Diverse Condensed Data Generation via Class Preserving Distribution Matching

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Abstract

Large-scale datasets for training many real-world machine learning models pose significant computational resource challenges. One approach to mitigate this is via data condensation, which aims at learning a small dataset but still sufficiently capturing the rich information in the original one. Most of existing approaches learn the condensed dataset and task-related model parameters (*e.g.*, classifier) in a bi-level meta-learning way. The recently proposed distribution matching (DM), however, avoids the expensive bi-level optimization but ignores task-related models. This work proposes a novel class preserving DM framework consisting of two key components. The first one is responsible for capturing the original data distribution of each class based on energy distance, which can encourage the diversity in the generated synthetic data. The other is classifier-critic constraint, which forces the learned synthetic samples to fit pre-trained task-related models, such as an off-the-shelf classifier. Designing the optimization loss in this way, we can generate more diverse and class preserving distilled data without the bi-level optimization. Extensive experiments reveal that our method can produce more effective condensed data for downstream tasks with less training cost and can also be successfully applied to de-biased dataset condensation.

1 Introduction

Deep neural networks (DNNs) have demonstrated unprecedented results in many applications and come with a cost: the training of DNNs heavily relies on the sheer amount of data, sometimes up to tens of millions of samples. Although it becomes easier than ever to construct large scale datasets with advanced data collection and labeling tools, the rapidly growing size of datasets not only posts challenges to data storage and preprocessing, but also makes it increasingly expensive to train a model on the given large-scale dataset. More importantly, designing new deep learning models or applying them to new tasks certainly require substantially more computations, as they involve to train multiple models on the same dataset for many times to verify the design choices, such as loss functions, architectures and hyperparameters (Cui et al., 2022; Ying et al., 2019; Lee et al., 2022). As a result, there is a strong demand for techniques that compress a large-scale dataset into a small subset of informative examples.

Numerous research endeavours have, therefore, focused on alleviating the cumbersome training process through constructing small training sets. One well-studied approach is referred to as coreset or subset selection (Agarwal et al., 2004; Sener & Savarese, 2018), which chooses important data points for training based on some heuristic criteria. However, most selection procedures incrementally and greedily select samples, which are shortsighted and do not guarantee any optimal solution for the downstream tasks. Besides, the presence of representative samples is not guaranteed (Zhao & Bilen, 2021; Zhao et al., 2021). Instead of selecting from existing data points, recently, *dataset condensation* (DC) (Wang et al., 2018a; Zhao et al., 2021; Nguyen et al., 2021a;b; Zhao & Bilen, 2021; 2023; Lee et al., 2022; Cazenavette et al., 2022; Wang et al., 2022) has emerged as a competitive alternative with promising results, which aims to condense a small training set $\mathcal{S}$ from a large-scale one $\mathcal{T}$ so that models trained on $\mathcal{S}$ can generalize to test data. Once the condensed dataset is learned, one can use it for various downstream classification applications, such as implementing neural architecture search (NAS) (Dong & Yang, 2020). Compared to performing NAS with the original large-scale dataset, using the small condensed dataset for capturing the rich information in the

original one does save computational resources, which is the goal of data condensation. Along this line, the pioneering approach of Wang et al. (2018a) proposed a meta-learning-based strategy, showing better performance than existing subset selection methods with a lower compression ratio. After that, many data condensation methods have been proposed based on various objectives, such as gradient matching (Zhao et al., 2021; Zhao & Bilen, 2021; Lee et al., 2022), training trajectory matching (Cazenavette et al., 2022), feature alignment (Wang et al., 2022; Zhao & Bilen, 2023; Zhao et al., 2023; Zhang et al., 2024) and kernel ridge regression (Nguyen et al., 2021a;b).

Among the existing methods, the learning of synthesized dataset is usually coupled with the optimization of the task-related model. That is to say, they require complex bi-level optimization of both models and datasets and fine-tuning of multiple hyper-parameters (Zhao et al., 2021; Zhao & Bilen, 2021; Lee et al., 2022; Cazenavette et al., 2022; Wang et al., 2022), which may lead to large computation and memory cost and inaccurate learning of the synthesized dataset. To overcome these drawbacks, Zhao & Bilen (2023); Zhao et al. (2023); Zhang et al. (2024) recently proposed a new framework named distribution matching (DM), which obtains a condensed dataset for each class c by minimizing its maximum mean discrepancy (MMD) (Gretton et al., 2012) to the real dataset of c and then combines the condensed datasets for all the classes. DM has shown promising performance and avoids bi-level optimization by using many randomly sampled embeddings without training.

In this paper, we stress that a condensed dataset should preserve the class information of the original dataset and its samples should be diverse enough to represent the original dataset, so that a model trained on the condensed dataset is generalizable to test data. The above two properties have not been carefully studied in previous DM approaches. Specifically, DM generates a condensed dataset independently for each class, without guaranteeing that the synthetic samples actually belong to the class. In other words, the synthetic samples might ignore some discriminative features of their corresponding class, which will harm their availability in various downstream classification tasks. Moreover, the MMD distance used in DM ignores the diversity of the generated samples. The diversity is a key point since we expect each condensed sample plays a different role for reducing redundancy. In this paper, we propose a new distribution matching framework to satisfy above two factors simultaneously. To enhance the class-preserving property of the condensed datasets, inspired by the idea of plug-and-play generative models (Nguyen et al., 2015; 2017), we propose to pre-train an off-the-shelf model to capture the class information in the training set and then use it to derive a classifier-critic regularization on the synthetic dataset. Designed in this way, the learning of classification model and synthetic set is disentangled, which avoids an expensive bi-level optimization like DM. However, moving beyond DM, we can preserve the class information of the original dataset by learning the condensed samples from the view of the classifier-critic constraint. To encourage the synthetic samples to be more diverse, we introduce to minimize the energy distance (Székely et al., 2007; Rizzo & Székely, 2016) between the synthetic samples and the real ones. In addition to making synthetic samples close to the real samples, energy distance enjoys an intrinsic mechanism to encourage the synthetic samples to be different from each other. In the comprehensive experiments, ours achieves the competing performance on standard dataset condensation tasks compared with other related methods. In addition, our proposed classifier-critic constraint enables our method to generate synthetic samples that preserve other necessary information from the original dataset, such as in the case of de-biased dataset condensation.

We summarize our contributions as follows: (1) We propose a class preserving distribution matching approach for optimizing the condensed dataset, where we introduce a classifier-critic regularization and combine it with DM loss. (2) We introduce energy distance to implement the DM between real and synthetic samples for generating more diverse synthetic samples. (3) We are the first to consider de-biased dataset condensation due to the flexibility of classifier-critic constraint.

2 Related Work

2.1 Synthesis-based Methods

Dataset Distillation aims to learn a small synthetic set $\mathcal{S}$ such that a model trained on it will have a similar performance on the real test set as a model trained on the large real training set. Previous work

formulates the dataset condensation as a bi-level meta-learning problem (Wang et al., 2018b). The inner loop optimizes the model parameters on the distilled data (meta-parameters) while the outer loop optimizes the distilled data $\mathcal{S}$ with meta-gradient updates. One key challenge in dataset distillation is computing the meta-gradient. To this end, some works (Nguyen et al., 2021a;b) adopt ridge regression which has a closed-form solution. However, computing analytical Neural Tangent Kernel (NTK) (Lee et al., 2019) for modern neural network can be extremely expensive (Zhou et al., 2022). An alternative way is to find the surrogate objectives, such as gradient matching (Zhao et al., 2021; Zhao & Bilen, 2021), feature alignment (Zhao & Bilen, 2023; Wang et al., 2022), and training trajectory matching (MTT) (Cazenavette et al., 2022). For example, dataset condensation (DC) (Zhao et al., 2021), differentiable Siamese augmentation (DSA) for DC (Zhao & Bilen, 2021), and DC with contrastive signals (DCC) (Lee et al., 2022) all aim to match the gradients of neural network weights computed on the real and distilled data. IDC (Kim et al., 2022) examines some of shortcomings of existing gradient matching methods, and introduces improved techniques based on the multi-formation process. MTT (Cazenavette et al., 2022) condenses a dataset from many expert training trajectories. FTD (Du et al., 2023) takes MTT as an instance of a gradient-matching method to explain the existence of the accumulated trajectory error, and then proposes a flat trajectory distillation technique to mitigate such an error. DREAM (Liu et al., 2023) proposes a novel matching strategy denoted as dataset distillation by representative matching. IDM (Zhao et al., 2023) designs to enhance DM through the partitioning and expansion strategy to enchance representations from synthetic dataset for fine-grained feature matching. Based on IDM, M3D (Zhang et al., 2024) further proposes to map image representations to a reproducing kernel Hilbert space (RKHS), where each order of moment can be aligned between the real and synthetic images.

Although our method falls into the group of surrogate objectives, it has the following differences. Compared with DM (Zhao & Bilen, 2023) that only solves the DM problem with MMD, we propose a novel class preserving DM, where we introduce energy distance for the DM problem and propose a classifier-critic constraint to assimilate the classification-related information into the condensed dataset. Different from other methods (Wang et al., 2022; Zhao & Bilen, 2021; Zhao et al., 2021; Lee et al., 2022), ours belong into the group of distribution matching, where we can avoid an expensive bi-level optimization. But, similar to them, we consider the classification performance of the synthetic samples when learning these samples, which is realized by our introduced classifier-critic constraint with an off-the-shelf classifier. Besides, our proposed method is flexible for being combined with other methods, such as the gradient matching-based methods and representation matching-based methods.

2.2 Selection-based Methods

The classic technique to compress the training set is coreset or subset selection (Agarwal et al., 2004; Chen et al., 2010; Wei et al., 2015). In addition to one naive method, which randomly picks data from the original dataset, most of these methods incrementally select important data points based on some heuristic selection criteria. For example, (Sener & Savarese, 2018) select data points such that the largest distance between a data point and its nearest center is minimized. (Aljundi et al., 2019) use the parameter gradient as the feature to maximize the diversity of samples in the replay buffer. However, these heuristic selection criteria cannot ensure that the selected subset is optimal for the downstream task, especially for training DNNs. Besides, the information in the dataset is usually uniformly distributed over all samples, thus finding such an informative coreset may not always be possible (Zhao & Bilen, 2021; Zhao et al., 2021).

2.3 Plug and Play Generative models

Our work is also closely related to generative models such as variational auto-encoder (Kingma & Welling, 2014) and generative adversarial networks (GANs) (Goodfellow et al., 2014). The difference is that image generation aims to generate real-looking and high-fidelity images that can fool human eyes. As explored by (Zhao et al., 2021), the images produced by GANs have similar performances to those randomly selected real images and are usually weaker than the dataset condensation methods. Another related work is plug-and-play generative networks (PPGN) (Nguyen et al., 2017; Graikos et al., 2022), which composes of 1) a generator for drawing a wide range of image types, and 2) a replaceable "condition" network that tells the generator what to draw, similar to our classifier-critic constraint. Despite of the recent community interest in

PPGN, the possibility of the constraint in dataset condensation has not been explored. Instead of producing "real-looking" samples under some constraints, we aim to generate a condensed informative training set for the target task.

3 Preliminary

3.1 Energy distance

Energy distance is a statistical distance between probability distributions (Rizzo and Szekely ,2016). For two independent random vectors $\mathbf{X}$ and $\mathbf{Y}$ in $\mathbb{R}^d$, the energy distance between them is defined as

$$\varepsilon(\mathbf{X},\mathbf{Y}) = 2E\|\mathbf{X}-\mathbf{Y}\| - E\,\|\mathbf{X}-\mathbf{X}'\| - E\,\|\mathbf{Y}-\mathbf{Y}'\|$$

where E means expectation operator, $\|\cdot\|$ is the Euclidean norm, $E\|\mathbf{X}\| < \infty, E\|\mathbf{Y}\| < \infty, \mathbf{X}'$ and $\mathbf{Y}'$ denote the independent and identically distributed (iid) copy of $\mathbf{X}$ and $\mathbf{Y}$, respectively.

A significant advantage of the energy distance is that $\varepsilon(\mathbf{X},\mathbf{Y}) = 0$ if and only if $\mathbf{X}$ and $\mathbf{Y}$ are identically distributed. Thus, the energy distance can be used for testing of equal distributions or multivariate goodness-of-fit measure. Another advantage of the energy distance is that it is distribution free. That is to say, the estimated value of energy distance does not depend on the distribution form of random vectors, although it can be represented as the form of characteristic function. Therefore, the energy distance can be estimated with the following surprisingly simple form. Denoting $\mathbf{x_1},\ldots,\mathbf{x_{n_1}}$ as the samples of $\mathbf{X}$ and $\mathbf{y_1},\ldots,\mathbf{y_{n_2}}$ as the samples of $\mathbf{Y}$, we can estimate the energy distance as follows

$$\varepsilon_{n_1,n_2}(\mathbf{X},\mathbf{Y}) = 2\frac{1}{n_1 n_2}\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\|\mathbf{x}_i-\mathbf{y}_j\| - \frac{1}{n_1^2}\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\|\mathbf{x}_i-\mathbf{x}_j\| - \frac{1}{n_2^2}\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\|\mathbf{y}_i-\mathbf{y}_j\| \tag{1}$$

3.2 Previous Works: Distribution Matching Methods for Dataset Condensation

Denote a large-scale dataset as $\mathcal{T} = \{\boldsymbol{x}_i, \boldsymbol{y}_i\}|_{i=1}^{|\mathcal{T}|}$ with $|\mathcal{T}|$ image and label pairs and C classes, and denote the small (synthetic) dataset as $\mathcal{S} = \{\boldsymbol{s}_j, \boldsymbol{y}_j\}|_{j=1}^{|\mathcal{S}|}$. We can further represent the real set and synthetic set in class c as $\mathcal{T}_c$ and $\mathcal{S}_c$, respectively. DM (Zhao & Bilen, 2023) learn $\mathcal{S}_c$ by minimizing the empirical estimate of MMD between the real and synthetic samples of the class c:

$$\mathbb{E}_{\theta\sim P_\theta}\left\|\frac{1}{|\mathcal{T}_c|}\sum_{i=1}^{|\mathcal{T}_c|} f_\theta\left(\boldsymbol{x}_i\right) - \frac{1}{|\mathcal{S}_c|}\sum_{j=1}^{|\mathcal{S}_c|} f_\theta\left(\boldsymbol{s}_j\right)\right\|^2, \tag{2}$$

where f_θ is the embedding function parameterized with θ sampled from P_θ, $|\mathcal{T}_c|$ and $|\mathcal{S}_c|$ is the number of samples of $\mathcal{T}_c$ and $\mathcal{S}_c$. DM obtains the whole synthetic dataset $\mathcal{S}$ by summarizing $\mathcal{S}_c$ in each class. Different from others (Zhao & Bilen, 2021; Zhao et al., 2021; Wang et al., 2022), DM avoids the expensive bi-level optimization and second-order derivative with promising performance. However, DM ignores whether the synthetic set $\mathcal{S}_c$ can actually be helpful to the classification of the class c when optimizing them. Moreover, DM ignores the diversity of distilled samples when using MMD and produces undesired performance with a slightly larger compression ratio.

4 Method

4.1 Proposed Method: Class-preserving Distribution Matching

In this work, we explore a novel efficient dataset condensation approach with the following intuition: a condensed synthetic set is desired if the generated samples can not only match the distribution of original training set but also can be confidently classified its corresponding labels by a well-trained classifier. Therefore, we introduce a distribution matching loss and a classifier-critic constraint to learn the condensed dataset $\mathcal{S}$, described below in detail.

4.2 Classifier-critic Constraint

Existing dataset condensation methods put their attention on the standard dataset condensation task, where they assume the original large-scale training set is high-quality without subpopulation shift or other problems. However, datasets in real world are usually not perfect. Below, we first introduce the classifier-critic constraint for standard (commonly-used) dataset condensation and then explain how to design the corresponding classifier-critic constraint for de-biased dataset condensation.

Standard dataset condensation. Specifically, we can assume g_ϕ is a classifier pre-trained on the training set $\mathcal{T}$, *e.g.*, based on the Cross-Entropy (CE) loss. Then we can use the given classifier g_ϕ to design a classifier-critic regularization with CE loss when optimizing each synthetic samples $\boldsymbol{s}$ with label $\boldsymbol{y}$, formulated as:

$$\ell = -\log \Pr(\boldsymbol{y} \mid \boldsymbol{s}; g_\phi) \tag{3}$$

where g_ϕ is frozen during the learning of $\mathcal{S}$ since it already has the acceptable classification ability. In addition to using a classifier pre-trained on $\mathcal{T}$, we can also adopt existing available classifers, such as the powerful Contrastive Language¨CImage Pre-training (CLIP) model (Radford et al., 2021), which efficiently learns visual concepts from natural language supervision based a sufficiently large dataset rather than $\mathcal{T}$. This constraint forces the to-be-learned synthetic samples $\boldsymbol{s}$ being classified into its corresponding label $\boldsymbol{y}$ using the pre-trained classifier g_ϕ. That is to say, we consider distilling the discriminative information beneficial for the classification task into the synthetic set $\mathcal{S}$.

De-biased dataset condensation. Subpopulation shift widely exists in many real-world applications, where subpopulations are seen but underrepresented in the training data (Han et al., 2022; Yao et al., 2022), such as fairness of machine learning and class imbalance (Hashimoto et al., 2018; Japkowicz, 2000). Then models may perform poorly when they falsely rely on the spurious correlation between the particular subpopulation and the label. Thus, a practical research is condensing a training dataset with biased demographic subpopulations into a small de-biased synthetic dataset. Here A denotes the number of spurious attributes like gender and ethnicity, where we can represent the attribute vector of synthetic sample $\boldsymbol{s}$ as $\boldsymbol{a}$. Now we can pre-train two classifiers on original dataset $\mathcal{T}$, where one is the label classifier g_{ϕ_1} for predicting class while other is spurious attribute classifier g_{ϕ_2}. To enforce the presence of label and absence of the spurious attribute, we can define the classifier-critic as follows:

$$\ell = -\log \Pr(\boldsymbol{y} \mid \boldsymbol{s}; g_{\phi_1}) + \log \Pr(\boldsymbol{a} \mid \boldsymbol{s}; g_{\phi_2}). \tag{4}$$

By minimizing the CE loss about label and maximizing the CE loss about the spurious attribute, we can encourage the synthetic image to be classified as its label and not be recognized as its spurious attribute. It reduces the spurious correlations of label on subpopulation group in condensed samples.

4.3 Energy Distance between $\mathcal{S}_c$ and $\mathcal{T}_c$

Enforcing the matching between $\mathcal{S}_c$ and $\mathcal{T}_c$ is also important, otherwise we might encounter "spurious" examples that have high classification performance only under pre-trained classifier but ignore the structure in data, which may cause poor generalization performance when using $\mathcal{S}$ to train downstream classification tasks. Different from generative models that aim to generates real-looking images, dataset condensation aims to accurately match the distribution of the real training data with limited synthetic data. Despite DM learning $\mathcal{S}$ with MMD and proving its effectiveness, it may ignore the diversity in the synthesized samples, which plays a key role in reducing the redundancy of condensed samples. To this end, we propose to learn $\mathcal{S}$ by minimizing the energy distance between the real and synthetic data distributions for each class. We first introduce the definition of energy distance (Rizzo & Székely, 2016). Denoting $\mathbf{X}$ and $\mathbf{Y}$ as two independent random vectors in $\mathbb{R}^d$, their energy distance is defined as:

$$\varepsilon(\mathbf{X}, \mathbf{Y}) = 2\mathbb{E}\|\mathbf{X} - \mathbf{Y}\| - \mathbb{E}\,\|\mathbf{X} - \mathbf{X}'\| - \mathbb{E}\,\|\mathbf{Y} - \mathbf{Y}'\|\,, \tag{5}$$

where $\|\cdot\|$ indicates the Euclidean norm, $E\|\mathbf{X}\| < \infty, E\|\mathbf{Y}\| < \infty$, $\mathbf{X}'$ and $\mathbf{Y}'$ denote the independent and identically distributed (iid) copy of $\mathcal{X}$ and $\mathcal{Y}$, respectively. A significant advantage of the energy distance is that $\varepsilon(\mathbf{X}, \mathbf{Y}) = 0$ if and only if $\mathbf{X}$ and $\mathbf{Y}$ are identically distributed.

Algorithm 1 Workflow of our method.

Require: Training set $\mathcal{T}$, synthetic set $\mathcal{S}$ randomly initialized from $\mathcal{T}$ with corresponding labels, off-the-shelf classifier g_ϕ, feature extractor f_θ, number of classes C, max-iter and learning rate η.
for t in maxiter **do**
  Randomly sample θ from P_θ and compute λ;
  Randomly sample a minibatch $B_c^{\mathcal{T}} \sim \mathcal{T}_c$ and $\mathcal{S}_c \sim \mathcal{S}$ for each class c in C;
  Compute $\mathcal{L} = \sum_c^C \left(\varepsilon(B_c^{\mathcal{T}}(\theta), \mathcal{S}_c(\theta)) - \lambda \mathbb{E}_{\boldsymbol{s}\sim\mathcal{S}_c}[\Pr(\boldsymbol{y} \mid \boldsymbol{s}; g_\phi)\right)$
  Update $\mathcal{S} \leftarrow \mathcal{S} - \eta \nabla_{\mathcal{S}} \mathcal{L}$;
end for
Output: a synthetic dataset $\mathcal{S}$

In this work, taking the class c as the example, we have samples $\boldsymbol{x}$ and $\boldsymbol{s}$ from $\mathcal{T}_c$ and $\mathcal{S}_c$, respectively. Now the energy distance between $\mathcal{T}_c$ and $\mathcal{S}_c$ can be estimated as follows:

$$\varepsilon(\mathcal{T}_c, \mathcal{S}_c) = \frac{2}{|\mathcal{T}_c||\mathcal{S}_c|} \sum_{i=1}^{|\mathcal{T}_c|} \sum_{j=1}^{|\mathcal{S}_c|} d\left(\boldsymbol{x}_i, \boldsymbol{s}_j\right) - \frac{1}{|\mathcal{S}_c|^2} \sum_{i,j=1}^{|\mathcal{S}_c|} d\left(\boldsymbol{s}_i, \boldsymbol{s}_j\right) - \frac{1}{|\mathcal{T}_c|^2} \sum_{i,j=1}^{|\mathcal{T}_c|} d\left(\boldsymbol{x}_i, \boldsymbol{x}_j\right), \tag{6}$$

where we compute the distance function in embedding spaces following DM (Zhao & Bilen, 2023), *i.e.* $d\left(\boldsymbol{x}_i, \boldsymbol{s}_j\right) = \|f_\theta(\boldsymbol{x}_i) - f_\theta(\boldsymbol{s}_j)\|$ with f_θ denoting the feature extractor sampled from P_θ. The main difference between Eq. equation 2 in DM and our adopted energy distance is the presence of a *repulsive* term between generated data, $d\left(\boldsymbol{s}_i, \boldsymbol{s}_j\right)$, which encourages the condensed samples to capture the full distribution and improve the representativeness.

Discussion about why adding the energy distance can help improve diversity: Recall that our adopted energy distance in Eq (5) is composed of $\sum_{i,j=1} d(x_i, s_j)$, $\sum_{i,j=1} d(s_i, s_j)$ and $\sum_{i,j=1} d(x_i, x_j)$, where the third term can be ignored for only computing the distance between real samples, i.e., x_i and x_j. The first term $\sum_{i,j=1} d(x_i, s_j)$ aims to minimize the distance between real and synthetic samples, which is the attractive term similar to the empirical estimate of MMD in Eq (1). The second term $\sum_{i,j=1} d(s_i, s_j)$ of ED in Eq (5) aims to maximize the distance between the condensed data points, playing the role of a repulsive term. Therefore, the main difference between the adopted ED in Eq (5) and the empirical estimate of MMD in Eq (1) is the repulsive term. And we had conducted the ablation study to compare the ED and MMD (i.e., DM) in Table 2 in experiments, where ED produces superior performance than that of DM. Especially, ED outperforms DM by large margins with larger IPCs. It is reasonable since ED not only minimizes the distance between synthetic data and real dataset, but also maximizes the distance between synthetic samples. Therefore, we stress that ED enjoys an intrinsic mechanism to encourage the synthetic samples to be different from each other, improving the diversity of synthetic samples.

4.4 Training Loss

Recall that we aim to learn the class preserving and diverse condensed samples, which can be solved by introducing a classifier-critic constraint and the energy distance between $\mathcal{T}_c$ and $\mathcal{S}_c$. Therefore, to learn the synthetic set $\mathcal{S}_c$ of class c for standard (or de-biased) dataset condensation, we can jointly minimize the energy distance in equation 6 and the CE loss in equation 3 (or in equation 4). Therefore, the optimization loss can be formulated as follows:

$$\min_{\mathcal{S}_c} \mathcal{L} = \mathbb{E}_{\theta\sim P_\theta}[\varepsilon(\mathcal{T}_c, \mathcal{S}_c)] - \lambda \mathbb{E}_{\boldsymbol{s}\sim\mathcal{S}_c}[\log \Pr(\boldsymbol{y} \mid \boldsymbol{s}; g_\phi)], \tag{7}$$

where the trade-off hyper-parameter $\lambda = \frac{\text{maxiter}-t}{\text{maxiter}}$ with t denoting the t-th iteration and maxiter denoting the maximum number of iterations. We summarize the training process of standard dataset condensation in Algorithm 1, which targets at image classification problems. Following Zhao & Bilen (2023; 2021), we initialize the synthetic images using randomly sampled real images with corresponding labels before training. At each training iteration, we can randomly sample embedding function θ from P_θ and a mini-batch of real samples for each class. Our method enforces the synthetic data to assimilate the task-related information by a very flexible classifier-critic constraint parameterized with already-learned ϕ, avoiding a bi-level optimization

Table 1: The performance (testing accuracy %) comparison to baselines on CIFAR10, CIFAR100 and TinyImageNet. $*$ means the results cited from Cui et al. (2022) and $\circ$ indicates the results reported by original paper.

Dataset	IPC	Coreset Selection		Method								Whole
		Random*	K-Center *	DC *	DSA *	DM *	CAFE °	OURS	IDM °	M3D °	OURS+M3D	
CIFAR10	1	15.40 ± 0.28	25.16 ± 0.45	29.34 ± 0.37	27.76 ± 0.47	26.45 ± 0.39	**31.6 ± 0.8**	29.30 ± 0.33	45.6 ± 0.7	45.3 ± 0.3	**45.9 ± 0.2**	85.95
	10	31.00 ± 0.48	41.49 ± 0.73	50.99 ± 0.62	52.96 ± 0.41	47.64 ± 0.55	50.9 ± 0.5	**53.42 ± 0.17**	58.6 ± 0.1	**63.5 ± 0.2**	63.2 ± 0.1	
	50	50.55 ± 0.32	56.00 ± 0.29	56.81 ± 0.44	60.28 ± 0.37	61.99 ± 0.33	62.3 ± 0.4	**65.92 ± 0.30**	67.5 ± 0.1	69.9 ± 0.5	**71.0 ± 0.3**	
CIFAR100	1	5.30 ± 0.23	10.89 ± 0.17	13.66 ± 0.29	13.73 ± 0.45	11.20 ± 0.27	**14.0 ± 0.3**	12.80 ± 0.20	20.1 ± 0.3	26.2 ± 0.3	**27.6 ± 0.2**	56.69
	10	18.64 ± 0.25	25.04 ± 0.30	28.42 ± 0.29	32.23 ± 0.35	29.23 ± 0.26	31.5 ± 0.2	**33.39 ± 0.22**	45.1 ± 0.1	42.4 ± 0.2	**43.2 ± 0.4**	
	50	34.66 ± 0.41	38.64 ± 0.43	30.56 ± 0.56	43.13 ± 0.33	42.32 ± 0.37	42.9 ± 0.2	**45.19 ± 0.18**	50.0 ± 0.2	50.9 ± 0.7	**52.5 ± 0.5**	
TinyImageNet	1	1.65 ± 0.11	3.03 ± 0.12	5.27 ± 0.10	**5.67 ± 0.14**	3.82 ± 0.21	-	5.60 ± 0.11	-	-	-	39.83
	10	6.88 ± 0.25	11.38 ± 0.26	12.83 ± 0.14	16.34 ± 0.21	13.51 ± 0.31	-	**16.94 ± 0.15**	-	-	-	
	50	18.62 ± 0.22	22.02 ± 0.40	12.66 ± 0.36	25.31 ± 0.22	22.76 ± 0.28	-	**28.10 ± 0.21**	-	-	-	

and second-order derivative. It is different from DM which ignores classifier-related model when learning the synthesized dataset, *i.e.* without considering the classification performance of the synthetic samples; and also different from other related work (Zhao et al., 2021; Wang et al., 2022; Zhao & Bilen, 2021) that usually couples the learning of the synthesized set with the learning of task-related model at each training iteration. Besides, it improves the diversity of the condensed samples by introducing the energy distance.

5 Experiments

5.1 Datasets and Implementation Details

Datasets. For standard dataset condensation, we consider three widely adopted image datasets - CIFAR10, CIFAR100, and TinyImageNet. **CIFAR10** (Krizhevsky et al., 2009) and **CIFAR100** (Krizhevsky et al., 2009) consists of tiny colored natural images from 10 and 100 categories, respectively. Each dataset has 50K training images and 10K test images with the size of 32×32. **TinyImageNet** (Le & Yang, 2015) contains 100K training and 10K testing images from 200 categories with a higher resolution 64×64.

Implementation Details. Following previous work (Zhao & Bilen, 2023), we adopt the commonly-used DSA augmentation (Zhao & Bilen, 2021) and learn 1/10/50 image(s) per class as (IPC) synthetic sets for all datasets using the ConvNet architecture. The ConvNet includes three repeated convolutional blocks, and each block involves a 128-kernel convolution layer, instance normalization layer (Ulyanov et al., 2016), ReLU activation function (Nair & Hinton, 2010) and average pooling. Note that four-block ConvNets are used to adjust to the larger input size of TinyImageNet. To evaluate the synthesis-based methods, we learn 5 synthetic datasets and train 20 randomly initialized classifiers on each synthetic dataset; then report the mean and standard deviation of 100 test accuracies. We set $\text{maxiter} = 40K$ and learn model parameters ϕ using the original training set, which adopts the same architecture and setting with the downstream classifier in the standard dataset condensation. Besides, we also utilize CLIP[1] to represent off-the-shelf model. We defer the learning rate, mini-batch size and training details for ϕ to Appendix A.

Baselines. We compare our method to coreset selection methods, including Random and K-Center (Sener & Savarese, 2018). We also consider recent condensation methods based on matching loss, namely DC (Zhao et al., 2021), DSA (Zhao & Bilen, 2021), DM (Zhao & Bilen, 2023), CAFE (Wang et al., 2022), IDM (Zhao et al., 2023) and M3D (Zhang et al., 2024). Following (Cui et al., 2022), DSA augmentation is enabled during evaluation for all base methods to make a fair comparison. For the implementation details of the baselines, please refer to dataset condensation Benchmark (Cui et al., 2022) for more details. Due to the flexibility of ours, we can also combine it with M3D by adding our proposed loss with the kernel-based matching loss in M3D, denoted as OURS+M3D, where we set the coefficient for two losses as 1 without exhausted adjustment.

5.2 Experimental Results on Standard Dataset Condensation

We adopt a common evaluation method for dataset condensation methods, *i.e.* measuring the test accuracy of the neural networks trained on the condensed data. Here we report the performance comparison with different IPCs in Table 1. While DSA and CAFE can condense more data-efficient samples when IPC=1, our method outperforms them at IPC=10/50. A similar phenomenon has been observed by DM. The possible reason is that DSA and CAFE on bigger synthetic data use limited iterations in the inner-loop model

[1]https://github.com/openai/CLIP

optimization to ensure scalability, making them less accurate. More importantly, in terms of TinyImageNet with higher image resolution and more diverse classes, our method with only 10% of data (IPC=50) can recover about 71% classification performance of the baseline, which is trained on the whole original training set. Despite both DM and ours adopting the distribution matching and avoiding the bi-level optimization, ours significantly outperforms DM on various settings. It is not surprising since we introduce the classifier-critic regularization and energy distance, whose effectiveness will be further explored in ablation studies. When compared with state-of-the-art (SOTA) methods, like IDM and M3D, our proposed loss function can be integrated with them as an additional supplement and obtain significant performance gains in most settings, which demonstrates the desired generalization and flexibility of ours.

Learning Larger Synthetic Sets. Prior works mainly evaluate the condensed dataset of sizes up to 50 IPCs, such as 1% compression ratio for CIFAR10, which is a rather extreme case. To get a more informative subset in most practical applications, we further explore the performance of condensation algorithms under larger compression ratios on CIFAR10, where we set IPC up to 1,000. As shown in Figure 1, the performance gain of ours over baselines is obvious for IPCs larger than 100 and less than 500. When IPC $>$ 500, the performance gap between data condensation methods and the random baseline narrows, where ours still outperforms other competitors. Especially, our method needs only about 500 images for each class to reach the performance of baselines trained for IPC = 1000. All methods achieve similar performance to the random baseline when IPC=1000. The possible reason is that randomly selecting more samples will approach the whole dataset, which we consider as the upper-bound. In summary, our method can be successfully applied into realistic settings, where the synthetic data of different classes can be learned independently and in parallel for avoiding bi-level optimization and second-order derivative.

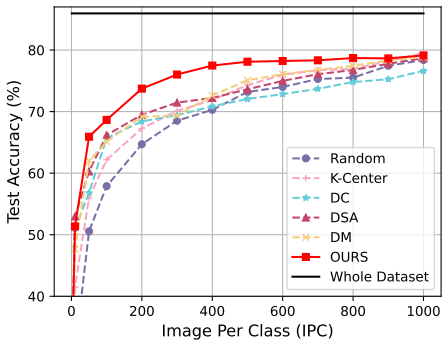

Figure 1: Performance comparison on CIFAR10 with different IPCs using Random, K-Center, DC, DSA, DM and OURS.

Training Cost. Considering the importance of training cost when optimizing the condensed dataset, without loss of generality, we compare the training time of ours and baselines when IPC=50 on CIFAR10, as shown in Figure 2. We can find that both DM and ours are more efficient than other methods for avoiding the bi-level optimization. Particularly, our method trained for about 35 minutes can produce comparable performance over DSA trained for about 15 hours and also outperforms DC and CAFE by a large margin. DM performs better than ours in the early stage since our method takes more time at each iteration. It is reasonable since DM computes the distance between the mean of the real and synthetic samples and energy distance additionally introduces a repulsive term and computes the distance between each real (synthetic) and synthetic sample. However, our method outperforms DM as a whole with an acceptable cost. Therefore, these observations prove the effectiveness of ours. Note that we run these methods in the same computation environment with one NVIDIA A100 GPU.

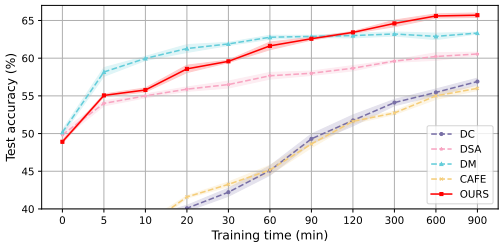

Figure 2: Training time-consuming comparison of DC, DSA, DM, CAFE and ours on CIFAR10 with IPC=50.

5.3 Ablation Studies

Energy distance. To illustrate the effectiveness of our introduced energy distance, we compare it with another distribution matching method, *i.e.* DM (Zhao & Bilen, 2023). Here we discard our proposed constraint for a fair comparison and denote ED as our decayed variant. As listed in Table 2, ED produces superior performance than that of DM. Especially, ED outperforms DM by large margins with larger IPCs. It is reasonable since ED not only minimizes the distance between synthetic data and whole-dataset, but also

Table 2: Ablation study on CIFAR10.

Method	10	50	100	200	300
DM	47.64±0.55	61.99±0.33	65.12±0.40	69.15±0.17	69.36±0.35
DM + KL	48.43±0.28	62.39±0.28	66.37±0.18	70.29±0.31	70.56±0.36
DM + CLIP	50.25±0.23	63.07±0.05	66.30±0.15	71.26±0.26	71.83±0.42
DM + Classifier	50.34±0.01	64.79±0.18	69.44±0.23	72.45±0.43	73.54±0.32
ED	50.71±0.30	63.30±0.33	67.88±0.25	71.92±0.31	73.85±0.34
ED + KL	51.12±0.25	65.19±0.12	68.43±0.02	72.88±0.25	74.01±0.37
ED + CLIP	50.34±0.22	64.02±0.24	69.01±0.29	72.27±0.63	75.11±0.33
ED + Classifier	**51.34±0.24**	**65.92±0.30**	**69.47±0.30**	**73.73±0.21**	**76.02±0.26**

maximizes the diversity of synthetic samples. These observations suggest that using the energy distance to match feature distributions of the synthetic and original training images is effective in dataset condensation.

Classifier-critic constraint. To evaluate the effectiveness of our proposed classifier-critic constraint for training the synthetic images, we consider different ways to build the pre-trained classifier, including traditional classifier trained on original training set and CLIP. Besides, we also implement the constraint following knowledge distillation (Hinton et al.) by minimizing the KL divergence $\mathrm{KL}\left(p_\phi(\boldsymbol{y}|B_c^{\mathcal{T}})\|p_\phi(\boldsymbol{y}|B_c^{\mathcal{S}})\right)$, where we keep the mini-batch size for real data in accord with the size of synthetic set. As listed in Table 2, adopting the CLIP, which is trained on a public dataset instead of the target large-scale training set, is inferior to using the traditional classifier. The reason behind this might be that the traditional classifier is optimized by the original training set. However, replacing the traditional classifier with CLIP only reduce the performance a little. Compared with ignoring the classification constraint, introducing CLIP to implement the classifier-critic can result in a remarkable performance improvement when IPC is equal or greater than 50, where DM+CLIP is better than DM, ED+CLIP is better than ED, and ED+CLIP still outperforms DM+CLIP. Thus, even using existing available classifiers trained on different dataset, such as CLIP model, ours can still achieve a desired performance. Besides, KL is inferior to our method. The possible reason is that KL provides the soft label predicated by classifier g_ϕ on the real dataset as the target of the synthetic set but we use the ground-truth label. To sum up, introducing KL, CLIP or traditional classifier as the constraint can usually result in a remarkable performance improvement. Thus, we claim that the classifier-critic constraint on synthetic images is a key-point in data condensation settings; this highlights the contribution and originality of our method.

Hyperparameter λ. The hyperparameter λ in Eq. equation 7 indicates the importance of the log-likelihood of the task-related information in synthetic set, *i.e.* the weight of task-related loss. To explore the effect of λ in our method, we design ablation study on CIFAR10 with IPC=10/50. As listed in Table 3, a small λ, such as 0.01, achieves slightly better or even worse performance than abandoning the task-related loss *i.e.*$\lambda = 0$. Both settings of $\lambda = 10$ and $\lambda = \frac{\text{maxiter}-t}{\text{maxiter}}$ can produce a better performance than others. Since we aim to use a much more general hyperparameter rather than searching a perfect λ for each dataset with each IPC jadedly, we set $\lambda = \frac{\text{maxiter}-t}{\text{maxiter}}$. That is to say, we wish to learn synthetic set with more emphasis on $p(\mathcal{C}|\mathcal{S})$ during the early training stage and pay more attention on $p(\mathcal{S})$ during the late training stage. Our proposed method achieves desired performance in various tasks using this hyperparameter varying with the training iterations.

Table 3: Ablation study of hyperparameter λ on CIFAR10 with IPC=10 and IPC=50.

λ	10	1	0.1	0.01	$\frac{\text{maxiter}-t}{\text{maxiter}}$	0
IPC=10	51.26±0.38	50.30±0.54	50.80±0.48	50.17±0.26	**53.42±0.17**	50.71±0.30
IPC=50	**66.72±0.24**	65.48±0.49	63.90±0.17	63.59±0.22	65.92±0.30	63.30±0.33

The efficiency of the CE loss during the training. Recall that the coefficient of the additional CE loss is set to be gradually reduced during the training. To further explore the efficiency of the CE loss during the whole training process, we compare the test performance learned by ours and DM with varying number of training iterations, where we consider ours with CE loss and ours without CE loss. As shown in Figure 3, we evaluate the performance of the condensed samples at each 2000 training iterations. We can find that ours with CE loss performs better than DM in the whole training stage. Besides, ours with CE loss outperforms

ours without CE loss all the time. Therefore, CE loss is not only beneficial in the later phase of the training but also useful in the beginning phase.

Combination with other methods. Considering our proposed method is flexible, we can also combine ours with gradient matching loss and representation matching loss to optimize the synthetic samples. For example, DREAM typically selects the training gradients as the matching objective and provides better results in most cases. Therefore, we can take DREAM as an example and explore where ours can be combined with it, where we use the official code provided by DREAM. Specifically we can add our proposed feature matching loss into the gradient matching loss in DREAM, denoted as OURS+DREAM, where we set the coefficient for two losses as 1 without exhausted adjustment. Besides, as discussed above, we can also add our proposed feature matching loss into the representation matching loss in M3D, denoted as OURS+M3D. As shown in Table 4, our method can improve DREAM and M3D when combining with them respectively in most settings; please see more results of OURS+M3D and M3D in Table 1. The possible reason for the improvement is that ours loss enforces the synthetic dataset to match the real large-scale dataset in the embedding space with energy distance and classifier-critic constraint, which might be complementary to the gradient matching in DREAM and the representation matching in M3D. Therefore, we think that it might provide an interesting direction about how to combine ours with the existing matching loss in dataset condensation.

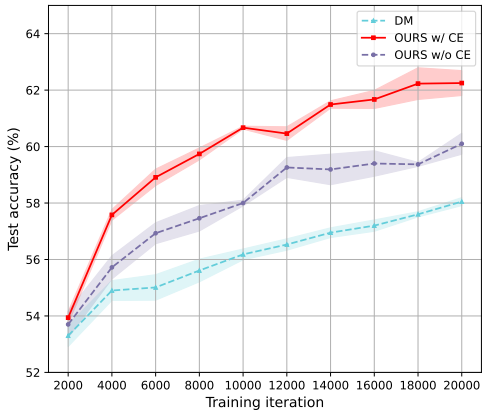

Figure 3: The test performance of ours (w/ CE loss or w/o CE loss) and DM on CIFAR10 (IPC=50) with the varying number of training iterations.

Table 4: The test performance (%) when combining with DREAM on CIFAR10.

IPC	DREAM	OURS+DREAM	M3D	M3D+DREAM
10	65.7	**66.5**	**63.5**	63.2
50	73.8	**74.2**	69.9	**71.0**

5.4 Transferability

Table 5: Testing accuracy of different methods on ConvNet versus transferred to other architectures, where IPC=10/50. The "Transfer" column records the average results on MLP, ResNet18, ResNet152 and ViT.

Dataset	CIFAR-10				CIFAR-100				TinyImageNet			
Method	IPC=50		IPC=10		IPC=50		IPC=10		IPC=50		IPC=10	
	ConvNet	Transfer	ConvNet	Transfer	ConvNet	Transfer	ConvNet	Transfer	ConvNet	Transfer	ConvNet	Transfer
Random	50.55	36.39	31.00	24.16	34.66	23.12	18.64	11.31	18.62	8.85	6.88	3.53
K-Center	56.00	41.45	41.19	31.01	38.64	26.17	25.04	15.31	22.02	10.77	11.38	5.42
DC	56.81	31.42	50.99	**32.22**	30.56	14.95	28.42	11.95	12.66	4.62	12.83	3.74
DSA	60.28	38.37	52.96	31.15	43.13	**27.11**	32.23	15.77	25.31	10.93	16.34	6.75
DM	61.99	40.04	47.64	30.66	42.32	25.22	29.23	13.59	22.76	9.75	13.51	4.08
OURS	**65.92**	**41.55**	**53.42**	31.74	**44.90**	26.83	**32.80**	**16.22**	**28.10**	**23.24**	**16.41**	**7.13**

Following (Cui et al., 2022), we explore the transferability of different condensation methods on the 3 datasets using 5 architectures, including ConvNet, MLP, ResNet18 (He et al., 2016), ResNet152 (He et al., 2016) and ViT (Dosovitskiy et al., 2020). More details about architectures and transfer results on each architecture can be found in the Appendix B. As shown in Table 5, although the performance of all methods drops when transferring to other architectures, ours still produces a desired balance between the seen ConvNet and unseen architectures. When IPC=10 on CIFAR10 and IPC=50 on CIFAR100, ours is slightly worse than DC or DSA. In addition to above-mentioned setings, ours achieves the better performance over the seen ConvNet and better-averaged generalization performance over the unseen architectures. Remarkably, when IPC=50 on TinyImageNet, the performance gain of ours over DSA is about 14%. The possible reason is that learning the synthetic data from TinyImageNet is very challenging and ours extract more class-relevant information with less architectural inductive bias than other baselines; see the visualization below. Thus,

our proposed method is reliable and generalizable whose condensed images using one architecture can be effectively used to train other unseen ones.

5.5 Neural Architecture Search (NAS)

NAS aims to automatically search for a top architecture from a vast search space, which typically requires expensive training of numerous architectures multiple times on the whole training set and picking the best performing ones on a validation set. It is beneficial that applying the smaller condensed dataset to NAS (Zhao et al., 2021). Following the implementations of (Cui et al., 2022), we randomly sample 100 networks from NAS-Bench-201 (Dong & Yang, 2020), which contains the ground-truth performance of 15,625 networks. We reduce the number of repeated blocks from 15 to 3 during the search phase, making the size of condensed dataset adapt the networks. All models are trained on CIFAR10 and IPC=50 for 50 epochs under 5 random seeds and ranked according to their average accuracy on a held-out validation set of 10k images. We consider two metrics 1) Spearman's rank correlation coefficient between the ranking of models trained on condensed dataset and the original dataset; 2) The ground-truth performance of the best architecture trained on the condensed dataset (Top 1). As we can see in Table 6, all methods produce negative correlation between the performance on condensed and full dataset except for K-Center, indicating the difficulty of utilizing the condensed dataset to guide model designs. Besides, ours still achieves a relatively better trade-off between correlation and ground-truth performance than other methods. It verifies that our condensed images can be used to efficiently train multiple networks to identify the best network for saving computation resources.

Table 6: We implement NAS on CIFAR10 using NAS-Bench-201. The correlation of the original dataset is lower than 1.0 as we use a small architecture and perform ranking based on the validation set.

	Random	K-Center	DC	DSA	DM	OURS	Whole
Correlation ↑	-0.06	**0.11**	-0.19	-0.37	-0.37	-0.17	0.7487
Top 1 (%) ↑	91.9	91.78	86.44	73.54	92.16	**92.47**	93.5

5.6 Continual Learning (CL)

CL (Kirkpatrick et al., 2017) aims to address the catastrophic forgetting problem when the model learns sequentially from a stream of tasks. Due to its highly condensed nature, distilled samples have been successfully applied to the CL. We evaluate the effectiveness of the condensed dataset in CL. Following (Zhao & Bilen, 2023), we set up the baseline based on GDumb (Prabhu et al., 2020) which stores class-balanced training samples in memory greedily and trains a model from scratch on the latest memory only. We randomly and evenly split the 100 classes from CIFAR100 dataset into 5 steps, *i.e.* 20 classes per step respectively. The memory budget is 20 images/class for all seen classes. We compare our method with other baselines. Figure 4 shows that the proposed method is superior to other baselines. It indicates that our method can produce a better and more informative condensed set for training models than those produced by competitors. Similarly, the results of 10 step class-incremental learning are shown in Figure 5.

5.7 Visualization of Data Distribution

Considering data condensation methods synthesize a small dataset, one natural question is whether it introduces any bias into the data distribution. Therefore, we visualize the data distribution of the real dataset and the synthetic datasets by different methods in Figure 6 for CIFAR10 with IPC=50. Specifically, we use the ResNet18 pretrained on the real dataset to extract features and visualize the features with T-SNE (Van der Maaten & Hinton, 2008), where we plot the samples from the first class, second class and fourth class. We find that the synthetic images learned by baselines are usually biased to real image distribution, especially for DC and CAFE. However, ours can obtain more evenly distributed samples and cover the real image distribution in a better way. It thus proves the effectiveness of ours in above-mentioned downstream tasks.

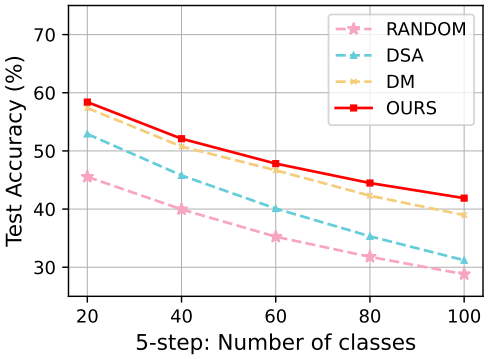

Figure 4: Test accuracy of 5 steps continual learning with condensed samples on CIFAR100 with IPC=20 for Random, DM, DSA and ours, respectively.

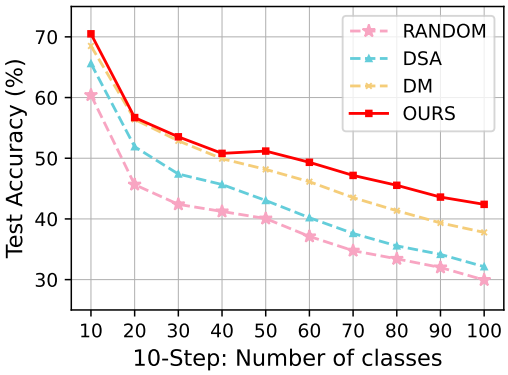

Figure 5: Test accuracy of 10 steps continual learning with condensed samples on CIFAR100 with IPC=20 for Random, DM, DSA and ours, respectively.

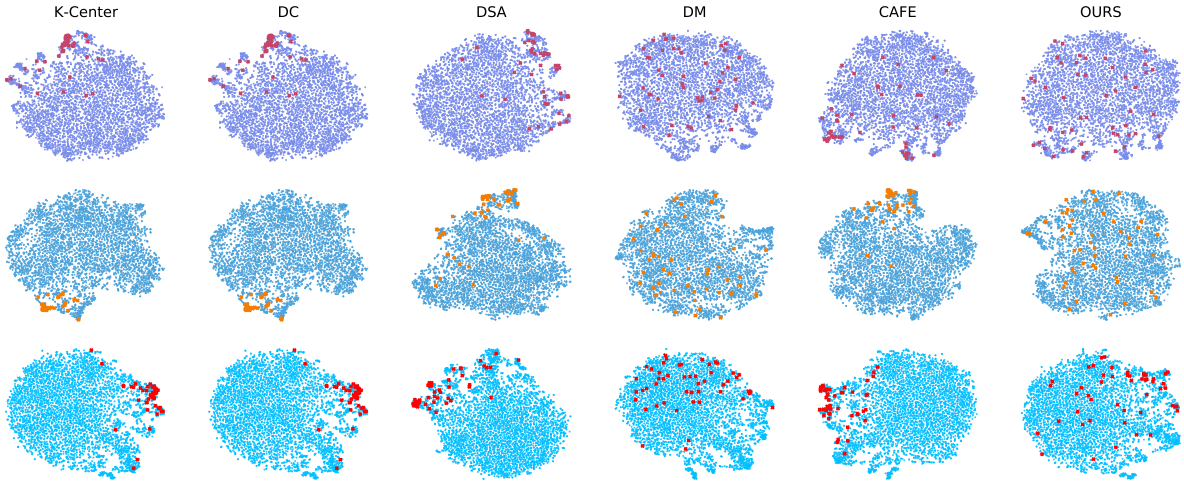

Figure 6: Visualization of synthetic dataset distribution (red points) and original dataset (blue points) on CIFAR10 with IPC=50, where the first row is the first class, and the second row is the second class and the third row is the fourth class.

5.8 Debiased Dataset Condensation

We examine whether ours can be used to compress a biased training set into a small de-biased synthetic dataset. Here we adopt the **CelebA** face dataset (Liu et al., 2015) that includes 40 attributes. We use the hair color ($blond, not-blond$) as the target and gender ($male, female$) as the spurious attribute, where the label is spuriously correlated with gender. We now have 4 groups: (blond, male), (not blond, female), (blond, female), (not blond, male). To reduce the computational resource, we resize the original image into 64×64 and keep the proportions of 4 groups in original dataset to randomly drop some samples, resulting in 10000 training examples with 85 in the smallest group (blond-haired males). See Appendix C for more details. We adopt the official test split to evaluate the methods and report the average and worst-group performance. We follow (Graikos et al., 2022) that uses the pretrained ResNet18 face attribute classifier on CelebA to realize both ϕ_1 and ϕ_2. We consider random selection, DM, ED, and ED+label as our baselines, where ED+label adds the minimization about the CE loss of label in the distilled image to the ED. Ours further adds the maximization about the CE loss of spurious attribute to the ED+label. As listed in Table 7, ours can generally achieve the better worst-group and average performance than its baselines, indicating the effectiveness of introduced constraints in de-biased dataset condensation. Besides, adding the constraint

Table 7: Performance on CelebA with different IPCs. The average and worst performance on the whole dataset are 94.53 ± 0.19 and 93.85 ± 0.27, respectively.

Method	10		50		100		200		300	
	Avg.	Worst	Avg.	Worst	Avg.	Worst	Avg.	Worst	Avg.	Worst
RANDOM	80.30±0.29	76.18±0.29	86.68±0.13	77.44±0.41	83.45±0.23	81.69±0.57	88.05±0.09	84.66±0.66	88.29±0.26	86.37±0.12
DM	86.68±5.31	66.71±2.64	87.56±1.48	84.44±1.44	88.51±0.56	87.89±0.04	89.74±0.40	88.64±0.28	89.40±0.36	88.36±0.38
ED	86.48±1.17	69.42±1.48	86.86±0.75	84.63±0.36	88.86±1.07	88.38±0.88	90.61±0.56	89.60±0.30	90.72±0.05	89.56±0.90
ED+label	**86.76±1.49**	75.64±0.95	87.85±1.07	86.48±1.68	89.31±0.25	87.04±0.80	90.33±0.31	89.66±0.25	91.09±0.32	89.84±0.13
OURS	86.66±1.38	**76.78±1.68**	**87.85±0.06**	**86.48±0.48**	**89.54±0.56**	**88.38±0.26**	**90.71±0.63**	**89.71±0.66**	**91.48±0.38**	**90.74±0.09**

about the label usually leads to improved average performance. It is reasonable since ED+label pays more attention to all groups. These observations claim that introducing classifier-critic constraint about the specific task is effective in dataset condensation and can provide more applications. We also perform a data condensation on point cloud classification task using our proposed method and report the results in Appendix 6, to validate the effectiveness of ours on different modalities.

5.9 Visualization of Condensed Dataset

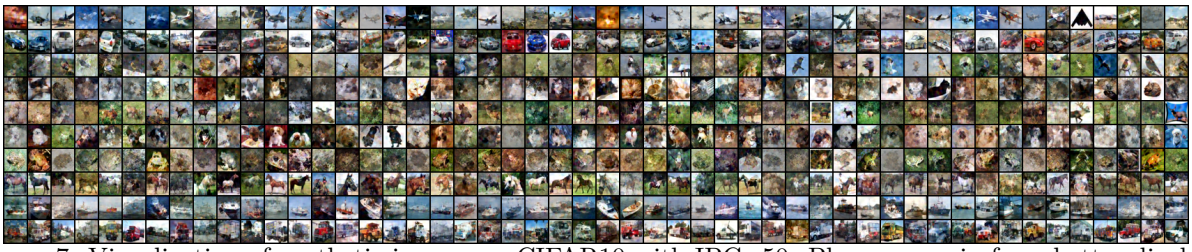

Figure 7: Visualization of synthetic images on CIFAR10 with IPC=50. Please zoom in for a better display.

We visualize the synthetic images on CIFAR10 with IPC=50 in Figure 7. Due to the limited space, we defer the generated synthetic sets of CIFAR10 with IPC=10 in Figure 8 of Appendix and the generated synthetic sets of CelebA with IPC=10 in Figure 9 of Appendix, respectively. With the development of IPC, it is easy to find that the synthetic images generated by our method is visually similar to original real CIFAR10 images but looks more class-representative. Taking the Figure 7 (the visualization of synthetic images CIFAR10 with IPC=50) as the example, the 50 images per class results are diverse which cover the main variations, such as the various airplanes in the first row ("airplane" class). For CelebA, we consider the debiased dataset condensation, where the hair color (blond or not-blond) is the label and the gender (male or female) is the spurious attributes. That is to say, we aim to obtain a condensed face dataset with two classes, where one class is composed of blond hair face images and another is composed of non-blond hair face images and both of them can not keep the gender information in the images as soon as possible. As shown in Figure 9 of Appendix, we can see that the condensed can meet the expected constraints to a great extent.

6 Results on Other Modalities

To validate the effectiveness of ours on different modalities, we perform a data condensation on point cloud classification task using our proposed method. Here we use preprocessed modelnet10 dataset (Wu et al., 2015), which contains 3981 training and 400 testing samples from 10 categories and each sample is composed of 10K points (x;y; z-coordinates;R;G;B). Researchers usually utilize random subsampling to achieve fewer points (e.g., 2048) for each object. We compare random subsampling, our proposed ED, and ED+classifier to achieve fewer points. We randomly sample 100 training samples and condense each sample with 10K 3D points (x;y; z-coordinates) into a synthetic sample with 64 3D points. We then utilize condensed datasets to train the pointnet++ for point cloud classification task, where we set epoch=50. We adopt a common evaluation method for dataset condensation methods, i.e. measuring the test accuracy of the neural networks trained on the condensed data, where we use all test samples and report instance accuracy (IA) and class accuracy (CA). The test performance is summarized in Table 8. Compared with random subsampling, data

condensation can generate more data-efficient samples, indicating the effectiveness of our proposed data condensation method in 3D.

Table 8: The accuracy comparison of IA and CA for point cloud classification.

Method	Random	ED	ED+classifier
IA	51.92	62.87	**67.24**
CA	53.43	65.01	**68.93**

7 Conclusion

This work aims to compress the original training dataset into a condense set via class preserving distribution matching. To this end, we design the optimization method of synthetic samples from two key points. One is responsible for capturing the original data distribution, where we introduce energy distance to increase the diversity. The other aims to introduces the task-related information, such as classification, into the learning of synthetic samples without a bi-level optimization. We introduce a classifier-critic constraint based on an off-the-shelf pre-trained classifier to implement the second goal. By minimizing the energy distance between real and synthetic samples and the classifier-critic constraint, we learn a diverse and class-preserving synthetic set with less training cost, avoiding an expensive bi-level optimization. Extensive experiments demonstrate the effectiveness of our proposed method in commonly-used downstream tasks and the de-biased dataset condensation task.

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A Detailed Implementation Settings

We list our used learning rate for each dataset with varying IPCs in Table 9 and the mini-batch size of the real dataset in Table. 10. We also report the pretraining details for the model parameters ϕ on different datasets in Table 11. For a standard dataset condensation task, ϕ indicates a classifier and its training details can be found in 11, where we adopt the same architecture and settings with the downstream classification task. For the di-biased dataset condenstaion task, ϕ represents a face attribute classifier, we use a well pretrained model following (Graikos et al., 2022).

Table 9: Learning rate on CIFAR-10, CIFAR-100, TinyImageNet and CelebA with different IPCs. To obtain the results in Figure 1, we set the learning rate as 6 with IPC $\geq$ 300 on CIFAR-10.

IPC	CIFAR-10	CIFAR-100	TinyImageNet	CelebA
1	0.1	0.1	0.2	-
10	0.1	0.1	0.2	10
50	0.1	0.1	0.2	10
100	1	-	-	10
200	1	-	-	10
300	5	-	-	10

Table 10: Mini-batch size of real dataset on CIFAR-10, CIFAR-100 and TinyImageNet with different IPCs.

Dataset	IPC												
	1	10	50	100	200	300	400	500	600	700	800	900	1000
CIFAR-10	512	512	512	512	512	512	512	512	256	256	64	64	64
CIFAR-100	512	512	512	-	-	-	-	-	-	-	-	-	-
TinyImageNet	512	512	256	-	-	-	-	-	-	-	-	-	-
CelebA	-	512	512	512	512	512	-	-	-	-	-	-	-

B More Details about Transfer Experiments

Following (Cui et al., 2022), the evaluated network architectures are as follows:

ConvNet: A standard convolutional network which is classical in vision tasks, is used to extract features from the synthetic and original datasets in previous works (Zhao & Bilen, 2023). The structure includes three 3×3 convolutional layers followed by a 2×2 avgpooling and instance normalization with hidden widths of 128 on CIFAR-10 and CIFAR-100. For TinyImagenet, a four layers network called ConvNetD4 is used for

Table 11: Training details when ϕ indicates a classifier on CIFAR-10, CIFAR-100 and TinyImageNet, where BS denotes Batch Size, E denotes Epoch, lR denotes Learning rate, Optim denotes Optimizer, Mome denotes Momentum and WD denotes Weight Decay.

Data	Model	BS	E	LR	Scheduler	Optim	Mome	WD
CIFAR10	ConvNet	256	1000	0.01	STEPLR	SGD	0.9	5e-4
CIFAR100	ConvNet	256	1000	0.01	STEPLR	SGD	0.9	5e-4
TinyImageNet	ConvNetD4	32	1000	0.01	STEPLR	SGD	0.9	5e-4
CelebA	ResNet18	256	1000	0.01	STEPLR	SGD	0.9	5e-4

better performance and ability in extracting features following (Cui et al., 2022). There are 0.32M trainable parameters in ConvNet and 0.45M in ConvNetD4, correspondingly.

MLP: A simple fully connected network with 3 layers and 128 neurons in each layer is considered to evaluate methods. There are 0.41M trainable parameters in MLP.

ResNet18/ResNet152: Standrad architecture includes 4/50 residual blocks, resepectively. A ReLU activation layer and batch normalization layer in each block follow two convolution layers. There are 11.17M trainable parameters in ResNet18 and 58.16M in ResNet152.

ViT: Vision Transformer is a new and competitive model architecture used in image processing, which is based on transformer structure. There are 10M trainable parameters in our ViT.

Previously, we show the transfer results in Table. 5 with IPC=50. We further report the detailed transfer results on each architecture in Table. 13 for easier references.

C Details about the De-biased Dataset Condensation

We extract features and evaluate the synthetic images in ResNet18, where we use the same architecture in transfer experiments. Refer to Table. 12 for details about dataset.

Table 12: Number of samples in different groups in training dataset and test dataset on CelebA.

Group	Blond Male	Not blond Male	Blond Famale	Not blond Famale	All
Original training dataset	1387	66874	22880	71629	162770
Our training dataset	85	4101	1405	4401	10000
Test dataset	180	2480	7536	9767	19963

D Visualization of the Learned Synthetic Samples

- We visualize the generated synthetic sets of CIFAR10 with IPC=10 in Figure8, respectively.
- We visualize the generated synthetic sets of CelebA with IPC=10 in Figure9 , respectively.

We can observe that the synthetic images can capture diverse appearances in the categories. Besides, the synthetic images in each class are less redundant.

Figure 8: Visualization of synthetic images on CIFAR10 with IPC=10.

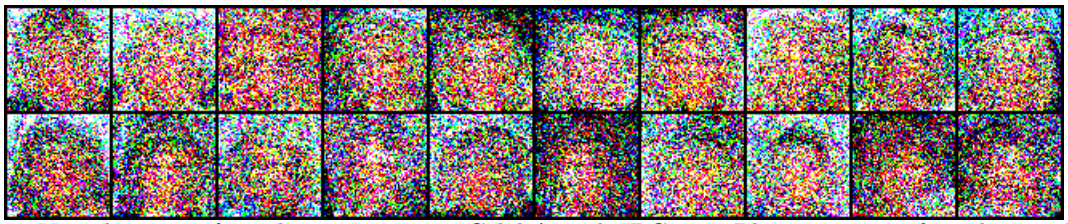

Figure 9: Visualization of synthetic images on CelebA with IPC=10. Please zoom in for a better display.

Table 13: Comprehensive transferability results of different methods tested on ConvNet, MLP, ResNet18, ResNet152 and ViT with IPC 1, 10 and 50. We report testing accuracy.

Dataset	Method	IPC	Network ConvNet	MLP	ResNet18	ResNet152	ViT
CIFAR-10	Random	1	15.40±0.28	14.37±0.38	16.56±0.46	12.15±1.80	14.19±0.99
		10	31.00±0.48	25.08±0.27	29.52±0.87	15.84±0.91	26.21±0.49
		50	50.55±0.32	25.21±0.44	47.26±0.27	23 36±2.31	39.73±0.52
	K-Center	1	25.16±0.45	24.01±0.32	25.99±0.57	14.64±1.30	21.54±0.55
		10	41.49±0.73	32.92±0.38	40.08±0.88	19.35±0.71	31.95±0.57
		50	56.00±0.29	40.61±0.34	52.69±0.70	27.84±1.07	44.65±0.39
	DC	1	29.34±0.37	29.02±0.52	27.43±0.71	15.31±0.36	28.14±1.11
		10	50.99±0.62	34.06±0.40	43.96±1.37	16.51±0.89	34.36±0.35
		50	56.81±0.44	31.63±0.55	45.94±1.41	17.98±1.06	30.14±0.51
	DSA	1	27.76±0.47	25.04±0.77	25.59±0.56	15.12±0.65	23.70±0.20
		10	52.96±0.41	34.49±0.47	42.11±0.56	16.10±1.03	31.88±0.35
		50	60.28±0.37	41.01±0.36	49.52±0.72	19.65±1.16	43.30±0.43
	DM	1	26.45±0.39	10.02±0.55	20.64±0.47	14.09±0.58	20.47±0.46
		10	47.64±0.55	34.44±0.30	38.21±1.05	15.60±1.51	34.37±0.49
		50	61.99±0.33	40.49±0.38	52.76±0.44	21.67±1.34	45.22±0.37
	CAFE	1	-	-	-	-	-
		10	50.68±0.17	35.94±0.65	42.40±0.88	25.51±0.41	35.22±0.16
		50	61.97±0.22	24.41±0.17	26.12±0.77	14.78±0.55	25.10±0.37
	OURS	1	29.30±0.33	24.58±0.10	24.56±0.34	13.60±0.21	20.74±0.30
		10	53.42±0.14	34.94±0.36	40.65±0.14	17.20±0.33	34.17±0.16
		50	65.92±0.24	41.40±0.67	55.49±0.21	23.12±0.42	46.20±0.29
CIFAR-100	Random	1	5.30±0.23	4.27±0.09	4.36±0.15	1.73±0.12	4.45±0.15
		10	18.64±0.25	10.20±0.18	15.77±0.24	5.19±0.46	14.07±0.21
		50	34.66±0.41	16.80±0.31	30.23±0.61	18.55±1.29	26.90±0.33
	K-Center	1	10.89±0.17	7.96±0.17	8.75±0.43	2.22±0.19	7.81±0.13
		10	25.04±0.30	13.92±0.20	22.18±0.59	7.14±0.79	17.98±0.44
		50	38.64±0.43	19.32±0.36	34.00±0.51	21.25±1.46	30.12±0.65
	DC	1	13.66±0.29	9.78±0.27	9.71±0.46	2.67±0.16	9.27±0.14
		10	28.42±0.29	12.36±0.20	17.94±0.59	5.28±1.05	12.22±0.17
		50	30.56±0.56	13.29±0.30	17.64±0.31	11.36±0.95	17.51±0.15
	DSA	1	13.73±0.45	10.56±0.22	9.95±0.55	2.95±0.44	9.48±0.27
		10	32.23±0.35	16.17±0.26	21.86±0.43	5.45±1.04	19.61±0.15
		50	43.13±0.33	21.42±0.31	34.34±0.44	20.79±1.76	31.89±0.49
	DM	1	11.20±0.27	8.17±0.21	5.36±0.31	2.11±0.13	4.59±0.26
		10	29.23±0.26	14.68±0.18	18.72±0.49	3.91±0.73	17.06±0.25
		50	42.32±0.37	20.14±0.24	33.34±0.40	17.29±2.41	30.11±0.25
	OURS	1	12.80±0.20	6.71±0.16	5.76±0.11	2.26±0.32	7.28±0.19
		10	33.39±0.22	16.06±0.08	22.58±0.33	6.73±0.17	19.51±0.23
		50	45.19±0.18	19.63±0.17	38.54±0.66	18.44±0.36	30.70±0.21
TinyImageNet	Random	1	1.65±0.11	1.37±0.08	1.27±0.08	0.63±0.08	1.71±0.03
		10	6.88±0.25	3.12±0.13	3.34±0.16	1.01±0.15	6.63±0.21
		50	18.62±0.22	5.28±0.2	10.35±0.33	2.90±0.40	16.87±0.20
	K-Center	1	3.03±0.12	2.53±0.13	2.29±0.10	0.82±0.10	2.27±0.02
		10	11.38±0.26	4.73±0.08	5.46±0.24	1.55±0.21	9.92±0.34
		50	22.02±0.40	5.99±0.17	13.51±0.34	3.91±0.49	19.70±04
	DC	1	5.27±0.10	2.67±0.17	3.17±0.21	0.90±0.14	2.00±0.12
		10	12.83±0.14	4.12±0.11	5.44±0.21	1.24±0.18	4.17±0.10
		50	12 66±0.36	3.81±0.17	7.05±0.21	2.39±0.21	5.22±0.23
	DSA	1	5.67±0.14	3.90±0.16	3.20±0.13	0.84±0.12	3.17±0.03
		10	16.34±0.21	6.31±0.21	7.60±0.36	1.90±0.21	11.17±0.15
		50	25.31±0.22	6.72±0.20	13.36±0.40	3.78±0.56	19.87±0.44
	DM	1	3.82±0.21	3.11±0.10	1.79±0.17	0.89±0.07	3.21±0.07
		10	13.51±0.31	4.24±0.13	3.57±0.20	1.06±0.19	7.46±0.20
		50	22.76±0.28	5.74±0.27	11.07±0.39	3.33±0.46	18.88±0.36
	OURS	1	4.41±0.37	2.20±0.17	1.64±0.07	0.81±0.16	3.74±0.11
		10	16.94±0.15	3.57±0.16	8.98±0.28	6.28±0.04	9.69±0.21
		50	28.10±0.21	25.04±0.12	22.44±0.31	23.74±0.42	21.72±0.37