SparsyFed: SPARSE ADAPTIVE FEDERATED TRAINING

Anonymous authors
Paper under double-blind review

ABSTRACT

Sparse training is often adopted in cross-device federated learning (FL) environments where constrained devices collaboratively train a machine learning model on private data by exchanging pseudo-gradients across heterogeneous networks. Although sparse training methods can reduce communication overhead and computational burden in FL, they are often not used in practice for the following key reasons: (1) data heterogeneity impacts more clients' consensus on sparse, compared to dense, models, requiring training for longer; (2) a lack of sufficient plasticity to adapt to never-seen data distributions, crucial in cross-device FL; (3) requiring additional hyperparameters, which are notably challenging to tune in FL. This paper presents *SparsyFed*, a practical federated sparse training method that critically addresses the aforementioned problems. Previous works have only managed to solve one, or perhaps two of these challenges, and at the expense of introducing new trade-offs, such as clients' consensus on masks versus sparsity pattern plasticity. We show that *SparsyFed* simultaneously (1) can produce 95% sparse models, with negligible degradation in accuracy, while only needing a single hyperparameter, (2) achieves a per-round weight regrowth 200 times smaller than previous methods, and (3) still offers plasticity under this sparse design, by outperforming all the baselines at adapting to never-seen data distributions.

1 INTRODUCTION

Federated Learning (McMahan et al., 2017) has become a standard technique for distributed training on private data (Yang et al., 2018; Ramaswamy et al., 2019; Pati et al., 2022; Wang et al., 2023; Huba et al., 2022; Bonawitz et al., 2019), particularly on edge devices. Given its application to constrained hardware, mitigating communication and computational overheads—significant in standard FL infrastructures (Kairouz et al., 2021; Bellavista et al., 2021)—remains a key focus of the field. Practical cross-device FL methods typically assume stateless clients with imbalanced, heterogeneous datasets and constrained, diverse hardware (Wang et al., 2021). Restricted client hardware and low communication bandwidth significantly increase training time compared to centralized methods, elongating hyperparameter tuning (Khodak et al., 2021). Additionally, unknown data distributions and dynamic client availability demand robust optimization methods that can handle these variations. Importantly, when device availability is constrained, the federated orchestrator may struggles to sample a representative client subset (Eichner et al., 2019; Cho et al., 2020; Li et al., 2020b), inducing trade-offs between sampling ratio and efficiency (Charles et al., 2021).

Sparse training methods improve computational and communication efficiency by reducing (a) memory usage and FLOPs during local training (Raihan & Aamodt, 2020), and (b) the payload size of communication messages (Bibikar et al., 2022). However, applying these methods in cross-device FL is challenging due to client availability and data heterogeneity, which can disrupt sparsity patterns (i.e., the binary mask structure) across clients (Qiu et al., 2021). Such inconsistencies hinder consensus on a shared sparsity pattern (i.e., the same binary mask), lowering global model performance (Babakniya et al., 2023). Recent approaches address these issues using fixed sparse masks (Huang et al., 2022; Qiu et al., 2021) or dynamic methods involving mask warm-up and refreshing (Babakniya et al., 2023). However, fixed masks reduce adaptability to unseen distributions, while dynamic methods require careful tuning of additional hyperparameters (e.g., warm-up duration, refresh interval). Fixed-mask methods also limit model plasticity—the ability to rewire and adapt to diverse distributions (Lyle et al., 2022; 2023). For example, it is known that in a multi-

task or continual learning setting, neural networks can be iteratively pruned to build task-specific sub-networks (Mallya & Lazebnik, 2018), a lost ability when adopting a fixed-mask method.

This lack of adaptability makes fixed-mask approaches unsuitable for cross-device FL, where unseen distributions frequently arise. Thus, we argue that sparse training methods for cross-device FL should (a) adopt dynamic masking and (b) remain agnostic to optimization and selection methods, while minimizing hyperparameter complexity.

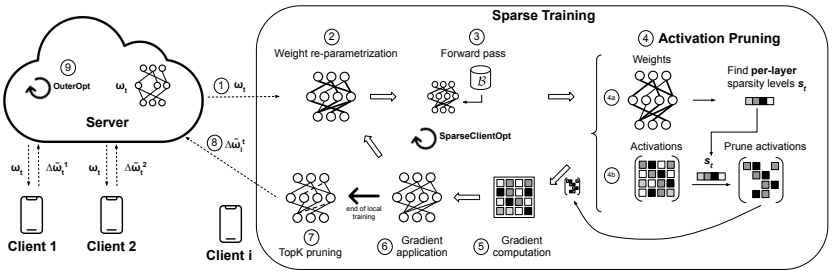

Figure 1: *SparsyFed* pipeline. (1) Server broadcasts the global model ω_t. (2) Client i re-parameterizes local weights. (3) Executes a forward pass on batch $\mathcal{B}$. (4a) Computes layer-wise sparsity s_t. (4b) Prunes activations using s_t and stores them. (5) Computes grads. (6) Applies grads. (7) Computes model updates and applies `TopK` pruning. (8) Sends sparse updates $\Delta\tilde{\omega}_i^t$ back to the server. (9) Apply server optimizer to obtain global model. Steps (2-6) repeat to convergence.

To address these challenges, our work introduces *SparsyFed*, a sparse training method for training sparse global models in cross-device federated learning. During local training, as illustrated in fig. 1, *SparsyFed* uses an easy-to-tune sparse training approach by: (1) pruning activations with an adaptive layer-wise method, and (2) pruning model weights before communication using the target sparsity. These strategies reduce communication costs by transmitting sparse updates and thus accelerate federated training. With suitable hardware support (Raihan & Aamodt, 2020), FLOPs and memory usage can also be reduced. Unlike previous methods (Babakniya et al., 2023; Qiu et al., 2021), which applied fixed global sparsity across layers, our layer-wise approach prunes based on each layer's parameter proportion. This removes more capacity from dense layers while preserving parameter-efficient ones. *SparsyFed* also prunes models at the end of each round, allowing full flexibility for local optimzation. **Crucially**, by starting from a dense model in the first round, *SparsyFed* provides robust initialization for crucial layers, such as embeddings. Earlier works (Qiu et al., 2021; Raihan & Aamodt, 2020) that pruned from the first step had to exclude layers to preserve performance.

Additionally, *SparsyFed* employs a sparsity-inducing weight reparameterization (Schwarz et al., 2021) reliant on a single parameter, enhancing the model's resilience to sparsity and improving plasticity, which facilitates adapting the global mask to new clients with diverse data distributions. *SparsyFed* is agnostic to the choice of the outer optimizer, treating sparse model updates as pseudo-gradients. It is also compatible with biased client selection policies, allowing training masks to remain plastic and adapt to the utilized training data.

The primary scientific contributions of this paper include:

1. We introduce *SparsyFed*, a method which accelerates on-device training in FL. *SparsyFed* achieves high sparsity (up to 95%) without sacrificing accuracy through a novel approach combining hyperparameter-free activation pruning and weight reparameterization.
2. We compare *SparsyFed* against the latest state-of-the-art methods, demonstrating superior accuracy over sparse training baselines (including those using fixed sparse masks) and, in some cases, surpassing non-pruned baselines at extreme sparsity levels.
3. *SparsyFed* quickly achieves consensus on client sparse model masks, enabling faster global convergence and significantly reducing downlink communication costs (up to $19.29\times$).
4. We evaluate *SparsyFed* with ablation studies on typical cross-device FL datasets, including CIFAR-10/100 (Krizhevsky, 2012) and Speech Commands (Warden, 2018), under various data heterogeneity conditions.

Finally, to facilitate result reproducibility and as a code base for the community of researchers in the field, we have made the developed code publicly available in this repository.

2 BACKGROUND

In the following, we describe typical sparse training and weight reparametrization techniques, which are key components of our work, while also discussing their relevance to cross-device FL.

2.1 CROSS-DEVICE FEDERATED LEARNING

Cross-device FL (Kairouz et al., 2021) involves the distributed training of a machine learning (ML) model across a population of edge devices exchanging model updates with a central server through heterogeneous networks (McMahan et al., 2017; Li et al., 2020a). Clients in these settings usually possess very limited data samples and very heterogeneous data distributions Kairouz et al. (2021). Given the constraints of edge devices (e.g., limited processing power and battery life) and the heterogeneity of networks (e.g., diverse bandwidth), computational- and communication-efficient training methods are crucial for practical FL (Bonawitz et al., 2019). The research community has proposed means to optimize FL for communication efficiency (Sattler et al., 2019; Jiang et al., 2023) and computational efficiency (Horvath et al., 2021; Niu et al., 2022; Mei et al., 2022). Our approach aims to optimize both by leveraging sparse training.

2.2 SPARSE TRAINING

In centralized settings, sparse training tries to learn a sparse model during training, typically to achieve model compression, lower computational demands during training, or faster inference. A typical sparse training pipeline tends to start with a random sparse network and follow a cycle of regular training, pruning, and regrowth (Mocanu et al., 2018; Mostafa & Wang, 2019; Dettmers & Zettlemoyer, 2019; Liu et al., 2020), acting only on model parameters. A more advanced technique relevant to our work, Sparse Weight Activation Training (SWAT) (Raihan & Aamodt, 2020), tailors the forward and backward passes, acting on both activations and model parameters, to induce a sparse weight topology and reduce the computational burden. In each forward iteration, SWAT selects the `TopK` weights based on magnitude, using only these as the *active* weights. During the backward pass, only the highly activated neurons associated with `TopK` sparsified activations are considered for backpropagation. Importantly, full gradients are still applied in the backward pass, allowing updates to both *active* and *non-active* weights. This mechanism enables the dynamic exploration of different network topologies throughout the training process.

2.3 WEIGHT REPARAMETRIZATION

Weight reparametrization (Salimans & Kingma, 2016; Li et al., 2019; Gunasekar et al., 2017; Miyato et al., 2018; Vaskevicius et al., 2019; Kusupati et al., 2020; Schwarz et al., 2021; Zhao et al., 2022) in neural network training involves restructuring how weights are represented to improve training dynamics, optimize convergence, or introduce specific properties such as sparsity, without changing the network's architecture. Particularly relevant to our work, Schwarz et al. (2021) propose a sparsity-inducing weight reparametrization that aims to shift the weight distribution towards higher density near zero, aiding in pruning low-magnitude weights. This is achieved by raising the model parameters to the power of $\beta > 1$ during the forward pass while preserving their sign. The reparametrized weight vector component w is computed as $w = \text{sign}(v) \cdot |v|^{\beta-1}$, where v represents the original weight vector component, and β is a scalar value. Due to the chain rule, small-valued parameters receive smaller gradient updates, while large-valued parameters receive larger updates, reinforcing a "rich get richer" dynamic.

3 SPARSE ADAPTIVE FEDERATED TRAINING

In the following, we present our *SparsyFed* method for sparse training in cross-device FL settings. *SparsyFed* reduces the computational and communication overhead of highly heterogeneous FL environments, adapting the sparse mask of the global model during the training. Our method introduces a novel approach based on activation pruning and weight parametrization, applicable to any cross-device FL setting for obtaining a sparse global model. The procedure is outlined in Algorithms 1 and 2.

Algorithm 1 Sparse federated training pipeline of *SparsyFed*.

Require: ω_0: initial model's weights, β: weight re-parametrization exponent, $\hat{s}$: target sparsity
Require: T: number of federated rounds, E: number of client local epochs per round
Require: P: clients population, $\eta_t = \eta(t)$: learning rate scheduler as function of round t
Require: $\{\mathcal{D}_i\}_{i \in P}$: clients' datasets, B: local batch size

1: **procedure** *SparsyFed*
2: **for** $t = 0, \ldots, T-1$ **do**
3: Server samples a subset of clients $S_t \subseteq P$
4: **for** each client $i \in S_t$ **in parallel do**
5: $\omega_{i,0} \leftarrow \omega_t$
6: **for** $k = 0, \ldots, E-1$ **do**
7: $\omega_{i,k+1} \leftarrow \texttt{SparseClientOpt}(\omega_{i,k}, \mathcal{D}_i, B, \beta, \eta_t)$ ▷ See Alg.2
8: $\Delta\omega_i^t \leftarrow \omega_{i,E} - \omega_t$ ▷ Compute pseudo-gradient
9: $\Delta\tilde{\omega}_i^t \leftarrow \texttt{TopK}(\Delta\omega_i^t, \hat{s})$ ▷ Prune $\Delta\omega_i^t$ w/ global unstructured `TopK` using target $\hat{s}$
10: $\omega_{t+1} \leftarrow \texttt{OuterOPT}(\omega_t, \{\Delta\tilde{\omega}_i^t\}_{i \in S_t})$ ▷ Server optimization, e.g., Reddi et al. (2021)
11: **return** ω_T

Assumptions on the FL setting. As usual in cross-device FL settings, the training is orchestrated by a parameter server (McMahan et al., 2017) that is in charge of initializing the global model, sampling a subset of clients every federated round, aggregating the pseudo-gradients after clients have trained on their local datasets. By following practical considerations (as extensively discussed in Bonawitz et al. (2019); Wang et al. (2021)), *SparsyFed* does not require any particular assumption on the client selection policy, nor on the server optimizer, nor the client optimizer. Thus, the design of our algorithm will have the opportunity to seamlessly benefit from any novel contribution from the research community on these topics, without losing its properties. Notably, *SparsyFed* only requires the addition of one hyperparameter compared to the standard dense training, whose sensitivity is discussed in appendix E.1, making it most suitable for cross-device FL where hyperparameter tuning is challenging (Khodak et al., 2021). After initialization, the parameter server iteratively samples clients, broadcasts the latest version of the global model, waits for pseudo-gradients to arrive from clients, and aggregates the updates for obtaining the new global model, as described in Algorithm 1. Let us notice that, in this way, we assume agnosticism to any server optimizer (line 10 in Alg. 1) to allow practitioners to use their preferred one, e.g., `ServerOpt` in Reddi et al. (2021).

Algorithm 2 Sparse Client Optimization of *SparsyFed*

Require: ω_0: initial model's weights, β: weight re-parametrization exponent, η: learning rate
Require: $\mathcal{D} = \{\mathbf{x}_i, y_i\}_{i=1,\ldots,N}$: dataset composed of N samples with inputs $\mathbf{x}_i$ and outputs y_i
Require: $\mathcal{B}$: batch of data samples, B: batch size, $T = \lceil \frac{N}{B} \rceil$: number of batches, F: cost function
Require: $s(\theta_{t,l})$: function computing the sparsity of the layer $\theta_{t,l}$ at time t
Require: $\mathit{size}(\theta)$: computes the number of layers in the model θ
Require: $\texttt{GetLayer}(a_t, l)$: function extracts the weights and activations for the current layer.
Require: $\texttt{SetLayer}(\tilde{a}_{t,l}, l)$: function updates the sparse weights and activations in the model.

1: **procedure** SPARSECLIENTOPT($\omega_0, \mathcal{D}, B, \beta, \eta$)
2: **for** each step $t = 0, \ldots, T-1$ **do**
3: $\mathcal{B}_t \leftarrow \texttt{GetNextMiniBatch}(\mathcal{D}, B, t)$
4: $\theta_t \leftarrow \text{sign}(\omega_t) \cdot |\omega_t|^\beta$ ▷ Point-wise weights re-parametrization (Schwarz et al., 2021)
5: $a_t \leftarrow \texttt{Forward}(\theta_t, \mathcal{B}_t)$ ▷ Forward pass to compute activations
6: **for** each layer $l = 0, \ldots, \mathit{size}(\theta_t) - 1$ **do**
7: $\theta_{t,l}, a_{t,l} \leftarrow \texttt{GetLayer}(\theta_t, l), \texttt{GetLayer}(a_t, l)$
8: $s_{t,l} = s(\theta_{t,l})$
9: $\tilde{a}_{t,l} \leftarrow \texttt{TopK}(a_{t,l}, s_{t,l})$ ▷ Prune $a_{t,l}$ w/ unstructured `TopK` using target $s_{t,l}$
10: $\tilde{a}_t \leftarrow \texttt{SetLayer}(\tilde{a}_{t,l}, l)$
11: $g_t \leftarrow \nabla F(\theta_{t,l}, \tilde{a}_t, \mathcal{B}_t)$ ▷ Compute gradients using sparse activations
12: $\omega_{t+1} = \texttt{ClientOpt}(\omega_t, g_t, \eta, t)$ ▷ Apply client optimizer, e.g., Reddi et al. (2021)
13: **return** ω_T

Sparsity-Inducing Weights Reparametrization. Before the local forward pass at step t, we apply a sparsity-inducing reparametrization to the local model weights ω_t, producing θ_t (line 4, Alg. 2). As discussed in section 2.3, reparametrization techniques have been widely studied for various purposes. In our case, we aim to enhance pruning effectiveness and efficiency by inducing sparsity directly through optimization, adapting to input data. This is particularly beneficial in cross-device FL settings, where datasets are highly imbalanced, and some clients may only have a few samples (section 2.1). From the available sparsity-inducing methods, we adopt Powerpropagation (Schwarz et al., 2021) due to its simplicity (introducing only one hyperparameter) and its ability to preserve the neural network's functional relationships during training. Furthermore, it avoids introducing non-uniform biases across layers, which improves its compatibility with pruning. Every parameter weight $w \in \omega$ is transformed into $v = \text{sign}(w) \cdot |w|^{\beta}$, where $\text{sign}(w)$ is the sign of w, $|w|$ is its L1 norm, and β is the Powerpropagation parameter. More broadly, this reparametrization enhances global training by promoting client consensus. The dynamics introduced by Powerpropagation naturally guide training toward the subset of non-zero weights in the global model. Weights transitioning from zero to non-zero during training typically have smaller magnitudes, limiting their impact on updates. This ensures that clients focus on a shared subset of weights, facilitating aggregation.

Pruning Activations During Local Training. The local training procedure of *SparsyFed* is outlined in Algorithm 2 and relies on two main pillars. First, it ensures that the edge devices benefit from the model's sparsity by reducing the FLOPs and memory footprint. Second, it guarantees the retention of as much information as possible during the training. Following Raihan & Aamodt (2020), we: (a) Use dense activation vectors (a_t, line 5) during the forward pass to retain all learned information. (b) Prune activations before the backward pass by aligning their sparsity with the weight vectors (lines 7–10). Specifically, activations are pruned layer wise using the `TopK` method with a target per-layer sparsity level ($s_{t,l}$, line 8), determined by the corresponding weight sparsity (line 9). This involves retrieving the weights and activations per layer (line 7) and updating the pruned activations before computing gradients. The weight parametrization preserves high sparsity throughout training (Figure 7, in the appendix), ensuring consistent patterns between weights and activations and seamless integration into the model's sparse structure. (c) Keep gradient vectors (g_t, line 11) dense to avoid losing crucial information during updates. Pruning activations before the backward pass reduces computational cost while maintaining dense gradients for robust updates. Notably, initial model weights remain unpruned to allow meaningful training initialization, as clients train a dense model in the first round. With these principles, the local training procedure is designed to be optimizer-agnostic, e.g., compatible with `ClientOpt` (Reddi et al., 2021).

Pruning model parameters before communication. The data-driven and hyperparameter-less pruning procedure described above requires a further step to ensure satisfaction of the communication requirements. Clients receive a model parameters target sparsity, $\hat{s}$, from the server, which must be met before communicating the pseudo-gradient updates. Thus, the client applies a global unstructured pruning step based on `TopK` using the target value $\hat{s}$ for the output sparsity, guaranteeing to save communication costs using a single parameter. Notably, this allows for non-uniform sparsity across layers, which has been proved to help maintain performance (Kusupati et al., 2020).

4 Experimental Design

Datasets. We selected three datasets to assess *SparsyFed*'s performance: CIFAR-10 (Krizhevsky, 2012), CIFAR-100 (Krizhevsky, 2012), and Speech Commands (Warden, 2018). CIFAR-10 and CIFAR-100 datasets contain 32×32 color images of 10 and 100 classes, respectively. The Speech Commands dataset includes audio samples of 35 predefined spoken words, and it is used for a speech recognition task. These datasets are commonly used in the federated learning literature, as their samples can be easily distributed across a pre-defined number of clients to simulate real-world heterogeneous data distributions. On all datasets, we trained for multi-label classification tasks.

Data partitioning and sampling. The datasets above are distributed among 100 clients and partitioned using the method in Hsu et al. (2019), simulating various levels of data heterogeneity. The distribution of labels across clients is controlled via a concentration parameter α that rules a Latent Dirichlet Allocation (LDA), where a lower α value translates to non-IID distribution and a higher value to the IID distribution of labels. Specifically, we refer to data distributions as IID for $\alpha = 10^3$ and non-IID for $\alpha = 1.0$ and $\alpha = 0.1$. To ensure reproducibility, we fixed the seed to 1337 for the

LDA partitioning process. The federated orchestrator randomly sampled 10 clients out of the 100 clients in the population every round.

Model and training implementation. We employed a ResNet-18 (He et al., 2016) backbone for all experiments, adapting the classification layer to each specific task depending on the number of classes. ResNet-18 was chosen for its popularity as part of the ResNet family, which allows for scalability and comparability with other works in the field. While our training pipeline is implemented with `PyTorch` (Paszke et al., 2019), we designed custom layers and functions for some of *SparsyFed*'s components, such as layer-wise activation pruning and weight parametrization. We used `Flower` (Beutel et al., 2022) to simulate the federated learning setting. All models were trained from scratch without relying on any pre-trained weights.

Sparsity ratios. In our experiments, we targeted different values for the sparsity ratio in the set $\{0.9, 0.95, 0.99, 0.995, 0.999\}$. We chose 0.9 as the minimum value because it has been shown to bring effective gains on edge devices for both memory footprints and FLOPs. Our investigation spans to the extreme value of 0.999 to fairly present the downsides of these sparsity ratios. We applied the same target sparsity for all devices across the federation. Introducing heterogeneous sparsity among clients would have required the addition of a well-motivated simulation framework for hardware heterogeneity based on real-world data. However, given its minimal design assumptions, we expect our method to extend straightforwardly to such a setting.

Communication costs. The measure of communication costs of *SparsyFed* is crucial for understanding its practical implications on cross-device FL settings. To make our analysis agnostic to any compression technique implementation, we report the costs as the number of non-zero parameters effectively exchanged during the FL. Therefore, the communication cost is derived from the effective sparsity of the transmitted model, considering both the downlink (server-to-clients) and uplink (clients-to-server) communication steps. For clarity, we calculate the communication cost as if only one client were participating, assuming all communication occurs in parallel. This ensures that our measurements reflect the total communication load without temporal delays between clients and avoids any bias introduced by the varying sampling proportion of clients in each round.

5 Evaluation

We discuss in this section the assessment we conducted to evaluate our method, *SparsyFed*, for adaptive sparse training in cross-device FL. The experimental results shown here aim to answer the following questions.

1. Does *SparsyFed* mitigate the expected accuracy degradation at high and very high sparsity levels? We compare against the baselines, i.e., `TopK`, ZeroFL (Qiu et al., 2021), and FLASH (Babakniya et al., 2023), for both the accuracy and the communication costs (sections 5.1 and 5.2, respectively).
2. How does our adaptive sparsity pattern interact with the heterogeneous data distributions compared to other sparse training methods? Similarly to Babakniya et al. (2023), we analyze the consensus across clients on the sparse pattern (section 5.3).
3. How do the main components of *SparsyFed* contribute to maintaining the accuracy at different sparsity levels? We ablate the activation pruning step (section 5.5) and vary the weight reparametrization (section 5.4) to answer this question.

To ensure a fair and comprehensive comparison, we re-implemented state-of-the-art methods in our experiments, including ZeroFL, FLASH, and `TopK` pruning. A detailed description of these baseline methodologies and their comparisons against *SparsyFed* is presented in appendix G.

5.1 Accuracy Degradation

Our approach demonstrated *SparsyFed*'s resilience to the performance degradation typically associated with pruning, allowing us to achieve competitive results even at high levels of sparsity. Compared to other competitive methods under the same sparsity constraints, table 1 shows that *SparsyFed* exhibits the lowest accuracy drop across all settings and target sparsity levels. A noticeable drop in performance compared to the dense model was observed only at 99% sparsity. This advantage arises from using weight reparametrization, which, in some cases, can even enhance the performance of

Dataset	Sparsity	$\alpha = 1.0$				$\alpha = 0.1$			
		ResNet-18	ZeroFL	FLASH	***SparsyFed***	ResNet-18	ZeroFL	FLASH	***SparsyFed***
CIFAR-10	dense	83.70 ± 1.70	-	-	-	73.81 ± 4.84	-	-	-
	0.9	80.56 ± 1.90	76.16 ± 1.30	81.15 ± 1.03	$\mathbf{82.13 \pm 1.53}$	69.79 ± 3.78	67.40 ± 4.11	71.87 ± 2.63	$\mathbf{75.00 \pm 2.78}$
	0.95	74.71 ± 3.29	75.53 ± 2.27	79.36 ± 1.03	$\mathbf{82.60 \pm 1.58}$	60.00 ± 4.66	61.55 ± 4.18	72.08 ± 2.09	$\mathbf{75.95 \pm 3.39}$
	0.99	66.27 ± 5.08	70.71 ± 0.15	73.45 ± 1.37	$\mathbf{77.71 \pm 1.69}$	43.96 ± 11.99	51.71 ± 3.54	56.91 ± 3.55	$\mathbf{63.69 \pm 3.90}$
	0.995	63.82 ± 2.41	56.02 ± 3.95	69.15 ± 1.60	$\mathbf{70.01 \pm 0.43}$	19.02 ± 10.77	41.33 ± 3.64	52.15 ± 3.87	$\mathbf{56.79 \pm 3.97}$
	0.999	31.79 ± 19.10	17.66 ± 8.34	36.07 ± 7.49	$\mathbf{51.39 \pm 3.19}$	11.50 ± 4.49	18.76 ± 4.28	29.31 ± 6.75	$\mathbf{43.68 \pm 7.61}$
CIFAR-100	dense	52.29 ± 1.14	-	-	-	48.34 ± 2.71	-	-	-
	0.9	46.57 ± 1.71	40.70 ± 4.72	51.99 ± 0.21	$\mathbf{53.08 \pm 0.90}$	41.96 ± 2.16	31.92 ± 7.65	45.59 ± 0.75	$\mathbf{48.37 \pm 1.73}$
	0.95	28.07 ± 23.27	38.82 ± 1.75	47.19 ± 1.88	$\mathbf{52.81 \pm 1.72}$	11.48 ± 17.51	34.21 ± 7.65	44.31 ± 2.14	$\mathbf{48.27 \pm 2.70}$
	0.99	19.65 ± 16.30	18.97 ± 2.08	42.76 ± 4.08	$\mathbf{46.64 \pm 1.59}$	0.14 ± 0.72	13.07 ± 2.26	34.75 ± 3.38	$\mathbf{41.03 \pm 2.14}$
	0.995	9.51 ± 14.81	6.01 ± 4.74	36.43 ± 4.97	$\mathbf{42.21 \pm 1.03}$	0.14 ± 0.72	7.04 ± 5.25	26.44 ± 17.35	$\mathbf{35.72 \pm 2.01}$
	0.999	3.81 ± 2.18	1.96 ± 0.66	5.80 ± 2.86	$\mathbf{15.96 \pm 0.64}$	0.14 ± 0.72	1.66 ± 0.97	3.56 ± 2.07	$\mathbf{13.84 \pm 3.69}$
Speech Commands	dense	91.49 ± 0.94	-	-	-	80.15 ± 2.69	-	-	-
	0.9	84.28 ± 0.88	87.79 ± 1.40	88.68 ± 1.72	$\mathbf{92.32 \pm 1.59}$	65.44 ± 0.97	70.35 ± 2.65	77.15 ± 0.77	$\mathbf{79.67 \pm 2.78}$
	0.95	78.58 ± 0.44	84.29 ± 1.50	84.89 ± 0.49	$\mathbf{89.14 \pm 1.15}$	57.39 ± 1.04	65.90 ± 1.88	71.28 ± 1.75	$\mathbf{75.46 \pm 2.24}$
	0.99	65.01 ± 0.84	57.79 ± 0.82	69.22 ± 1.59	$\mathbf{75.82 \pm 3.72}$	50.42 ± 6.26	41.42 ± 1.60	53.55 ± 2.00	$\mathbf{56.69 \pm 4.56}$
	0.995	56.73 ± 1.00	37.16 ± 2.71	58.23 ± 1.84	$\mathbf{68.02 \pm 3.14}$	34.20 ± 1.43	22.61 ± 3.45	43.16 ± 3.47	$\mathbf{48.30 \pm 5.39}$
	0.999	21.56 ± 12.79	10.10 ± 4.01	17.70 ± 2.58	$\mathbf{47.43 \pm 1.66}$	19.25 ± 6.01	8.85 ± 3.76	17.14 ± 2.97	$\mathbf{29.24 \pm 2.34}$

Table 1: Aggregated results for CIFAR-10, CIFAR-100, and Speech Command datasets, with ResNet-18, ZeroFL, FLASH, and SparsyFed implementations.

the dense model. Additionally, the minimal performance drop at lower sparsity levels ($90 - 95\%$) allows *SparsyFed* to outperform the dense model in certain scenarios. To completely assess our method capabilities, we increased the target sparsity to the point (99.9%) where *SparsyFed* is no longer able to retain sufficient accuracy. Our results show that other methods struggle to train effectively under such conditions. The plasticity of *SparsyFed* promotes more consistent performance across clients, even in highly sparse settings, leading to a more synchronized and globally pruned model.

5.2 Communication costs

The promising accuracy achieved at very high sparsity ratios makes *SparsyFed* particularly suitable for cross-device FL settings, where communication costs are a critical concern. As illustrated in fig. 2 (left), *SparsyFed* significantly outperforms the baselines regarding both communication savings ($19.29\times$ less communication costs compared to the dense model and $1.66\times$ compared to ZeroFL) and maintaining required accuracy (consistently above 45%). FLASH has comparable communications costs, $0.97\times$ ours, but results in lower accuracy. Notably, *SparsyFed* consistently achieves higher accuracy for any given communication cost (x-value) compared to the baselines. This advantage arises from our ability to prune weights during client training and maintain a close-to-target sparsity ratio during server aggregation, effectively reducing uplink and downlink communication costs. Importantly, FLASH does not see an increase in model density after aggregation due to its fixed local training regime, which prevents any regrowth of weights. This characteristic results in stable communication costs. In contrast, ZeroFL experiences a substantial increase in model density after aggregation, leading to a systematic and significant rise in downlink communication costs.

5.3 Consensus on the sparse masks across clients

Achieving consensus on the sparse masks of the clients' updates after each training round is crucial for ensuring minimal accuracy drop. This consensus is evident in the global model's progressively more stable sparsity level across federated rounds. As shown in fig. 2 (right), *SparsyFed* demonstrates a minimal deviation from the target sparsity (90%), unlike ZeroFL and TopK, whose values drop below 47% and 83%, respectively, during the first stage of training. *SparsyFed*'s consistency implies that clients effectively collaborate to train the same subset of parameters, which reduces the need for excessive pruning after local training and helps retain more useful information. For all methods, local updates maintain consistent sparsity due to post-training pruning; therefore, any increase in the global model's density indicates that local weight regrowth during training alters the local masks. Once clients' updates are aggregated, such alteration results in a denser global model on the server. This misalignment among clients' masks can negatively impact communication efficiency and accuracy. Global training seems to take advantage of this consensus in terms of training

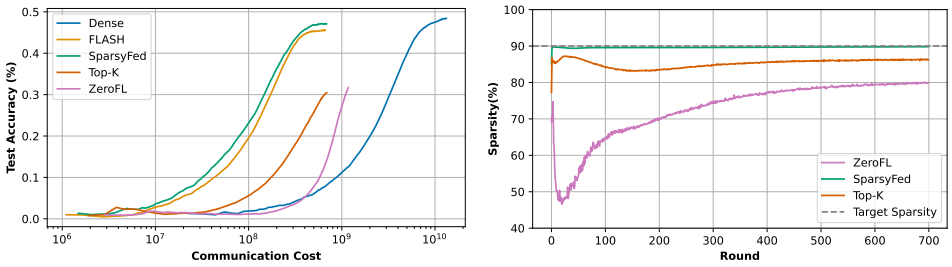

Figure 2: **(a)** The plot on the left compares accuracy versus communication cost for four implementations: ZeroFL, Top-K, FLASH, and *SparsyFed*, with the dense model as a reference. The test is conducted on CIFAR-100 with LDA $\alpha = 0.1$ and 95% sparsity. Our implementation outperforms the others, achieving high accuracy quickly. **(b)** The plot on the right shows the global model sparsity level, measured on the server after aggregating local updates (CIFAR-100, $\alpha = 0.1$). The density gain reflects mismatches between client updates, causing the aggregated model to regain density, which can degrade performance and increase downlink communication. **Note**: FLASH maintains target sparsity after the first round with a fixed mask.

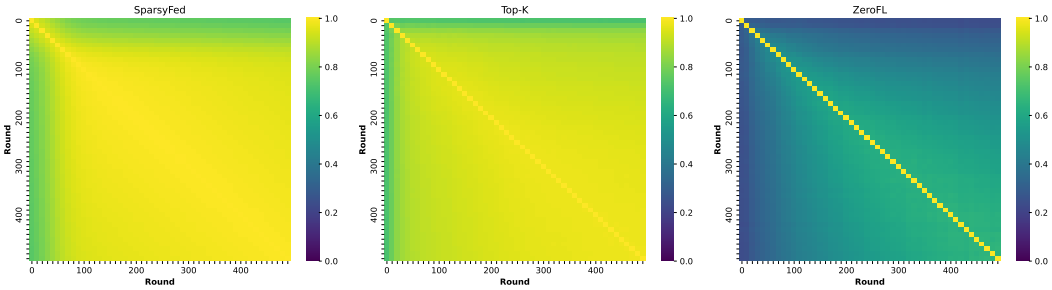

Figure 3: Intersection over Union (IoU) of global model binary masks between training rounds for *SparsyFed*, TopK, and ZeroFL (CIFAR-100, $\alpha = 0.1$, 95% target sparsity). The IoU is calculated between each mask and all other masks across rounds to show mask movement over time. Both the x and y axes represent training rounds, with the diagonal indicating identical comparisons. Higher IoU values (close to 1.0) signify stronger similarity between masks, while lower values indicate significant changes. *SparsyFed* shows consistent mask movement with minimal variation, suggesting strong consensus on weight usage among clients. ZeroFL struggles the most to find mask consensus, with masks continuing to shift even in later rounds. **Note:** FLASH is not present since the global mask is fixed for the entire federate training.

convergence, as shown in fig. 2 (left), where both *SparsyFed* and FLASH outperform `TopK` despite similar overall communication costs.

5.3.1 Sparsity pattern dynamics

A deeper analysis of the global model's sparse mask evolution during training highlights *SparsyFed*'s ability to maintain a stable sparse pattern, driven by two key factors: focused weight utilization and plasticity. The weight reparameterization method targets a critical subset of weights, concentrating information where it is most impactful. This targeted approach improves training efficiency, enhances client collaboration, and reduces communication costs, as shown in fig. 2.

Unlike fixed-mask methods like FLASH, *SparsyFed* incorporates dynamic mask adaptation, promoting plasticity throughout the training process. This allows a natural warmup phase during which the sparsity pattern emerges organically, as shown in fig. 3.left. Clients rapidly converge on a stable shared mask, ensuring consistent performance. In contrast, `TopK` (fig. 3.center), while faster in its initial rounds, never fully settles on a stable mask, resulting in continuous variation across rounds.

ZeroFL (fig. 3.right), though more robust initially, struggles to maintain performance as it forces weight regrowth to adjust the mask, leading to instability in later rounds.

FLASH is not considered in this comparison, as it fixes its mask after the first round, eliminating any plasticity but reducing mask variability entirely. While this strategy ensures that the model adheres to a fixed sparse structure, it introduces potential drawbacks. Specifically, since the mask is determined based on the data distribution observed in the first round, FLASH becomes highly dependent on the initial client data. This lack of plasticity contrasts with our method, which allows continuous adaptation, enabling it to handle shifts in data distribution across rounds more effectively.

5.4 Ablation on Weight Reparametrization

In this ablation study, we assessed various weight reparameterization techniques for their ability to sustain a sparsity-driven model while preserving dense-like performance. Since the sparse activations during the backward pass rely on a sparse weight model, it is essential for the model to maintain high sparsity levels throughout training while minimizing accuracy loss.

We evaluated three approaches: fixed-mask training, spectral reparameterization Miyato et al. (2018), and Powerpropagation Schwarz et al. (2021). Each technique was applied to a ResNet-18 model trained with sparse activations for the backward pass and `TopK` unstructured pruning at the end of each training round. This setup enabled direct comparison with a baseline model that lacked any reparameterization. Among the methods, Powerpropagation proved most effective for our use case (fig. 4), demonstrating superior resilience in preserving accuracy with minimal degradation. By leveraging a "rich get richer" dynamic, Powerpropagation naturally induces sparsity during training. This enables the model to remain sparse during the training complementing our sparse activation backward pass, improving the model's overall efficiency and effectiveness.

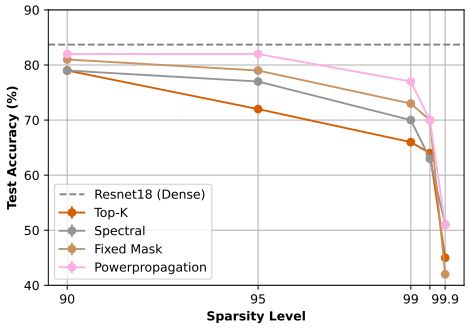

Figure 4: Test accuracy of different reparameterization methods with sparse activations during backpropagation. We deployed a ResNet-18 trained on the CIFAR-10 dataset using LDA with $\alpha = 1$. This plot illustrates the model's performance under different proposed reparameterization techniques on different sparsity levels. Powerpropagation exhibited superior robustness to the applied sparsity levels, achieving the best overall performance among these methods.

5.5 Ablation on Activation Pruning

The activation pruning step in *SparsyFed* (lines 6–10 in algorithm 2) is designed to reduce computational costs on edge devices. This decision is motivated by prior studies, as discussed in the related work section. As part of our ablation studies, we analyzed the impact of this step on the model's accuracy and overall performance. Since sparsifying activations during the backward pass do not directly affect weight density, this analysis specifically focuses on its influence on test accuracy.

To evaluate this, we compared *SparsyFed* with and without activation pruning during the backward pass. Our results show that activation pruning has a minimal impact on test accuracy at higher density levels. Significant performance degradation occurred only under extreme sparsity, where excessive pruning in specific layers substantially reduced activations, leading to diminished overall performance. Ultimately, the computational speedup achieved through activation pruning validates its inclusion in *SparsyFed*, as it balances efficiency with model accuracy.

6 Related Work

Sparse training in centralized settings. Methods to enforce sparsity in neural networks can be grouped into two main categories: (1) dense-to-sparse methods that train a dense model and achieves

Sparsity	$\alpha = 10^3$ (IID)		$\alpha = 1.0$ (non-IID)	
	no Act. Pr.	Act. Pr.	no Act. Pr.	Act. Pr.
0.90	83.92 ± 1.58	$\mathbf{84.31 \pm 0.86}$	$\mathbf{82.27 \pm 2.21}$	82.13 ± 1.23
0.95	83.80 ± 0.90	$\mathbf{84.25 \pm 1.38}$	81.53 ± 2.10	$\mathbf{82.6 \pm 1.58}$
0.99	$\mathbf{77.54 \pm 1.98}$	77.16 ± 0.85	75.76 ± 1.78	$\mathbf{77.71 \pm 1.69}$
0.995	$\mathbf{74.6 \pm 1.01}$	72.71 ± 0.65	$\mathbf{70.89 \pm 2.22}$	70.01 ± 0.43
0.999	$\mathbf{62.12 \pm 1.74}$	55.24 ± 2.09	$\mathbf{62.67 \pm 2.19}$	51.39 ± 3.191

Table 2: Accuracy comparison between *SparsyFed* with and without the pruning of the activation, on CIFAR-10 whit LDA $\alpha = 10^3$ and $\alpha = 1.0$.

a sparse model after training (Molchanov et al., 2017; Louizos et al., 2017), (2) sparse-to-sparse methods where pruning happens during training (Louizos et al., 2018; Dettmers & Zettlemoyer, 2019; Evci et al., 2020; Jayakumar et al., 2021; Raihan & Aamodt, 2020), thus theoretically reducing computational requirements (Bengio et al., 2015) and speeding up training. Our work introduces a novel method to implement sparse training directly within FL clients, inspired by sparse-to-sparse approaches in centralized settings such as Raihan & Aamodt (2020).

Pruning model updates in FL. After-training pruning, where clients first train dense models and then prune updates, is a common approach in FL (Sattler et al., 2019; Wu et al., 2020; Malekijoo et al., 2021). Sparse Ternary Compression (Sattler et al., 2019) combines `TopK` pruning with ternary quantization to compress client updates, while FedZip (Malekijoo et al., 2021) uses layer-wise pruning, and FedSCR (Wu et al., 2020) employs patterns in client updates for more aggressive compression. However, these methods primarily improve communication efficiency without reducing computational overhead, as they still train dense models. In contrast, our method trains sparse models from the start, resulting in sparse updates and maintaining sparsity even after server aggregation, ensuring efficient upstream and downstream communication.

Sparse training in FL. Several studies have explored sparse learning in federated settings (Bibikar et al., 2022; Huang et al., 2022; Jiang et al., 2023; Qiu et al., 2021; Babakniya et al., 2023), but each has limitations. FedDST (Bibikar et al., 2022) applies RigL (Evci et al., 2020) to train sparse models, focusing on heterogeneous data without addressing extreme sparsity levels. FedSpa (Huang et al., 2022) uses a fixed sparse mask throughout training without a clear rationale behind it. PruneFL (Jiang et al., 2023) computes a sparse mask using biased client data and requires full gradient uploads, increasing communication costs. ZeroFL (Qiu et al., 2021) integrates SWAT (Raihan & Aamodt, 2020) for local training but struggles with weight regrowth, requiring pruning after each round, which can cause information loss. FLASH (Babakniya et al., 2023) introduces a fixed mask after a warmup phase, but it depends heavily on the chosen clients during the first sampling and does not adapt to concept drift. In contrast, our method uses a dynamic sparse mask, offering more flexibility and better performance in highly non-IID FL settings.

7 Conclusions

This work presents *SparsyFed*, an adaptive sparse training method tailored for cross-device federated learning (FL). We show that *SparsyFed* can achieve impressive sparsity levels while minimizing the accuracy drop due to the compression. *SparsyFed* outperforms in accuracy three federated sparse training baselines, `TopK`, ZeroFL, and FLASH, using adaptive and fixed sparsity for three typical datasets used in cross-device FL. We were able to ensure a limited drop in accuracy at sparsity levels of up to 95%, achieving up to a $19.29\times$ reduction in communication costs compared to dense baselines. The results presented in this work make our proposal particularly suitable for cross-device FL settings, which may require extreme communication cost reductions and the capability to adapt to heterogeneous distributions across federated rounds.

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A IMPLEMENTATION

We implemented all experiments using a ResNet-18 backbone (He et al., 2016), which was adjusted to accommodate the number of classes for each task. The training pipeline was developed using `PyTorch` (Paszke et al., 2019), and specific components of the various methods, such as layer-wise activation pruning and weight reparameterization, were implemented through custom layers and functions.

To simulate the federated learning environment, we used the `Flower` framework (Beutel et al., 2022). All models were trained from scratch, without pre-trained weights, to ensure a fair evaluation of *SparsyFed*'s performance under different sparsity settings.

To structure the code and ensure easy reproducibility, we utilized a template from CaMLSys Lab. The complete code for the implementation is available in this repository.

B RELATED WORK

Sparse training in centralized settings. Methods to enforce sparsity in neural networks can be grouped into two main categories: (1) dense-to-sparse methods that train a dense model and achieves a sparse model after training (Molchanov et al., 2017; Louizos et al., 2017), (2) sparse-to-sparse methods where pruning happens during training (Louizos et al., 2018; Dettmers & Zettlemoyer, 2019; Evci et al., 2020; Jayakumar et al., 2021; Raihan & Aamodt, 2020), thus theoretically reducing computational requirements (Bengio et al., 2015) and speeding up training. In our work, we introduce a novel method, inspired by sparse-to-sparse approaches in centralized settings such as Raihan & Aamodt (2020), to implement sparse training directly within FL clients.

Pruning model updates in FL. After-training model pruning, where clients regularly train their dense model and then apply pruning to the resulting updates, has been widely explored in the FL literature (Sattler et al., 2019; Wu et al., 2020; Malekijoo et al., 2021). Sattler et al. (2019) introduced Sparse Ternary Compression, which combines `TopK` pruning with ternary quantization on client updates. Similarly, FedZip (Malekijoo et al., 2021) applies a layer-wise pruning approach, while FedSCR (Wu et al., 2020) leverages patterns in client updates for more aggressive compression. However, these methods primarily focus on enhancing communication efficiency without tackling the issue of reducing computational overhead during training, as they conduct regular training on a dense model before pruning weight updates. Our approach simultaneously addresses both concerns, as training a sparse model naturally yields sparse updates. Furthermore, existing after-training pruning methods typically focus solely on upstream communications or experience performance degradation when applied downstream. In contrast, our method effectively maintains sparsity after server-side aggregation, as clients rapidly converge on a shared sparsity mask, ensuring that server-to-client payloads remain sparse as well.

Sparse training in FL. Few studies have investigated the benefits of sparse learning in federated settings (Bibikar et al., 2022; Huang et al., 2022; Jiang et al., 2023; Qiu et al., 2021; Babakniya et al., 2023). Specifically, FedDST (Bibikar et al., 2022) used RigL (Evci et al., 2020) to train sparse models on clients but mainly focused on highly heterogeneous data distributions without considering extreme sparsity levels. FedSpa (Huang et al., 2022) employed a randomly initialized sparse mask that remained fixed throughout training, providing no clear justification for this approach. PruneFL (Jiang et al., 2023) computes the sparse mask during the initial round on a designated client, using its potentially biased data. Qiu et al. (2021) empirically found that, after local training, the `TopK` weights differ across clients, particularly at higher sparsity levels, leading to divergent sparse masks. This divergence makes aggregation inefficient and results in downstream dense models. In response, Qiu et al. (2021) proposed ZeroFL, which integrates unstructured SWAT during local client training. However, this alone does not guarantee achieving the desired level of sparsity, as SWAT often leads to weight regrowth with each optimizer step. To address this, ZeroFL applies `TopK` pruning before sending the model back to the server, ensuring the model meets the targeted sparsity. It is important to highlight that ZeroFL applies the same level of sparsity uniformly across all layers of the model, regardless of their sensitivity. Additionally, it suffers significantly from weight regrowth, necessitating pruning after training, which may lead to the loss of valuable information. The recent work in Babakniya et al. (2023) introduces FLASH, which employs a fixed binary mask throughout the

entire training process. This mask is established during a warmup phase where a subset of clients ([10, 20]) train their model for a non-negligible number of epochs ([10, 20, 40]), and compute their per-layer sensitivity. The server aggregates the client's sensitivities to determine the mask. The fixed nature of this mask means that no weight regrowth is allowed, and no further pruning is required after the warmup phase.

However, the success of FLASH heavily depends on the warm-up phase, which is influenced by the client pool used to determine the mask and requires tuning an additional hyperparameter (the number of epochs for computing sensitivity). Also, due to the mask's fixed nature, FLASH does not adapt to concept drift in client data during subsequent FL rounds.

In contrast to these studies, we utilize a dynamic sparse mask, allowing for greater flexibility and enabling the model to achieve extreme sparsity even with continual settings and highly non-IID data suffering limited performance degradation.

Cross-device federated learning. Federated Learning (FL) has emerged as a paradigm shift from traditional machine learning approaches improving data privacy and moving the computation load to the network's edge. FL enables collaborative model training across decentralized devices while keeping data localized, thereby mitigating risks associated with centralized data aggregation (McMahan et al., 2017). In FL, client devices participate in model training via iterative rounds, aggregating local updates to build a global model. This distributed approach is particularly advantageous for scenarios involving edge devices with limited computational resources and intermittent connectivity (Kairouz et al., 2021). Cross-device Federated Learning (FL) is particularly challenging due heterogeneous and resource-constrained nature of client devices, such as smartphones and IoT devices. The variability in hardware capabilities and data distributions across devices necessitates specialized techniques to optimize both computation and communication techniques such as quantization and model pruning have shown promise in reducing the amount of data that needs to be transmitted during each communication round. However, these methods often face challenges related to maintaining model performance and achieving consensus on the sparsity patterns among clients.

Model sparsification. Sattler et al. (2019) propose Sparse Ternary Compression (STC), a lossy compression scheme able to extremely reduce the per-round communication cost of FL iterations. STC first applies top-k sparsification after unaltered on-device training and then further compresses the weight updates using a ternary quantization. Other types of work have followed the idea proposed in Rigging the Lottery Ticket (Evci et al., 2020) where an initial sparse mask is fixed at the beginning of the training and remains mostly unchanged throughout. This approach allows clients to train only the initially fixed weights (Jiang et al., 2023; Babakniya et al., 2023).

Weight parametrization. Weight reparameterization in machine learning refers to the process of altering the parameterization of a model's weights to achieve various goals or facilitate certain training strategies (Li et al., 2019; Gunasekar et al., 2017; Zhao et al., 2022; Vaskevicius et al., 2019). This approach can be particularly useful for enhancing the model's robustness to the application of sparsity. In this work, we consider a parameterization based on Weight Spectral Normalization (Miyato et al., 2018) and Powerpropagation (Schwarz et al., 2021). The former reparameterizes the weights based on the proportion of each weight relative to the one with the highest magnitude, while the latter applies an alpha power to the weights inducing a "rich get richer" mechanism.

$$W_{\text{refactor}} = W \cdot \frac{W}{\sigma(W)}$$

C Additional explanations on *SparsyFed* components

Model Preparation for Sparsification. The first step is to prepare the model for sparsification by applying a parametrization to the weights at the layer level. Several parametrizations have been proposed in the past, but we opted for the one proposed in Schwarz et al. (2021). This approach leverages the information already present in the model itself. The idea is to apply a power to a specific value, β, to the weights of the network. While prior work aimed to aid model convergence,

this parametrization induces a sparse representation. This transformation is applied only to the weights of the model, leaving other parameters untouched.

Following the original approach, each weight w_i is replaced by w_i^β. This transformation is only applied to the weights of the neural network, leaving other parameters unchanged. Given the reparameterized loss function $L(\mathbf{w}^\beta)$, the gradient with respect to $\mathbf{w}$ becomes:

$$\nabla_{\mathbf{w}} L(\mathbf{w}^\beta) = \nabla_{\mathbf{w}^\beta} L(\mathbf{w}^\beta) \cdot \beta \mathbf{w}^{\beta-1}$$

Here, $\nabla_{\mathbf{w}^\beta} L(\mathbf{w}^\beta)$ is the gradient with respect to the reparameterized weights. This gradient is scaled element-wise by $\beta \mathbf{w}^{\beta-1}$, which adjusts the step size proportionally to the magnitude of each weight. This update is distinct from simply scaling the gradients in the original parameterization, as it directly modifies the reparameterized weights, not the original weights. This modified gradient step enables a "rich-get-richer" dynamic where the gap between high and low-magnitude weights increases, leading to a natural separation between them. This makes the model relatively insensitive to pruning.

While the original work utilized this approach to implement a package for various tasks in a centralized environment, we leveraged these new properties to implement an extensive sparsification method to reduce training and communication costs.

Weight Sparsification. Subsequent to the reparametrization, the model is then ready to be pruned. Our first concern was to reduce the model's size during communication rounds to speed up FL training. In this context, a compression mechanism is typically used to reduce the payload size that has to be exchanged. To achieve this, it is necessary to reduce the number of parameters in the network, inducing a high level of sparsity. We decided to use a top-k sparsification method to remove all low-magnitude weights from the network. This pruning operation is implemented at the end of each local training on the client. This decision is based on several observations: (a) since the parametrization affects gradient descent, pruning before training would not be beneficial, (b) the server is not aware of the features of the data on the clients, and initializing the sparsity mask on the server side could cause a significant drop in performance.

We also decided to induce sparsity from the very first round. Using a dense model for initial rounds of training and then pruning has proven ineffective and can even degrade performance. This is because the clients would only use a small portion of the weights during training. It is crucial that they start training with the restricted fraction from the beginning to maximize their effectiveness. Learning on a wider network that is later pruned does not help them converge.

Sparsity is then induced after the first round of training on each device, allowing the local updates to be sent back to the server for aggregation into a new global model. From this communication round onwards, a high level of compression can be performed to reduce the model's size.

From the next round onward, a second quality of Powerpropagation comes into play, drastically reducing the regrowth of new weights during training. This results in minimal shifts of the model's sparsity mask from the global one received from the server. Thus, the sparsity mask of the new model update sent to the server will differ very little from the global one. In other words, using Powerpropagation, we are forcing all clients to converge towards a shared sparsity mask, similar to Evci et al. (2020), without totally inhibiting the regrowth of new paths. The mask is decided by the clients in the very first round of training based on their local data. This means that the global model will remain highly sparse even after aggregating new local updates from the clients. This result allows for significant compression during the downlink communication from the server to the clients in subsequent training rounds.

Activation Sparsification. To further reduce memory consumption during the training phase, we implemented a layer-wise pruning similar to the one proposed in Raihan & Aamodt (2020). The original idea was to speed up inference and training by reducing the number of computational operations, inducing a fixed level of sparsity on the weights of each layer during both the forward and backward passes. However, this could cause a significant drop in performance with high levels of sparsity, as not all layers retain the same level of information. Applying the same sparsity to all layers could negatively impact those that naturally retain more information. In our implementation, we

heavily modified the approach, retaining only key concepts. Since we keep the model sparse during almost all training rounds, except for the first, sparsifying the weights during the forward pass is not necessary.

For the sparsification of activations during the backward pass, instead of applying the same level of sparsity to all layers of the model, the global sparsity mask is used to determine the pruning sensitivity of each layer. The level of sparsification applied to the activations is directly proportional to the level of sparsity of the weights in the same layer, allowing layers that retain more information to maintain denser activations while drastically reducing the activations in layers that reach a high level of sparsity.

Following this approach, we were able to utilize effective and extensive sparsification while minimizing the degradation typically induced by pruning.

D Additional Background

D.1 Powerpropagation

Powerpropagation is a weight reparameterization technique that induces sparsity in neural networks. Essentially, it causes gradient descent to update the weights in proportion to their magnitude, leading to a "rich get richer" dynamic where small-valued parameters remain largely unaffected by learning. As a result, models trained with Powerpropagation exhibit a weight distribution with significantly higher density at zero, allowing more parameters to be pruned without compromising performance. Powerpropagation involves raising model parameters to the power of β (where $\beta > 1$) in the forward pass, while preserving their sign. This reparameterization can be expressed as

$$w = v \cdot |v|^{\beta-1}$$

where w represents the weights, and v are the reparameterized parameters. Due to the chain rule, this reparameterization causes the magnitude of the parameters (raised to $\beta - 1$) to appear in the gradient computation. Consequently, small-valued parameters receive smaller gradient updates, while large-valued parameters receive larger updates, thus amplifying the "rich get richer" dynamic. Powerpropagation leads to intrinsically sparse networks, meaning that a significant portion of the weights converge to values near zero during training. This property allows for the pruning (removal) of a large number of weights without significantly compromising model performance. Powerpropagation can also be easily integrated with existing sparsity algorithms to further enhance performance. Studies show the benefits of combining Powerpropagation with popular methods such as Iterative Pruning and TopKAST (Gale et al., 2019; Jayakumar et al., 2021).

D.2 Fixed mask sparse training (FLASH)

Training a fixed subset of weights can be an effective approach for sparse model training in Federated Learning (FL), offering several significant benefits. By limiting the number of active (non-zero) weights, this approach reduces the computational and memory demands compared to dense models, which is particularly advantageous given the resource constraints often found in client devices within FL. In addition to these resource savings, sparse models also enhance communication efficiency between clients and servers. Since only the active weights need to be transmitted, the message size is reduced, leading to faster training and lower bandwidth usage.

Another advantage of this method lies in its potential to uncover "winning tickets" within neural networks Frankle & Carbin (2019). Research indicates that dense, randomly initialized networks often contain sparse sub-networks, known as "winning tickets," which, when trained independently, can achieve similar performance to the full model. Training a fixed subset of weights promotes this sparsity and encourages the model to learn more efficient representations, potentially revealing these sub-networks.

Despite these benefits, certain challenges arise when using a fixed subset of weights in FL. If the mask is not initialized correctly or fails to adapt during training, the model's performance may be compromised. To mitigate these issues, strategies such as sensitivity-based pruning and selective

mask updates are crucial for fully leveraging the advantages of sparse learning in FL Babakniya et al. (2023). The training process for sparse models can be formalized as:

$$W_{\text{sparse}} = M \odot W$$

where W_{sparse} represents the sparse weights, M is the binary mask (with values 0 or 1), and $\odot$ denotes element-wise multiplication.

D.3 Spectral Normalization

Spectral Normalization (SN) is a weight normalization technique that aims to limit the largest singular value of each weight matrix, thereby controlling the Lipschitz constant of the function represented by the network. This is achieved by constraining the spectral norm of each layer $g : h_{in} \to h_{out}$. Formally, given a weight matrix W, its spectral norm is defined as the largest singular value $\sigma(W)$, i.e.,

$$\sigma(W) = \max_{h \neq 0} \frac{\|Wh\|_2}{\|h\|_2}$$

SN normalizes the weight matrix W by dividing it by its spectral norm:

$$\bar{W} = \frac{W}{\sigma(W)}$$

This ensures that the Lipschitz constant of each layer remains bounded, promoting stability during training, particularly in scenarios where gradients may explode. Originally proposed as a regularization technique to stabilize discriminator training in Generative Adversarial Networks (GANs) Miyato et al. (2018), SN has since found broader applications, including improvements in generative neural networks Zhao et al. (2018), and has been theoretically linked to enhanced generalization and adversarial robustness Farnia et al. (2018); Sokolic et al. (2017); Cisse et al. (2017).

In the context of federated learning and sparse training, SN can significantly improve the resilience of models to sparsification. SN regularizes the model mappings by enforcing the Lipschitz constraint, reducing sensitivity to high sparsity levels. This effect has been observed in prior work on pruning, where SN was used to prune redundant mappings and enforce a spectral-normalized identity prior Lin et al. (2020).

However, by strictly constraining the Lipschitz constant, SN may reduce the model's flexibility during training, potentially affecting convergence in some cases. To address this, following an approach similar to those proposed in other parameterization works Vaskevicius et al. (2019), we propose a slightly different method where the spectral norm is modified to enhance the distribution of the weights. Specifically, we redefine the weight update rule as:

$$w = w \cdot \frac{|w|}{\sigma(W)}$$

This approach allows us to maintain most of the model's performance while improving its resilience to pruning.

E ADDITIONAL EXPERIMENTAL RESULTS

E.1 SENSITIVITY ANALYSIS OF POWERPROPAGATION EXPONENT

The primary concern with introducing Powerpropagation reparameterization is the addition of another hyperparameter that needs to be tuned for federated training. However, preliminary tests suggest that the sensitivity of this hyperparameter is not as critical as initially expected. We tested several values proposed in the original paper to assess their performance in sparse training within a federated environment. As shown in the figure, any value between 1 and 2 results in a significant performance improvement in dynamic sparse training compared to the baseline without parameterization.

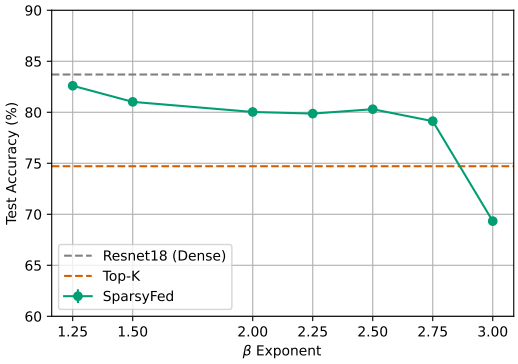

Figure 5: Test Accuracy with different β values, with 95% sparsity on CIFAR-10 LDA $\alpha = 1.0$.

E.2 DISTRIBUTION OF THE SPARSITY THROUGH THE LAYERS.

Sparsity distribution is crucial as the sparsity achieved in the weights of each layer is used to determine the sparsity level applied to the activations during the backward pass. Each layer is pruned with a distinct sparsity level based on the information it contains, leveraging layer sensitivity to implement an effective pruning strategy for activations. This ensures that the sensitivity observed in the weights is reflected in the activation pruning, allowing for dynamic sparsity for both weights and activations based on the natural sensitivity of each layer. Empirical evidence, shown in fig. 6, supports this approach, showing that in ZeroFL implementations, the sparsity of the layers remains uniform across all layers of the model, while in others, the sparsity levels vary significantly between layers. The first layers tend to be nearly fully dense, while the deeper layers exceed the global sparsity target, indicating that the first layers retain more information than the deeper layers.

A key factor in this behavior is the limited weight regrowth observed in *SparsyFed*, where only a small number of weights transition from zero to non-zero values after each training round. As

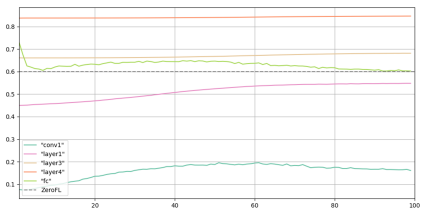

Figure 6: Different sparsity ration of some relevant layer, ZeroFL (red) vs *SparsyFed* with Powerpropagation and sparsity ratio of 60%. The first layers are not shown as they are kept dense in ZeroFL. Empirical observation regarding the nature of the pruning procedure (constant across layers for ZeroFL, variable unstructured across layers for *SparsyFed*). We could use this in the main paper only if we make the case that our method performs better because of this. We consider this appendix material for now.

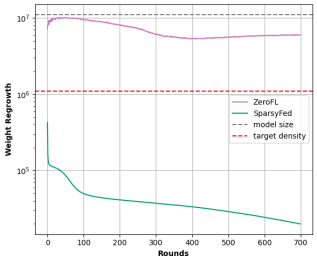

Figure 7: Number of weights regrown (weights switching from zero to non-zero) after one epoch of local training, compared between *SparsyFed* and ZeroFL. ZeroFL performs the worst of all tested implementations in this regard, with 50-80% of weights regrowing after local training. In contrast, *SparsyFed* shows near-zero regrowth (200× lower than ZeroFL). This ensures that the movement of the mask focuses on important weights rather than being distributed across many. **Note**: FLASH does not allow weight regrowth due to its fixed-mask training approach, so it is excluded from this graph.

depicted in 7, *SparsyFed* exhibits minimal regrowth, which stabilizes quickly over a few rounds. This is directly attributable to the use of Powerpropagation during training, which drastically reduces the impact of smaller weights. Although this behaviour was not highlighted in the original paper, it represents empirical evidence supporting the effectiveness of an inherited sparse model training procedure.

Our implementation, as illustrated in 3, shows that the global model's mask shifts slightly during the initial training rounds. This suggests that overly rigid approaches, such as those proposed in FLASH, may negatively impact performance by failing to accommodate necessary flexibility. On the other hand, *SparsyFed*'s approach maintains flexibility while still converging toward a consistent global mask.

E.3 Full Tables Accuracy Results

Dataset	Sparsity	Resnet18	ZeroFL	FLASH	SparseFedPP
CIFAR10 ($\alpha = 10^3$)	dense	85.14 ± 1.18	-	-	-
	0.9	82.16 ± 0.80	78.67 ± 1.52	82.57 ± 2.05	$\mathbf{84.31 \pm 0.86}$
	0.95	77.92 ± 0.97	76.16 ± 1.28	82.22 ± 0.14	$\mathbf{84.25 \pm 1.38}$
	0.99	68.11 ± 3.50	72.40 ± 1.08	$\mathbf{77.48 \pm 2.86}$	77.16 ± 0.85
	0.995	54.49 ± 8.70	60.31 ± 4.11	71.85 ± 1.63	$\mathbf{72.71 \pm 0.65}$
	0.999	17.87 ± 6.22	21.91 ± 1.11	46.43 ± 16.73	$\mathbf{55.24 \pm 2.09}$
CIFAR100 ($\alpha = 10^3$)	dense	53.79 ± 1.63	-	-	-
	0.9	46.47 ± 1.74	44.11 ± 0.93	53.06 ± 1.10	$\mathbf{54.44 \pm 1.33}$
	0.95	13.45 ± 21.45	33.67 ± 3.34	49.25 ± 1.63	$\mathbf{54.33 \pm 1.48}$
	0.99	1.54 ± 1.22	10.59 ± 2.48	44.40 ± 1.77	$\mathbf{47.62 \pm 1.67}$
	0.995	0.97 ± 0.64	4.04 ± 3.43	36.82 ± 1.72	$\mathbf{42.05 \pm 1.21}$
	0.999	0.97 ± 0.64	0.87 ± 0.21	9.61 ± 3.61	$\mathbf{13.85 \pm 1.01}$
Speech Commands ($\alpha = 10^3$)	dense	92.98 ± 0.67	-	-	-
	0.9	85.53 ± 0.84	89.12 ± 1.11	89.89 ± 0.74	$\mathbf{91.34 \pm 0.52}$
	0.95	80.19 ± 1.92	85.63 ± 1.12	88.16 ± 1.37	$\mathbf{89.79 \pm 0.21}$
	0.99	67.67 ± 2.32	60.79 ± 2.44	76.41 ± 1.31	$\mathbf{77.00 \pm 0.73}$
	0.995	36.69 ± 1.45	41.16 ± 3.10	67.54 ± 1.15	$\mathbf{70.03 \pm 1.14}$
	0.999	63.63 ± 3.24	16.77 ± 6.23	31.83 ± 2.18	$\mathbf{49.27 \pm 0.50}$

Table 3: Aggregated results for CIFAR10, CIFAR100, and Google Speech Command datasets.

Dataset	Sparsity	Resnet18	ZeroFL	FLASH	SparseFedPP
CIFAR10 ($\alpha = 1.0$)	dense	83.70 ± 1.70	-	-	-
	0.9	80.56 ± 1.90	76.16 ± 1.30	81.15 ± 1.03	$\mathbf{82.13 \pm 1.53}$
	0.95	74.71 ± 3.29	75.53 ± 2.27	79.36 ± 1.03	$\mathbf{82.60 \pm 1.58}$
	0.99	66.27 ± 5.08	70.71 ± 0.15	73.45 ± 1.37	$\mathbf{77.71 \pm 1.69}$
	0.995	63.82 ± 2.41	56.02 ± 3.95	69.15 ± 1.60	$\mathbf{70.01 \pm 0.43}$
	0.999	31.79 ± 19.10	17.66 ± 8.34	36.07 ± 7.49	$\mathbf{51.39 \pm 3.19}$
CIFAR100 ($\alpha = 1.0$)	dense	52.29 ± 1.14	-	-	-
	0.9	46.57 ± 1.71	40.70 ± 4.72	51.99 ± 0.21	$\mathbf{53.08 \pm 0.90}$
	0.95	28.07 ± 23.27	38.82 ± 1.75	47.19 ± 1.88	$\mathbf{52.81 \pm 1.72}$
	0.99	19.65 ± 16.30	18.97 ± 2.08	42.76 ± 4.08	$\mathbf{46.64 \pm 1.59}$
	0.995	9.51 ± 14.81	6.01 ± 4.74	36.43 ± 4.97	$\mathbf{42.21 \pm 1.03}$
	0.999	3.81 ± 2.18	1.96 ± 0.66	5.80 ± 2.86	$\mathbf{15.96 \pm 0.64}$
Speech Commands ($\alpha = 1.0$)	dense	91.49 ± 0.94	-	-	-
	0.9	84.28 ± 0.88	87.79 ± 1.40	88.68 ± 1.72	$\mathbf{92.32 \pm 1.59}$
	0.95	78.58 ± 0.44	84.29 ± 1.50	84.89 ± 0.49	$\mathbf{89.14 \pm 1.15}$
	0.99	65.01 ± 0.84	57.79 ± 0.82	69.22 ± 1.59	$\mathbf{75.82 \pm 3.72}$
	0.995	56.73 ± 1.00	37.16 ± 2.71	58.23 ± 1.84	$\mathbf{68.02 \pm 3.14}$
	0.999	21.56 ± 12.79	10.10 ± 4.01	17.70 ± 2.58	$\mathbf{47.43 \pm 1.66}$

Table 4: Aggregated results for CIFAR10, CIFAR100, and Google Speech Command datasets.

Dataset	Sparsity	Resnet18	ZeroFL	FLASH	SparseFedPP
CIFAR 10 ($\alpha = 0.1$)	dense	73.81 ± 4.84	-	-	-
	0.9	69.79 ± 3.78	67.40 ± 4.11	71.87 ± 2.63	**75.00** $\pm$ **2.78**
	0.95	60.00 ± 4.66	61.55 ± 4.18	72.08 ± 2.09	**75.95** $\pm$ **3.39**
	0.99	43.96 ± 11.99	51.71 ± 3.54	56.91 ± 3.55	**63.69** $\pm$ **3.90**
	0.995	19.02 ± 10.77	41.33 ± 3.64	52.15 ± 3.87	**56.79** $\pm$ **3.97**
	0.999	11.5 ± 4.494	18.76 ± 4.28	29.31 ± 6.75	**43.68** $\pm$ **7.61**
CIFAR100 ($\alpha = 0.1$)	dense	48.34 ± 2.71	-	-	-
	0.9	41.96 ± 2.16	31.92 ± 7.65	45.59 ± 0.75	**48.37** $\pm$ **1.73**
	0.95	11.48 ± 17.51	34.21 ± 7.65	44.31 ± 2.14	**48.27** $\pm$ **2.70**
	0.99	0.14 ± 0.72	13.07 ± 2.26	34.75 ± 3.38	**41.03** $\pm$ **2.14**
	0.995	0.14 ± 0.72	7.04 ± 5.25	26.44 ± 17.35	**35.72** $\pm$ **2.01**
	0.999	0.14 ± 0.72	1.66 ± 0.97	3.56 ± 2.07	**13.84** $\pm$ **3.69**
Speech Commands ($\alpha = 0.1$)	dense	80.15 ± 2.69	-	-	-
	0.9	65.44 ± 0.97	77.15 ± 1.77	77.15 ± 0.77	**79.67** $\pm$ **2.78**
	0.95	57.39 ± 1.04	71.28 ± 2.75	71.28 ± 1.75	**75.46** $\pm$ **2.24**
	0.99	50.42 ± 6.26	53.55 ± 4.00	53.55 ± 2.00	**56.69** $\pm$ **4.56**
	0.995	34.20 ± 1.43	43.16 ± 3.47	43.16 ± 3.47	**48.30** $\pm$ **5.39**
	0.999	19.25 ± 6.01	17.14 ± 2.97	17.14 ± 2.97	**29.24** $\pm$ **2.34**

Table 5: Aggregated results for CIFAR10, CIFAR100, and Google Speech Command datasets.

F Additional Experimental Configuration Details

Learning rate scheduler. The learning rate follows a scheduled pattern defined by the function:

$$\eta_t = \eta_{\text{start}} \exp\left(\frac{t}{T} \ln\left(\frac{\eta_{\text{end}}}{\eta_{\text{start}}}\right)\right) \tag{1}$$

Reproducibility. Seeds were used for client sampling, while others were fixed for reproducibility purposes. All simulations were conducted using three different client sampling seeds: 5378, 9421, and 2035.

Experimental Setting. Each round consisted of one local epoch with a local batch size of 16. The initial learning rate was set to 0.5, gradually decreasing to a final value of 0.01 following eq. (1). For SparsyFed, the exponent for parameterization was set to $\beta = 1.25$ for the CIFAR-10/100 experiments and $\beta = 1.15$ for the Speech Commands experiment. The CIFAR experiments were run for 700 rounds, while the Speech Commands experiment was run for 500 rounds.

G Comparative analysis on algorithms and baselines

Here is a brief description of the implementation used during the experiments for `TopK`, ZeroFL Qiu et al. (2021), FLASH Babakniya et al. (2023), and *SparsyFed*:

G.1 TopK

1. **Sparsification**: The model is pruned per round at clients after executing the (local) training on their own data, i.e., just before sending the updated model to the server. The pruning method used is global unstructured Top-k, which prunes all the model parameters except for the k largest values.
2. **Aggregation strategy**: FedAvg. Alternatively, other aggregation strategies acting on the pseudo-gradients can also be applied straightforwardly.

G.2 ZEROFL

1. **Sparsification**: Pruning is performed during the local training at clients and before sending the model update back to the central server. During training, the SWAT unstructured (per-layer, in contrast to standard global unstructured TopK) approach is used to prune the weights before the forward pass and the activations during the forward pass. This alone doesn't ensure obtaining a model with the targeted degree of sparsity because SWAT often results in weight regrowth per optimizer step. To achieve the target sparsity, the model is pruned again before sending it back to the server, using the same approach as TopK, i.e., global unstructured. In the original work, three levels of masks ([0.0, 0.1, 0.2]) have been proposed to increase the density of the model before the uplink communication. For our experiments, we used the smaller one (0.0) since it is more in line with
2. **Aggregation strategy**: A slight variation of FedAvg is used, where the averaging is executed among the non-zero weights to avoid excessive dilution of information in the presence of highly sparse models. Adapting the aggregation function to act only on non-zero weights supports alternative aggregation strategies acting on the pseudo-gradients.

G.3 FLASH - SPDST (SENSITIVITY-DRIVEN PRE-DEFINED SPARSE TRAINING)

(THE ONE USED IN THE EXPERIMENTS)

1. **Sparsification**: Pruning is performed at the end of the first round of training (similar to TopK), which, in the original paper, is referred to as a warmup phase. Thus, the first round of training is executed using the dense model, producing the initial mask. The binary mask obtained during the warmup phase is fixed for the subsequent federated rounds, and only the non-zero weights are trained. This means there is no need for further pruning of the weights in subsequent rounds since no regrowth is allowed (in this version of Flash).
2. **Aggregation strategy**: The aggregation is performed only among the non-zero weights, similar to ZeroFL. During the first aggregation, at the end of the initial training round, the model is further pruned on a per-layer basis to counter the regained density caused by the mismatch in local masks. The pruning is done using the average sparsity level achieved by all clients for each layer. For example, if the average sparsity of layer l is d_l, adjusted by a factor r, then layer l of the pruned global model will have sparsity d_l. The factor r helps to maintain the target global sparsity by ensuring that the sum of individual layer sparsities meets the overall goal. The resulting binary mask becomes the final one, maintained for all subsequent training rounds. This process, defined in the original paper as sensitivity analysis, is applied only in the first round, as fixed mask training prevents any further modification of the mask in later rounds.

G.4 FLASH - JMWST (JOINT MASK WEIGHT SPARSE TRAINING)

1. **Sparsification**: A normal training procedure is applied (NO FIXED MASKS). Pruning is performed at the end of each local training session.
2. **Aggregation strategy**: The server aggregates and then prunes the model, applying the same sensitivity analysis introduced in SPDST to address the regained density. This is done every r round, as the training method allows for the regrowth of the clients. The original paper proposed two values for r: $r = 1$ and $r = 5$.

G.5 SPARSYFED

1. **Sparsification**: Powerpropagation is applied to re-parametrize the weights during the forward pass executed at clients. Pruning during training is applied only to the activations during the backward pass. At the end of the local training, the model is pruned and returned to the server. The first round of training is executed using a full-size model.
2. **Aggregation strategy**: FedAvg. Alternatively, other aggregation strategies acting on the pseudo-gradients can also be applied straightforwardly.