount of -1. The input to the agent is the last two states of the go board and one color plane, where each state is encoded relative to color of the player with 3 planes, own stones, opponent's stones and empty stones.

The encoder consists of a single convolution layer with 128 channels, followed by 6 residual 3×3 convolutional blocks with 128 channels (each block consists of two convolutional layers with a skip connection). The network is size-preserving so hidden states are of size 9x9. The transition model takes as input the last hidden state and the action encoded as a one-hot plane, and also consists in 6 residual convolutional blocks with 128 channels. The representation model consists in one 1×1 convolutional layer with 2 channels, followed by an MLP with a single layer of 256 units.

Table 6: Hyperparameters for Go

Hyperparameters	Value
Learning rate	4×10^{-4}
Discount factor	-1
Batch size	16
λ	0.99
Learner steps	10^5
Policy loss weight	1.0
Value loss weight	0.25
Num simulations	150
Dirichlet alpha	0.25
Exploration fraction	0.4

D ADDITIONAL RESULTS AND ANALYSIS

D.1 EXTENSIONS TO FIGURES IN MAIN PAPER

Figure 10 shows the same information as Figure 3 (contributions of search to performance) but split into separate groups and with error bars shown. Figure 11 shows the same information as Figure 5 (effect of search at evaluation as a function of the number of simulations) but using breadth-first search with a learned model. Figure 12 shows the same information as Figure 6 (effect of search on generalization to new mazes) but for the in-distribution mazes instead of the out-of-distribution mazes. Figure 13 presents learning curves for Go for different values of D_{UCT} and numbers of simulations.

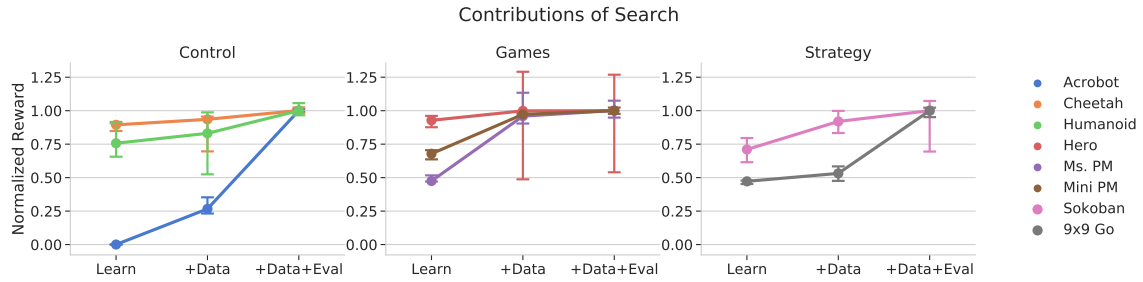

Figure 10: Contributions of the use of planning to performance. A breakdown containing the same information as Figure 3 with error bars showing the maximum and minimum seeds.

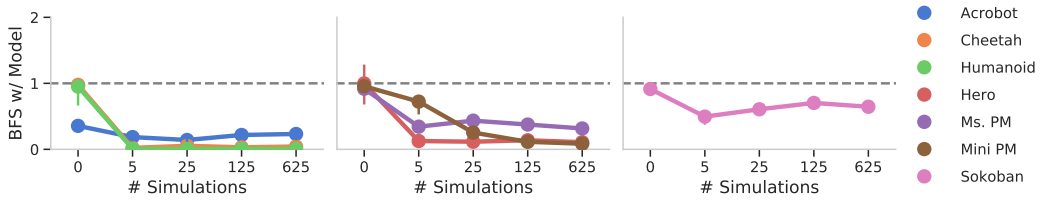

Figure 11: Effect of search at evaluation as a function of the number of simulations for breadth-first search (BFS) with the learned model. All colored lines show medians across seeds, with error bars indicating min and max seeds.

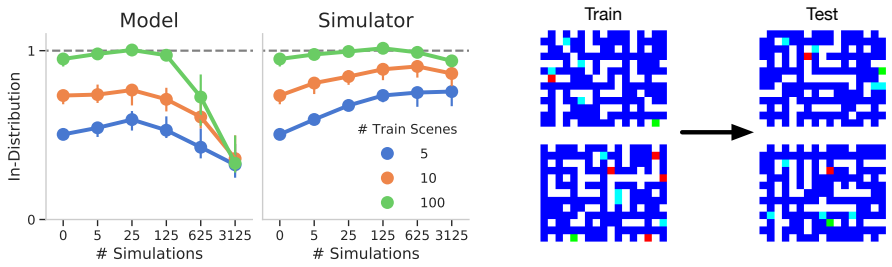

Figure 12: Effect of search on generalization to new in-distribution mazes in Minipacman. All points are medians across seeds, with error bars showing min and max seeds. Colors indicate agents trained on different numbers of unique mazes. The dotted lines indicate equivalent performance to the baseline. The maps on the right give examples of the types of mazes seen during train and test.

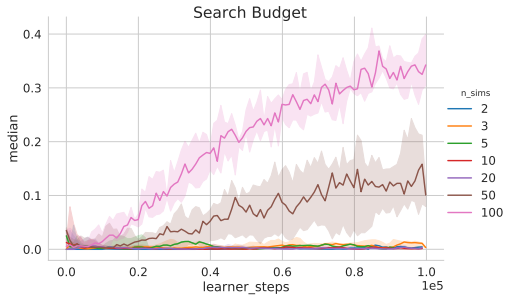

Figure 13: Further experiments on search budget in Go.

D.2 Baseline Learning Curves

Figure 14, 15, and 16 show the learning curves for each way of using planning, from Section 4.2.

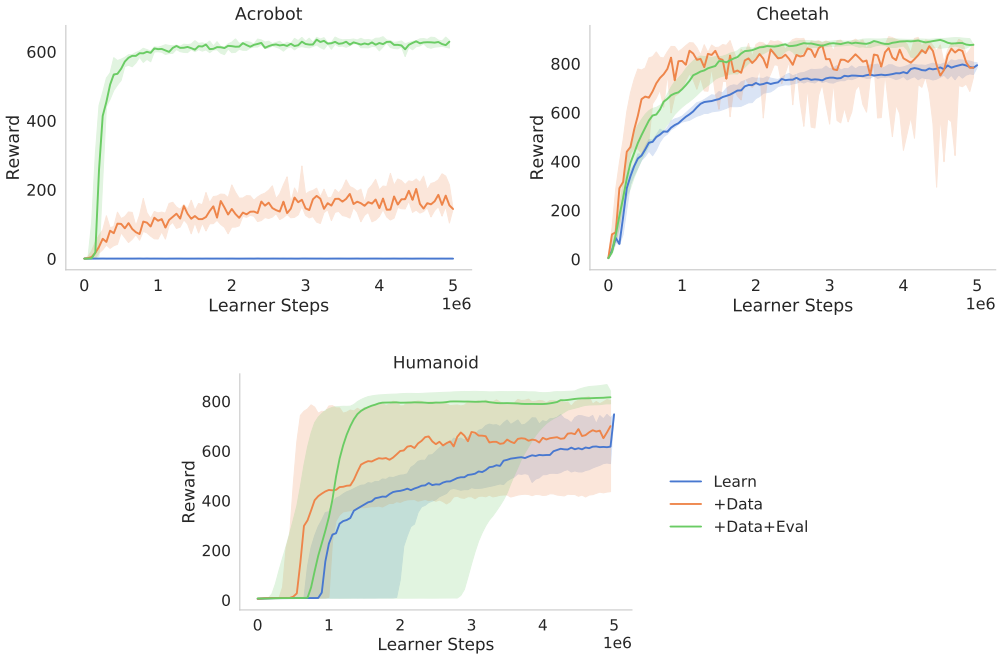

Figure 14: Learning curves for baseline results for control environments.

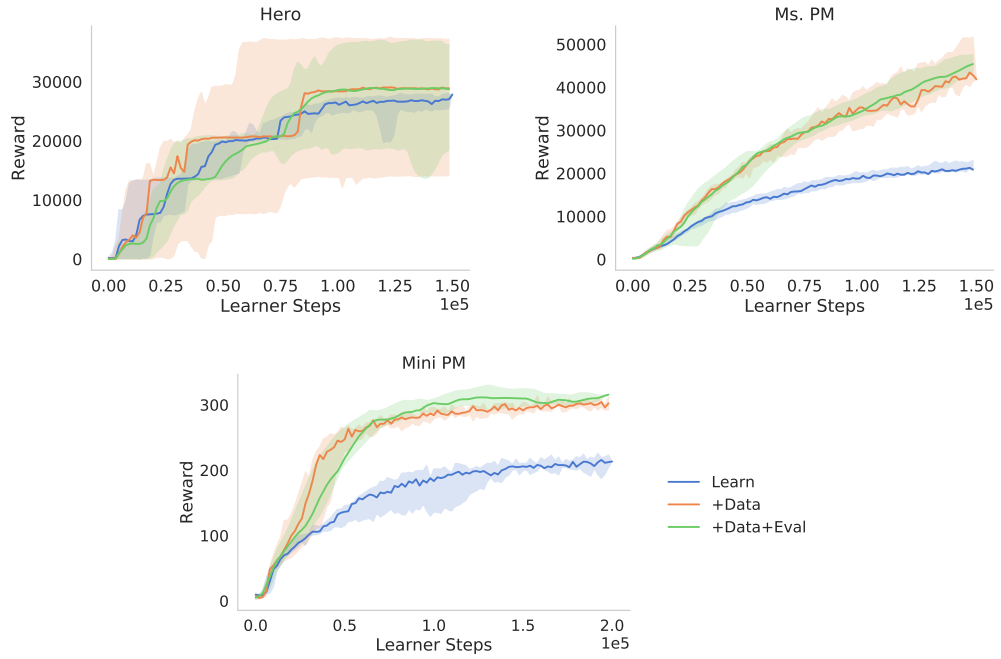

Figure 15: Learning curves for baseline results for game environments.

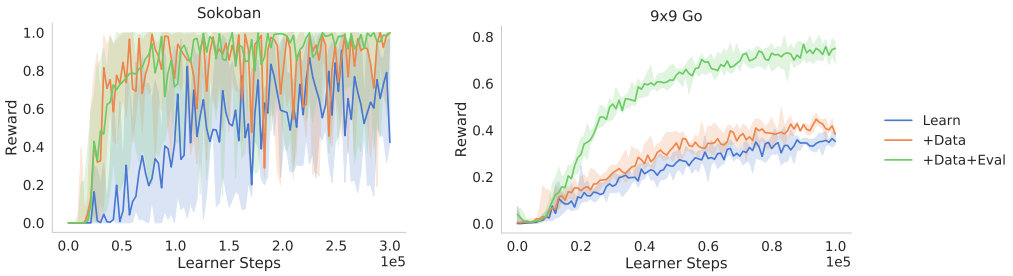

Figure 16: Learning curves for baseline results for strategy environments.

D.3 BASELINE VALUES

Table 7: Values obtained by the baseline vanilla MuZero agent (corresponding to the "Learn+Data+Eval" agent in Figure 3), computed from the average of the last 10% of scores seen during training. Shown are the median across five seeds, as well as the worst and best seeds. Median values are used to normalize the results in Figure 3.

	Median	**Worst Seed**	**Best Seed**
Acrobot	626.31	620.67	634.52
Cheetah	882.61	865.93	904.28
Humanoid	813.12	784.94	859.0
Hero	28843.06	15955.64	36608.31
Ms. Pacman	43735.44	41412.6	46966.5
Minipacman	309.68	303.07	316.8
Sokoban	0.93	0.64	0.99
9x9 Go	0.75	0.71	0.76

Table 8: Values obtained by a version of MuZero that uses no search at evaluation time (corresponding to the "Learn+Data" agent in Figure 3). Shown are the median across five seeds, as well as the worst and best seeds. Median values are used to normalize the results in Figure 4.

	Median	**Worst Seed**	**Best Seed**
Acrobot	166.86	145.32	221.12
Cheetah	822.63	646.41	845.95
Humanoid	675.59	426.51	802.88
Hero	28808.93	14057.64	37231.58
Ms. Pacman	41955.58	39539.66	49592.5
Minipacman	299.67	298.05	301.26
Sokoban	0.85	0.77	0.92
9x9 Go	0.4	0.35	0.44

Table 9: Values obtained by a baseline vanilla MuZero agent, evaluated offline from a checkpoint saved at the very end of training. For each seed, values are the average over 50 (control tasks and Atari) or 1000 episodes (Minipacman and Sokoban). These values are used to normalize the results in Figure 5 and Figure 6. Note that for Minipacman, the scores reported here are for agents that were both trained and tested on either the in-distribution mazes or the out-of-distribution mazes. Shown are the median across five seeds, as well as the worst and best seeds.

	Median	**Worst Seed**	**Best Seed**
Acrobot	526.76	494.6	625.72
Cheetah	893.78	806.24	905.49
Humanoid	809.9	786.24	867.8
Hero	28975.0	19350.8	37234.2
Ms. Pacman	43888.6	42776.6	50607.8
Minipacman (In-Distribution)	315.24	312.87	331.37
Minipacman (Out-of-Distribution)	497.64	494.05	504.19
Sokoban	0.96	0.95	0.97

D.4 Overall contributions of planning

Table 10: Values in Figure 3. The "Learn" column shows percent of the baseline value. The "+Data" column shows gain in percentage points over "Learn", and the "+Data+Eval" column shows gain in percentage points over "+Data".

	Learn	**+Data**	**+Data+Eval**
Acrobot	0.0	26.6	73.5
Cheetah	89.4	4.1	6.6
Humanoid	75.7	7.4	16.9
Hero	92.8	7.1	0.1
Ms. Pacman	47.5	48.4	4.2
Minipacman	67.8	29.4	2.8
Sokoban	71.0	20.9	8.0
9x9 Go	47.2	5.9	46.8
Median	69.4	14.2	7.3

D.5 Planning for learning

Table 11: Effect of the different contributions of search, modeled as `Reward` $\sim$ `Environment * Use of Search`. This ANOVA indicates that both the environment and the type of search (Learn, +Data, +Data+Eval) each are significant predictors of reward, and that there is an interaction between them.

Variable	**Statistic**	**Strength of Evidence**
Environment	$F(7, 101) = 38.06, p < 0.001$	***
Use of Search	$F(2, 101) = 115.82, p < 0.001$	***
Environment:Use of Search	$F(14, 101) = 12.24, p < 0.001$	***

Table 12: Effect of tree depth, D_{tree}, modeled as `Reward` $\sim$ `Environment` $\star$ $\log(D_{\text{tree}})$. Where $D_{\text{tree}} = \infty$, we used the value for the maximum possible depth (i.e. the search budget). Top: this ANOVA indicates that both the environment and tree depth are significant predictors of reward, and that there is an interaction between environment and tree depth. Bottom: individual Spearman rank correlations between reward and $\log(D_{\text{tree}})$ for each environment. p-values are adjusted for multiple comparisons using the Bonferroni correction.

Variable	**Statistic**	**Strength of Evidence**
Environment	$F(7, 184) = 18.27, p < 0.001$	***
$\log(D_{\text{tree}})$	$F(1, 184) = 13.61, p < 0.001$	***
Environment:$\log(D_{\text{tree}})$	$F(7, 184) = 6.70, p < 0.001$	***
Acrobot	$\rho = 0.55, p = 0.04$	*
Cheetah	$\rho = -0.44, p = 0.24$	
Humanoid	$\rho = 0.15, p = 1.00$	
Hero	$\rho = -0.28, p = 1.00$	
Ms. Pacman	$\rho = 0.54, p = 0.04$	*
Minipacman	$\rho = -0.89, p < 0.001$	***
Sokoban	$\rho = 0.45, p = 0.20$	
9x9 Go	$\rho = 0.96, p < 0.001$	***

Table 13: Effect of exploration vs. exploitation depth, D_{UCT}, modeled as `Reward` $\sim$ `Environment` $\star$ $\log(D_{\text{UCT}})$. Where $D_{\text{UCT}} = \infty$, we used the value for the maximum possible depth (i.e. the search budget). Top: this ANOVA indicates that neither the environment nor exploration vs. exploitation depth are significant predictors of reward. Bottom: individual Spearman rank correlations between reward and $\log(D_{\text{UCT}})$ for each environment. p-values are adjusted for multiple comparisons using the Bonferroni correction.

Variable	**Statistic**	**Strength of Evidence**
Environment	$F(6, 161) = 1.03, p = 0.40$	
$\log(D_{\text{UCT}})$	$F(1, 161) = 0.50, p = 0.48$	
Environment:$\log(D_{\text{UCT}})$	$F(6, 161) = 0.36, p = 0.91$	
Acrobot	$\rho = 0.10, p = 1.00$	
Cheetah	$\rho = -0.51, p = 0.07$	.
Humanoid	$\rho = 0.10, p = 1.00$	
Hero	$\rho = -0.16, p = 1.00$	
Ms. Pacman	$\rho = 0.03, p = 1.00$	
Minipacman	$\rho = -0.20, p = 1.00$	
Sokoban	$\rho = -0.12, p = 1.00$	

Table 14: Effect of the training search budget, B, on the strength of the policy prior, modeled as `Reward` $\sim$ `Environment` $\star$ $\log(B)$ + $\log(B)^2$. Top: this ANOVA indicates that the environment and budget are significant predictors of reward, and that there is a second-order effect of the search budget, indicating that performance goes down with too many simulations. Additionally, there is an interaction between environment and budget. Bottom: individual Spearman rank correlations between reward and $\log(B)$ for each environment. p-values are adjusted for multiple comparisons using the Bonferroni correction. Note that the correlation for Go does not include values for $B > 50$ (and thus is largely flat, since Go does not learn for small values of B).

Variable	**Statistic**	**Strength of Evidence**
Environment	$F(7, 214) = 25.00, p < 0.001$	***
$\log(B)$	$F(1, 214) = 261.60, p < 0.001$	***
$\log(B)^2$	$F(1, 214) = 80.99, p < 0.001$	***
Environment:$\log(B)$	$F(7, 214) = 7.00, p < 0.001$	***
Acrobot	$\rho = 0.69, p < 0.001$	***
Cheetah	$\rho = 0.73, p < 0.001$	***
Humanoid	$\rho = 0.89, p < 0.001$	***
Hero	$\rho = 0.77, p < 0.001$	***
Ms. Pacman	$\rho = 0.43, p = 0.14$	
Minipacman	$\rho = 0.69, p < 0.001$	***
Sokoban	$\rho = 0.86, p < 0.001$	***
9x9 Go	$\rho = 0.64, p = 0.01$	*

D.6 PLANNING FOR GENERALIZATION

Table 15: Effect the evaluation search budget, B, on generalization reward when using the learned model with MCTS, modeled as `Reward` $\sim$ `Environment` $\star$ $\log(B)$. Top: this ANOVA indicates that the environment and budget are significant predictors of reward, and that there is an interaction between environment and budget. Bottom: individual Spearman rank correlations between reward and $\log(B)$ for each environment. p-values are adjusted for multiple comparisons using the Bonferroni correction.

Variable	**Statistic**	**Strength of Evidence**
Environment	$F(6, 161) = 3.97, p < 0.001$	***
$\log(B)$	$F(1, 161) = 7.06, p = 0.009$	**
Environment:$\log(B)$	$F(6, 161) = 17.70, p < 0.001$	***
Acrobot	$\rho = 0.80, p < 0.001$	***
Cheetah	$\rho = 0.14, p = 1.00$	
Humanoid	$\rho = 0.30, p = 1.00$	
Hero	$\rho = -0.14, p = 1.00$	
Ms. Pacman	$\rho = -0.33, p = 0.75$	
Minipacman	$\rho = -0.33, p = 0.75$	
Sokoban	$\rho = 0.74, p < 0.001$	***

Table 16: Effect the evaluation search budget, B, on generalization reward when using the simulator with MCTS, modeled as `Reward` $\sim$ `Environment` $\star$ $\log(B)$. Top: this ANOVA indicates that the environment and budget are significant predictors of reward, and that there is an interaction between environment and budget. Bottom: individual Spearman rank correlations between reward and $\log(B)$ for each environment. p-values are adjusted for multiple comparisons using the Bonferroni correction.

Variable	**Statistic**	**Strength of Evidence**
Environment	$F(6, 161) = 35.63, p < 0.001$	***
$\log(B)$	$F(1, 161) = 113.77, p < 0.001$	***
Environment:$\log(B)$	$F(6, 161) = 38.46, p < 0.001$	***
Acrobot	$\rho = 0.75, p < 0.001$	***
Cheetah	$\rho = 0.71, p < 0.001$	***
Humanoid	$\rho = 0.54, p = 0.04$	*
Hero	$\rho = 0.06, p = 1.00$	
Ms. Pacman	$\rho = 0.98, p < 0.001$	***
Minipacman	$\rho = 0.42, p = 0.24$	
Sokoban	$\rho = 0.98, p < 0.001$	***

Table 17: Rank correlations between the search budget, B, and generalization reward in Minipacman for different types of mazes and models. p-values are adjusted for multiple comparisons using the Bonferroni correction.

Scene Type	**Model Type**	**Correlation**	**Strength of Evidence**
In-distribution	Learned model	$\rho = -0.51, p < 0.001$	***
Out-of-distribution	Learned model	$\rho = -0.48, p < 0.001$	***
In-distribution	Simulator	$\rho = 0.25, p = 0.04$	*
Out-of-distribution	Simulator	$\rho = 0.34, p = 0.002$	**

Table 18: Effect the evaluation search budget (B), the number of unique training mazes (M), and test level on generalization reward in Minipacman when using the simulator with MCTS, modeled as `Reward` $\sim \log(M)$ `*` $\log(B)$ `+ Test Level`. This ANOVA indicates that the both the number of training mazes and the search budget are significant predictors of reward, and that there is an interaction between them.

Variable	**Statistic**	**Strength of Evidence**
Test Level	$F(1, 175) = 18.83, p < 0.001$	***
$\log(S)$	$F(1, 175) = 403.00, p < 0.001$	***
$\log(B)$	$F(1, 175) = 116.05, p < 0.001$	***
$\log(B)$:$\log(M)$	$F(1, 175) = 59.41, p < 0.001$	***