Unsupervised Causal Generative Understanding of Images

Abstract

We present a novel framework for unsupervised object-centric 3D scene under-standing that generalizes robustly to out-of-distribution images. To achieve this, we design a causal generative model reflecting the physical process by which an image is produced, when a camera captures a scene containing multiple objects. This model is trained to reconstruct multi-view images via a latent representation describing the shapes, colours and positions of the 3D objects they show. It ex-plicitly represents object instances as separate neural radiance fields, placed into a 3D scene. We then propose an inference algorithm that can infer this latent representation given a single out-of-distribution image as input – even when it shows an unseen combination of components, unseen spatial compositions or a radically new viewpoint. We conduct extensive experiments applying our approach to test datasets that have zero probability under the training distribution. These show that it accurately reconstructs a scene's geometry, segments objects and infers their positions, despite not receiving any supervision. Our approach significantly out-performs baselines that do not capture the true causal image generation process.

1 Introduction

Most machine learning approaches – both supervised and unsupervised – make the assumption that at test time, they are applied to data drawn from the same distribution as seen during training [12, 55, 17, 26]. This means the powerful generalization guarantees of statistical learning theory apply [101]. Indeed, learning-based methods for computer vision do *not* generalize to observations that are statistically different from those seen in their training set – recent works have demonstrated this for images taken from unfamiliar viewpoints [3, 38, 39, 7, 53], shifted by few pixels [6], and showing scenes with an unseen composition of objects [9, 86, 91, 29, 30]. It has been suggested that this is because they learn spurious correlations – or *shortcuts* [29] – to achieve low training loss, but which do not capture the true *causal* relationships that remain universally valid. However, in the real world, we do commonly encounter observations that are outside of the distribution from which we learnt – indeed, there are exponentially many possible combinations of objects, viewpoints and environments, so we cannot hope to gather data showing every one.

In this work, we consider the task of transforming a single observed image into a detailed rep-resentation of the scene it depicts, providing explicit information about its 3D structure such as object locations, shapes and appearances. We focus on the challenging – yet practically important – setting where at test time, we see images depicting scenes that have zero probability in the training distribution (Fig. 1). We adopt an *unsupervised* approach to learning, thus avoiding the need for expensive manual annotation of object masks, 3D positions, and similar – we require only a dataset of posed multi-view images.

Our approach is to build a generative model (Sec. 3) jointly over image pixels and the 3D world they depict, that can be robustly inverted to infer the latent factors that gave rise to an input image

Figure 1: Our generative model is trained to reconstruct multi-view images drawn from its training distribution (red), in terms of a latent object-centric 3D scene representation. After training, it can sample plausible scenes containing multiple objects ('Generation', top center). At test time, it takes as input single images drawn from different distributions (e.g. with novel compositions of objects), but is still able to infer segmentations, depth-maps, and novel views.

[90, 10, 103, 58, 18]. If such a model is to support generalization to out-of-distribution (OOD) data, it should reflect the underlying causal model of the environment from which the data arose [81, 90]. In practice this means its graphical model structure (i.e. conditional independencies) should match those present in the environment, with conditional distributions in the model corresponding (as far as possible) to *independent mechanisms* [89, 44] in the world. In this case, when certain mechanisms or physical processes in the environment change, we still expect much of our model to remain applicable; this has been called *sparse mechanism shift* [90]. For example, even if objects are no longer placed according to the same patterns as at training time, we still expect the appearances of object types to remain consistent – even if chairs are no longer placed near tables, both should still have four legs.

However, in general it is impossible to recover such a causal model purely from observational data [81, 90]. We therefore embed in our model universally-applicable physical knowledge that the world is composed of 3D objects of different shapes, which may appear at different locations, and which are imaged by a camera subject to the laws of 3D geometry and perspective projection. Each object is associated with separate latent variables describing its position and appearance; the use of separate latents ensures disentanglement, so the same object can be represented invariantly in different locations. The final image is synthesized by volumetric rendering of the latent 3D shapes placed in a 3D scene space according to their latent positions. This structure means the model is *compositional*, in the sense that having learnt about one type of object in one context or environment, it can also represent and perform inference about a similar object even in different contexts.

We perform inference on OOD data by *intervening* on our model [81, 83] – i.e. replacing certain conditional distributions (or mechanisms) that are no longer appropriate – then doing posterior inference over the updated model. For example, if object positions no longer follow the distribution seen during training, we replace the learnt prior on locations with a uniform prior (leaving unchanged the conditional distribution on object appearances), then perform inference over this updated model. We must also ensure that the inference method itself supports OOD generalization. This means we cannot employ the common approach of amortized variational inference [50, 85], as an encoder network is in general not robust to changes of distribution [71, 29], and in our setting we do not have a dataset of images from the test-time distribution to learn from. Instead, we develop a novel Markov chain Monte-Carlo (MCMC) inference scheme, that finds posterior samples for a given test image without any encoder network.

Targeting OOD generalization imposes several technical constraints on our model compared with other recent compositional generative models of 3D scenes (e.g. [77, 35, 97]), none of which address this setting. Firstly, we cannot include a learnt neural renderer (mapping features to pixels in image space) [77, 15, 74] in the generative model, as these are not guaranteed to generalise to novel camera viewpoints nor compositions of objects. Similarly, spatial mixture models [26, 73, 97] cannot perform

inference on scenes with objects at unseen positions because their representation of object appearance is entangled with that of position [71, 63].

To evaluate our approach (Sec. 4), we create challenging test datasets that have zero probability under the corresponding training distribution yet share aspects of its structure, such as featuring the same object classes composed in different arrangements. We show that our model can generalize to unseen numbers of objects, unseen compositions, and radically new camera viewpoints – all significantly better than existing works. It successfully localizes objects in the 3D scene, reconstructs their 3D shapes, and hence predicts depth-maps and instance segmentation masks.

To summarise, our core contribution is **the first unsupervised framework for inference of explicit object-centric 3D scene representations, that generalizes to out-of-distribution scenes**. Our secondary contributions are: (i) experimental evidence that current methods do not generalize to images with a radically new camera viewpoint, a different number of objects, or an unseen composition of objects; (ii) a novel generative model of 3D scenes based on multi-object radiance fields with explicit object positions and volumetric rendering; and (iii) a novel MCMC inference scheme exploiting the structure of our model, that allows inferring 3D scenes from a single out-of-distribution image.

2 Related Work

Recently numerous works have observed that modern learning-based computer vision methods fail to generalise on OOD data, and have analysed this [38, 3, 39, 53, 6, 9, 86, 91, 29, 30, 72, 69] as well as constructing various benchmarks [58, 7, 32, 48, 109, 105]. Others have tried to improve OOD generalisation, the most relevant by aiming to learn relations that are invariant across datasets [5, 94, 2, 1, 80, 56, 93, 61]. These methods assume that multiple differently-distributed datasets are available during training, and only address the supervised setting (mainly image classification). Like the latter, our focus is on enabling OOD generalisation, but in contrast to them, we address unsupervised object-centric generative modelling. Moreover, we do not rely on multiple *training* datasets to discover causal factors, but instead directly incorporate universal knowledge such as separation of the world into objects.

Our work is also connected to the *vision as inverse graphics* paradigm [31, 8, 87]. In this setting, it is assumed that we have access to (maybe parametric) 3D models of objects, and wish to find suitable pose and other parameters to explain an input image [65, 57, 41, 87, 88, 40]. Like our work, these typically use a test-time optimisation; unlike ours, they do not attempt to learn priors on object layout nor shapes – instead assuming these are known *a priori*.

Neural implicit scene representations [98] aim to a learn continuous representation of a 3D scene from 2D images using neural rendering, either by explicit volumetric rendering [70, 67, 106, 108, 78] or with CNN post-processing of rendered features [96, 95]. These initial works fitted individual scenes without learning characteristics common across them, therefore requiring many images as input. This was addressed by sharing models across different scenes [110, 99, 59, 28, 82, 76, 43], allowing inference of novel views from one or few images. All these methods model a scene as a single monolithic entity, without decomposing it into individual objects – meaning manipulating the scene, e.g. moving single objects – is not possible. In contrast, [79] divides a scene into objects, but requires detailed manual annotations to do so, and does not support inference from few images; [22] relies on weaker supervision in the form of ground-truth object masks. [111, 97] discover such a decomposition automatically (though with depth supervision for [97]), however they rely on spatial mixtures to assign each point in space to an object, without any explicit, controllable representation of object positions. This entanglement of latent position and appearance means they are not guaranteed to generalise to OOD combinations of position and appearance [63]. Finally, [33] supports composition of multi-object scenes using neural scattering functions – but these must be learnt from multiple views of single objects (a form of weak supervision), with no probabilistic model over appearances nor layouts.

Other approaches have extended neural rendering to the generative setting [92, 77, 74, 75, 54, 16, 21, 20], allowing sampling objects or scenes *a priori*. However, these do not allow us to perform inference in the OOD setting, as they have components that do not reflect the causal, compositional structure of the world – they either include a monolithic latent space lacking object-centric representations, or a learnt neural renderer that will not generalise to OOD viewpoints nor compositions. In contrast, there are object-centric generative models that can sample plausible images and perform inference – but

only in 2D, without reasoning over a latent 3D scene representation, and therefore without supporting 3D tasks such as reconstruction depth prediction. Some use a full-image spatial mixture model [26, 73, 51, 24, 25, 47] or alpha stacking [100, 102]; others model images as composed of smaller patches or sprites [27, 45, 4], learning both the distribution of appearances and compositions/layouts; others use compositional energy-based models [23, 62]. A 3D extension of these latter is proposed by [35], using a voxel representation, but this requires videos for inference at test time, and does not support OOD generalisation. In the supplementary, we provide a table comparing the capabilities of our method to closely related works.

3 Method

Our goal is to infer an explicit object-centric representation of a 3D scene from one or multiple images, even when they show scenes lying outside the distribution observed during training. We achieve this by learning a compositional generative causal model jointly over multi-view images and the scenes they depict, as described in Sec. 3.1. For training (Sec. 3.2) we use only posed multi-view images $(\mathbf{x}_1, \mathbf{v}_1), ..., (\mathbf{x}_N, \mathbf{v}_N)$ drawn from a training distribution $P_X^{train}(\mathbf{x}, \mathbf{v})$ without any annotations such as depth-maps, bounding-boxes or segmentation masks. At test time, input images are drawn from a distribution $P_X^{OOD}(\mathbf{x}, \mathbf{v})$ with support disjoint from the training distribution. Performing inference over the generative model using our proposed framework (Sec. 3.3) yields an explicit object-centric representation, including 3D object shapes, positions and appearances.

3.1 Compositional Generative Causal Model

We model a multi-view set of N images $\{\mathbf{x}_1 \ldots \mathbf{x}_N\}$ as caused by a single 3D scene $\mathcal{S}$ being rendered from viewpoints $\{\mathbf{v}_1 \ldots \mathbf{v}_N\}$, by a function $C(\mathcal{S}, \mathbf{v})$. We now describe this scene representation, then the generative process by which it is sampled.

Scene representation. The scene $\mathcal{S} = \left(\mathbf{s}_{bg}, \{(\mathbf{s}_i^{app}, \mathbf{s}_i^{pos})\}_{i=1}^{O} \right)$ is composed of a 3D background component $\mathbf{s}_{bg}$ describing the background's shape and color, and 3D objects indexed $i = 1 \ldots O$ with shape and color described by $\mathbf{s}_i^{app}$ and explicit 3D positions $\mathbf{s}_i^{pos}$. Each $\mathbf{s}_i^{app}$ explicitly represents the 3D appearance of an object as a neural radiance field (NeRF) [70], placed in a *canonical space* (e.g. with the object centered at the origin). The positions $\mathbf{s}_i^{pos}$ specify where the objects are placed in a global 3D *scene space*. In contrast to most prior work that parametrized 3D object positions as coordinates, we represent them by 1-hot vectors choosing from a set of plausible candidate locations to use as the center position of the object. As we explain in Sec. 3.3, this makes gradient-based optimization easier.

Generative process for $\mathcal{S}$. We first sample a high-level latent scene embedding $\mathbf{z}^g \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$, that will model correlations between objects and learn the typical composition of a scene [45, 4]. The individual object appearances are specified by Gaussian latent variables $\mathbf{z}_i^{shape}$ and $\mathbf{z}_i^{col}$ that respectively encode the shape and color of the ith object; they are conditioned on $\mathbf{z}^g$, with mean and log-variance given by a fully-connected network $\zeta_\theta(\mathbf{z}^g)$ with weights θ.[1] The position $\mathbf{z}_i^{pos}$ is specified by a categorical variable, with logits given by $\xi_\theta(\mathbf{z}^g)$. We similarly introduce latents $\mathbf{z}_{bg}^{shape}$ and $\mathbf{z}_{bg}^{col}$ to encode the shape and color of the background. For brevity, we will write $\mathbf{z}^s = \{\mathbf{z}_{bg}^{shape}, \mathbf{z}_{bg}^{col}, \mathbf{z}_{1...O}^{shape}, \mathbf{z}_{1...O}^{col}, \mathbf{z}_{1...O}^{pos}\}$.

The latents $\mathbf{z}^s$ are mapped to the scene $\mathcal{S}$ by a function S_θ. This sets $\mathbf{s}_i^{pos}$ equal to $\mathbf{z}_i^{pos}$, and derives the object NeRF representations $\mathbf{s}_i^{app}$ from $\mathbf{z}_i^{shape}$ and $\mathbf{z}_i^{col}$ as described in the next paragraph. The probability of an image $\mathbf{x}_n$ given its camera viewpoint $\mathbf{v}_n$ is then

$$p_\theta(\mathbf{x}_n \,|\, \mathbf{v}_n) = \iint f_\mathcal{N}(\mathbf{x}_n;\, C(S_\theta(\mathbf{z}^s),\, \mathbf{v}_n),\, \sigma^2)\, p_\theta(\mathbf{z}^s \,|\, \mathbf{z}^g)\, p_\theta(\mathbf{z}^g)\, \mathrm{d}\mathbf{z}^s \mathrm{d}\mathbf{z}^g \qquad (1)$$

where $C(\mathcal{S}, \mathbf{v})$ renders the scene described by $\mathcal{S}$ from viewpoint $\mathbf{v}$, and $f_\mathcal{N}$ represents a factored Gaussian likelihood over the $H \times W \times 3$ pixels of the image, with fixed standard deviation σ. The probability of a composition $\mathbf{z}^s$ of objects and background in a scene is given by

$$p_\theta(\mathbf{z}^s \,|\, \mathbf{z}^g) = p_\theta(\mathbf{z}_{bg}^{shape}, \mathbf{z}_{bg}^{col} \,|\, \mathbf{z}^g) \prod_{i=1}^{O} p_\theta(\mathbf{z}_i^{shape}, \mathbf{z}_i^{col}, \mathbf{z}_i^{pos} \,|\, \mathbf{z}^g) \qquad (2)$$

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[1]See the supplementary for all network architectures

173 **Rendering the scene $\mathcal{S}$.** The rendering process $C(\mathcal{S}, \mathbf{v})$ outputs an image $\mathbf{x}$ for a camera viewpoint
174 $\mathbf{v}$, given our explicit compositional representation of a scene $\mathcal{S}$. Recall $\mathcal{S}$ contains a 3D background
175 component $\mathbf{s}_{bg}$ and a set of object components $\{\mathbf{s}_i^{app}, \mathbf{s}_i^{pos}\}_{i=1}^{O}$; for brevity we will identify the
176 background as component $i = 0$, with $\mathbf{s}_0^{pos}$ fixed to the origin. We extend multi-component neural
177 radiance fields (NeRF) [70, 67] to support explicit placement of objects in the 3D scene according to
178 the position variables $\mathbf{s}_i^{pos}$. Specifically, the latent codes $\mathbf{z}_i^{shape}$ and $\mathbf{z}_i^{col}$ for the ith object parametrize
179 a learnt function $f_\theta^*(\mathbf{q}^*; \mathbf{z}_i^{shape}, \mathbf{z}_i^{col})$, that maps points $\mathbf{q}^*$ in the canonical space of the object to a
180 color $c \in [0, 1]^3$ and density $\sigma \in \mathbb{R}^+$. We place each object at its 3D position $\mathbf{s}_i^{pos}$ by convolving its
181 density and color functions with a one-hot location indicator:

$$f_i(\mathbf{q}) = \iiint_{\mathbf{q}^*} s_i^{pos}(\mathbf{q}^*) \cdot f_\theta^*\big(\mathbf{q} - \mathbf{q}^*; \mathbf{z}_i^{shape}, \mathbf{z}_i^{col}\big) \, \mathrm{d}\mathbf{q}^* \equiv \big(c_i(\mathbf{q}), \sigma_i(\mathbf{q})\big) \tag{3}$$

182 where $\mathbf{q}$ is a position in scene space, and $s_i^{pos}(\mathbf{q})$ is an indicator function with a unit impulse if point
183 $\mathbf{q}$ is chosen as the center position of the object by the 1-hot indicator $\mathbf{s}_i^{pos}$. Similar to [111], we
184 divide the scene space into foreground and background regions, and only render the corresponding
185 components in each.

186 Given the placed object densities σ_i and colors c_i, we calculate the color of each pixel in the image $\mathbf{x}$
187 by casting a ray $\mathbf{r}(t) = \mathbf{x_0} + t\mathbf{d} \in \mathbb{R}^3$ from the pixel in direction $\mathbf{d}$ through a camera at position $\mathbf{x_0}$,
188 summing the contributions from different objects [67, 68]:

$$C(\mathcal{S}, \mathbf{v})[\mathbf{r}] = \int_0^\infty T(t) \sum_{i=0}^{O} \sigma_i(\mathbf{r}(t)) \cdot \mathbf{c}_i(\mathbf{r}(t)) \mathrm{d}t, \quad \text{where} \quad T(t) = \exp\left(-\int_0^t \sum_{i=0}^{O} \sigma_i(\mathbf{r}(t')) dt'\right) \tag{4}$$

189 **Continuous relaxation of object placement.** To allow gradient-based training and inference, our
190 generative process must be differentiable. We therefore relax the categorical position variable to a
191 Gumbel-Softmax [42, 66]. The object placement operation Eq. 3 can then be efficiently implemented
192 as a convolution operation in the Fourier domain – exploiting the Fourier transform's *sifting property*
193 [14, 4]. This approach ensures we always receive non-zero gradients of the image with respect to
194 every possible object position, easing optimisation. This is in contrast to models based on spatial
195 transformers [77, 107, 60], which can get stuck in local minima if the model has a poor initial
196 prediction, as the gradient of pixels wrt position is zero if the predicted and true positions do not
197 overlap.

3.2 Training

199 We train our generative model from a dataset of images containing K views for each of T scenes. The
200 model includes three learnable components, with parameters θ: (i) $f_\theta^*(\mathbf{q}; \mathbf{z}^{shape}, \mathbf{z}^{col})$ that represents
201 a 3D object as a function from position to color and density conditioned on the object appearance
202 embedding; (ii) $f_\theta^{bg}(\mathbf{q}; \mathbf{z}_{bg}^{shape}, \mathbf{z}_{bg}^{color})$ that similarly represents the 3D background; (iii) ζ_θ and ξ_θ,
203 that map the global scene latent $\mathbf{z}^g$ to parameters of the object and background latents $\mathbf{z}^s$. We train
204 these components using autoencoding variational Bayes [50, 85]. The posteriors over Gaussian latent
205 variables are all diagonal Gaussians (parametrized by mean and log-variance), whilst for positions the
206 posterior is Gumbel-Softmax (parametrized by logits). We use two encoder networks to parametrize
207 these variational posteriors. $\text{enc}_\phi^s(\{\mathbf{x}_n, \mathbf{v}_n\}_{n=1}^M)$ parametrizes $q(\mathbf{z}^s|\{\mathbf{x}_n, \mathbf{v}_n\}_{n=1}^M)$; for efficiency, we
208 pass it only a subset of $M < K$ images. It encodes each observed image and its viewpoint $(\mathbf{x}_n, \mathbf{v}_n)$
209 independently then sums the results (as in [28]) before outputting the posterior parameters; this
210 ensures the encoder is invariant to the ordering of images. $\text{enc}_\phi^g(\mathbf{z}^s)$ parametrizes $q(\mathbf{z}^g|\mathbf{z}^s)$, and takes
211 the lower-level latent code $\mathbf{z}^s$ as input.

212 For stable training, we adopt a two-stage approach. We first train the model to reconstruct $\mathbf{x}_{1\ldots K}$,
213 via the object-level latent space $\mathbf{z}^s$, ignoring the scene-level latent $\mathbf{z}^g$ and placing standard Gaussian
214 priors on $\mathbf{z}^s$ (c.f. [45]), i.e. maximizing the following evidence lower-bound (ELBO):

$$\mathcal{L}^s = \mathbb{E}_{q_\phi(\mathbf{z}^s|\{\mathbf{x}_n, \mathbf{v}_n\}_{n=1}^M)} \left[\sum_{n=1}^{K} \log f_\mathcal{N}(\mathbf{x}_n; C(S_\theta(\mathbf{z}^s), \mathbf{v}_n), \sigma^2) \right] - D_{\text{KL}}\left[q_\phi(\mathbf{z}^s \mid \{\mathbf{x}_n, \mathbf{v}_n\}_{n=1}^M) \parallel \mathcal{N}(\mathbf{0}, \mathbf{1}) \right] \tag{5}$$

215 After this has converged, we learn the scene-level latent space by maximizing

$$\mathcal{L}^g = \mathbb{E}_{q_\phi(\mathbf{z}^g|\mathbf{z}^s)} \left[\mathbb{E}_{q_\phi(\mathbf{z}^s|\{\mathbf{x}_n, \mathbf{v}_n\}_{n=1}^M)} \log p_\theta(\mathbf{z}^s \mid \mathbf{z}^g) \right] - D_{\text{KL}}\left[q_\phi(\mathbf{z}^g \mid \mathbf{z}^s) \parallel p_\theta(\mathbf{z}^g) \right] \tag{6}$$

	GQN						ARROW					
	per-image				per-scene		per-image				per-scene	
	PSNR ↑	D.MRE ↓	ARI ↑	mSC ↑	PSNR ↑	D.MRE ↓	PSNR ↑	D.MRE ↓	ARI ↑	mSC ↑	PSNR ↑	D.MRE ↓
Test												
Ours	24.1	0.031	0.81	**0.88**	20.8	0.058	27.1	**0.100**	**0.71**	**0.82**	26.8	**0.107**
Slot Att.	30.5	–	**0.94**	0.67	–	–	28.3	–	0.48	0.17	–	–
uORF*	27.0	0.027	0.74	0.59	24.2	0.049	**35.1**	0.176	0.64	0.44	**33.8**	0.202
NeRF-VAE*	**32.0**	**0.016**	–	–	**27.8**	**0.033**	25.3	0.991	–	–	25.3	0.991
OOD												
Ours	**21.8**	**0.034**	**0.68**	**0.89**	**18.3**	**0.069**	26.7	0.139	**0.57**	**0.81**	26.2	0.137
Slot Att.	20.3	–	0.66	0.56	–	–	22.8	–	0.26	0.14	–	–
uORF*	14.7	0.287	0.45	0.45	14.1	0.308	22.7	**0.132**	0.38	0.41	22.6	**0.131**
NeRF-VAE*	15.9	0.271	–	–	14.9	0.301	19.4	0.992	–	–	19.4	0.992

Table 1: Quantitative results on discriminative tasks, comparing performance for different methods on an in-distribution test set and OOD data. Dashes indicate the method does not support the task. Best results are shown in **bold**.

We use Adam for optimization [49], β-weighting of KL terms [37], and approximate each of the above expectations by a single sample. We also further approximate $\mathcal{L}^s$ by rendering only a random subset of pixels per minibatch. More implementation details are in the supplementary material.

3.3 Inference for out-of-distribution (OOD) images

At test time, we assume images are sampled from a distribution disjoint from the training distribution. This means that directly performing posterior inference under our model (which has learnt the training distribution, and ideally assigns zero probability to OOD test images) is not sound. We therefore make appropriate *interventions* on our model [81], taking advantage of its causal nature. For example, when the distribution of object arrangements is different at test time, we replace the learnt prior $p_\theta(\mathbf{z}^s)$ on object arrangements with an uninformative uniform prior. Moreover, the variational encoder networks enc_ϕ^s and enc_ϕ^g used during training are not suitable for use at test time, due to domain shift in their inputs. Therefore, our framework instead directly samples the posterior distribution of latent variables given an observed image, using Markov chain Monte-Carlo (MCMC) inference [52].[2]

Our novel MCMC scheme alternates Langevin dynamics (LD) [11, 104] and Metropolis-Hastings (MH) [34] steps, to infer the latent scene variables $(\mathbf{z}^s, \mathbf{z}^g)$ from a *single* observed image $\mathbf{x}^*$ with viewpoint $\mathbf{v}^*$. The MH steps encourage the Markov chain to make large jumps between modes of the posterior, while the LD steps generate high-probability samples with less exploration. Each LD step ascends the gradient of

$$\log f_\mathcal{N}(\mathbf{x}^*; C(S_\theta(\mathbf{z}^s), \mathbf{v}^*), \sigma^2) + \log p_\theta(\mathbf{z}^s \mid \mathbf{z}^g) + \log p(\mathbf{z}^g) \propto \log p(\mathbf{z}^g, \mathbf{z}^s \mid \mathbf{x}^*, \mathbf{v}^*) \quad (7)$$

Each MH step first picks an object slot i uniformly at random. It then samples a new latents for that object from a proposal distribution $\tilde{p}(\mathbf{z}_i^{shape}, \mathbf{z}_i^{col})$, and decides whether to accept this transition according to the usual Metropolis-Hastings acceptance criterion [34]. The proposal distribution $\tilde{p}$ approximates $\frac{1}{J} \sum_{i=1}^J p(\mathbf{z}_i^{shape}, \mathbf{z}_i^{col} \mid \mathbf{z}_g)$ using a Gaussian mixture model fitted by expectation-maximisation [19]; it thus captures the distribution of object latent codes while disregarding the ordering of object indices. Note that the compositionality of our model increases sampling efficiency of the chain, as we can consider proposals for each object independently – in contrast to a monolithic latent embedding, which would require accepting or rejecting global modifications to the entire scene. In practice we perform ten LD steps, followed by a single MH step, and repeat this overall process until convergence. When evaluating results, we take the posterior mode, i.e. the single sample from each chain that had maximum probability.

4 Experiments

We conduct experiments on two synthetic datasets, using our model and three baselines. We first evaluate performance in the standard setting where the test-set distribution matches the training

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[2]We also conducted early experiments with black-box variational inference [84], but this performed poorly due to becoming trapped in local minima. MCMC is more able to explore diverse modes of the posterior

distribution. Then, we evaluate generalisation to OOD data, by using several OOD test splits for each dataset, which have zero probability under the training distribution. We first describe these datasets and test splits (Sec. 4.1), the tasks we address and metrics used (Sec. 4.2), baselines we compare against (Sec. 4.3), and finally the performance of our proposed model and the baselines (Sec. 4.4). Implementation details for our model and baselines (hyperparameters, hardware, etc.) are given in the supplementary material.

4.1 Datasets

GQN. We render images of rooms containing several objects (cubes, cylinders, spheres), based on the 'rooms ring camera' dataset of [28]; similar datasets were used in [35, 26], but in all cases without OOD test splits. In the training split, the camera viewpoints are on a circular path around the center of the room, with the camera pointed at the center at fixed elevation angle. Textures for the walls and colors for the objects are selected randomly from a finite set, with some combinations held out. Three walls have the same texture as each other, with the fourth different. There are 3–4 objects present; these are placed near the side of the room identified by an odd texture. We define the following OOD test splits: (i) *position*: the 3–4 objects are now placed near a different wall; (ii) *composition*: unseen combinations of background and object textures; (iii) *number of objects*: 1/5/6 objects instead of 3–4; (iv) *viewpoint*: camera positions and elevations are randomly sampled. Exact details of the data generation process are given in the supplementary material.

ARROW. We render images using a modified version of the CLEVR dataset [46], similar to those in [45]. These have four objects, of which one is always an arrow, two of which are the same as each other, and a fourth that is different. The arrow always points at the odd (fourth) object. Object colors are randomly sampled. The camera has a random azimuth and shallow elevation. We define the following OOD test splits: (i) *position*: the four objects are positioned in a line, and the arrow no longer points to an odd object; (ii) *composition*: all objects are the same shape and color, with no arrow present; (iii) *number of objects*: 1/5/6 objects instead of four; the arrow still points at one odd object; (iv) *viewpoint*: camera looks down steeply on the objects.

4.2 Tasks and Metrics

We consider the following discriminative tasks, all taking a single image as input:

- instance segmentation, measured by adjusted Rand index (**ARI**) and mean segmentation covering (**mSC**) [26]; in the PER-SCENE setting, this measures how well the model infers segmentation maps for novel viewpoints
- depth prediction, measured by the mean relative error between predicted and true depths (**D.MRE**)
- pixel reconstruction, measured by peak signal-noise ratio (**PSNR**); in the PER-IMAGE setting, this measures faithfully our latent representation can autoencode OOD images; in the PER-SCENE setting, it measures how well the model performs novel view synthesis (NVS)

We report these metrics according to two different protocols: (i) PER-IMAGE, where we calculate the metrics only on the input image passed to the model; and (ii) PER-SCENE, where we calculate the metrics jointly over 10 images of the scene, taken from different viewpoints, but still having received only one of these as input. Thus, the latter setting measures how well the model predicts appearance, depth and segmentation from novel viewpoints. Finally, we evaluate *a priori* image generation, measured by the Fréchet Inception distance (**FID**) [36] and kernel Inception distance (**KID**) [13] between sampled and ground-truth image distributions.

4.3 Baselines

We compare our approach to three existing works. **Slot Attention [64]** is a recent unsupervised object segmentation model, with a spatial mixture representation. It is purely 2D and not generative; however, it does support generalisation to differing numbers of objects at test time. **uORF [111]** decomposes 2D images into 3D components represented as NeRFs. Unlike ours, it is not generative, and does not explicitly represent position separate from appearance. **NeRF-VAE [54]** is a generative method over 3D NeRFs, but which does not separate individual objects in its latent space. We used the original implementation of [64], with hyperparameters re-tuned on our datasets. We reimplemented [111, 54] in our own framework, and denote these as uORF* and NeRF-VAE* respectively. For a fair comparison, we use the same number of samples along each ray, the same or larger number of parameters, and similar encoder/decoder architectures as for our own model. Full details are in the

Figure 2: Qualitative results on various tasks, using our model and the baselines, on out-of-distribution (OOD) data. Each row shows the input image, then (col. 2–6) outputs from our model: the reconstruction, instance segmentation, depth map, and two novel viewpoints. We see that our model predicts high-quality segmentations and depth-maps, and that the new viewpoints are plausible and consistent with the input. The final three columns show reconstructions (the easiest task) by the baselines. Here we see that they fail to generalise to OOD data – they have specialised to their training dataset, and learnt shortcuts [29] that interpret the input images as if they were drawn from it. Thus, they fail to map objects in unfamiliar viewpoints or contexts to appropriate latent representations.

supplementary. We also modify NeRF-VAE to take just one view as input, and use non-iterative amortized inference for computational tractability.

4.4 Results

In-distribution data. We first evaluate how each model performs on the distribution of images it was trained on. Results on the discriminative tasks are given in Tab. 1; the top four rows show performance on the test split, which is drawn from the same distribution as the training data. Qualitative results are displayed in Fig. 4.4, and standard deviations on all results are given in the supplementary. We see that all methods successfully reconstruct input images (high per-image PSNR), with NeRF-VAE* slightly out-performing other approaches. The 3D-aware methods also predict depths with a small relative error (low per-image D.MRE), again with NeRF-VAE* best. A similar pattern holds for novel views (per-scene setting), albeit with slightly poorer scores overall. For GQN, Slot Attention performs best on segmentation under ARI scores, with ours better according to mSC and for both metrics on ARROW. Results for image generation are given in Tab. 2 and Fig. 3 (see the supplementary for images from NeRF-VAE*). Note that Slot Attention and uORF* are excluded, as they cannot generate images *a priori*. Our model has clearly learnt the distribution of both datasets, including the space of likely object and background appearances. Quantitatively, ours performs similarly to NeRF-VAE* on GQN (with FID slightly better and KID slightly worse), and out-performs it on both metrics for ARROW. In Fig. 4, we also show that our semantically-interpretable latent space allows users to edit the scene shown in an image, by counterfactual inference over the model.

OOD data. We now consider the more challenging setting where test data is drawn from a different distribution than the training set. Quantitative results in this setting are given in the bottom half of Tab. 1. This shows the mean over all OOD splits; a full breakdown is given in the supplementary. We see that in general, our method significantly out-performs the baselines on these splits – showing the benefit of our causal model and inference scheme. In particular, our method performs best on

| | GQN | | ARROW | |
	FID	KID	FID	KID
Ours	80.3	0.053	141.4	0.167
NeRF-VAE*	84.2	0.047	182.7	0.190

Table 2: Quantitative results on image generation. We report FID and KID scores for our method and baseline NeRF-VAE.

Figure 3: Examples of images sampled from our model for GQN (top) and ARROW (bottom)

Figure 4: Editing scenes using our model, by counterfactual inference. We condition on the input scene, then modify (intervene on) certain latent variables and re-render the result

all combinations of tasks and datasets, except for depth estimation on ARROW where uORF* is slightly better. Thus, the best methods on the in-distribution test split are therefore *not* the best on OOD data. They fit the statistics of the training distribution very well, but since they do not reflect its causal structure, they learn shortcuts [29] that do not generalise well on the OOD splits. Qualitative results (Fig. 4.4) reinforce this interpretation – we see that our method still predicts accurate segmentations and depth-maps for OOD data, and novel viewpoints look plausible. Notably, in all cases, it successfully reconstructs the input image via its latent space and synthesises new viewpoints, in spite of never having seen such an image during training. In contrast, the baselines struggle with images that are different to their training distribution – e.g. mispredicting object colors (1st row, uORF* and NeRF-VAE*), predicting an over-smoothed average scene (3rd row, NeRF-VAE*), or failing to separate walls and ceiling when they are in an unfamiliar pose (5th row, Slot Attention).

4.5 Limitations and Societal Impacts

Currently our method has the following limitations.

- The scene-level prior distribution $p_\theta(\mathbf{z}^s \mid \mathbf{z}^g)$ uses an unstructured latent space and decoder – thus, this particular component of the model is not interpretable. It would be worthwhile to use a structured prior, that explicitly models sparse relations among objects.

- We have only tested it on synthetic data; it would be interesting to conduct experiments on natural images. This would require modelling the image-formation process more faithfully, e.g. by modelling light transport and non-Lambertian materials.

- Our MCMC inference scheme is computationally expensive, as we render a complete image at each step. The efficiency could be increased by using a stochastic gradient for the LD transitions.

- Our model lacks variables to explicitly model object presence – if an object slot is unused, it learns a latent representation of a zero-density object.

We do not anticipate any negative societal impacts from this work. While all generative models have the potential to be used for creating fake content, our method requires further development to work on realistic images. On discriminative tasks, support for OOD inference should reduce the susceptibility of models to dataset bias; thus, we see a significant long-term benefit to such methods.

4.6 Conclusion

We have presented a new object-centric generative model that captures the causal process by which images are produced, and incorporates universal physical knowledge such as 3D geometry. We have shown that it is possible to intervene on our model then perform Monte-Carlo inference, in order to process out-of-distribution images. On instance segmentation and novel view synthesis in this OOD setting, it significantly out-performs three state-of-the-art approaches.

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