C^2P: Featuring Large Language Models with Causal Reasoning

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Abstract

Causal reasoning is one of the primary bottlenecks that Large Language Models (LLMs) must overcome to attain human-level intelligence. Recent studies indicate that LLMs display near-random performance on reasoning tasks. To address this, we introduce the Causal Chain of Prompting (C^2P), a reasoning framework that aims to equip current LLMs with causal reasoning capabilities as the first framework of its kind operating autonomously without relying on external tools or modules during both the causal learning and reasoning phases. To evaluate the performance of C^2P, we first demonstrate that reasoning accuracy improved by over 30.7% and 25.9% for GPT-4 Turbo and LLaMA 3.1, respectively, when using our framework, compared to the same models without C^2P on a synthetic benchmark dataset. Then, using few-shot learning of the same LLMs with C^2P, the reasoning accuracy increased by more than 20.05% and 20.89%, respectively, with as few as ten examples, compared to the corresponding LLMs without C^2P on the same dataset. To evaluate C^2P in realistic scenarios, we utilized another benchmark dataset containing natural stories across various fields, including healthcare, medicine, economics, education, social sciences, environmental science, and marketing. The results show improved reasoning when C^2P is applied, compared to cases where our framework is not used, which often leads to random and hallucinated responses. By showing the improved performance of few-shot learned GPT-4 Turbo and LLaMA 3.1 with C^2P, we demonstrate the generalizability of our framework.

1 Introduction

Recent advancements in Large Language Models (LLMs) have impacted existing AI paradigms and raised expectations regarding AI's capabilities (Achiam et al., 2023; Brown et al., 2020). LLMs generally produce outputs based on the most likely results learned from vast amounts of training data (Vaswani et al., 2017). This enables them to acquire extensive knowledge, ranging from common sense to specialized domains such as mathematics and science (Jiralerspong et al., 2024). In the training of LLMs, numerous examples of interventions, outcomes, and explanations are included in their training to reduce hallucinations and improve reasoning. However, despite significant architectural differences, hallucinatory responses still occur and true causal reasoning remains lacking (Kalai & Vempala, 2023; Xu et al., 2024). As a result, although models may appear to reason causally, they do not engage in a genuine causal reasoning process (Zečević et al., 2023). This deficiency represents a fundamental limitation of LLMs as AI systems compared to human intelligence, which is based on causal reasoning rather than simple associations for decision making (Penn & Povinelli, 2007; Anwar et al., 2024). Judea Pearl introduced "The Ladder of Causation" in (Pearl & Mackenzie, 2018) that easily addresses the reason for this deficiency. This ladder includes three main levels. At the first level, association, only patterns and dependencies are identified, and why things are related cannot be answered. The second level, intervention, involves understanding cause-and-effect by predicting outcomes of actions or changes, asking "What happens if I do this?" The highest level, counterfactuals, deals with imagining alternative scenarios, asking "What would have happened if...?". This framework illustrates how reasoning evolves from pattern recognition to understanding and predicting causal relationships, highlighting the differences at each level.

Recently, studies on the causal reasoning capabilities of LLMs have garnered significant interest. Most

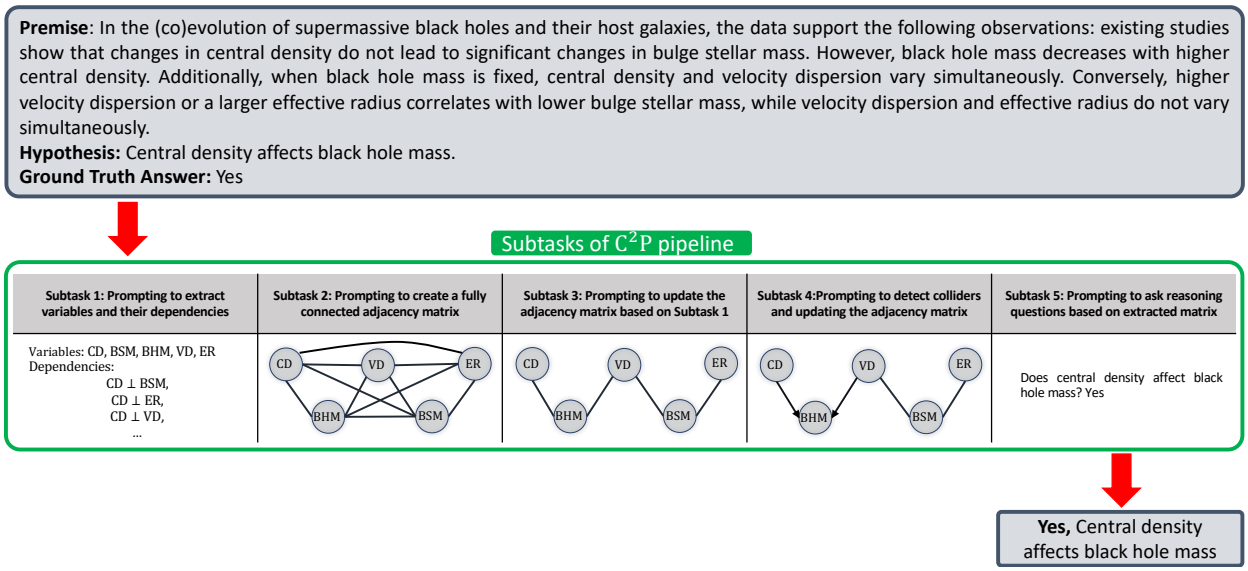

Figure 1: The 5 successive subtasks of C^{2}P applied to Black Holes and their Host Galexis example presented in (Pasquato et al., 2023). Central Density(CD), Black Hole Mass(BHM), Velocity Density(VD), Bulge Stellar Mass(BSM), Effective Radius(ER)

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of these focus on evaluating the reasoning abilities of LLMs, while a smaller number aim to enhance this feature. Additionally, some studies explore the application of LLMs in the field of causality.(Kıcıman et al., 2023; Zhang et al., 2023; Feder et al., 2024; Khatibi et al., 2024). The inefficiency of LLMs in reasoning has been extensively studied from various perspectives, as demonstrated in works such as (Xu et al., 2023; Romanou et al., 2023; Jin et al., 2023a; Liu et al., 2024a; Hobbhahn et al., 2022). Similarly, Jin et al. (2023b) introduces the CORR2CAUSE dataset, revealing that current models often perform no better than random chance when answering causal questions. Most recently, simple tasks have been shown to completely break down the reasoning abilities of state-of-the-art LLMs (Nezhurina et al., 2024). Additionally, studies such as (Petroni et al., 2019; Jiang et al., 2020) aimed to reason causally based on the knowledge already present in the training data of LLMs, which is why LLMs are referred to as "causal parrots" in (Zečević et al., 2023). The reason for these behaviors is discussed in (Wang et al., 2023; Imani et al., 2023). As one of the initial attempts to enhance reasoning in LLMs, chain-of-thought prompting is presented in (Wei et al., 2022b), showing improvement based on the data from the given query. Despite CoT's potential to enhance reasoning performance, the analysis in Bao et al. (2024) reveals a pattern: correct answers often follow incorrect CoTs, and vice versa. As another approach, LLMs have been utilized in conjunction with external tools to extract causal structures, as demonstrated in (Jiralerspong et al., 2024). More recently, in (Ashwani et al., 2024), a novel architecture called the Context-Aware Reasoning Enhancement with Counterfactual Analysis (CARE-CA) framework is presented to enhance causal reasoning and explainability. Their proposed framework incorporates an external explicit causal detection module with ConceptNet (Speer et al., 2017) and counterfactual statements, as well as implicit causal detection through LLMs, showing progress in causal reasoning in short and simple queries. Several other works at the intersection of causal inference and LLMs are discussed in an extensive survey by Liu et al. (2024b). The main drawbacks of the existing frameworks aiming to equip LLMs with reasoning are their reliance on external modules, the need for extensive information to function, and their very low accuracy.

In this paper, we propose the Causal Chain of Prompting (C^{2}P), a reasoning framework for LLMs designed to enhance reasoning skills by climbing the causality ladder and answering reasoning questions. Unlike existing methods, C^{2}P operates autonomously, without relying on external tools or modules during the learning and reasoning phases. It can be easily implemented in the few-shot to improve reasoning in causal questions. Importantly, C^{2}P extracts the causal relation based on associations mentioned in a given premise. C^{2}P

is inspired by Pearl's foundational work, which argues that causal Directed Acyclic Graphs (DAGs), along with d-separation, enable the investigation of cause-and-effect relationships without relying on structural equation models in computational studies (Pearl, 1995). Based on this, we demonstrate that by extracting the adjacency matrix of causal relationships among variables in the premise—equivalent to the use of the causal DAG in Pearl's framework—a reasoning query can be effectively answered. This framework includes five simple main subtasks, as follows: (1) Prompting to extract random variables from the provided data. (2) Prompting to extract all conditional and unconditional associations, as well as cause-and-effect relations specifically mentioned among the random variables. (3) Prompting to create the initial adjacency matrix with values of 1 for all elements except the diagonal elements and those corresponding to effect-cause relations (the cause-and-effect elements are also set to 1). (4) Prompting conditional and unconditional independencies and identification of colliders, step by step, to extract the causal adjacency matrix. (5) Prompting for reasoning questions or hypotheses. (see Fig. 1). To evaluate the accuracy and reliability of implementing the C^2P on LLMs, we initially assess it using publicly available benchmark synthetic and "Natural Story" datasets, such as those in (Jin et al., 2023b). To demonstrate the practical applicability of C^2P, we present the results of few-shot learned LLMs with C^2P in both synthetic and real-world scenarios. Subsequently, we evaluate it in more realistic and complex scenarios found in real-world problems presented in (Pasquato et al., 2023).

Contributions. In this work, we present several contributions to facilitate causal reasoning in large language models. Concretely,

1. We introduce the C^2P framework as the first reasoning framework to equip LLMs with causal reasoning within real-world scenarios, *without relying on external tools*.
2. Through extensive experiments with our framework, we demonstrate *significant improvements* in the causal reasoning abilities of LLMs across various benchmark datasets, as well as in more complex, real-world scenarios across multiple domains.
3. We demonstrate the generality of C^2P by implementing few-shot learning for both GPT-4 Turbo and LlaMA 3.1, obtaining significant improvements in reasoning.

The code is publicly available at `https://anonymous.4open.science/r/C2P-5C2A`.

2 Preliminaries on Causal Learning and Reasoning

In this section, we first introduce the concept of the "Ladder of Causation" to clarify the distinction between observation and intervention, helping to understand how dependencies relate to causation and reasoning. We then present the key concepts essential for understanding causality, along with the necessary assumptions and methods. Finally, we introduce the PC algorithm, which serves as the methodological foundation of C^2P.

2.1 Ladder of Causation (Pearl & Mackenzie, 2018):

To perform any level of causal reasoning, the Ladder of Causation proposes three main levels: "seeing or association," "doing or intervention," and "imagining or counterfactual". The questions that can be answered in Association (Seeing) are mostly similar to "What is happening?" This is the most basic level where we observe patterns and correlations between variables. At this level, we can only say that two things are related or tend to occur together, but we cannot explain why. Machine learning models and statistical methods that rely on pattern recognition (like LLMs) often operate at this level. In the second level, Intervention (Doing), questions such as "What happens if I do something?" can be addressed. At this level, we can go beyond mere association and ask about the effect of an action or intervention. This requires understanding the cause-and-effect relationships. To reach the third level of the ladder, more information is needed on the causal structure, which can mainly be provided with structural causal models (SCMs), assuming that all assumptions are satisfied (Bareinboim et al., 2022). However, studies such as (Spirtes et al., 2001) have shown that Level 2 can be reached (up to equivalence classes) using only observational data.

2.2 Prerequisite Concepts of Causality

To effectively understand and perform the reasoning steps of LLMs, it is essential to grasp several fundamental concepts that underpin the field. Table 1 includes the nature of observational data, foundational causal assumptions, graphical representations, and formal tools for causal reasoning. These concepts collectively enable the identification and interpretation of causal relationships, distinguishing causation from correlation. More details on these concepts are presented in Appendix A.1.

Concept	Role in Causal Reasoining	How It Relates to Others
Observational Data	Provides correlations but not causation; the starting point of causal discovery.	Motivates the need for causal models since correlation alone is insufficient for causation.
Common Cause Principle	Explains how correlations arise: either through direct causation or a common cause.	Justifies the use of **DAGs** to represent causal structures.
Directed Acyclic Graphs (DAGs)	Represent causal relationships using directed edges and nodes.	Encodes causality, forming the foundation for **d-separation, do-calculus, and causal inference**.
Markov Property	Ensures that each variable is independent of its non-descendants given its parents.	Defines the **independence structure** of DAGs, allowing us to use **d-separation** to test relationships.
d-Separation	A graphical criterion for determining conditional independence in a DAG.	Provides testable implications from a DAG and is **crucial for do-calculus and causal inference**.
Causal Sufficiency	Assumes all relevant confounders (common causes) are measured.	If violated, DAG-based methods may fail as **unobserved confounders distort causal inference**.
Faithfulness	Assumes that statistical independencies in data reflect the true DAG structure.	If violated, observed dependencies may **mislead causal discovery algorithms**.
PC Algorithm	A constraint-based algorithm for learning causal structures from observational data.	Relies on **d-separation, faithfulness, and causal sufficiency** to iteratively remove edges and infer causal relationships.
Markov Equivalence	Shows that multiple DAGs can represent the same independence relationships.	Limits what can be inferred from observational data alone, impacting methods like the **PC Algorithm**.
Do-Calculus	Provides rules for estimating causal effects from observational data under certain conditions.	Extends **d-separation and DAGs** to determine when interventions can be estimated from non-experimental data.

Table 1: Key concepts in causal discovery and their relationships.

2.3 The PC algorithm

The PC algorithm is developed based on Reichenbach's common cause principle and the Markov property, its steps can be described as follows:

(i) Form a complete undirected graph

(ii) Eliminate edges between variables that are unconditionally independent

(iii) For each pair of variables (X_1, X_2) having an edge between them, and for each variable, X_3 with an edge connected to either of them, eliminate the edge between X_1 and X_2 if $X_1 \perp\!\!\!\perp X_2 | X_3$

(iv) For each pair of variables X_1, X_2 having an edge between them, and for each pair of variables $\{X_3, X_4\}$ with edges both connected to X_1 or both connected to X_2, eliminate the edge between X_1 and X_2 if $X_1 \perp\!\!\!\perp X_2 | \{X_3, X_4\}$.

(v) For each triple of variables (X_1, X_2, X_3) such that X_1 and X_2 are adjacent, X_2 and X_3 are adjacent, and X_1 and X_3 are not adjacent, orient the edges $X_1 - X_2 - X_3$ as $X_1 \rightarrow X_2 \leftarrow X_3$, if X_2 was not in the set conditioning on which X_1 and X_3 became independent and the edge between them was accordingly eliminated. We call such a triple of variables a V-structure.

(vi) For each triple of variables such that $X_1 \rightarrow X_2 - X_3$, and X_1 and X_2 are not adjacent, orient the edge $X_2 - X_3$ as $X_2 \rightarrow X_3$. This is called orientation propagation.

All the mathematical background and proofs for the PC algorithm are provided in (Spirtes et al., 2001). Note that the current causal discovery methods are primarily divided into two groups: (i) Constraint-based algorithms such as the PC algorithm (Spirtes et al., 2001), which has a PDAG as an output, which represents the MEC of the true underlying graph and is the best outcome that these methods can achieve; (ii) Score-based methods such as GES (Chickering, 2002), NOTEARS (Zheng et al., 2018), GOLEM (Ng et al., 2020), DAGMA (Bello et al., 2022), and TOPO (Deng et al., 2023), among many others, which extract a DAG that mostly fits the data and demonstrated high accuracy in extracting Bayesian networks. Nevertheless, score-based methods require the solution of a numerical optimization problem, making them impossible to integrate with LLMs.

3 The Causal Chain of Prompting

To develop the Causal Chain of Prompting framework, our goal is to extract the adjacency matrix of variables from a given premise, serving as an equivalent alternative to the causal DAG in Pearl's work and enabling causal reasoning. To achieve this, the Causal Chain of Prompting is designed to replicate the exact steps of the PC algorithm for the given premise.

3.1 Causal Chain of Prompting (C^2P)

The C^2P framework consists of five main subtasks as follows, for learning and reasoning about cause and effect relations for the given premise:

- **Subtask 1:** Prompting to extract the random variables in the provided data.
- **Subtask 2:** Prompting to extract all the cause-and-effect relations along with all conditional and unconditional relations among the random variables specifically mentioned in the given premise.
- **Subtask 3:** Prompting to create an initial adjacency matrix where all elements are 1, except for the diagonal elements and those corresponding to the cause-and-effect relationships specifically mentioned in the given premise (extracted in subtask 2).
- **Subtask 4:** Prompting of conditional and unconditional independence evaluation and identification of colliders to extract the causal PDAG.
- **Subtask 5:** Prompting for cause-and-effect questions or hypotheses.

Each subtask in C^2P can include one or multiple steps (prompts). In general, to execute the framework, 9 main steps must be completed. **Subtask 1** is completed in Step 1. **Subtask 2** is accomplished with Step 2. **Subtask 3** is achieved with Step 3. To perform **Subtask 4**, 5 steps must be applied. Step 4 first

eliminates all unconditional independencies achieved in Subtask 2 and Step 5 then removes all conditional independencies extracted in Subtask 2. Step 6 identifies potential nodes that can act as colliders. Step 7 confirms whether the nodes identified in Step 6 are colliders. Step 8 updates the adjacency matrix, resulting in the final adjacency matrix of a causal structure. To perform **Subtask 5**, the causal question is asked in Step 9. **The exact prompts for all the steps are provided in Appendix A.3 and code repository.**

3.2 Few-shot learning

LLMs have powerful zero-shot capabilities, yet they struggle with complex tasks because of their engineering designs and insufficient examples in the training process. In such situations, few-shot learning is a versatile and efficient technique for in-context learning, which can be used to quickly adapt LLMs to new tasks and significantly enhance their performance (Min et al., 2022; Touvron et al., 2023). This approach involves providing several examples with desired answers to condition the LLMs to produce correct responses for new instances with similar patterns. The few-shot learning process of C^2P, as described in the previous subsection, is based on more abstract prompts (due to token limitations). The prompts and an example of the given story are included in Appendix A.4. Depending on the token limitations of the employed LLMs, the number of examples (shots) may vary. For GPT-4 Turbo, which has a 30,000 token limit, we were able to include ten examples. For LLaMA 3.1, we used the same examples in the few-shot learning process as well.

It is important to note that steps from other constraint-based methods can either replace or complement the steps of the PC algorithm in C^2P methods, as comprehensively discussed in (Glymour et al., 2019). For instance, the steps in the FCI method can be used in cases where the causal sufficiency assumption is violated, i.e., where latent variables and selection bias may be present (Spirtes et al., 1995). Additionally, the classifiers proposed by Ceraolo et al. (2024) formalize the definition of causal questions and establish a taxonomy for finer-grained classification. These classifiers can be used before our framework to identify the causal question as a prerequisite of our method.

4 Experiments

4.1 Datasets

The experiments are divided into two data groups: a synthetic dataset and a real-world scenarios dataset.

The CORR2CAUSE dataset: Introduced in (Jin et al., 2023b), the CORR2CAUSE dataset serves as a benchmark to assess the ability of LLMs to respond to reasoning queries. Tab. 2 shows the details of the samples in this dataset. The process of creating CORR2CAUSE is as follows: First, select the number N of variables (Step 1) and generate all unique DAGs with N nodes (Step 2). Next, gather all d-separation sets from these graphs to identify MECs (Step 3). In Step 4, formalize the data by associating each MEC with its corresponding causal graphs. For each MEC, construct a correlation statement based on the statistical relations within the MEC, hypothesize a causal relationship between two variables, and assign a validity "Yes" if the hypothesis holds for all causal graphs in the MEC, or "No" if the hypothesis does not necessarily apply to all graphs in the MEC. Finally, introduce the verbalization process. An example of a premise and its corresponding hypothesis in the CORR2CAUSE dataset is as follows:

Premise: Suppose that there is a closed system of 3 variables, A, B and C. All statistical relations among these 3 variables are as follows: A correlates with C. B correlates with C. However, A is independent of B.

Hypothesis: *A directly affects C.*

The Natural Stories dataset: The Natural Stories dataset is also introduced in (Jin et al., 2023b) to assess the reasoning capabilities of LLMs in realistic scenarios. This dataset builds upon the CORR2CAUSE

dataset as a foundation for future extensions in various contexts, such as instantiating variables with real-world phenomena and placing the narratives in more natural settings. For instance, the rule "correlation does not imply causation" can be illustrated using ice cream sales and swimming pool attendance as variables, by arguing that ice cream sales do not necessarily impact swimming pool attendance, as both could be influenced by a third factor, such as hot weather. The Natural Stories data presented in (Jin et al., 2023b) is not open-source. However, generating such data is straightforward. By providing an example from the CORR2CAUSE dataset along with the corresponding Natural Stories created by a human or existing example on the web, GPT-4 can generate realistic, everyday narratives. For more detailed examples of generating such stories, refer to the code directory accompanying this paper. A natural story generated by GPT-4, inspired by the previous example, could be:

Premise: Let's consider three factors: eating junk food, obesity, and watching television. There is a correlation between eating junk food and obesity, and between watching television and obesity. However, eating junk food and watching television are independent from each other.

Hypothesis: *Eating junk food directly affects obesity.*

More details and examples are provided in the code repository of this study.

An example on supermassive black holes. The data and results on the coevolution of supermassive black holes (SMBHs) and their host galaxies, presented in (Pasquato et al., 2023), are used as a real-world example in its verbalized form. The verbalized information is as follows:

Premise: In the (co)evolution of supermassive black holes (SMBHs) and their host galaxies, the data supports the following observations. Existing studies show that with a change in central density, there is no significant change in bulge stellar mass. However, there is a decrease in black hole mass with higher central density. Additionally, when black hole mass is fixed, central density and velocity dispersion change simultaneously. Conversely, higher velocity dispersion or effective radius results in lower bulge stellar mass, while velocity dispersion and effective radius do not change simultaneously.

Hypothesis: *Does central density affect black hole mass?*

4.2 Experimental setup

To test existing LLMs on the first synthetic data, we first include two BERT-based NLI models in the transformers library (Wolf et al., 2020): BART (Lewis et al., 2019), DistilBART (Shleifer & Rush, 2020). We evaluate LLaMA3-8B (Touvron et al., 2023) and LLaMA3-70B (Taori et al., 2023). We also evaluate the latest, more efficient models, LLaMA 3.1 8B and LLaMA 3.1 70B released in late September 2024. We assess the GPT-3.5 (i.e., ChatGPT), and the GPT-4 (Achiam et al., 2023), using the OpenAI API (https://openai.com/api/). We also used the latest GPT-4o model, released on October 2, 2024. Then, we evaluate the reliability of C^2P on various queries, each with a different number of random variables. Due to the limitation of tokens for different versions of GPTs, we employed GP4-Turbo which has a 30000 maximum token limit. We also used C^2P with LLaMA 3.1 70B as well. In all of the models, we set the temperature to 0. Note that various types of LLMs were tested in (Jin et al., 2023b), and the results showed that existing LLMs perform worse than random, completely random, or only slightly better than random in responding to such queries. From the models tested in (Jin et al., 2023b), we selected those with the highest accuracies to avoid duplicating the same results and to stay focused on the main objective of this study.

According to Tab. 2, the majority of responses in the CORR2CAUSE test are "No" (81.5%). As reported in Jin et al. (2023b) that duplicated in Tab. 4, even when responses from their test dataset are selected randomly, the resulting accuracy is higher than that of random selection. Additionally, most samples involved in reasoning have six variables (96.06%), a challenging task even for humans. As a result of these biases, the F1 score is used as the primary metric for evaluating accuracy in CORR2CAUSE simulations. We

Table 2: Information on CORR2CAUSE samples, different variable numbers, and responses.

Scenario \ Varible	3 varibles	4 varibles	5 varibles	6 varibles	Sum
CORR2CAUSE test data					
All samples	45	72	518	522	1157
Samples with "No" answers	30	57	449	402	938
Samples with "Yes" answers	15	15	69	120	338
CORR2CAUSE train data					
All samples	0	576	7524	197634	205734
Samples with "No" answers	0	529	6542	160615	167686
Samples with "Yes" answers	0	47	982	37019	38048

Table 3: Comparison of applying C^2P frameworks in LLMs compared to the existing LLMs with CoT

Models	F1	Precision	Recall	Accuracy
Random Baselines				
Random (Proportional)	13.5	12.53	14.62	71.46
Random (Uniform)	20.38	15.11	31.29	62.78

resolve these biases, by using 420 samples. First, to ensure a balanced distribution of samples, we select a population with 60 "Yes" and 60 "No" answers and an equal number of samples for each number of variables. This sample size is determined based on two main considerations. First, to ensure significant improvements in both step-by-step prompting and few-shot experiments, we applied the sample size formula for comparing two proportions as described by (Chow et al., 2017) and the minimum required sample size is 117. Additionally, to address the limited number of "Yes" responses (only 15 samples), we selected 30 samples for each variable number to minimize bias, resulting in a total of 120 samples. To ensure that the improvement of the results remains significant for the higher number of samples, we use 300 samples with 5 random variables with equal "Yes" and "No" answers. This experimental design enhances the realism and comprehensibility of the computational metrics, providing a more accurate reflection of the model's performance. As a result, all four key metrics—F1 score, precision, recall, and accuracy—are practical, each offering a unique perspective on the model's capabilities. These samples explore reasoning within three causal scenarios: direct cause-and-effect relationships (Fig. 2 i), indirect cause-and-effect relationships (Fig. 2 ii), and the presence of an effect due to two causes (Fig. 2 iii).

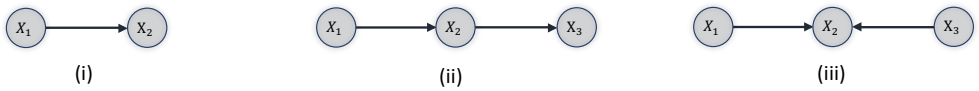

Figure 2: Three cause and effect scenarios. i. X_1 directly causes X_2 (X_2 is directly effect of X_1). ii. X_1 indirectly causes X_3 (X_3 is indirectly effect of X_1). iii. X_1 and X_3 are causes of X_2 (X_2 is common effect of X_1 and X_3)

To evaluate the proposed framework for natural stories, we used GPT-4, which excels at story generation. We crafted instructions in the prompts and generated 30 stories for our case study in fields such as healthcare and medicine, economics, social sciences, environmental science, and marketing, all highlighting the importance of causality. This process is similar to the one presented in (Jin et al., 2023b). Our approach can be tested using the examples of Simpson's paradox discussed in (Pearl & Mackenzie, 2018). However, since these examples are already included in the training data of current LLMs, the models simply repeat the correct answers based on that training data, similar to the parrot study in (Zečević et al., 2023). Consequently,

our generated natural stories replicate these examples in a manner that the LLM cannot address within its existing training data. We aimed to demonstrate how the symbolic expressions in the CORR2CAUSE dataset can affect the reasoning of our proposed framework.

Finally, we assess the coevolution of supermassive black holes (SMBHs) and their host galaxies using our proposed framework, replicating the results from (Pasquato et al., 2023). The goal in (Pasquato et al., 2023) was to extract a PDAG of SMBHs based on numerical data and then infer causal reasoning questions from the graph. We use verbalized information on SMBHs to evaluate whether the LLM, enhanced with C^2P, can answer reasoning questions such as, "Does central density affect black hole mass?"

It is important to note that Chain-of-Thought (CoT) prompting is implemented in most existing LLMs, enhancing their reasoning accuracy, as discussed in (Chung et al., 2024). Consequently, in our experiments, LLMs are prompted to think step by step when responding to reasoning questions, ensuring that the CoT mechanism is activated.

4.3 Evaluation of the C^2P on synthetic dataset

Results of the C^2P on CORR2CAUSE dataset: In Tab. 4, we show the performance of LLMs in the cause-effect task with and without employment of C^2P framework.

Table 4: Comparison of applying C^2P frameworks in LLMs compared to the existing LLMs with CoT

Models	F1	Precision	Recall	Accuracy
BERT-Based Models				
DistilBART MNLI	26.74	15.92	83.63	30.23
BART MNLI	33.38	31.59	35.38	78.50
LLaMA-Based Models				
LLaMA 3-8B	43.37	48.6	43.15	47.5
LLaMA 3-70B	49.41	53.84	62.23	55.77
LLaMA 3.1-8B	53.96	45.94	61.41	46.66
LLaMA 3.1-70B	57.14	48.64	49.15	48.07
C^2P with LLaMA 3.1-70B	**81.63**	**83.3**	**83.3**	**81.66**
C^2P Few-shot learned LLaMA 3.1-70B	**76**	**79.1**	**73.07**	**76.66**
GPT-Based Models				
GPT-3.5	47.5	56.2	43.2	52.5
GPT-4 Turbo	51.9	51.92	45	54.2
GPT-4	50.2	54.1	47.1	55
GPT-4O	59.86	61.46	58.33	60.83
C^2P with GPT-4 Turbo	**91.72**	**93.47**	**89.2**	**91.66**
C^2P Few-shot learned GPT-4 Turbo	**81.6**	**79.21**	**77.21**	**79.28**

According to Tab. 4, performing causal reasoning tasks remains a significant challenge for existing LLMs, even when prompted to think step by step, similar to (Wei et al., 2022b). These results show that even more updated models can sometimes perform worse than older ones in some metrics, for instance, LlaMa 3.1 acts worse than LlaMa 3. Based on the responses presented in the code repository for this study, model hallucination is one of the major factors contributing to this poor performance, aligning with the findings in Jin et al. (2023b). Tab. 4 shows that by applying 5 consecutive tasks of C^2P on GPT-4 Turbo and LlaMa 3.1, the reasoning accuracy improved by over 30.7% and 25.9%, respectively, compared to the corresponding LLMs without C^2P on a synthetic benchmark dataset. Then, using few-shot learning of GPT-4 Turbo and LLaMA 3.1 with C^2P, reasoning accuracy increased by over 20.05% and 20.89%, respectively, with as few as ten examples, compared to the corresponding LLMs without C^2P on the same dataset. To ensure that the improvements are significant in both step-by-step prompting and few-shot experiments, we applied the sample size formula for comparing two proportions from (Chow et al., 2017). This formula indicates that

even with a sample size of 117, the observed differences are statistically significant we assume a confidence level of 99% and 80% power. It is important to note that the primary factor contributing to the discrepancy between the results of LLaMA and GPT, and those reported in (Jin et al., 2023b), is the distribution of the provided premise. This table shows that when reasoning questions are posed to GPT models, their responses tend to slightly favor the "No" answer. Interestingly, LLaMA models slightly tend to answer "Yes" to reasoning questions more frequently. It is important to note that even when LLMs provide the correct final answer, the reasoning process that leads to that answer can still be incorrect, as illustrated in A.2, Tab. 8, for the results of GPT-4. This highlights the randomness of their responses more clearly which is also investigated and shown in Bao et al. (2024) as well. Other prompts that can execute the algorithm can be used in place of ours, as long as they perform the same subtasks. This supports the idea presented in the Lu et al. (2024). Furthermore, this becomes particularly evident in the few-shot learning process, where providing more examples has a significant impact; even more abstract prompts can achieve the same or even better results.

Robustness and reliability analysis: To assess the robustness of the proposed prompts for each of the 5 subtasks, Tab. 5 shows the computed accuracy computed in each subtask of implementing C^2P (subsection 3.1) on GPT-4 Turbo for different numbers of variables in the given premise. The sample size formula for

Table 5: Accuracy by number of variables and subtasks

Number of variables	**Accuracy**				
	First subtask	Second subtask	Third subtask	Fourth subtask	Fifth subtask
3 variables	100%	100%	100%	99.12%	98.7%
4 variables	100%	100%	100%	97.5%	84.1%
5 variables	100%	100%	100%	87.5%	75%
6 variables	100%	100%	100%	78.3%	70%

comparing two proportions indicates that fewer than 120 samples are sufficient to demonstrate a significant improvement in reasoning performance when using C^2P. Figure 3 shows results for datasets with sample sizes ranging from 30 to 300. This figure demonstrates that LLMs, such as GPT-4 Turbo, consistently achieve higher performance across all four metrics when C^2P is applied, compared to scenarios where C^2P is not used. According to this figure, the C^2P framework enhances all metrics across varying sample sizes.

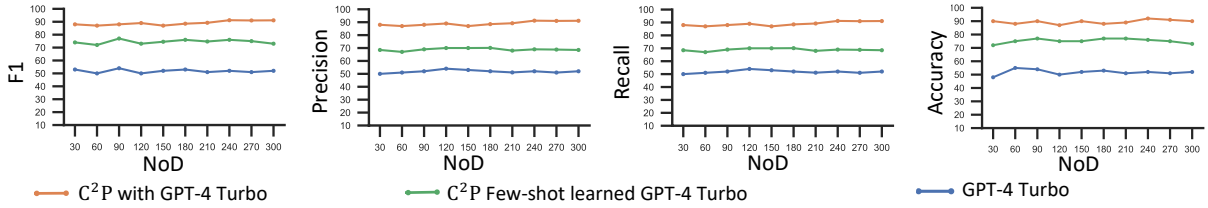

Figure 3: The accuracy metrics for different numbers of samples with 3 different methods.

4.4 Evaluation of the C^2P on natural stories

The results of applying the C^2P framework, both step-by-step and few-shot learned for GPT-4 Turbo and LLaMA 3.1, are provided in Tab. 6 and compared to the results of GPTs and LLaMA 3.1. This table, along with the highlighted sections, clearly demonstrates that the existing methods respond randomly to reasoning prompts. In contrast, GPT-4 Turbo and LLaMA 3.1 with step-by-step prompting of the C^2P framework and few-shot learned C^2P, significantly improve accuracy in scenarios involving natural stories.

Additionally, Tab. 7 provides an example of such natural stories and demonstrates the result of applying few-shot learned GP4-Turbo and LLaMA 3.1 with C^2P to them. This showcases how GPT models (including

Table 6: Comparison of applying C^2P frameworks with existing LLMs with CoT in responding to queries within natural stories, TP: True Positives, FP: False Positives, TN: true negatives, FN: False Negatives.

Models	**TP**	**FP**	**TN**	**FN**
LLMs without C^2P				
GPT-3.5	5	8	7	10
GPT-4 Turbo	6	7	8	9
GPT-4	7	8	7	8
GPT-4O	8	6	9	7
LLaMA 3-70B	9	9	6	6
LLaMA 3.1-70B	9	10	5	6
Featured LLMs with C^2P				
C^2P on LLaMA 3.1-70B	**11**	**3**	**12**	**4**
C^2P Few-shot learned LLaMA 3.1-70B	**10**	**3**	**12**	**5**
C^2P on GPT-4 Turbo	**13**	**1**	**14**	**2**
C^2P Few-shot learned GPT-4 Turbo	**11**	**3**	**12**	**4**

versions 3.5, 4, 4 Turbo, and 4O) fail to respond correctly. It is important to note that the few-shot learning process of GPT-4 Turbo and LLaMA 3.1 was performed using only ten examples from the CORR2CAUSE dataset, without including any natural story examples.

Table 7: Example of a natural story and the result of few-shot learned GPT-4-Turbo with C^2P

Example of true story	**Results of few-shot learned GPT-4 Turbo with C^2P**
Premise: eating junk food (A), obesity (C), and watching television (B) have relations with each other. There is a correlation between eating junk food and obesity, and between watching television and obesity. However, eating junk food and watching television are independent from each other. **Hypothesis**: Eating junk food directly affects obesity	Step 1: `Number of random variable:3` `Names of random variable: A, B, C`

	Step 2: `All of Statistical Relations:{`
	`Dependencies: [[A, C], [B, C]],`
	`Unconditional Independencies: [[A, B]],`
	`Conditional Independencies: []}`

	Step 3: `A: {A: 0, B: 1, C: 1},`
	`B: {A: 1, B: 0, C: 1},`
	`C: {A: 1, B: 1, C: 0}`

	Step 4: `A: {A: 0, B: 0, C: 1},`
	`B: {A: 0, B: 0, C: 1},`
	`C: {A: 1, B: 1, C: 0}`

	Step 5: `A: {A: 0, B: 0, C: 1},`
	`B: {A: 0, B: 0, C: 1},`
	`C: {A: 1, B: 1, C: 0}`

	Step 6: `C: [A, B]`
	Step 7: `C: [A, B]`

	Step 8: `A: {A: 0, B: 0, C: 1},`
	`B: {A: 0, B: 0, C: 1},`
	`C: {A: 0, B: 0, C: 0}`

	Step 9: `Checking matrix[A][C] = 1 and matrix[C][A] = 0.` `According to rule 2, this suggests` A is a direct cause of C, or C is a direct effect of A.

4.5 Evaluation of the C^{2}P framework on the (co)evolution of supermassive black holes and their host galaxies

In astrophysics, interventional experiments are impossible—for example, determining how changing one variable, such as central density, affects another variable, such as black hole mass. Thus, causal reasoning must rely exclusively on observational data. Pasquato et al. (2023) computationally studied the coevolution of supermassive black holes based on numerical data, and a causal graph for the underlying mechanism is extracted. Then, the causal hypothesis, "central density affects black hole mass", is answered. Similar to their study, we (co)evolute supermassive black holes (SMBHs) and their host galaxies based on the verbalized information in (Pasquato et al., 2023). We GPT-4 Turbo with C^{2}P to process the verbalized information provided in subsection 4.1, obtain a PDAG, and then reason about the causal questions.

Fig. 4 illustrates the results of the subtasks of C^{2}P on SMBHs data presented. The first four subtasks aim

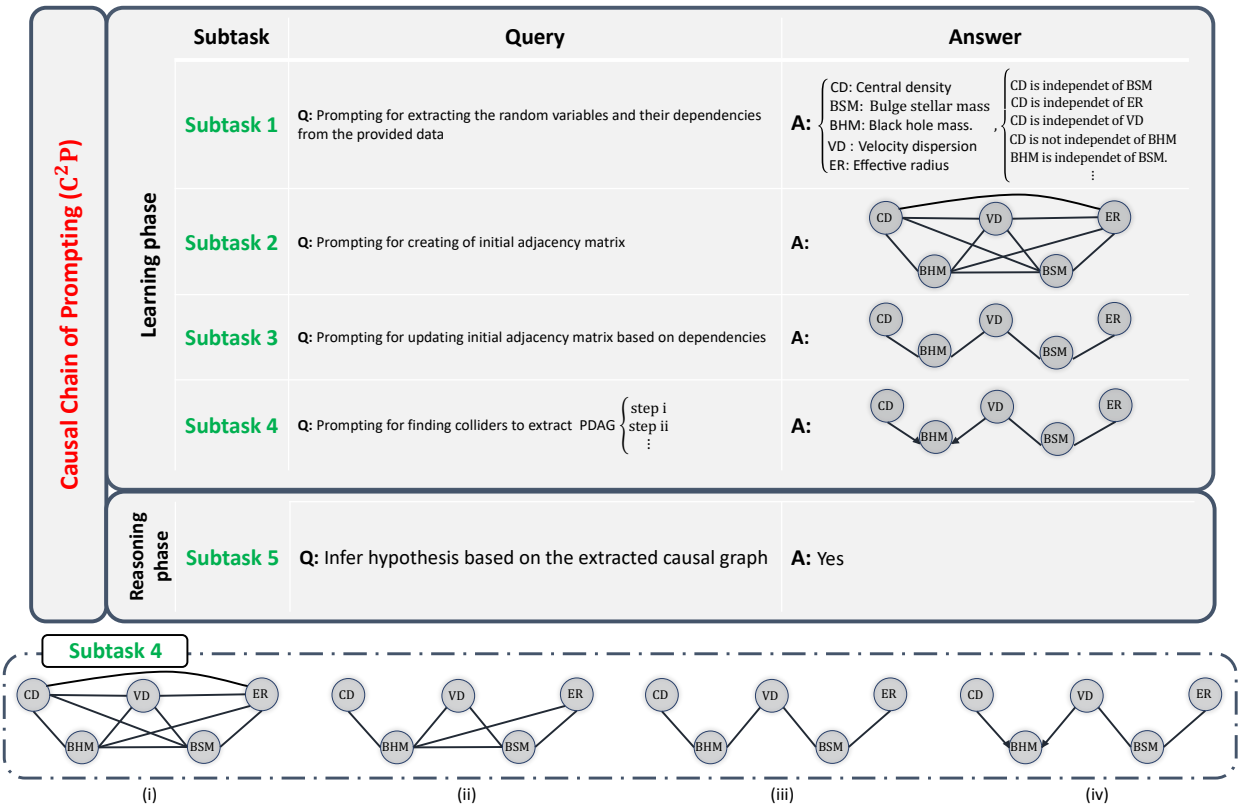

Figure 4: Prompts (**Q**) and results (**A**) of subtasks application of the C^{2}P framework to real-world complex scenarios and steps of subtask 3 for the given premise.

to extract the PDAG from the given data. Then, the hypothesis that central density affects black hole mass is evaluated in subtask 5 and answered. According to the extracted PDAG, many other reasoning questions questions can be answered that are too complicated for the existing LLMs to answer. It is important to note that by implementing C^{2}P, not only many causal questions can be answered with this approach, but it is easy to show which causal questions can not be answered based on the given premise and which more information has to be given to be able to causally reason. The discussion on which questions can and cannot be answered based on PDAG is presented in detail in studies such as (Hernán & Robins, 2006; Hauser & Bühlmann, 2012; Perkovic, 2020). Importantly, the inability to answer certain causal reasoning questions based on the PDAG, or its corresponding adjacency matrix, does not reflect a limitation of the C^{2}P framework. Rather, the identifiablity limitation and any rational agent needs additional information to effectively address all causal inquiries.

5 Discussion, Challenges and Future Work

Practical insights on simulations and results of C^2P: Each part of our study has a specific aim. The goal of the simulations, where we applied the 9 steps of C^2P step by step, is to show what the expected output would be and how it leads to causal reasoning. However, our investigations demonstrate sensitivity to the prompts. In few-shot learning, where examples of expected results are provided for each prompt, the sensitivity to prompts was not significant; instead, the results were primarily dependent on the number of examples. Interestingly, while the prompt is lengthy —due to the fact that all steps are given to LLMs as a single prompt —- the results remained highly accurate with as few as ten examples, as reported in Tab.s 6 and 7. By comparing the prompts in Appendices A.3 and A.4, it is clear that the prompts in the few-shot learning section do not need to be as detailed and are more abstract than those used in the step-by-step application of C^2P. To generate the prompts to achieve the goal of each step in C^2P, we used GPT4. However, it is important to note that there are countless prompts that can achieve the same goal for each step, and to the best of our knowledge, there is no systematic method to identify the best prompt or automate the process. However, GPT itself is quite helpful in generating prompts to accomplish the goal of each step. However, the number of samples is more important than the prompt, as long as the prompt correctly takes the output of the previous step and generates the required format output. For instance, even for CoT that has no rules to break a task, as stated in the technical report of the DeepSeek model, more than 600K examples are used to ensure the model enhances reasoning, while we used only 10 examples in few-shot learning. Therefore, we see some sensitivity to the prompts; however, if the number of examples increases, similar to what DeepSeek has done, this sensitivity should decrease. Additionally, it is important to note that the results for natural stories using few-shot learned C^2P were highly accurate (see Table 7), even though the examples in the few-shot learning process were synthetic “cause-effect" examples from the CORR2CAUSE dataset rather than real story examples.

The comparison of results between C^2P and CoT provides new insights into how LLMs aim to reason and why C^2P and similar frameworks can guide a model to "think" step by step. As demonstrated in the examples provided in the code repository—under "Sample Responses of LlaMa 3.1 with CoT" for LlaMa 3.1 and "Sample Responses of GPT-4 Turbo with CoT" for GPT-4 Turbo— both models attempt to break tasks into multiple subtasks when they are asked to think step by step (sometimes with more than nine consecutive subtasks). However, they still lack rationality in how they structure these subtasks, as it is discussed in Wei et al. (2022a). In other words, they struggle to design the necessary subtasks for reasoning, a task that C^2P successfully performs to enhance reasoning. This challenge is also highlighted in Wei et al. (2022b), where CoT is introduced, and its logical inconsistencies and poor step alignment are discussed as key limitations of the approach. The Deepseek-R1 model has also demonstrated improvements in reasoning with addressing the exact same challenge, as mentioned in their technical report, Guo et al. (2025), "We have filtered out chain-of-thought responses containing mixed languages, long paragraphs, and code blocks. For each prompt, we sample multiple responses and retain only the correct ones. In total, we collect approximately 600k reasoning-related training samples".

Causal reasoning and identifiability: Extracting a PDAG using only observational data is a key step in learning the true underlying causal mechanism. Based on the extracted PDAG, two main questions arise: Given a PDAG, under what conditions can we make causal reasoning? This involves determining the necessary assumptions and data required to address a causal question. This issue is known as identifiability Spirtes et al. (2001). Essentially, it refers to the limitations on what can be inferred solely from observational data, regardless of the method used for learning causal structure Peters et al. (2017). It is generally possible to answer some cause-effect questions based on the PDAG, represented by a causal graph, and lower-level data. These questions are mainly the ones that are related to the part where we have extracted a collider, or if the question is just about the existence of a cause-effect relation; in some cases, these questions can be answered as well. However, it is almost impossible to answer all the causal questions based on the given information, as discussed in (Bareinboim et al., 2022; Pearl & Bareinboim, 2022). As a result, the second question is, what else is needed to answer all the causal questions? In such cases, interventional data is necessary to explicitly extract all the directions of causal relations in a mechanism. To do so, while a naive

baseline approach would require $O(n^2)$ interventions, various methods have been proposed, such as those in (Kocaoglu et al., 2017; Choo & Shiragur, 2023; von Kügelgen et al., 2024; Squires & Uhler, 2023), for cases in different situations. However, one clear thing is that it is not possible to reason all the causal questions. For instance,

- Directions cannot be determined: PDAGs typically include both directed and undirected edges. The undirected edges represent uncertainty about the direction of causality. It is important to note that this is not a limitation of the reasoning method; rather, due to identifiability limits, it is not possible to determine the causal direction for these relationships without additional data.

- Full causal path analysis: While PDAGs can indicate possible paths between variables, they may not fully reveal which paths are indeed causal and which are due to confounding or indirect effects. Questions about specific causal pathways can thus be hard to answer definitively.

- Predictions under interventions: Questions about the outcomes of hypothetical or actual interventions on one or more variables (do-calculus questions) often require a fully specified causal model. PDAGs, with their partial specification, might not support detailed predictions under interventions without resolving the ambiguities in causal direction.

Next Steps for Reasoning in LLMs: The human brain possesses an intrinsic drive to understand causality. Whether driven by curiosity or the pursuit of specific goals, we continuously seek to understand why events occur and how they are interconnected. Causal reasoning is a broad and complex task in AI and LLMs. While current machine learning methods find it difficult to extract causal structures and subsequently reason causally, this problem can be even more intricate within the context of LLMs. The primary reason for this complexity is the distinction between association and cause-effect relationships. The inherent structure of LLMs relies on the attention mechanism presented in (Vaswani et al., 2017), which is akin to what Pearl refers to as "association" in the ladder of causality (Pearl & Mackenzie, 2018). As a result, advancing up the ladder of causality is essential for enabling true reasoning. To enable LLMs to address this fundamental quest for causality, they must be capable of sequentially performing two main tasks. As demonstrated in our study, C^2P has significantly enhanced the reasoning capabilities of LLMs on the provided datasets, which can be the main task in the reasoning process. However, the first task is to be able to understand the causal questions, which involves formalizing the definition of causal questions and establishing a taxonomy for finer-grained classification. The initial effort on this is presented in Ceraolo et al. (2024). Then, C^2P can be employed. However, the first subtask of this framework is to extract random variables, which requires the models to understand the definition of a random variable to ensure that the variables in the reasoning process are correctly identified. Failure in this process could undermine our proposed framework. Another important concept in this process is extracting the dependencies provided in the premise. In the datasets used for our experiments, these dependencies are explicitly stated in the given premise; however, in reality, they are often implicit. As a result, LLMs should be able to accurately extract these dependencies. Additionally, more comprehensive examples and scenarios need to be generated to aid in the learning process of an LLM. By overcoming these challenges, the integration of C^2P with LLMs can provide these models with causal reasoning capabilities, similar to the transformative impact of "Chain-of-Thought" (Wei et al., 2022b), as highlighted by Chung et al. (2024). The main difference between CoT and C^2P is that CoT does not have a specific design for breaking down a task, which can lead to hallucinations. In contrast, C^2P enables the LLM to learn how to break down a reasoning task effectively. It is also important to note that while our experimental results with few-shot learned LLMs show a significant increase in accuracy for tasks involving direct cause-and-effect relationships or common cause questions, the models still struggle with indirect cause-and-effect questions. Based on our experiments, for cases with five or more variables, there was no significant improvement in reasoning with LLM using C^2P for indirect cause-and-effect questions, even though the extracted adjacency matrix was correct. Moreover, the accuracy of our framework decreases as the number of random variables increases. As Pearl demonstrated that DAGs and d-separation are complementary in causal reasoning tasks, more structured subtasks can be introduced to improve performance in answering indirect causal questions and handling more complex scenarios with numerous variables. These subtasks would help guide LLMs in performing reasoning tasks more effectively.

The key benefit of few-shot learning is that it demonstrates that pre-trained models can perform causal reasoning without needing to be retrained from scratch or necessarily fine-tuned, making it more cost-effective and compute-efficient. This leads to reduced costs and less demanding infrastructure requirements. Therefore, a model fully trained or fine-tuned with C^2P would probably perform even better than few-shot learned ones, as it can be trained or fine-tuned on thousands of examples instead of the ten examples used during few-shot learning. This highlights how integrating C^2P during the training and fine-tuning of LLMs can revolutionize existing models. This is also discussed in Chung et al. (2024) as the "scaling law". Additionally, Studies like Kaplan et al., 2020 showed that language models benefit from increased data and computational resources. More data during fine-tuning allows the model to capture better representations, leading to improved performance in various tasks. This is one of the core principles behind why larger datasets, would likely boost accuracy compared to just 10 examples. In Brown et al. (2020) with GPT-3, the authors highlight how performance increases as the model is trained on more examples. In few-shot learning, the model can generalize from a small set of examples, but fine-tuning with larger datasets usually leads to more substantial improvements. Devlin (2018) also demonstrates that fine-tuning with more task-specific examples improves the model's performance. As a result, by addressing the mentioned challenges, employing C^2P in the fine-tuning process is expected to improve the reasoning accuracy of the models.

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A Appendix

A.1 Details on Preliminaries of Causal Learning and Reasoning

Common cause principle (Reichenbach, 1991): If two random variables X_1 and X_2 are statistically dependent, i.e., $X_1 \not\perp\!\!\!\perp X_2$, then there exists a third variable X_3 that causally influences both. (As a special case, X_3 may coincide with either X_1 or X_2.) Furthermore, this variable X_3 screens X_1 and X_2 from each other in the sense that given X_3, they become independent, $X_1 \perp\!\!\!\perp X_2 | X_3$.

Directed Acyclic Graphs: A graph G is called a Partially Directed Acyclic Graph (PDAG) if there is no directed cycle, that is, if there is no pair (X_j, X_k) with directed paths from X_j to X_k and from k to j. G is called a Directed Acyclic Graph (DAG) if it is a PDAG and all edges are directed.

Markov Property: The Markov property in a DAG G states that each node X_i is conditionally independent of its non-descendants, given its parents. In other words, $X_i \perp\!\!\!\perp \text{NonDe}(X_i)|\text{Pa}(X_i)$, where $\text{NonDe}(X_i)$ represents the non-descendants of X_i, excluding itself, and $\text{Pa}(X_i)$ represents the parents of X_i. It helps factorize the distribution of all graph nodes as $P(X_1, \ldots, X_N) = \prod_{i=1}^{N} P(X_i|\text{Pa}(X_i))$.

d-separation (Pearl, 1995): d-Separation is a criterion used in Bayesian network analysis to determine whether a set of variables X_1 is independent of another set of variables X_2, given a third set of variables X_3. This concept is foundational in understanding the flow of causal effects in graphical models and helps in deciding whether a path between two variables is "blocked" or not by conditioning on other variables. According to the d-separation, a path between two variables is blocked if it includes an intermediate variable that is a collider and is not conditioned on or a non-collider that is conditioned on. Here, a **collider** is a variable that has arrows inward from two other nodes (i.e., $X_1 \rightarrow X_2 \leftarrow X_3$), whereas a **non-collider** does not meet this criterion.

Causal Sufficiency: A set of observed variables V is causally sufficient if, for any pair of variables $X_1, X_2 \in V$, every common cause X_3 of X_1 and X_2 is also included in V.

Faithfulness: This assumption ensures that all the d-separation sets in the graph can be inferred from the independence relations in the distribution. In the sequel, we assume faithfulness, a widely used assumption in causal discovery (Spirtes et al., 2001).

Markov Equivalence of Graphs: Two DAGs are Markov equivalent if they generate the same joint distribution, $P(\mathbf{X})$. A set of DAGs that are Markov equivalent is referred to as a Markov equivalence class (MEC). Causal graphs within the same MEC are easily recognizable because they share the same skeleton (i.e., the same undirected edges) and the same V-structures (i.e., configurations like $X_1 \rightarrow X_2 \leftarrow X_3$, where X_1 and X_3 are not directly connected).

do calculus (Pearl, 1995): do-calculus, developed by Judea Pearl et al., is a set of rules used to transform and manipulate causal expressions within causal diagrams (or graphical models). do-calculus is a formal tool used to reason in causal relationships from a mixture of experimental and observational data. do-calculus consists of three main rules that allow one to rewrite expressions involving interventions (typically represented as $do(x)$, indicating an intervention to set variable X to value x). These rules are crucial for determining the identifiability of causal effects from data, allowing researchers to reason about causal relationships using a combination of experimental and observational data. The three main rules of the do-calculus are as follows:

1. **Insertion/deletion of observations**: If a variable X_2 is independent of X_1 given a set of other variables X_3, then the conditional distribution of X_2 given $do(X_1)$ and X_3 is the same as the conditional distribution of X_2 given X_3 alone. This rule can be applied when X_1 and X_2 are d-separated by X_3 in the graph modified by removing all arrows pointing into X_1.

2. **Action/observation exchange**: If X_2 is independent of X_1 given X_3 and another set W where X_1 is intervened, then the graph can be modified to reflect this exchange.

3. **Insertion/deletion of actions**: This rule states that if intervening on X_1 does not affect X_2 after accounting for X_3, the intervention can be ignored in the expression.

A.2 Example of comparison in responding to causal queries

For the given example in Subsection 4.1, Tab. 8 demonstrates that while GPT-4 arrives at the correct final answer, the reasoning process is flawed, providing further evidence of the random behavior of existing models in causal reasoning.

Table 8: An example of the correct final answer with GPT-4 Turbo while the reasoning process leading is incorrect and results of C^2P Few-shot learned GPT-4 Turbo

GPT-4 Turbo with CoT	C^2P Few-shot learned GPT-4 Turbo
The hypothesis that A directly affects C could be possible. The premise states that A correlates with C, which means there is a relationship between them. However, correlation does not necessarily imply causation. It could be that A affects C, C affects A, or that both A and C are affected by a separate, unmentioned variable. However, since the premise does not provide any information contradicting the hypothesis, it is possible that A directly affects C. Final Answer: "Yes"	According to the adjacency matrix, matrix["A"]["C"] = 1 and matrix["C"]["A"] = 0 indicates that A directly cause C. Therefore, the hypothesis that A causes C or C is effect of A is correct. Final Answer: "Yes"

A.3 Prompts for C^2P frameworks

The proposed steps (prompts) for step-by-step proceeding with 5 main subtasks of C^2P are as follows:

1. **Prompting for extracting the random variables from the provided data.**
   - **Step 1:** Please give the number of random variables in the given premise and write the names of all random variables.
2. **Prompting to extract all the cause and effect relations along with all conditional and unconditional relations among the random variables specifically mentioned in the given premise**
   - **Step 2:** If 2 random variables, for instance, R_1 and R_2, are independent, write it in this form: "R_1 is independent of R_2". If there exist 2 random variables, for instance, R_1 and R_2, are conditionally independent given a third random variable, for instance, R_3, write it in this form: "R_1 and R_2 are independent given R_3". If two random variables, for instance, R_1 and R_2, are specially mentioned to have cause and effect relation, write it in this form: "R_1 is the cause of R_2".
3. **Prompting to create an adjacency matrix where all elements are 1, except for the diagonal elements and the elements corresponding to the cause-and-effect relationships specifically mentioned in the given premise.**
   - **Step 3:** In this phase, each random variable is treated as a node within a fully connected undirected graph. Then, for each pair, for instance, R_1 and R_2, presented in the form: "R_1 is the cause of R_2", set the element in ["R_2", "R_1"] in the adjacency matrix to 0.
4. **Prompting of the conditional and unconditional independency valuation and identifying the colliders, step by step, to extract the causal PDAG.**
   - **Step 4:** Update the adjacency matrix based on the specified unconditional independencies between random variables. Each pair of variables that is declared independent should have

their corresponding value set to zero in the adjacency matrix. The initial adjacency matrix and the list of independencies are provided below. Please ensure all independencies are correctly reflected in the updated matrix. Instructions: - For each pair of variables listed as independent, set their corresponding entries in the adjacency matrix to 0.

- **Step 5:** Update the adjacency matrix based on the specified conditional independencies between random variables. Each pair of variables that is declared independent should have their corresponding value set to zero in the adjacency matrix. The initial adjacency matrix and the list of independencies are provided below. Please ensure all independencies are correctly reflected in the updated matrix. Instructions: - For each pair of variables listed as independent given other variable(s), set their corresponding entries in the adjacency matrix to 0.
- **Step 6:** Task: Given an initial adjacency matrix, follow these steps: Step 1: Identify all rows (key values) in the matrix where there are two or more than two columns with the value "1" in them. For each identified row, find all pairs of different columns where the values are "1".Ensure to exclude rows that do not contain any pairs from the results. Step 2: Display these pairs, "All Pairs", where each row name is key, and the value is a list of column names that are identified in Step 1.
- **Step 7:** Given the "All Pairs" and the list of independencies, follow these instructions step by step: Instruction: For each key in "All Pairs", delete all the pairs that are not mentioned as independent in the "independencies" list and return other with all their values. The "All Pairs" contains pairs of elements associated with each key. The goal is to update this by removing pairs that are not mentioned as independent. The list of independencies provides information about which pairs are independent of each other.
- **Step 8:** Given the initial adjacency matrix represented and the "All Pairs" list, for each key-value pair ("R") in "All Pairs", modify the initial adjacency matrix as follows: 1- Set the value in the "C_1" row and "R" column to 0: ("C_1", "R") = 0. 2- Set the value in the "C_2" row and "R" column to 0: ("C_2", "R") = 0.

5. **Prompting for cause-and-effect questions or hypotheses**

- **Step 9:** To extract and understand causal relations in the adjacency matrix: For each specified variable "R" and "C", for instance, that are listed in the adjacency matrix: - If matrix entry at ["R", "C"] = 1 and ["C", "R"] = 1, then the causal direction between "R" and "C" is undetermined. - If matrix entry at ["R", "C"] = 1 and ["C", "R"] = 0, then "R" is a direct cause of "C" or "C" is a direct effect of "R". - If matrix entry at ["R", "C"] = 0 and ["C", "R"] = 1, then "C" is a direct cause of "R" or "R" is a direct effect of "C". If two variables directly affect a third variable, the first two variables are common causes, and the third variable is a collider. Evaluate the hypothesis based on the given partially presented as an adjacency matrix with the given Instruction.

A.4 Prompts for few-shot learning of C^{2}P

An example of given prompts for the few-shot learning process of C^{2}P is as follows:

Premise: Suppose there is a closed system of 5 variables, A, B, C, D, and E. All the statistical relations among these 5 variables are as follows: A correlates with C. A correlates with D. A correlates with E. B correlates with D. B correlates with E. C correlates with D. C correlates with E. D correlates with E. However, A is independent of B. A and B are independent given C. B is independent of C. B and C are independent given A. C and E are independent given A, B, and D.

Hypothesis: There exists at least one collider (i.e., common effect) of A and B.

- **"Subtask 1"**- The number of random variables and their names in the given premise in JSON format:

  Output: "number of random variables: 5, "names of random variables": ["A", "B", "C", "D", "E"]

- **"Subtask 2"**- All the dependencies, conditional and unconditional independencies between all random variables extracted in "subtask 1":

  Output: {"All of Statistical Relations": {"Dependencies": [["A", "C"], ["A", "D"], ["A", "E"], ["B", "D"], ["B", "E"], ["C", "D"], ["C", "E"], ["D", "E"]], "Unconditional Independencies": [["A", "B"], ["B", "C"]], "Conditional Independencies": [["A", "B"], ["B", "C"], ["C", "E"]]}}

- **"Subtask 3"**- The adjacency matrix of all random variables extracted in "subtask 1" where each random variable is treated as a node within a fully connected undirected graph:

  Output: {"A": {"A": 0, "B": 1, "C": 1, "D": 1, "E": 1}, "B": {"A": 1, "B": 0, "C": 1, "D": 1, "E": 1}, "C": {"A": 1, "B": 1, "C": 0, "D": 1, "E": 1}, "D": {"A": 1, "B": 1, "C": 1, "D": 0, "E": 1}, "E": {"A": 1, "B": 1, "C": 1, "D": 1, "E": 0}}

- **"Subtask 4"**- Update the adjacency matrix extracted in the output of "subtask 3" based on the specified unconditional independencies between random variables. Each pair of variables that are declared independent should have their corresponding value set to zero in the adjacency matrix. - For each pair of variables listed as unconditional independent in "subtask 2", we set their corresponding entries in the adjacency matrix to 0. - We do not change any other entries except those specified by the independence.

  Output: {"A": {"A": 0, "B": 0, "C": 1, "D": 1, "E": 1}, "B": {"A": 0, "B": 0, "C": 0, "D": 1, "E": 1}, "C": {"A": 1, "B": 0, "C": 0, "D": 1, "E": 1}, "D": {"A": 1, "B": 1, "C": 1, "D": 0, "E": 1}, "E": {"A": 1, "B": 1, "C": 1, "D": 1, "E": 0}}

- **"Subtask 5"**- Update the adjacency matrix in the output of "Subtask 4" based on the specified conditional independencies between random variables extracted in "subtask 2". Each pair of variables that are declared conditional independent should have their corresponding value set to zero in the adjacency matrix. - For each pair of variables listed as conditionally independent given other variable(s), we set their corresponding entries in the adjacency matrix to 0. - We do not change any other entries except those specified by the conditional independence.

  Output: {"A": {"A": 0, "B": 0, "C": 1, "D": 1, "E": 1}, "B": {"A": 0, "B": 0, "C": 0, "D": 1, "E": 1}, "C": {"A": 1, "B": 0, "C": 0, "D": 1, "E": 0}, "D": {"A": 1, "B": 1, "C": 1, "D": 0, "E": 1}, "E": {"A": 1, "B": 1, "C": 0, "D": 1, "E": 0}}

- **"Subtask 6"**: In this subtask, for each key value in the "Adjacency Matrix" mentioned in the output of "Subtask 5", the task is to extract only pairs of columns with values "1" step by step according to the provided steps and put it in "Candidates" dictionary: follow these steps: Step 1: Identify all rows (key values) in the matrix where there are at least two or more columns with the value "1". For each identified row, find all pairs of different columns where the values are "1". Ensure to exclude rows that do not contain any pairs from the results. Make sure to check each row individually and include all valid pairs for every row. Step 2: Display these pairs in a simplified JSON format, where each row name is a key, and the pair values are a list of column names that are identified in step 1. If there are no such rows, show an empty JSON object. Step 3: Do not include any rows with one or zero columns with "1" values in the output. Example output format: Candidates: { "row1 name": [["1value column1 name", "1value column2 name"], ["1value column1 name", "1value column3 name"], ["1value column2 name", "1value column3 name"]], "row2 name": [["1value column5 name", "1value column6 name"]], ... } The values in each row have to be pairs and the output cannot be as follows: { "row1 name": [["1value column1 name"], ["1value column1 name"], ["1value column2 name", "1value column3 name"]], "row2 name": [["1value column5 name"]], ... } Please provide only the desired output formatted exactly as shown in the example without any further explanation.

  Output: {"A": [["C", "D"], ["C", "E"], ["D", "E"]], "B": [["D", "E"]], "C": [["A", "D"]], "D": [["A", "B"], ["A", "C"], ["A", "E"], ["B", "C"], ["B", "E"], ["C", "E"]], "E": [["A", "B"], ["A", "D"], ["B", "D"]]}

- **"Subtask 7"**- In this subtask, given the output of "Subtask 6", "Candidates", and "Unconditional Independencies" in "Subtask 2", the task is to identify and extract all the pairs in the "Candidates" that are also present in the "Unconditional Independencies" step by step according to the provided

steps. follow these steps: 1- For each pair in the "Candidates" list, check if it is present in the "Unconditional Independencies" list. 3-Only keep all the pairs from "Candidates" that are also present in "Unconditional Independencies". If a pair in "Candidates" is found in "Conditional Independencies", keep it. 4-Remove any pairs in "Candidates" that are not found in "Conditional Independencies". If a pair in "Candidates" is not found in "Unconditional Independencies", remove it. 5-Output the result as the modified "Candidates" dictionary without any additional text or explanation. Only the updated "Candidates" dictionary and nothing else.

Output: {"D": [["A", "B"], ["B", "C"]], "E": [["A", "B"]]}

- **"Subtask 8"**- Given the adjacency matrix in the output of "Subtask 5" and the "All Pairs" list in the output of "Subtask 7", for each key-value pair ("R") in "All Pairs", we modify the adjacency matrix as follows: -For each key "R" and pair ("C1", "C2") in the candidates, change the values in key "R" and pairs ("C1", "C2") to zero. -Ensure that only the specified modifications are made, and all other entries in the adjacency matrix remain unchanged.

  Output: {"A": {"A": 0, "B": 0, "C": 1, "D": 1, "E": 1}, "B": {"A": 0, "B": 0, "C": 0, "D": 1, "E": 1}, "C": {"A": 1, "B": 0, "C": 0, "D": 1, "E": 0}, "D": {"A": 0, "B": 0, "C": 0, "D": 0, "E": 1}, "E": {"A": 0, "B": 0, "C": 0, "D": 1, "E": 0}}

- **"Subtask 9"**- In this subtask, given the "Adjacency Matrix" mentioned in the output of "Subtask 8", evaluate the correctness of the hypothesis step by step according to the provided rules. Rules: 1- If matrix["R"]["C"] = 1 and matrix["C"]["R"] = 1, then the causal direction between "R" and "C" is undetermined. 2- If matrix["R"]["C"] = 1 and matrix["C"]["R"] = 0, then "R" is a direct cause of "C", or "C" is a direct effect of "R". 3- If matrix["R"]["C"] = 0 and matrix["C"]["R"] = 1, then "C" is a direct cause of "R", or "R" is a direct effect of "C". 4- Test each variable in the matrix, for instance, "R", "R" is a collider if the matrix entries in ["R", "C1"] = 0, ["C1", "R"] = 1, ["R", "C2"] = 0, and ["C2", "R"] = 1 Perform it step by step and provide the final "Yes" or "No" answer.

  Output: Step 1: According to the rules, a node is a collider (common effect) if the matrix entries in ["R", "C1"] = 0, ["R","C2"] = 0, ["C1", "R"] = 1 and ["C2", "R"] = 1.

  Step 2: According to rule 4, "R" is a collider if the matrix entries in ["R", "D"] = 0, ["R", "E"] = 0, ["D", "R"] = 1 and ["E", "R"] = 1.

  Step 3: Checking each variable in the matrix

  Node C: ["C", "A"] = 1 and ["A", "C"] = 1, so C is not a common effect of A and B.

  Node D: ["D", "A"] = 0 and ["A", "D"] = 1, and ["D", "B"] = 0 and ["B", "D"] = 1, so D is a common effect of A and B.

  Node E: ["E","A"] = 0 and ["A","E"] = 1, and ["E","B"] = 0 and ["B", "E"] = 1, so E is a common effect of A and B.

  Step 4: So, the hypothesis that there exists at least one collider (i.e., common effect) of A and B is correct. The answer is "Yes".