VITERBI-BASED PRUNING FOR SPARSE MATRIX WITH FIXED AND HIGH INDEX COMPRESSION RATIO

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ABSTRACT

Weight pruning has proven to be an effective method in reducing the model size and computation cost while not sacrificing the model accuracy. Conventional sparse matrix formats, however, involve irregular index structures with large storage requirement and sequential reconstruction process, resulting in inefficient use of highly parallel computing resources. Hence, pruning is usually restricted to inference with a batch size of one, for which an efficient parallel matrix-vector multiplication method exists. In this paper, a new class of sparse matrix representation utilizing Viterbi algorithm that has a high, and more importantly, fixed index compression ratio regardless of the pruning rate, is proposed. In this approach, numerous sparse matrix candidates are first generated by the Viterbi encoder, and then the one that aims to minimize the model accuracy degradation is selected by the Viterbi algorithm. The model pruning process based on the proposed Viterbi encoder and Viterbi algorithm is highly parallelizable, and can be implemented efficiently in hardware to achieve low-energy, high-performance index decoding process. Compared with the existing magnitude-based pruning methods, index data storage requirement can be further compressed by 85.2% in MNIST and 83.9% in AlexNet while achieving similar pruning rate. Even compared with the relative index compression technique, our method can still reduce the index storage requirement by 52.7% in MNIST and 35.5% in AlexNet.

1 INTRODUCTION

Deep neural networks (DNNs) demand increasing number of parameters as the required complexity of tasks and supporting number of training data continue to grow (Bengio & Lecun, 2007). Correspondingly, DNN incurs considerable amount of computations and memory footprint, and thus, requires high performance parallel computing systems to meet the target response time. As an effort to realize energy-efficient DNN, researchers have suggested various low-cost hardware implementation techniques. Among them, pruning has been actively studied to reduce the redundant connections while not degrading the model accuracy. It has been shown that pruning can achieve $9\times$ to $13\times$ reduction in connections (Han et al., 2015).

After pruning, remaining parameters are often stored in sparse matrix formats. Different ways of representing indices of non-zero values constitute the different sparse matrix format, and have a significant impact on the level of achievable computational parallelism when a sparse matrix is used as an input operand (Bell & Garland, 2009). If the format is not properly designed, then the performance of DNN with sparse matrix can be even lower than the case with dense matrix (Yu et al., 2017). The two most important characteristics of a hardware-friendly sparse matrix format are 1) reducing index storage footprint and 2) parallelizable index decoding process. As a compromise between index size reduction and index decoding complexity, numerous formats have been proposed (Bell & Garland, 2009).

DNN after pruning heavily involves sparse matrix-vector and matrix-matrix multiplications (SpMV and SpMM, respectively). Despite the sparse content, the computation time for SpMM is longer than that of dense matrix multiplication in modern graphic processing unit (GPU), due to its serialized index decoding process and irregular memory access patterns. For example, the inference latency of AlexNet and VGG16 with SpMM can be increased by $2\times$ to $5\times$ on GPUs or CPUs (Han et al., 2016a). Traditional pruning technique, therefore, is only attractive in the case where SpMV can be

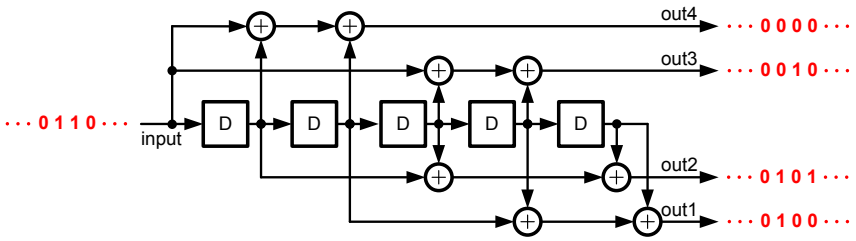


Figure 1: Viterbi decompressor (VD) structure.

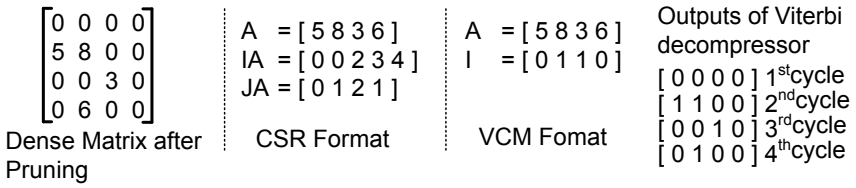


Figure 2: CSR Format and the proposed sparse matrix format comparison.

utilized (i.e., batch size of 1) (Han et al., 2016b) (Yu et al., 2017). Therefore, a sparse matrix representation associated with parallelizable dense-matrix reconstruction in a wide range of computing operations is the key in extending the usage of pruning.

We propose a new DNN-dedicated sparse matrix format and a new pruning method based on error-correction coding (ECC) techniques. An unique characteristic of this sparse matrix format is the fixed, yet high (as shown in Section 3) index compression ratio, regardless of the pruning rate. Moreover, sparse-to-dense matrix conversion employing the proposed format becomes a parallel process and is no longer the performance bottleneck. Notice that conventional sparse matrix formats entail at least one column or row index value for each non-zero parameter such that the amount of index data is larger than that of non-zero values. On the other hand, the proposed approach compresses the locations of non-zero values with a convolutional code which is a type of ECC code. Consequently, the size of sparse matrix index becomes negligible.

Conventional pruning approaches first identify the parameter candidates to be pruned, then construct a matrix (often sparse) using formats such as Compressed Sparse Row (CSR) to represent the survived parameters. On the contrary, in the proposed scheme, pruning is performed in a restricted manner since a specific sparse matrix format is first constructed. A DNN-specific Viterbi encoder takes an input pattern and generates a sequence of random-number, where a "1" indicates the parameter is survived, and pruned otherwise. Depending on the length of the input pattern, a vast (but limited) number of output patterns (hence candidates of the final sparse matrix representations) are considered. In this case, the input pattern is used as the sparse matrix index. The content of the input pattern, which generates a deterministic output random number sequence, is chosen such that the accuracy degradation is minimized based on a user-defined cost function (more details on Section 2). Both Viterbi encoder and algorithm have shown to be computationally efficient with inherent parallelizable characteristic, as demonstrated in digital communication applications (Viterbi, 1998). In this work, we further extend its application and demonstrate how Viterbi algorithm can be modified to perform energy-efficient DNN pruning.

2 Pruning using Viterbi-based Approach

Figure 1 illustrates the proposed Viterbi decompressor (VD), which is based on a Viterbi encoder widely used in digital communication. VD has a simple structure consisting of FlipFlops (FFs) and XOR gates only. In this configuration, VD takes one input bit and produces four output bits every clock cycle. Notice that FFs and XOR gates intermingle input bits and generate pseudo random number outputs. Suppose that there is a dense matrix after pruning, as shown in Figure 2, and an input sequence of $\{0, 1, 1, 0\}$ is applied to VD through four clock cycles to generate the outputs, where '1' implies that the corresponding parameter is survived. In this case, the proposed Viterbi-

Compressible Matrix (VCM) format's overhead in index is significantly less than that of CSR. In the VCM format, the input sequence to the VD becomes the index information. This index size is independent of the number of non-zero values and can be determined beforehand based on the target index compression ratio[1]. Unlike CSR format, the available VD-compressible dense matrix representation is limited, meaning that not all possible dense matrix representation after conventional magnitude-based pruning (such as (Han et al., 2015)) can be reconstructed by VD. Pruning method considering VCM, therefore, may result in a matrix that contains different survived parameters compared to a pruning method using CSR format. Thus, the key to the success of VCM is to design a VD allowing diversified parameters to be survived, and to efficiently search for the optimal VD input sequence that minimizes the accuracy degradation[2].

2.1 Viterbi Decompressor (VD) Design Considerations

If the input sequence length and the total output sequence length of a VD are denoted as p and q, respectively, then the index compression ratio can be calculated as q/p. Achieving a high index compression ratio (i.e., $q >> p$) implies that the possible 2^p VD-compressible dense matrix representations need to be uniformly distributed inside the 2^q space to maximize the likelihood of finding a dense matrix representation that is closely matched to the optimal case.

In other words, the goal of VD is to act as a random number generator using the input sequence. It would be interesting to note that such an effort has already been studied in ECC design (Morelos-Zaragoza, 2006). Since "random coding" has been introduced by C. Shannon to prove his channel capacity model (Shannon, 1948), practical ECC techniques with fixed encoding rate have been proposed to simulate random coding with an allowed decoding complexity. We choose Viterbi encoder, which is the base model of VD, as a controllable random number generator because of its simplicity and flexible design in increasing the number of outputs. The randomness of VD outputs is determined by the number of FFs and XOR gates configuration. We present the details of VD design methodology in Appendix A.1.

The basic structure of VD is similar to the design introduced in (Lee & Roy, 2012). VD targeting DNN applications, however, requires the number and/or distribution of 1 (i.e., pruning rate) to be a user-defined parameter, whereas in the typical applications that require random number generation, such as ECC and VLSI testing, the number of 1 and 0 should be approximately the same. In order to control the pruning rate, the VD outputs are connected to binary number comparators. For instance, in Figure 1, one input of the comparator takes a two-bit number $\{out2, out1\}$, while the other input takes a user-defined threshold value (TH_c). If $\{out2, out1\}$ (or $\{out4, out3\}$) is larger than TH_c, the comparator produces a "1", and a "0" otherwise. There is a trade-off between the granularity of the pruning rate and the index compression ratio. If the number of VD ouputs, the number of comparator input bits, and the number of comparators (i.e., index compression ratio) are denoted as NUM_v, NUM_c, and R, respectively, then $NUM_v = NUM_c \times R$ (see Figure 10). The proposed index decoding operation utilizing VD is inherently a parallel process with a small hardware overhead. Unlike CSR or other similar formats employing irregular index structure, decoding VCM using VD does not incur significant buffer/memory overhead for indices and/or non-zero values, and most importantly, can be done with a fixed and predictable rate. Fixed index compression ratio is also desirable for efficient memory bandwidth utilization and for applying tiling technique to further improve the level of parallelism.

2.2 Viterbi Algorithm for Pruning

The basic idea of our proposed pruning method is to assign a cost function to each pruning case enabled by VD and evaluate all possible (2^p) pruning cases with a "Branch-and-Cut" algorithm. The pruning case that has the optimal (i.e., lowest) cost function should lead to minimal accuracy degradation. Viterbi algorithm computes the maximum-likelihood sequence in a hidden Markov model (Forney, 1973), and can be utilized as a fast and efficient pruning exploration technique for our pruning method. Pruning using Viterbi algorithm follows the next 3 steps.

[1] As an example, the structure shown in Figure 1 provides four output bits per one input bit, achieving an index compression ratio of four

[2] In the context of magnitude-based pruning, the objective of pruning using the VD is to identify a set of VD input sequence that preserves maximum number of larger value weights

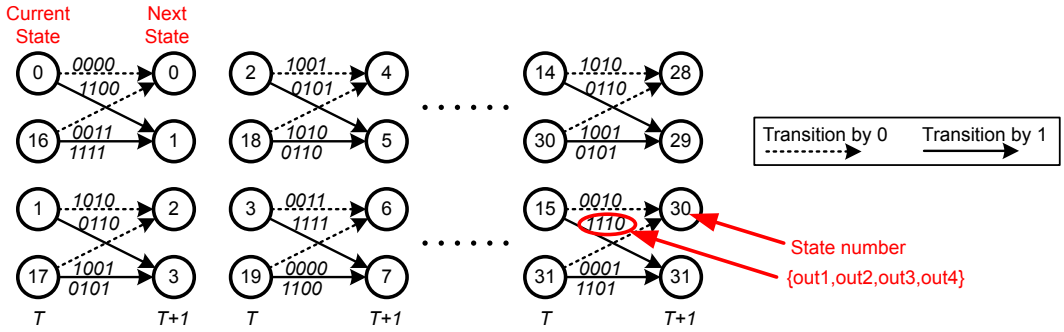


Figure 3: Trellis diagram of VD shown in Figure 1.

The first step is to construct a trellis diagram which is a time-indexed version of a state diagram. A state of VD can be represented using FF values, where the leftmost FF value becomes the least significant bit. If VD has k FFs, the total number of states is 2^k. Hence, VD of Figure 1 has a total of 32 states as shown in Figure 3, where T is the time index. Each possible transition with an input bit (0 or 1) produces multiple corresponding output bits. A trellis diagram holds the entire operations inside VD in a compact fashion.

The next step is to compute a cost function for possible transitions using the branch metric and the path metric. The branch metric is expressed as $\lambda_t^{i,j}$ where t is a time index and i is a predecessor state of j. $\lambda_t^{i,j}$ denotes the cost of traversing along a transition from i to j at the time index t. By accumulating the branch metrics and selecting one of two possible transitions reaching the same state at the same time index, the path metric is defined as

$$\Gamma_{t+1}^{j} = \max\left(\Gamma_t^{i1} + \lambda_t^{i1,j}, \Gamma_t^{i2} + \lambda_t^{i2,j}\right), \tag{1}$$

where $i1$ and $i2$ are two predecessor states of j. In practice, path metrics can be normalized to avoid overflow. Note that we use $\max$ function for the path metric instead of $\min$ function in Eq. (1) because the metric values in our method describe a degree of 'reward' rather than 'cost'. For all the "survived path" selections during the path metric update, the decisions are stored in the memory and the old path metrics can be discarded. The objective of this Viterbi algorithm is to find a path maximizing the accumulation of the branch metrics ($\lambda_t^{i,j}$), which is expressed as:

$$\begin{aligned} D_t^{i,j,m} &= \left(W_t^{i,j,m} - TH_p\right)/S_1,\ 0 \le W_t^{i,j,m}, TH_p \le 1 \\ \beta_t^{i,j,m} &= \begin{cases} \tanh\left(D_t^{i,j}\right) \times S_2, & \text{when survived} \\ -\tanh\left(D_t^{i,j}\right) \times S_2, & \text{when pruned} \end{cases}, \ \lambda_t^{i,j} = \sum_{m=1}^{R} \beta_t^{i,j,m}, \end{aligned} \tag{2}$$

where $W_t^{i,j,m}$ is the magnitude of a parameter at m^{th} comparator output and time index t, normalized by the maximum magnitude of all parameters inside the dense matrix to be pruned, and TH_p is the pruning threshold value. Intuitively, $\beta_t^{i,j,m}$ favors(discourages) the survival(pruning) of parameters with larger magnitude through the skewed $\tanh$ function. Pruning with Viterbi algorithm is flexible such that different cost function can be assigned to the branch metric, depending on the type of pruning approach, as long as pruning algorithm follows a hidden Markov model (Lou, 1995)[3]. The two constants, S_1 and S_2, are the scaling factors, and are empirically determined to be 5.0 and 10^4 for our experiments. Note that exploring diversified states (and hence, various pruning cases) is achieved by maintaining approximately 50% of '1' and '0' distributions for both inputs and outputs of VD (Forney, 1973). Consequently, the target pruning rate is mainly controlled by the comparator threshold value, TH_c (e.g., if TH_c is a 4-bit number and TH_c=3, then $25\%(=(3+1)/2^4)$ is the target pruning rate). TH_p is determined by considering the distribution of parameters and the given target pruning rate (e.g., if the parameters follow a Gaussian distribution and the target pruning rate is 68.3%, TH_p corresponding to one sigma is recommended).

[3]Eq. (2) in this work is related to magnitude-based pruning

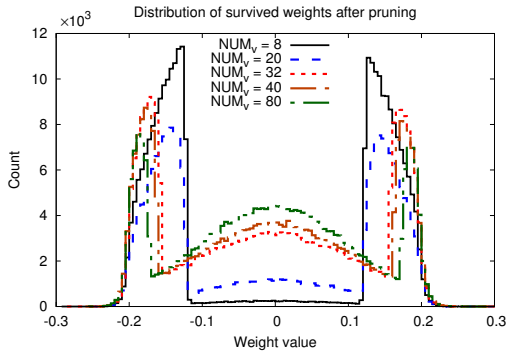


Figure 4: Distribution of FC1's weights after pruning with different NUM_v.

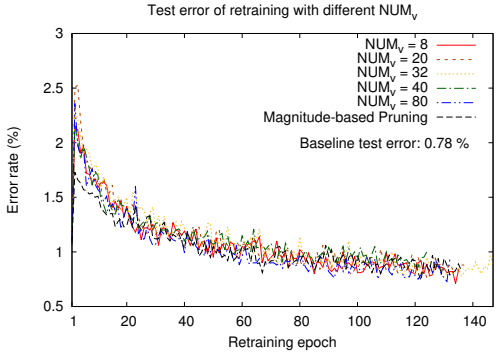


Figure 5: Test error of retraining with different NUM_v.

Once the final time index is reached, then as the last step of Viterbi pruning, the state with the maximum path metric is chosen, and the previous state is traced by reading the survived path selection data. We continue this trace-back procedure to the first time index of a trellis diagram. Note that if the initial state of FFs are all 0s, then the number of available states, hence the number of sparse matrix representations, in the first few time indices may be limited. As an alternative, a dummy input sequence having the length equal to the number of FFs[4] in VD can be inserted such that every state of VD is reachable (refer to Figure 11). In this case, the VCM's compressed input index is a combination of the survived dummy sequence and input sequence. It should be noted that Viterbi algorithm is a dynamic programming. The time complexity to find the best pruning method becomes $O(l \cdot 2^f)$ where l is the length of the input sequence and f is the number of FFs. As can be seen in Appendix A.1, f is small even with a large number of VD outputs.

3 Experimental Results

In this section, the impact of different VD configurations and branch metric selections on model accuracy and the index compression ratio is analyzed. We empirically study the weight distribution after pruning and the sensitivity of accuracy using MNIST. Then, the observations from MNIST are applied to AlexNet to validate the scalability of our proposed method.

3.1 VD Design and Branch Metric Exploration Using MNIST

We perform experiments using LeNet-5-like convolutional MNIST model[5]. For simplicity, both the minimum Hamming distance and the XOR taps (introduced in Appendix A.1) are fixed to be 4, and NUM_c is 4 (i.e., $NUM_v = 4 \times R$). These parameters are selected for fast design exploration, and increasing them will enhance VD output randomness and target pruning rate resolution which are critical to improving pruning rate with minimal accuracy degradation.

Number of VD outputs (NUM_v): Right after training, we prune weights with different NUM_v for VD. Figure 4 shows weight distributions after pruning in FC1 layer with fixed TH_c and TH_p. Lower NUM_v (i.e, lower index compression ratio) leads to a sharper pruning around the weight determined by TH_p. Hence, NUM_v provides a trade-off between accuracy and the index compression ratio. Extensive experiments indicate that for the Conv layer, a low NUM_v is desired, while for the FC layer, a wide range of NUM_v can lead to minimal accuracy degradation as shown in Figure 5 (magnitude-based pruning is from (Han et al., 2015)). For MNIST, NUM_v=8 for Conv layers and NUM_v=40 for FC layers have been chosen to achieve optimal trade-off between the index compression ratio and accuracy.

Pruning threshold value (TH_p): Even when the parameters before pruning follow a known distribution (e.g., Gaussian), it may still be an iterative task to search for an optimal TH_p that results in

[4]The storage overhead of this dummy input sequence is negligible compared to the index data storage

[5]https://github.com/tensorflow/tensorflow/blob/r1.3/tensorflow/examples/tutorials/mnist/mnist_deep.py

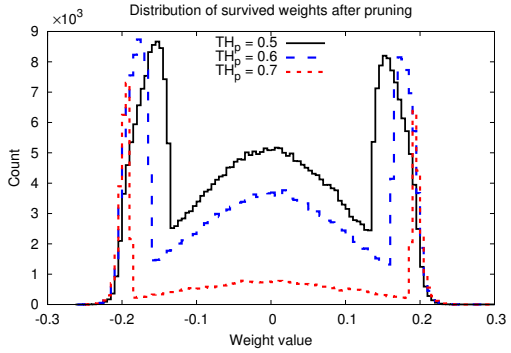


Figure 6: Distribution of FC1's weights after pruning with different TH_p.

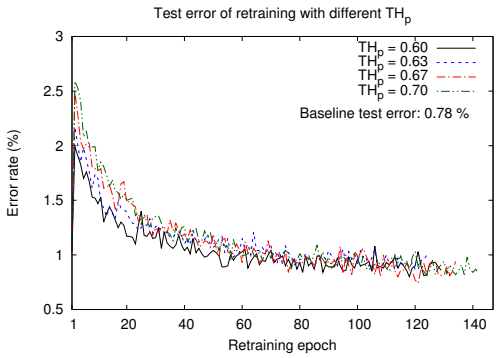


Figure 7: Test error of retraining with different TH_p.

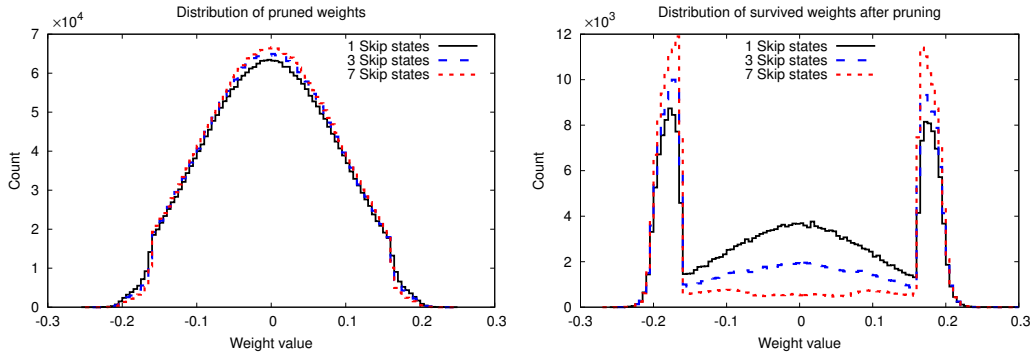


Figure 8: Distributions of pruned (Left) and survived (Right) FC1 weights with different skip state.

the target pruning rate, especially with high NUM_v, as evident from Figure 4. Thus, it is necessary to investigate the sensitivity of accuracy to TH_p. In Figure 6, TH_p affects distributions of survived weights and pruning rates given the same TH_c. Note that if the actual pruning rate is different from the target pruning rate, then VD outputs exhibit skewed supply of '1's or '0's to comparators and the trellis diagram path exploration is also biased. In Figure 7, however, it is clear that all the retraining processes converge, despite the minor discrepancy between the target and actual pruning rate (target pruning rate is 93.75%).

Skip state (Appendix A.2): So far we have only considered the case where one input bit is supplied to VD at every clock cycle. However, if n input bits are provided to VD at every clock cycle, then $n-1$ time indices in a trellis diagram are skipped. While this results in a lower index compression ratio, which is defined as R / (skip state + 1), the skip state allows for more diverse state exploration and improves the pruning quality. As can be seen in Figure 8, more larger magnitude weights are preserved with increasing number of skip states while fixing both TH_p and NUM_v. In this work, the default skip state is one.

Branch Metric: For the branch metric, a variety of functions, such as e^x and sigmoid function $\sigma(x)$, has been investigated, as shown in Figure 9. Among them, "$\tanh$" function is chosen due to its pruning sharpness and low sensitivity to TH_p and NUM_v.

Based on the observations discussed above, we conducted pruning and retraining process, and compared test errors of the magnitude-based pruning method (Han et al., 2015) and the proposed Viterbi-based pruning method. For every round of pruning, all the weights, including the ones pruned in the previous run, are considered. Table 1 illustrates the comparator threshold values TH_c (MIN=0, MAX=15 with NUM_c=4) used for each pruning round and test error results. Since Conv1 is close to the input nodes, we choose a smaller TH_c to lower target pruning rate of Conv1. From Table 1, it

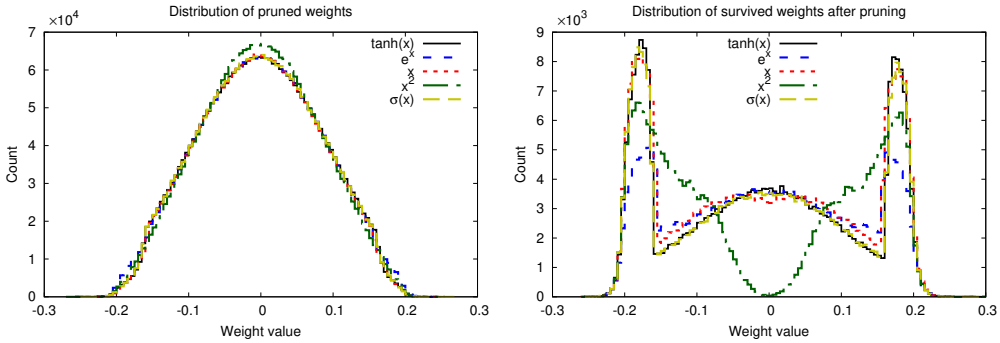
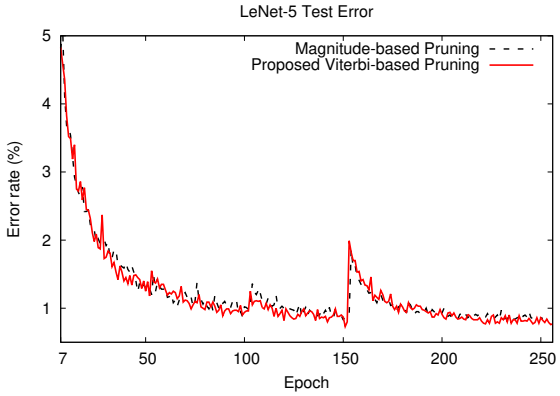


Figure 9: Left: Distribution of pruned (Left) and survived (Right) FC1 weights with different branch metric equations.

Table 1: MNIST test error and comparator threshold values with gradual pruning. Pruning is performed at 50th epoch ($\sim$ 50% target pruning rate), 100th epoch ($\sim$ 70% target pruning rate), and 150th epoch (final). 40 VD outputs are used for FC1 and 8 VD outputs for the others.



Layer	comparator threshold value		
	50th Epoch	100th Epoch	150th Epoch
Conv1	4	4	4
Conv2	7	10	12
FC1	7	10	14
FC2	7	10	12

Table 2: Sparse matrix comparison with MNIST using magnitude-based pruning (Han et al., 2015) and our proposed Viterbi-based pruning. We assume that non-zero values and index for magnitude-based pruning use 16 bits.

Layer	Weight Size	Magnitude-Based		Viterbi-Based		Matrix Size Reduction
		Pruning Rate	Sparse Matrix (CSR) Size	Pruning Rate	Sparse Matrix (VCM) Size	
Conv1	0.8K	34.4%	2.12KB	32.3%	1.16KB	45.3%
Conv2	51.2K	87.4%	25.41KB	81.3%	24.98KB	1.7%
FC1	3211.3K	91.0%	1125.54KB	93.1%	512.82KB	54.4%
FC2	10.2K	81.1%	7.62KB	80.4%	5.17KB	32.2%
Total	3273.5K	**90.9%**	1160.69KB	**92.8%**	544.13KB	**53.1%**
Test Error		0.77%		0.78%		

is clear that the proposed pruning method successfully maintains accuracy during the entire training process.

The final pruning rate and memory requirement for CSR and VCM for each layer are summarized in Table 2. Notice that the sparse matrix represented using VCM format leads to a significant memory footprint reduction (by **53.1%**) compared to the sparse matrix represented with CSR with a similar pruning rate. This is because VCM's index storage is reduced by **85.2%** compared to CSR's index

Table 3: Pruning and sparse matrix size comparison for AlexNet on ImageNet using magnitude-based pruning (Han et al., 2015) and our proposed Viterbi-based pruning. We assume that non-zero values and index for magnitude-based pruning use 16 bits.

Layer	Weight Size	Magnitude-Based		Viterbi-Based		Matrix Size Reduction
		Pruning Rate	Sparse Matrix (CSR) Size	Pruning Rate	Sparse Matrix (VCM) Size	
Conv1	34.8K	16%	69.70KB*	-	69.70KB*	0.0%
Conv2	307.2K	62%	467.46KB	62.5%	268.99KB	42.5%
Conv3	884.7K	65%	1239.40KB	62.3%	777.21KB	37.3%
Conv4	663.6K	63%	982.82KB	62.0%	586.73KB	40.3%
Conv5	442.4K	63%	655.22KB	56.0%	444.83KB	32.1%
FC1	37.7M	91%	13597.74KB	90.3%	8284.93KB	39.1%
FC2	16.8M	91%	6047.99KB	90.8%	3505.43KB	42.0%
FC3	4.1M	75%	4098.00KB	73.7%	2670.18KB	34.8%
Total	61.0M	**89%**	27158.31KB	**88.2%**	16607.99KB	**38.1%**
Test Error (Top-1)		42.73%		42.68%		
Test Error (Top-5)		19.77%		19.78%		

*Dense matrix size is considered in this layer because both CSR and VCM representation result in a larger memory footprint due to the low pruning rate.

size. Even if the CSR is represented with relative index using 5 bits (Han et al., 2016b), at the expense of increased index decoding complexity, VCM index size is still smaller by **52.7%**[6].

In summary, VCM is superior to CSR due to its encoded index format requiring a smaller storage requirement and parallel dense matrix reconstruction process through VD while maintaining a comparable model accuracy.

3.2 AlexNet on ImageNet Results

We verified the scalability of VCM and Viterbi-based pruning method using AlexNet model on ImageNet. The number of VD outputs is 50 for both FC1 and FC2 layer (NUM_v=50, NUM_c=5, R=10) and 8 for the other layers (NUM_v=8, NUM_c=4, R=2). Similar to the MNIST results, higher index compression ratio is set for layers with larger number of weights. Since the skip state is one, index compression ratio becomes $R/2$. The minimum Hamming distance and the XOR taps are 4. Table 3 presents the pruning rates and matrix sizes assuming that non-zero weights and CSR index are stored with 16-bit format.

The achieved **38.1%** reduction in matrix size using VCM is mainly due to the significant reduction in index storage requirement (**83.9%**). Compared with 4-bit relative index scheme introduced in (Han et al., 2016b), index size of VCM is reduced by **35.5%**. The advantage in index compression ratio of the proposed technique is largely attributed to the VD's limited search space out of all possible encodable index formats, while pruning methods employing traditional sparse matrix formats do not consider such restriction. Despite such limitation, both methods achieve similar top-1 and top-5 classification accuracy with the same retraining time.

4 Related Work

Denil et al. (2013) demonstrated that most neural networks parameters have significant redundancy. The redundancy increases the system complexity, and causes overfitting with small training dataset. Several approaches have been suggested to prune deep neural networks and increase the sparsity of parameters in order to minimize both the memory overhead and computation time, and avoid overfitting.

Chauvin (1989) and Hanson & Pratt (1989) introduced additional cost biases to the objective function to decay the unimportant parameters. LeCun et al. (1990) and Hassibi et al. (1993) suggested

[6]Additional size reductions techniques, such as quantizing non-zero weights and Huffman coding (Han et al., 2016b), can also be applied to our methods

pruning parameters while minimizing the increase of error approximated by Hessian matrix. Optimal Brain Damage (OBD) (LeCun et al., 1990) restricted the Hessian matrix to be diagonal to reduce the computational burden, at the cost of additional performance degradation. Optimal Brain Surgeon (OBS) (Hassibi et al., 1993) used a full Hessian matrix with additional computation cost to improve the pruning performance.

Han et al. (2015) proposed to prune deep neural networks by removing parameters based on the magnitude of their absolute values and then iteratively retrain the pruned network. The paper achieved $9\times$ and $13\times$ pruning rate for AlexNet and VGG-16 without loss of accuracy on ImageNet dataset. A follow-up paper compressed the pruned network further with weight sharing and Huffman coding (Han et al., 2016b). Although impressive compression rate is achieved by these suggested methods, the irregular sparsity of the survived parameters and the associated complicated index decoding process prevent common hardware such as GPUs from achieving noticeable speed-up improvement. Alternatively, Han et al. (2016a) designed a dedicated hardware accelerator to circumvent this problem.

Recently, several papers have suggested iterative hardware-efficient pruning methods to realize a faster inference speed and smaller model size. Molchanov et al. (2017c) suggested iterative pruning on a feature-map level based on a heuristic approach to evaluate the importance of parameters. This paper, which shares a similar idea as OBS, uses first-degree Taylor polynomial to estimate the importance of each parameter with reduced computational burden. Since the method prunes feature maps rather than each parameter, sparse matrix format is not required at the cost of a lower pruning rate. Li et al. (2017) suggested to prune the entire convolution kernels together with corresponding feature maps in CNN. Similar to Molchanov et al. (2017c), this coarse-level pruning avoids the use of sparse matrix format, at the expense of a lower pruning rate. Park et al. (2017) introduced a high-performance sparse convolution algorithm, where the sparse convolution was formulated as sparse-matrix-dense-matrix multiplication with the dense matrix generated on the fly. The paper shows that this method can improve the inference speed of pruned networks with moderate sparsity, and prune each parameter independently, leading to a better pruning rate. However, the paper demonstrated its results only on CPUs; it was not shown whether the proposed method also can be applied to throughput-oriented hardware such as GPUs.

Ardakani et al. (2017) proposed a scheme to generate a masking matrix using linear-feedback shift registers (LFSRs) to randomly prune some of synaptic weights connections. Even though the hardware structure for pruning can be simplified, it is not possible to selectively prune connections to improve the pruning quality. In addition, it can be applied to the fully-connected layer only, not the convolution layer.

Kingma et al. (2015) explained Gaussian Dropout is a special case of Bayesian regularization. Unlike Gaussian Dropout which considers dropout rates as a hyperparameter, Variational Dropout theoretically allows training dropout rates layer-wise, or even weight-wise. However, the paper did not include any experimental result on weight-wise variational dropout. Molchanov et al. (2017a) extended Kingma et al. (2015) and showed the working case of weight-wise Variational Dropout. Molchanov et al. (2017a) suggested to use this characteristic of Variational Dropout to prune deep neural networks. By pruning out weights with high dropout rate, the paper achieved high sparsity on a deep neural network for CIFAR-10 classification task. Molchanov et al. (2017b) and Louizos et al. (2017) suggested to prune deep neural networks in structured format with new bayesian models. The papers could prune deep neural networks neuron-wise or channel-wise, keeping the weight matrices in dense format. Both papers showed state-of-the-art sparsity on deep neural networks for CIFAR-10 classification task.

Multiple works tried to reduce the redundancy with popular lossy compression methods. Denton et al. (2014) applies low rank approximations to pre-trained weights. Gong et al. (2014) uses vector quantization to compress deep convolution neural networks. Chen et al. (2015) suggests HashedNets, which applies hashing tricks to reduce the model sizes. Iandola et al. (2016) achieves AlexNet-level accuracy using 50x fewer parameters with SqueezeNet, which is comprised of custom convolution filters called Fire modules. These methods are orthogonal to the network pruning, and can be combined together to achieve further model compression. For example, SqueezeNet combined with Deep Compression (Han et al., 2016b) achieves $510\times$ compression ratio compared to the original AlexNet.

5 FUTURE WORK

There exist many other ECC techniques which can also be potentially used to search for sparse matrix forms with high index compression (Morelos-Zaragoza, 2006). Efficient parallel ECC decoding and encoding implementation have also been proposed and realized (Zhang, 2015). We believe that efforts to combine existing and new ECC techniques/algorithms with DNN pruning methods create a new dimension in realizing energy-efficient and high-performance DNN. Even though the proposed approach is best for dedicated ASIC or FPGA, the inherent parallel characteristics of VD and Viterbi algorithm can also be utilized in GPUs through the construction of new kernels and libraries. We have not considered quantization of non-zero weight values or entropy-related coding design in this paper. In the future, such considerations can be embedded into the branch metric or path metric equations.

6 CONCLUSION

We proposed a new DNN-dedicated sparse matrix format and pruning method using Viterbi encoder structure and Viterbi algorithm. Unlike previous methods, we first consider only limited choices of pruning results, all of which have advantage of significant index compression ratio by our proposed index decompressing structures. One particular pruning result is selected from the limited pruning solution space based on Viterbi algorithm with user-defined branch metric equations that aim to minimize the accuracy degradation. As a result, our proposed sparse matrix, VCM, shows noticeable index storage reduction even compared with the relative index scheme. Fixed index compression ratio and inherently parallel reconstruction scheme allows a wide range of applications, such as SpMM, since sparse matrices can be converted into dense matrices efficiently.

REFERENCES

Arash Ardakani, Carlo Condo, and Warren J. Gross. Sparsely-connected neural networks: towards efficient VLSI implementation of deep neural networks. *International Conference on Learning Representations (ICLR)*, 2017.

Nathan Bell and Michael Garland. Implementing sparse matrix-vector multiplication on throughput-oriented processors. In *Proceedings of the ACM/IEEE Conference on High Performance Computing*, 2009.

Yoshua Bengio and Yann Lecun. Scaling learning algorithms towards AI, 2007.

Yves Chauvin. A back-propagation algorithm with optimal use of hidden units. In *Advances in Nneural Information Processing Systems*, pp. 519–526, 1989.

Wenlin Chen, James Wilson, Stephen Tyree, Kilian Weinberger, and Yixin Chen. Compressing neural networks with the hashing trick. In *International Conference on Machine Learning*, pp. 2285–2294, 2015.

Misha Denil, Babak Shakibi, Laurent Dinh, Nando de Freitas, et al. Predicting parameters in deep learning. In *Advances in Neural Information Processing Systems*, pp. 2148–2156, 2013.

Emily L Denton, Wojciech Zaremba, Joan Bruna, Yann LeCun, and Rob Fergus. Exploiting linear structure within convolutional networks for efficient evaluation. In *Advances in Neural Information Processing Systems*, pp. 1269–1277, 2014.

G. D. Forney. The Viterbi algorithm. *Proc. of the IEEE*, 61:268 – 278, March 1973.

Yunchao Gong, Liu Liu, Ming Yang, and Lubomir Bourdev. Compressing deep convolutional networks using vector quantization. *arXiv preprint arXiv:1412.6115*, 2014.

Song Han, Jeff Pool, John Tran, and William Dally. Learning both weights and connections for efficient neural network. In *Advances in Neural Information Processing Systems (NIPS)*, pp. 1135–1143, 2015.

Song Han, Xingyu Liu, Huizi Mao, Jing Pu, Ardavan Pedram, Mark A Horowitz, and William J Dally. EIE: efficient inference engine on compressed deep neural network. *International Conference on Computer Architecture (ISCA)*, 2016a.

Song Han, Huizi Mao, and William J Dally. Deep compression: Compressing deep neural networks with pruning, trained quantization and Huffman coding. *International Conference on Learning Representations (ICLR)*, 2016b.

Stephen José Hanson and Lorien Y Pratt. Comparing biases for minimal network construction with back-propagation. In *Advances in neural information processing systems*, pp. 177–185, 1989.

Babak Hassibi, David G Stork, and Gregory J Wolff. Optimal brain surgeon and general network pruning. In *Neural Networks, 1993., IEEE International Conference on*, pp. 293–299. IEEE, 1993.

Forrest N Iandola, Song Han, Matthew W Moskewicz, Khalid Ashraf, William J Dally, and Kurt Keutzer. Squeezenet: Alexnet-level accuracy with 50x fewer parameters and¡ 0.5 mb model size. *arXiv preprint arXiv:1602.07360*, 2016.

Diederik P Kingma, Tim Salimans, and Max Welling. Variational dropout and the local reparameterization trick. In *Advances in Neural Information Processing Systems*, pp. 2575–2583, 2015.

Yann LeCun, John S. Denker, and Sara A. Solla. Optimal brain damage. In D. S. Touretzky (ed.), *Advances in Neural Information Processing Systems 2*, pp. 598–605. Morgan-Kaufmann, 1990. URL `http://papers.nips.cc/paper/250-optimal-brain-damage.pdf`.

Dongsoo Lee and Kaushik Roy. Viterbi-based efficient test data compression. *IEEE Trans. on CAD of Integrated Circuits and Systems*, 31(4):610–619, 2012.

Hao Li, Asim Kadav, Igor Durdanovic, Hanan Samet, and Hans Peter Graf. Pruning filters for efficient convnets. In *International Conference on Learning Representations*, 2017.

Hui-Ling Lou. Implementing the Viterbi algorithm. *IEEE Signal Processing Magazine*, 12(5): 42–52, 1995.

Christos Louizos, Karen Ullrich, and Max Welling. Bayesian compression for deep learning. In *Advances in Neural Information Processing Systems 30 (NIPS 2017)*. 2017.

Dmitry Molchanov, Arsenii Ashukha, and Dmitry Vetrov. Variational dropout sparsifies deep neural networks. *arXiv preprint arXiv:1701.05369*, 2017a.

Dmitry Molchanov, Arsenii Ashukha, Dmitry Vetrov, and Kirill Neklyudov. Structured bayesian pruning via log-normal multiplicative noise. In *Advances in Neural Information Processing Systems 30 (NIPS 2017)*. 2017b.

Pavlo Molchanov, Stephen Tyree, Tero Karras, Timo Aila, and Jan Kautz. Pruning convolutional neural networks for resource efficient inference. In *International Conference on Learning Representations*, 2017c.

Robert H. Morelos-Zaragoza. *The art of error correcting coding*. John Wiley & Sons, 2nd edition, 2006.

Jongsoo Park, Sheng Li, Wei Wen, Ping Tak Peter Tang, Hai Li, Yiran Chen, and Pradeep Dubey. Faster cnns with direct sparse convolutions and guided pruning. In *International Conference on Learning Representations*, 2017.

Claude E. Shannon. A mathematical theory of communication. *Bell system technical journal*, 27, 1948.

Andrew J. Viterbi. An intuitive justification and a simplified implementation of the MAP decoder for convolutional codes. *IEEE Journal on Selected Areas in Communications*, 16(2):260–264, 1998.

Jiecao Yu, Andrew Lukefahr, David Palframan, Ganesh Dasika, Reetuparna Das, and Scott Mahlke. Scalpel: Customizing DNN pruning to the underlying hardware parallelism. In *Proceedings of the 44th Annual International Symposium on Computer Architecture*, pp. 548–560, 2017.

Xinmiao Zhang. *VLSI architectures for modern error-correcting codes*. CRC Press, 1st edition, 2015.

A APPENDIX

A.1 VD DESIGN METHODOLOGY

In Figure 1, each VD output is generated by a series of 2-input XOR gates which accept input bits from either input of VD or FF outputs. Hence, there are 6 possible input candidates in total for XOR gates and each candidate is called an XOR tap. Using input as x^0 and n^{th} FF output (from the left) as x^n, $out2$ can be represented as a polynomial of $x^5 + x^3 + x$ or equivalently, a vector $[101010]$. By combining such vectors of all 4 outputs, we can construct a VD Matrix to represent VD (of Figure 1) in a compact manner as the following:

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}. \tag{3}$$

The number of 1s (i.e., the XOR taps) is 3 in every row of VD Matrix and the Hamming distance[7] of any pair of two rows is 4. Increasing the number of XOR taps and minimum Hamming distance in VD Matrix improves the randomness of VD outputs (Lee & Roy, 2012). Given the number of XOR taps, the minimum Hamming distance, and the number of VD outputs, VD Matrix can be generated as Algorithm 1.

Algorithm 1: VD Matrix generation

```
input  : number of outputs N, number of XOR taps t,
         minimum Hamming distance h
output: VD Matrix S
i = 0, S = φ ;
while (number of vectors of S) < N do
    i++ ;
    a = binary representation of i ;
    if (number of 1s' of a) == t then
        isValid = true ;
        for (j=0; j <number of vectors of S; j++) do
            d = Hamming distance between S(j) and a;
            if d < h then
                isValid = false ;
            end
        end
        if (isValid == true) then
            put a in S;
        end
    end
end
```

Table 4 shows the minimum number of FFs generated by Algorithm 1, given the number of XOR taps, the minimum Hamming distance, and the number of VD outputs. Note that the number of VD outputs increases exponentially, while the number of FFs increases linearly. Thus, hardware resource implementing VD is not expensive even with high index compression ratio. Note that the number of XOR taps for pruning should always be an even number (otherwise, Viterbi algorithm chooses a trivial input sequence of all '1's to make all the weights survived to maximize the path metric.

In Figure 10, as more bits are consumed for two inputs of comparators (NUM_c bits) in order to enhance controllable target pruning rates (i.e., sparsity in *R* outputs) resolution, the number of VD outputs (NUM_v) needs to be increased.

[7]Hamming distance between two vectors is the number of positions where two values are different

Table 4: Various configurations of the number of XOR taps, the minimum Hamming distance, the number of FFs, and the number of VD outputs.

# of VD outputs	# of taps	Hamming	# of FFs
8	5	2	6
	5	4	8
	5	6	10
	4	6	12
32	4	2	6
	5	4	10
	5	6	15
	6	8	18
128	6	2	9
	7	4	13
	8	6	17
	6	6	19

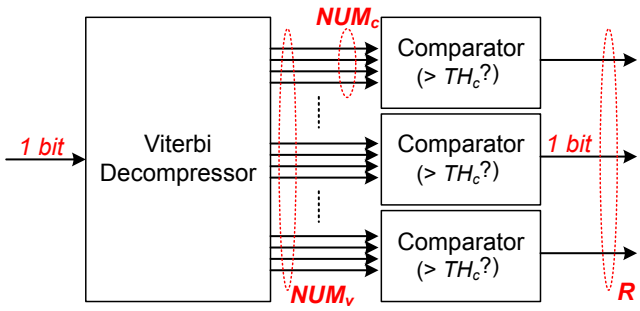


Figure 10: Index decompressing using VD and comparators to control the sparsity. A comparator threshold value TH_c can have a range of 0 to $2^{NUM_c} - 1$.

A.2 Dummy Inputs and Skip States

Accuracy degradation can be reduced by increasing the number of states to be explored in trellis diagram, primarily because of the increased search space dimension. In Figure 11, inserting dummy inputs as an initial input sequence increases the number of available states from which we start index encoding with weight parameters (i.e., the number of available states increases from 1 to 4 with 2 dummy input bits). The maximum size of dummy inputs is the number of FFs and all the dummy paths exhibit the same preference with the same branch metrics. The size of dummy inputs is negligible if the number of FFs in VD is much smaller than the number of weight matrix elements divided by R.

Besides dummy input sequence, skip state, which is defined as the number of times the time index in the trellis diagram is skipped, can also lead to reduced accuracy degradation. Similar to the idea of the dummy input, skip state increases the number of available states in the trellis diagram search. Figure 12 describes a case of (Skip State=1) where at every even-number time index, the output of VD is discarded. If the branch metrics are set to 0, following the Eq. (1), Viterbi algorithm will select the paths that lead to increased number of states with a higher path metric value. In the case of magnitude-based pruning, this implies that a larger magnitude weight has a higher chance to be preserved. In case of k skip states, VD outputs are discarded for k consecutive time indices. The entire length of time index is increased by $(k + 1)$ times and the index compression ratio is reduced by $(k + 1)$ times.

A.3 An Example of Selecting Functions for the Branch Metric

Suppose that the normalized magnitudes of parameters before pruning are given as $\{0.1, 0.2, 0.6, 1.0\}$ at a certain time index and TH_p is 0.3. We calculate the branch met-

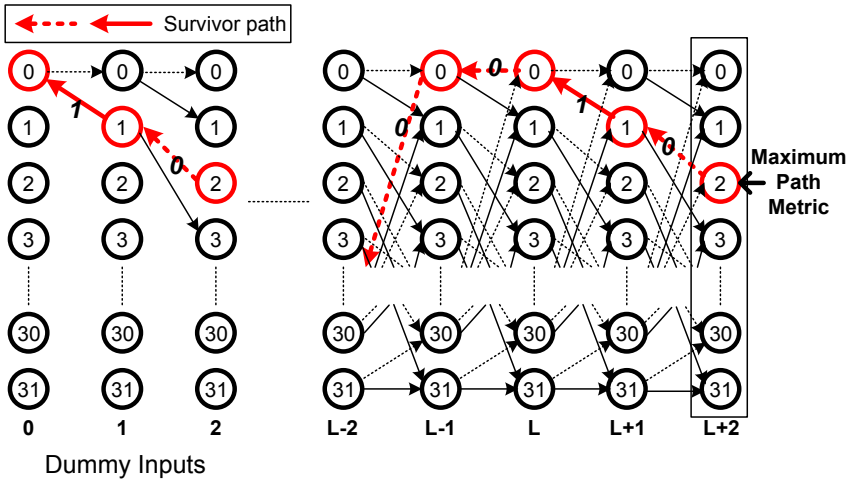


Figure 11: Backward survivor path finding procedure with 2 dummy input bits to increase the number of reachable states from 1 to 4.

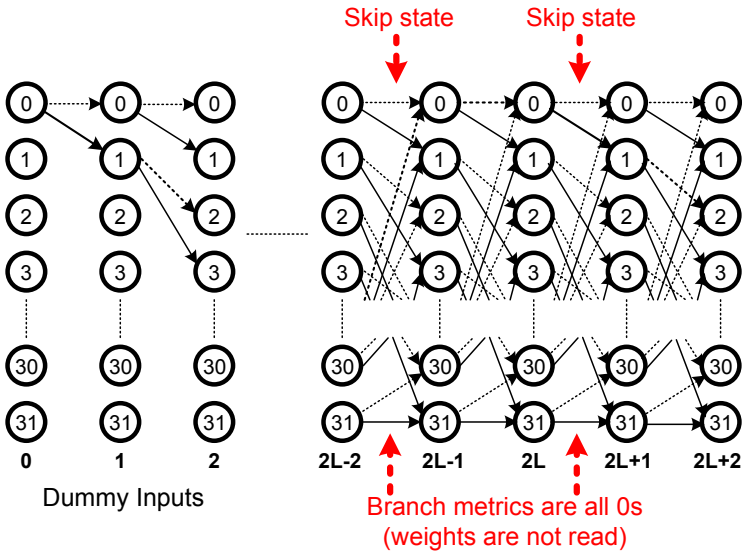


Figure 12: Trellis diagram exploration with (skip state=1). For every even-number time index after dummy input sequence, branch metrics are 0 while the path metrics are still updated. The entire time index length is increased by (skip state + 1) times.

rics with two different sets of comparator outputs $V1 = \{0, 1, 1, 0\}$ and $V2 = \{1, 0, 0, 1\}$ using Eq. (2). In the case of $\tanh(x)$, the branch metric with $V1$ is given as $S_2 \times (-\tanh((0.1 - 0.3)/S_1) + \tanh((0.2 - 0.3)/S_1) + \tanh((0.6 - 0.3)/S_1) - \tanh((1.0 - 0.3)/S_1)) = 10^4 \times (-(-0.04) + (-0.02) + (0.06) - (0.20)) = -1200$, while the branch metric with $V2$ is $10^4 \times ((-0.04) - (-0.02) - (0.06) + (0.20)) = 1200$. On the other hand, if we use x instead of $\tanh(x)$, then the branch metrics with $V1$ and $V2$ are -600 and 600, respectively. Compared with x, $\tanh$ assigns greater importance to parameters with higher magnitude. Note that in this example, $\tanh$ has been chosen in the context of the magnitude-based pruning method, and the branch metric equations can be made differently depending on the underlying pruning principle.

Table 5: Weight size and VD parameters per each layer for Viterbi-based pruning.

Network	Weight size per layer		VD parameters			
	Original	Pruning neurons	NUM_v	NUM_c	R	Skip state
LeNet-300-100	784×300	465×96	40	5	8	1
	300×100	96×100	20	5	4	1
	100×10	100×10	10	5	2	1
LeNet-5-Caffe	$5 \times 5 \times 1 \times 20$	$5 \times 5 \times 1 \times 20$	10	5	2	1
	$5 \times 5 \times 20 \times 50$	$5 \times 5 \times 20 \times 25$	30	5	6	1
	800×500	400×69	40	5	8	1
	500×10	69×10	10	5	2	1

Table 6: Sparse matrix comparison with LeNet-300-100 on MNIST using Variational Dropout-based pruning (Molchanov et al., 2017a) and our proposed Viterbi-based pruning. We assume that non-zero values and CSR index use 16 bits.

Layer	Weight Size	Variational Dropout		Additional Viterbi-Pruning		Matrix Size Reduction
		Pruning Rate	Sparse Matrix (CSR) Size	Pruning Rate	Sparse Matrix (VCM) Size	
FC1	89.28K	95.7%	7.67KB	94.8%	5.89KB	23.2%
FC2	19.2K	94.6%	2.23KB	94.0%	1.71KB	23.6%
FC3	20.0K	78.0%	0.88KB	68.0%	0.75KB	15.2%
Total	128.48K	**95.2%**	10.78KB	**94.2%**	8.34KB	**22.6%**
Test Error		1.96%		1.96%		

Table 7: Sparse matrix comparison with LeNet-5-Caffe on MNIST using Variational Dropout-based pruning (Molchanov et al., 2017a) and our proposed Viterbi-based pruning. We assume that non-zero values and CSR index use 16 bits.

Layer	Weight Size	Variational Dropout		Additional Viterbi-Pruning		Matrix Size Reduction
		Pruning Rate	Sparse Matrix (CSR) Size	Pruning Rate	Sparse Matrix (VCM) Size	
Conv1	1.0K	69.6%	0.63KB	69.6%	0.36KB	43.6%
Conv2	25.0K	96.4%	1.80KB	95.8%	1.52KB	15.4%
FC1	55.2K	97.5%	2.83KB	97.0%	2.49KB	12.2%
FC2	1.38K	66.1%	0.94KB	66.7%	0.53KB	43.0%
Total	82.58K	**96.7%**	6.20KB	**96.1%**	4.90KB	**21.0%**
Test Error		0.96%		0.98%		

A.4 Viterbi Algorithm on Highly Sparse DNNs

We test our proposed VCM and Viterbi-based pruning method with highly sparse DNNs by combining Variational dropout-based pruning method (Molchanov et al., 2017a). LeNet-300-100 and LeNet-5-Caffe architectures[8] on MNIST dataset are used in the test. Viterbi-based pruning based on the Eq. (2) is additionally performed to obtain VCM data, after pruning weights and neurons using the Variational dropout method (Molchanov et al., 2017a) as shown in Table 5. The main VD parameters for each layer are also presented in Table 5. The minimum Hamming distance and XOR taps are 4. Table 6 and 7 describe the pruning rate and the memory footprint comparisons for each layer. Storage requirement for VCM is reduced by **22.6 %** and **21.0 %**, respectively, compared with CSR, due to the reduction in index size (**63.0 %** and **53.6 %**, respectively). The almost same classification accuracy is achieved with a little retraining time in both LeNet-300-100 and LeNet-5-Caffe. Note that this result demonstrates that the proposed Viterbi-based techniques can be combined with existing pruning methods to extract VCM formats even without modifying Eq. (2).

[8] https://github.com/ars-ashuha/variational-dropout-sparsifies-dnn