NeuroChains: Extract Local Reasoning Chains of Deep Neural Nets

Abstract

We study how to explain the main steps/chains of inference that a deep neural net (DNN) relies on to produce predictions in a local region of data space. This problem is related to network pruning and interpretable machine learning but the highlighted differences are: (1) fine-tuning of neurons/filters is forbidden: only exact copies are allowed; (2) we target an extremely high pruning rate, e.g., $\geq 95\%$; (3) the interpretation is for the whole inference process in a local region rather than for individual neurons/filters or on a single sample. In this paper, we introduce an efficient method, NeuroChains, to extract the local inference chains by optimizing a differentiable sparse scoring for the filters and layers to preserve the outputs on given data from a local region. Thereby, NeuroChains can extract an extremely small sub-network composed of filters exactly copied from the original DNN by removing the filters/layers with small scores. We then visualize the sub-network by applying existing interpretation technique to the retained layer/filter/neurons and on any sample from the local region. Its architecture reveals how the inference process stitches and integrates the information layer by layer and filter by filter. We provide detailed and insightful case studies together with three quantitative analyses over thousands of trials to demonstrate the quality, sparsity, fidelity and accuracy of the interpretation within the assigned local regions and over unseen data. In our empirical study, NeuroChains significantly enriches the interpretation and makes the inner mechanism of DNNs more transparent than before.

1 Introduction

Deep neural networks (DNNs) greatly reshape a variety of tasks — object classification, semantic segmentation, natural language processing, speech recognition, robotics, etc. Despite its success on a vast majority of clean data, DNNs are also well-known to be sensitive to small amounts of adversarial noises. The lack of sufficient interpretability about their success or failure is one major bottleneck of applying DNNs to important areas such as medical diagnosis, public health, transportation systems, financial analysis, etc.

Interpretable machine learning has attracted growing interest in a variety of areas. The forms of interpretation vary across different methods. For example, attribution methods (Bach et al., 2015; Sundararajan et al., 2017; Shrikumar et al., 2017; Montavon et al., 2017; Kindermans et al., 2017; Smilkov et al., 2017) produce the importance score of each input feature to the output prediction for an given sample, while some other methods (Zeiler & Fergus, 2014; Simonyan et al., 2013; Erhan et al., 2009) aim to explain the general functionality of each neuron/filter or an individual layer regardless of the input sample. Another line of works (Ribeiro et al., 2016; Wu et al., 2018; Hou & Zhou, 2018) explain DNNs in a local region of data space by training a shallow (e.g., linear) and easily interpretable model to approximate the original DNN on some locally similar samples. Thereby, they reduces the problem to explaining the shallow model. These methods essentially reveal the neuron to neuron correlations (e.g., input to output, intermediate layer/neuron to output, etc), but they cannot provide an overview of the whole inference process occurring inside the complicated structure of DNNs.

In this paper, we study a more challenging problem: *Can we unveil the major hidden steps of inference in DNNs and present them in a succinct and human-readable form?* Solving this problem helps to answer many significant questions, e.g., which layer(s)/neuron(s) plays the most/least im-

portant role in the inference process? Do two similar samples really share most inference steps? Do all samples need the same number of neurons/layers to locate the key information leading to their correct predictions? How/when/where does a failure happen during the inference on a DNN? Which neuron(s)/layer(s)/feature(s) are shared by different samples from the same region even when their labels differ? Are DNNs using entirely different parts for data from different local regions? Some of them are related to other problems such as network pruning (Han et al., 2015; Li et al., 2016) and neural architecture search (NAS) (Zoph & Le, 2017). For example, are winning tickets (Frankle & Carbin, 2018; Liu et al., 2018) universal over different local regions and classes? Does the weight sharing scheme in recent NAS methods (Pham et al., 2018; Liu et al., 2019; Ying et al., 2019) limit the searching space or quality?

We develop an efficient tool called NeuroChains to extract the underlying inference chains of a DNN for a given local data region. Specifically, we aim to extract a much smaller sub-network composed of a subset of neurons/filters exactly copied from the original DNN and whose output for data from the local region stays consistent with that of the original DNN. In experiments, we assume the data from the same classes reside in a local region. While the selected filters explain the key information captured by the original DNN when applied to data from the local region, the architecture of the sub-network stitches these information sequentially, i.e., step by step and layer by layer, and thus recover the major steps of inference that lead to the final outputs. Despite its combinatorial nature, we parameterize the sub-network as the original DNN with an additional score multiplied to each filter/layer's output featuremap. Thereby, we formulate the above problem of sub-network extraction as optimizing a differentiable sparse scoring of all the filters and layers in order to preserve the outputs on all given samples.

Figure 1: Inference chain by NeuroChains for ResNet-50 (pre-trained on ImageNet) when applied to 20 test images of "dalmatian" and "strawberry". **Top:** The sub-network retains only 13/67 layers and 75/26560 filters of the ResNet-50. The scores for selected filters are represented using the colormap on the top-right. **Middle:** The per-layer featuremaps generated by SMOE (Mundhenk et al., 2019) show a clear trends of firstly extracting the local patterns (dots on dalmatian and strawberry) and gradually covering a global shapes of the classes. **Bottom:** Filters with the largest scores are visualized using the method by Erhan et al. (2009). In shallower layers, L4B1C2_48 and L4B1C3_524 capture a more local black spot pattern of the dalmatian, L3B1C3_683 captures the eyes and nose patterns; L3B1C3_818 extracts the local color pattern of strawberry. In the last bottleneck layer, L4B3SC_769 and L4B3SC_1457 capture the global patterns of dalmatian's black and white fur; L4B3SC_456 and L4B3SC_511 captures the main shape and color of strawberry. It shows an inference chain for strawberry: L1B1SC_177 → L2B1SC_75 and L2B1SC_342 → L3B1C3_818 → L4B1C2_328 → L4B3C3_456 and L4B3SC_511.

The above problem can be solved by an efficient back-propagation that only updates the scores with fixed filter parameters. The objective is built upon the Kullback–Leibler (KL) divergence between the sub-network's output distribution and that of the original DNN, along with an ℓ_1 regularization for sparse scores over filters. We further use a sigmoid gate per layer to choose whether removing the entire layer. The gate plays an important role in reducing the sub-network size since most local regions do not rely on all the layers. In practice, we further apply a thresholding to the sparse scores to obtain an even smaller sub-network and employ an additional fine-tuning to the filter scores on the sub-network. We illustrate the sub-network's architecture and visualize its filters and intermediate-layers' featuremaps by existing methods (Mundhenk et al., 2019; Erhan et al., 2009).

NeuroChains is a novel pruning technique specifically designed for interpreting the local inference chains of DNNs. As aforementioned, it potentially provides an efficient tool to study other problems in related tasks. However, it has several **fundamental differences to network pruning and exist-**

ing interpretation tasks, which deter methods developed for these two problems from addressing our problem. **Comparing to network pruning**: (1) fine-tuning is not allowed in NeuroChains; (2) it targets a much larger pruning rate for succinct visualization, e.g., $\geq 95\%$ for VGG-19 and $\geq 99\%$ for ResNet-50 on ImageNet with ≤ 200 filters remained; (3) it is for data from a local region instead of the whole data distribution. **Comparing to mainstream interpretation tasks**: (1) NeuroChains produces an interpretation of the entire inference process in a local region (which may contain different classes) rather than of one neuron/filter or output on/around a single sample; (2) the sub-network's architecture provides orthogonal information to the importance of individual neurons/filters.

2 RELATED WORKS

Interpretable machine learning methods can be mainly categorized into the ones aiming to evaluate the importance of each input feature of a single sample and the ones explaining individual neurons/filters. Approaches in the first category usually rely on certain back-propagation from the DNN's output to derive an importance score for each input feature or hidden node. Earlier works are based on the back-propagated gradients, e.g., deconvolution (Zeiler & Fergus, 2014), back-propagation (Simonyan et al., 2013) and guided back-propagation (Springenberg et al., 2014). Sundararajan et al. (2017) proposed to (approximately) calculate the integral of the gradients along a path between a baseline point and the input sample, which ensures the sensitivity and implementation invariance lacking in some previous methods. More recent methods propose novel back-propagation rules to directly derive the attribution scores of neurons from output to input, e.g., DeepLIFT (Shrikumar et al., 2017), deep Taylor decomposition (Montavon et al., 2017), and layer-wise relevance propagation (LRP) (Bach et al., 2015).

Methods in the second class treat DNNs as black boxes and seek a simple model to explain how the DNN's output changes in a local region. For example, they add perturbations to different parts of the input to evaluate how the perturbations change the output (Ancona et al., 2017), which reflect the importance of different parts. Zeiler & Fergus (2014) covered different parts of the input image with a gray square, which led to different prediction probabilities on the true class. Instead, Zintgraf et al. (2017) replaced each patch in the input image with the surrounding patch and tracked the induced changes in the output. In LIME (Ribeiro et al., 2016), Ribeiro et al. trained a sparse linear model on noisy input-output pairs as a local surrogate approximating the original DNNs, where the sparse weights are used to explain the importance of input features. As mentioned before, our main difference to the above methods are we explain DNNs for a local region of multiple samples and we further explain how DNNs step by step integrate the information of important filters/neurons.

Network pruning (Han et al., 2015; Li et al., 2016) remove redundant neurons/nodes or connections/weights from a pre-trained DNN and fine-tune the sub-network. Structural pruning removes whole layers/channels/filters/neurons according to a certain norm of the associated weights (Li et al., 2016) or sparsity (Hu et al., 2016). In contrast, Frankle & Carbin (2018); Liu et al. (2018) prune a DNN during its training. Luo et al. (2017) apply pruning to two adjacent convolution layers at each time to take the dependency between the two layers into account. Several recent works empirically verify "lottery ticket hypothesis", i.e., there exists sub-networks (i.e., winning tickets) that can reach comparable generalization performance as the original DNN if re-trained. In contrast, the sub-network extracted by NeuroChains cannot be fully re-trained since it has to preserve the original DNN's filters, and our goal is to retain the generalization performance only for a local region.

3 NEUROCHAINS

3.1 PROBLEM: EXTRACT LOCAL INFERENCE CHAINS AS SUB-NETWORKS

Although the DNNs widely used nowadays are usually composed of hundreds of layers and millions to billions of hidden nodes. *When applied to samples from a local region in data space, it is plausible that its inference process mainly relies on a small subset of layers/neurons/filters.* In this paper, we verify this conjecture by developing an efficient and practical algorithm, i.e., NeuroChains, to extract the subset and its underlying architecture as a sub-network whose weights/filters are selected and exactly copied from the original DNN while its outputs in a given local region retain the ones

produced by the original DNN. Although DNNs are usually non-smooth in definition if using a non-smooth piece-wise activation such as ReLU, when trained with the commonly used techniques, e.g., data augmentation, mix-up, dropout, the resulted DNNs are relatively smooth in a sufficiently small local region.

In order to preserve the original inference chains, we do not allow any fine-tuning or re-training on any filter or the the weight vector corresponding to any neuron: they can only be exactly copied from the original DNN. Let $F(\cdot; \{W^\ell\}_{\ell=1:L})$ (a mapping from input to output) denote the original DNN, W^ℓ represents the set of filters/weight vectors in layer-ℓ, and $W^\ell[i]$ represents the ith filter/weight vector in layer-ℓ. Any sub-network fulfilling our above requirement can be defined and parameterized by an indicator vector M^ℓ per layer, whose each entry is a $\{0, 1\}$ value indicating whether retaining the associated filter/neuron in W^ℓ. We further define operator $\circ$ as

$$(W^\ell \circ M^\ell)[i] \triangleq \left\{ \begin{array}{ll} W^\ell[i], & M^\ell[i] = 1; \\ \mathbf{0}, & M^\ell[i] = 0. \end{array} \right. \tag{1}$$

Thereby, $\{M^\ell\}_{\ell=1:L}$ defines a qualified sub-network for inference chain and its weights are $\{W^\ell \circ M^\ell\}_{\ell=1:L}$, where we extend the operator $\circ$ to make $W \circ M = \{W^\ell \circ M^\ell\}_{\ell=1:L}$ given the original DNN's weights $W = \{W^\ell\}_{\ell=1:L}$. Given a set of samples $\mathcal{X}$ drawn from a local region of data space, we can formulate the problem of finding an inference chain as the following combinatorial optimization, which aims to find the most sparse indicator M (i.e., the sub-network with the fewest filters retained) that does not change the outputs of the original DNN for $\forall x \in \mathcal{X}$, i.e.,

$$\min_{\{M^\ell\}_{\ell=1:L}} \sum_{\ell=1}^{L} \|M^\ell\|_1 \quad \text{s.t. } F(x; W) = F(x; W \circ M), \forall x \in \mathcal{X}. \tag{2}$$

However, directly solving this combinatorial optimization is impractical since the possible choices for M^ℓ is of exponential number. In this paper, we relax the 0-1 indicator vector M^ℓ to a real-valued score vector S^ℓ of the same size. We define an operator $\odot$ applied to W^ℓ and its associated scores S^ℓ as

$$(W^\ell \odot S^\ell)[i] \triangleq S^\ell[i] \cdot W^\ell[i]. \tag{3}$$

Note we do not limit entries in S^ℓ within $[0, 1]$ due to the possible redundancy among filters in the original DNN, i.e., there might be filters of similar functionality for the given samples and a preferred pruning should be able to only preserve one of them and multiply it by the number of those redundant filters in the sub-network. In addition, less constrains are easier to handle in optimization and helpful to find sub-network whose outputs are closer to that of the original DNN, since the class of sub-networks with parameters $W \odot S$ includes all the sub-networks with parameters $W \circ M$. Hence, we relax the challenging combinatorial optimization to the following unconstrained continuous optimization, i.e.,

$$\min_{\{S^\ell\}_{\ell=1:L}} \frac{1}{|\mathcal{X}|} \sum_{x \in \mathcal{X}} l(F(x; W), F(x; W \odot S)) + \lambda \sum_{\ell=1}^{L} \|S^\ell\|_1, \tag{4}$$

where $l(\cdot, \cdot)$ is a loss function aiming to minimize the distance between the original DNN's output $F(x; W)$ and the sub-network's output $F(x; W \odot S)$. In our experiments, for classification, we use KL-divergence between the output distributions over classes, where the two output distributions are computed by applying softmax to $F(x; W)$ and $F(x; W \odot S)$ respectively, i.e.,

$$l(F(x; W), F(x; W \odot S)) = D_{KL}(\text{softmax}(F(x; W))\|\text{softmax}(F(x; W \odot S))). \tag{5}$$

In addition, empirical evidences (Krueger et al., 2017; Singh et al., 2016) show that for most samples there exist some layers that can be entirely removed without changing the final prediction. Hence, only a few hard and confusing samples need more delicate features, while most other samples can be correctly classified based on simple patterns from shallower layers. Therefore, in NeuroChains, we apply a sigmoid function with input score α^ℓ as a gate G^ℓ determining whether removing the entire layer-ℓ during pruning, i.e.,

$$G^\ell = 1 / \left[1 + \exp(-\alpha^\ell / T) \right], \tag{6}$$

where T is a temperature parameter. With a gate G^ℓ applied after each layer-ℓ whose input and output has the same size (which is common in many DNNs), we can recursively define the input

$H^{\ell+1}(\cdot)$ to the next layer-$(\ell+1)$, i.e., $H^{\ell+1}(x; \{W^{\ell'} \odot S^{\ell'}, \alpha^{\ell'}\}_{\ell'=1:\ell}) =$

$$\begin{cases} G^\ell \cdot F^\ell(x; W^{1:\ell}) + (1 - G^\ell) \cdot H^\ell(x; \{W^{\ell'} \odot S^{\ell'}, \alpha^{\ell'}\}_{\ell'=1:\ell-1}), & \text{if input size = output size} \\ F^\ell(x; W^{1:\ell}) & \text{otherwise} \end{cases}$$
(7)

where $F^\ell(x; W^{1:\ell})$ denotes the output of layer-ℓ. The reason to use a gate here is that we expect to either remove the whole layer or retain it without adding an extra shortcut (which will change the original DNN's architecture). We apply a $T < 1$ to sharpen the gate values. Since we prefer to remove non-informative layers, in the objective, we add another regularization α^ℓ to encourage the removal of entire layers (because decreasing α reduces G^ℓ and thus increase the chance of layer removal). Therefore, the final optimization for NeuroChains is

$$\min_{\{S^\ell, \alpha^\ell\}_{\ell=1:L}} \frac{1}{|\mathcal{X}|} \sum_{x \in \mathcal{X}} l(F(x; W), H^{L+1}(x; \{W^{\ell'} \odot S^{\ell'}, \alpha^{\ell'}\}_{\ell'=1:L})) + \lambda \sum_{\ell=1}^{L} \|S^\ell\|_1 + \lambda_g \sum_{\ell=1}^{L} \alpha^\ell, \quad (8)$$

Our objective above is similar to the one used in Network Slimming (Liu et al., 2017) but we optimize it for a local region (so we can consider to remove layers) and we do not allow fine tuning on weights W.

3.2 ALGORITHM

Our algorithm is simply a standard back-propagation for the optimization problem in Eq. equation 8, which produces sparse scores for filters and gate values for layers. Note the weights in W are fixed and the backpropagation only updates S. We initialize the filter scores

Figure 2: Information of pre-trained DNNs in this paper.

STATISTICS	RESNET-50	VGG-19
TOP-1 TEST ACCURACY	76.5%	72.9%
TEST IMAGES/SUB-NETWORKS	10000/1688	10000/1746
CONVOLUTIONAL FILTERS	26560	4480
PARAMETERS OF CONV-LAYERS	23454912	20018880
PARAMETERS OF FC-LAYERS	2048000	123633664

$S = \mathbf{1}$ so $W \odot S = W$ at the beginning of optimization. We initialize the gate score $\alpha^\ell = 0$ for all $\ell = 1 : L$ so $G^\ell = 0.5$ at the beginning, i.e., the probabilities to remove or to retain a layer is equal. For classification, we set loss $l(\cdot, \cdot)$ to be the KL-divergence between the output distributions of the original DNN and the sub-network. After convergence of the optimization, we then apply a simple thresholding to these scores to further remove more filters and layers: (1) we remove the filters with score under a threshold τ; (2) we remove layer-ℓ if $\alpha^\ell < 0$ and $G^\ell < 1 - G^\ell$. This yields a sufficiently small sub-network architecture but the scores might be sub-optimal in preserving the original DNN's outputs within the local region. Therefore, we further fine-tune the nonzero scores in S (note filters are always fixed), i.e, the scores for the retained filters/layers, by minimizing Eq. equation 8 without the two regularizations, if the first term in Eq. equation 8 exceeds a threshold t. Given a sub-network produced by NeuroChains, we then visualize its architecture and scores as the structure of the inference chains. Moreover, we visualize the retained filters and layers on the chains by their activation patterns and featuremaps, respectively, using existing interpretation methods (Mundhenk et al., 2019; Erhan et al., 2009).

4 EXPERIMENTS

In experiments, we apply NeuroChains to extract the inference chains of two widely-adopted CNNs pre-trained on ImageNet, i.e., VGG-19 and ResNet-50. We provide the basic information of the two DNNs in Figure 2. In the following, We will present **three quantitative analysis over** 1500 **case studies**, which show that (1) NeuroChains is capable to produce sub-networks retaining only $< 5\%$ of filters and meanwhile preserve the outputs of the original DNN in most cases; (2) every filter selected by NeuroChains is important to preserving the outputs since removing one will leads to considerable drop in performance; (3) the sub-network extracted based on a finite number of samples can be generalized to unseen samples in nearby regions. We will then provide several detailed and insightful case studies and visualizations of extracted sub-networks for different local regions.

4.1 IMPLEMENTATION DETAILS

Figure 3: **Left: Size and fidelity (how well the sub-networks preserve the original DNN's outputs)** of 1500 sub-networks extracted by NeuroChains for ResNet-50 in different case studies (local regions). The x-axis refers to the number of retained filters, while the y-axis is the induced decrease of probability on the original-DNN predicted class. It shows that NeuroChains can usually find extremely small sub-networks and meanwhile preserve the original DNN's outputs. **Right: Faithfulness (filter score and degeneration by removing the filter)** of 1500 sub-networks extracted by NeuroChains for ResNet-50. The x-axis refers to the filter scores, while the y-axis denotes the decrease of the sub-network's probability on the original-DNN predicted class after removing a filter from the sub-network. It shows that the sub-networks suffer from more degeneration if removing a filter with higher score (magenta line implies strong correlation between the two). Hence, the scores faithfully reflect the importance of filters in explaining the original DNNs.

We implement NeuroChains by Py-Torch (Paszke et al., 2017). In every case study, we firstly randomly sample 2 classes and then randomly sample 10 images from each class's images that the original DNN has high confidence, e.g., $\geq 95\%$. Note high confidence does not guarantee the correctness since it can be associated with a wrong class. We choose images with high confidence since the inference on them mostly represent the original DNN's inference chains in the region. We apply inference on those 20 images and their outputs are used in solving the optimization of Eq equation 8 in order to extract the local inference chain in the form of a sub-network. For models with shortcuts, e.g., ResNet-50, the sigmoid gate is applied to prune a bottleneck block rather than a layer. A layer inside a block will be removed if the scores of all filters in the layer are nearly 0.

We use Adam optimizer for both the optimization of Eq equation 8 and fine-tune phases for filter/layer scores. We use a fixed learning rate 0.005 in the former and 0.01 in the latter. We set temperature $T = 0.2$ in the sigmoid gate

Figure 4: **Averaged test fidelity (stability)** of 1500 sub-networks extracted by NeuroChains for VGG-19 when applied to images in nearby local regions. For each sub-network and every image used to extract it, we rank images from the validation set of Deng et al. (2009) by their distance to the image (in VGG-19 last-layer's input space) and obtain its K-nearest neighbors (K=20). We evaluate the test fidelity (accuracy of preserving the original DNN's prediction) of the sub-network over these KNN images for all the images used to extract it (duplicates removed), whose total number is $N < 400$. We average this test fidelity over all the 1500 sub-networks. The x-axis is N and the y-axis is the test accuracy on KNN images.

(Eq. equation 6) to encourage the gate value G^ℓ close to either 0 or 1, and the threshold τ to goal scores is set to 0.1 so that the outputs of sub-networks are as consistent as possible. For different models, the weights of two penalties in Eq. equation 8 are different. For VGG-19, we use $\lambda = 0.007$ and $\lambda_g = 2$. While we choose $\lambda = 0.0065$ and $\lambda_g = 0.9$ for ResNet-50. We fine-tune S in the pruned network if the KL-divergence after training is greater than threshold $t = 0.1$. The iteration steps of training and fine-tune is 300 and 50 respectively. We stop training or fine-tuning when the loss difference is less than 0.02. It costs only $\sim 90s$ for VGG19 and $\sim 55s$ for ResNet50 to extract a sub-network on a single GPU since we only optimize a few number of scores

4.2 QUANTITATIVE ANALYSES

In Melis & Jaakkola (2018), they propose three criteria to evaluate the interpretation methods for DNNs, i.e., faithfulness, stability and explicitness. In this paper, we extend some of their notations and present three quantitative analyses of NeuroChains over 1500 case studies for different local regions, i.e., (1) **Fidelity**: does the sub-network preserve the original DNN's outputs on the given samples? how does it change for sub-networks of different sizes? (2) **Faithfulness**: how much degeneration on the fidelity will be caused if removing one filter from the sub-network? (3) **Stability**: what is the fidelity (the accuracy of preserving the original DNN's prediction) of the sub-network on unseen test images from nearby regions? In this paper, we evaluate the fidelity and faithfulness by the decreasing amount of probability on the original DNN's predicted class when using the sub-network for inference; we evaluate the stability by evaluating the test fidelity of the sub-network over an image's K-nearest neighbor images from an validation set (details below Figure 4). All the above metrics are averaged over 1500 sub-networks and across all the images used to extract each sub-network.

Figure 5: Inference chain by NeuroChains for VGG-19 when applied to images of "Indigo bunting" and "sorrel". **Left:** The sub-network retains only 9/16 layers and 118/4480 filters of the VGG-19. **Middle:** In the SMOE featuremaps, the eyes of both indigo bunting and sorrel, the feathers of indigo bunting, and the legs of sorrel are gradually located as the key features. **Right:** In shallow layers, filters extract local patterns like eyes (L23_198, L23_205 and L25_506) and feathers (L25_294 and L25_413). In the last two layers, L32_30 captures the eyes of sorrel (big and with eyelids) while L32_352 shows the eyes and the whole head of the bird; L32_455 captures the pattern of sorrel legs; L34_490 captures the contour of sorrel's main body. It implies an inference chain for Indigo bunting: L10_105 → L19_103 → L23_205 → L25_506 and L25_294 → L32_352 → L34_483.

The left plot of Figure 3 shows the fidelity for sub-networks of different sizes (measured by the number of filters) that are extracted by NeuroChains. Note most sub-networks only retain ≤ 1% of ResNet-50 for succinct visualization but they preserve the outputs of ResNet-50 with high fidelity.

The right plot in Figure 3 reports the faithfulness of extracted sub-networks, i.e., how a sub-network's performance in preserving the original DNN's output degrade if removing one filter from it, and what is the relationship between this degeneration and the score of the removed filter. The statistics on 1500 sub-networks in Figure 3(right) show that removing even only one filter from the sub-network can significantly degrade the explanation performance. Hence, NeuroChains usually find the smallest sub-networks without redundancy among retained filters/layers, i.e., every critical inference step is retained. Moreover, the degeneration and the score are strongly and positively correlated, indicating that our optimized scores faithfully reflect the importance of filters in explaining the original DNN.

We report the average stability of 1500 sub-networks extracted by NeuroChains in Figure 4. It shows that each sub-network can still achieve a high fidelity (the accuracy of preserving the original DNN's output) within a relatively large local region around its samples, e.g., when the total number of KNN images from an unseen validation set is under 100. It therefore indicates that the inference chains extracted for a finite number of samples by NeuroChains can be applied and generalized to other unseen samples from the same region or even the nearby regions. So NeuroChains is an effective solution to the problem raised in the introduction. More and complete quantitative analyses for both VGG-19 and ResNet-50 can be found in the Appendix.

4.3 CASE STUDIES

We present three case studies of the sub-networks extracted by NeuroChains . For Figure 1 and Figure 5, the data points are from classes which are easy to tell apart while in Figure 6 images are always mis-classified. More case studies are given in Appendix. The visualization in each case study is composed of five parts: (1) the sub-network's architecture and filter scores; (2) the 20 images from 2 classes used to extract the sub-network. The sub-network is supposed to explain the inference chains on images from the same local region of those 20 images; (3) one image from each class and the visualization of the image's per-layer featuremaps in the sub-network, which is produced by SMOE (Mundhenk et al., 2019); (4) for comparison, we also present the interpretations produced by two recent works (Wang et al., 2019; Nam et al., 2019) on the two images; (5) visualization of each filter retained in the sub-network using Erhan et al. (2009) after zoomed-in. In the caption below each case study, we highlight some filters/featuremaps and their related class. We also provide examples of inference chains composed of filters stitched by the sub-network's architecture. They show that NeuroChains considerably enrich the explanation details of DNN's inference process in a local data region. By connecting the important filters from different layers, the extracted sub-network highlights the main steps leading to the output prediction.

Figure 6: Inference chain by NeuroChains for VGG-19 when applied to images of "Castle" and "Stone Wall" which confuse DNN models. **Left:** The sub-network retains only 10/16 layers and 138/4480 filters of the VGG-19. **Middle:** In the SMOE featuremaps, the wall of castle and the windows of the wall are gradually located as the wrong key features. **Right:** For castle: L14_96 → L19_145 → L23_216 and L23_220 → L28_283 → L32_351; For stone wall: L14_20 → L19_364 → L23_449 → L28_378 → L32_45. Filters like L19_364, L23_216 and L28_378 confuse the model and cause wrong predicts since they capture the patterns of both classes.

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