Risk-Averse Zero-Order Trajectory Optimization

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Abstract: We introduce a simple but effective method for managing risk in zero-order trajectory optimization that involves probabilistic safety constraints and balancing of optimism in the face of epistemic uncertainty and pessimism in the face of aleatoric uncertainty of an ensemble of stochastic neural networks. Various experiments indicate that the separation of uncertainties is essential to performing well with data-driven MPC approaches in uncertain and safety-critical control environments.

Keywords: zero-order trajectory optimization, CEM, aleatoric, epistemic, MPC, safety, uncertainty

1 Introduction

Data-driven approaches to sequential decision making are becoming increasingly popular [1, 2, 3, 4]. They hold the promise of reducing the number of prior assumptions about the system that are imposed by traditional approaches that are based on nominal models.

Learning-based approaches come in several different flavors [5]. Model-free approaches attempt to extract closed-loop control policies directly from data, while model-based approaches rely on a learned model of the dynamics to either generate novel data to extract a policy, or to be used in a model-predictive control fashion (MPC). This study belongs to the latter line of work.

Model-based methods have several advantages over pure model-free approaches. Firstly, humans tend to have a better intuition on how to incorporate prior knowledge into a model rather than into a policy or value function. Secondly, most model-free policies are bounded to a specific task, while models are task-agnostic and can be applied for optimizing arbitrary cost functions, given sufficient exploration.

Nevertheless, learning models for control comes with certain caveats. Traditional MPC methods require the model and cost function to permit a closed form solution which restricts the function class prohibitively. Alternatively, gradient-based iterative optimization can be employed, which allows for a larger class of functions but typically fails to yield satisfactory solutions for complicated function approximators such as deep neural network models. In addition, calculating first-order or even second-order information for trajectory optimization tends to be computationally costly, which makes it hard to meet the time constraints of real-world settings. This motivates the usage of zero-order methods, such as the Cross-entropy Method (CEM) that do not rely on gradient information but are efficiently parallelizeable.

Many methods relying on a learned model and zero-order trajectory optimizers have been proposed [6, 7, 8], but all share the same problem: compounding of errors through auto-regressive model prediction. This naturally brings us to the question of how can we effectively manage model errors and uncertainty to be more data-efficient and safe. Arguably, this is one of the main obstacles to applying data-driven model-based methods to the real-world, e.g. to robotics settings.

In this work, we provide an improved management of errors and uncertainty in zero-order MPC and test it on challenging scenarios (Fig. 1). We argue that it is essential to differentiate between the two types of uncertainty in the model-predictive setting: the aleatoric uncertainty arising from inherent noise in the system and epistemic uncertainty arising from the lack of knowledge [9, 10]. We measure these uncertainties by making use of stochastic ensembles. Our contributions can be summarized as

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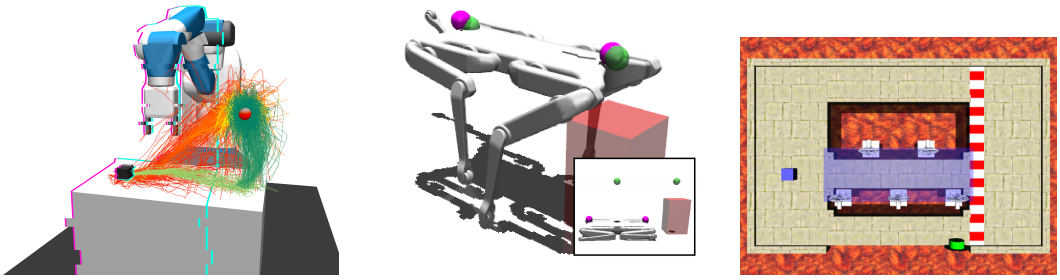


(a) Noisy-FetchPickAndPlace (b) Solo8-LeanOverObject (c) BridgeMaze

Figure 1: Environments considered for uncertainty aware planning.

follows: (i) effective use of aleatoric and epistemic uncertainty in model-based zero-order trajectory optimizers. (ii) A simple but practical approach to probabilistic safety constraints in zero-order MPC.

2 Related Work

Uncertainty Estimation. In the typical model-based reinforcement learning (MBRL) setting, the true transition dynamics function is modelled through an approximator. Impressive results have been achieved by both parametric models [11, 12, 13, 14], such as neural networks, and nonparametric models [15, 16, 17, 18], such as Gaussian Processes (GP). The latter inspired seminal work on incorporation of the dynamics model's uncertainty for long-term planning [18, 19]. However, their usability is limited to low-data, low-dimensional regimes with smooth dynamics [20, 21], which is not ideal for robotics applications. Alternative parametric approaches include ensembling of deep neural networks, used both in the MBRL community [6, 22], and outside [23, 24]. In particular, ensembles of *probabilistic* neural networks established state-of-the-art results in the MBRL community [6], but focus mainly on estimating the expected cost and disregard the underlying uncertainties. In comparison, we propose a treatment of the resulting uncertainties of the ensemble model.

Zero-order MPC. The learned model can be used for policy search like in PILCO [25, 18, 19, 26] or for online model-predictive control (MPC) [27, 28, 6]. In this work, we do planning in an MPC fashion and employ a zero-order method as trajectory optimizer, since less sensitive to hyperparameter tuning and less likely to get stuck in local minima of complex objective functions. Specifically, we consider a sample-efficient implementation of the Cross-Entropy method [29, 30] introduced in [31].

Safe MPC. Separating the sources of uncertainty is of particular importance for AI applications directly affecting humans' safety, as self-driving cars, elderly care systems, or in general any application that involves a physical interaction between the AI system and humans. Disentangling epistemic from aleatoric uncertainty allows for separate optimization of the two, as they represent semantically different objectives: efficient exploration and risk-awareness. Extensive research on uncertainty decomposition has been done in the Bayesian setting and in the context of safe policy search [32, 33, 34, 35], MPC planning [36, 37, 38], and distributional RL [39, 40]. On the other side, a state-of-the-art baseline for ensemble learning like PETS [6], despite estimating both uncertainties, only optimizes for the *expected* cost during action evaluation. Our work aims at filling this gap by explicitly integrating the propagated uncertainty information in the zero-order MPC planner.

3 Method

Our approach concerns itself with the efficient usage of uncertainties in zero-order trajectory optimization and is therefore generally applicable to such optimizers. We are interested in modeling noisy system dynamics $x_{t+1} = f(x_t, u_t, w(x_t, u_t))$ where f is a nonlinear function, x_t the observation vector, u_t applied control input and $w(x_t, u_t)$ a noise term sampled from an arbitrary distribution.

Consequently, in the absence of prior knowledge about the function f, the system needs to be modeled by a complex function approximator such as a neural network. Furthermore, we are interested in managing uncertainties based on our fitted model, which is erroneous. To this end, we use stochastic ensembles of size K, where the output of each model $\boldsymbol{\vartheta}^k(x_t, u_t)$ are parameters

of a normal distribution depending on input observation x_t and control u_t. As a by-product, our auto-regressive model prediction based on a sequence of controls $\boldsymbol{u}$ becomes a predictive distribution over trajectories τ, $\psi_\tau(x_t, \boldsymbol{u}) := p(\tau|x_t, \boldsymbol{u}; \theta)$ where θ denotes the parameters of the ensemble. For convenience, from this point onward we will differentiate between multiple usages of ψ_τ. We denote with ψ_t^x the distribution $p(x|x_t, \boldsymbol{u}_{t:t+h}; \theta)$ over states at time step t and $\psi_t^{\boldsymbol{\vartheta}}$ the distribution over the Gaussian parameter outputs $p(\boldsymbol{\vartheta}|x_t, \boldsymbol{u}_{t:t+h}; \theta)$ at time step t of the planner.

3.1 Planning and Control

To validate our hypothesis that accounting for uncertainty in the environment and in model prediction is essential to develop risk-averse policies, we use the Cross-Entropy Method (CEM) with improvements suggested in Pinneri et al. [31]. Accordingly, at each time step t we sample a finite number of control sequences $\boldsymbol{u}$ for a finite horizon H from an isotropic Gaussian prior distribution which we evaluate from the state x_t using an auto-regressive forward-model and the cost function. The sampling distribution is refitted in multiple rounds based on good-performing (elite) trajectories. After this optimization step, the first action of the mean of the fitted Gaussian distribution is executed. Since this approach utilizes a predictive model for a finite horizon at each time step, it naturally falls into the category of Model Predictive Control (MPC) methods.

Although we use CEM, our approach of managing uncertainty can generically be applied to other zero-order trajectory optimizers such as MPPI [28], by a simple modification of the trajectory cost function.

3.2 The Problem of Uncertainty Estimation

Since we have a stochastic model of the dynamics, at the model prediction time step t we observe a distribution over potential outcomes. Indeed, since our model outputs are parameters of a Gaussian distribution, with auto-regressive prediction we end up with a distribution over possible Gaussians for a certain time step t.

Given a sampled action sequence $\boldsymbol{u}$ and the initial state $\boldsymbol{x}_t$ we observe a distribution over trajectories ψ_τ. To efficiently sample from the trajectory distribution ψ_τ we use the technique introduced by Chua et al. [6] which involves prediction particles that are randomly mixed between ensemble members at each prediction step. In this way, the sampled trajectories are used to perform a Monte Carlo estimate of the expected trajectory cost $\mathbb{E}_{\tau\sim\psi_\tau}[c(\tau)]$. However, this does not take the properties of ψ_τ into account, which might be a high-entropy distribution and may lead to very risky and unsafe behavior. In this work, we alleviate this by looking at the properties of ψ_τ, i.e. different kinds of uncertainties arising from the predictive distribution.

3.3 Learned Dynamics Model

We learn a dynamics model f_θ that approximates the true system dynamics $x_{t+1} = f(x_t, u_t, w(x_t, u_t))$. As a model class, we use an ensemble of neural networks with stochastic outputs as in Chua et al. [6]. Each model k, parameterizes a multivariate Gaussian distribution with diagonal covariance, $f_\theta^k(x_t, u_t) = \mathcal{N}(\mu_\theta^k(x_t, u_t), \Sigma_\theta^k(x_t, u_t))$ where $\mu_\theta^k(\cdot,\cdot)$ and $\Sigma_\theta^k(\cdot,\cdot)$ are model functions outputting the respective parameters.

Iteratively, while interacting with the environment, we collect a dataset of transitions $\mathfrak{D}$ and train each model k in the ensemble by the following negative log-likelihood loss on the Gaussian outputs:

$$\mathcal{L}(\theta, k) = \mathbb{E}_{x_t, u_t, x_{t+1}\sim\mathfrak{D}}\left[\left\|x_{t+1} - \left(\mu_\theta^k(x_t, u_t) + x_t\right)\right\|_2^2 \cdot \Sigma_\theta^k(x_t, u_t)^{-1} + \log\left|\Sigma_\theta^k(x_t, u_t)\right|\right] \quad (1)$$

In addition, we use a number of regularization terms to make the model training more stable. We provide more details on this in Suppl. A.

3.4 Separation of Uncertainties

In the realm of parametric estimators, there are two uncertainties that are of particular interest. *Aleatoric* uncertainty is the kind that is irreducible, and results from inherent noise of the system, e.g. sensor noises in robots. On the other hand we have *epistemic* uncertainty resulting from lack of

data, i.e. lack of knowledge and is thus reducible. This begs the question, how can we separate these uncertainties given an auto-regressive dynamics model f_θ? The way that we efficiently sample from ψ^τ is by mixing sampled prediction particles, similarly as in PETS[6]. This process is illustrated by the red lines in Fig. 2.

It is clear that simple model prediction disagreement is not a good measure for *aleatoric* uncertainty, since it can be entangled with the epistemic uncertainty. Given our assumptions about the system dynamics, we measure *aleatoric* uncertainty as the entropy of the predicted normal distributions of the ensemble models. More concretely, given a sampled particle state $\tilde{x}_t$, we define the estimated aleatoric uncertainty for ensemble model associated to particle b at time step t as:

$$\mathfrak{A}^b(x|\tilde{x}_t, u_t) = \mathcal{H}_{x\sim\psi_t^{x,b}}(x) \tag{2}$$

Where $\psi_t^{x,b}$ is the output distribution of ensemble model based on inputs $\tilde{x}_t$, u_t. Since in the end we are interested in the aleatoric uncertainty incurred from applying the action sequence $\boldsymbol{u}$ from initial state x_t, the quantity of interest for us is the expected aleatoric uncertainty for time slice t:

$$\mathfrak{A}(x|u_t) = \mathbb{E}_{\tilde{x}_t^b\sim\psi_t^x}\left[\mathfrak{A}^b(x|\tilde{x}_t^b, u_t)\right] \tag{3}$$

Intuitively, because we only have access to the ensemble for sampling, we take a time-slice in the sampled trajectories from ψ_τ and compute the output entropies. Moreover, since we assume a Gaussian 1-step predictive distribution this is an expectation over differential Gaussian entropy. An alternative way of computation which we also explore in this work is calculating the expected particle variance for time slice t of the prediction horizon:

$$\mathrm{Var}^{\mathfrak{A}}_{t+1} = \frac{1}{B}\sum_{b=1}^{B}\Sigma_\theta^k(\tilde{x}_t^b, u_t) \tag{4}$$

For estimating the *epistemic* uncertainty, one would be tempted to look at the disagreement between ensemble models in parameter space $\mathrm{Var}[\theta]$, but this is not completely satisfying, since neural networks tend to be over-parametrized and variance within the ensemble still may exist albeit the optimum has been reached by all ensemble models. An alternative would be to calculate the Fisher information metric $\mathcal{I} := \mathrm{Var}[\nabla_\theta \log \mathcal{L}(x_{t+1}|x_t, u_t)]$ where $\mathcal{L}$ denotes the likelihood function, but this tends to be expensive to compute.

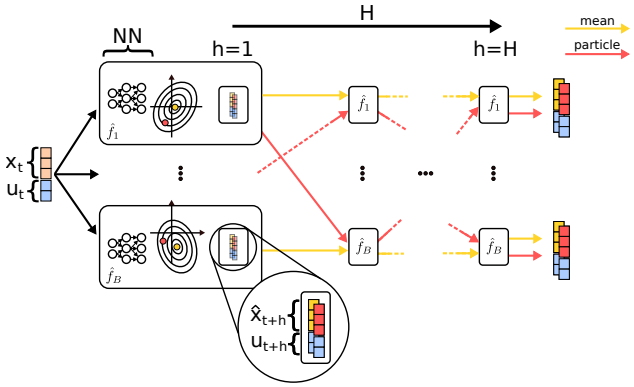


Figure 2: Probabilitic Ensembles with Trajectory Sampling and Uncertainty Separation (PETSUS)

Given the assumption of local Gaussianity, the true epistemic uncertainty for this case is the predictive entropy over the Gaussian parameters $\boldsymbol{\vartheta}_t$ at time step $t+h$.

$$\mathfrak{E}(x_t, \boldsymbol{u}_{t:t+h}) = \mathcal{H}_{\psi_t^{\boldsymbol{\vartheta}}}(\boldsymbol{\vartheta}|x_t, \boldsymbol{u}_{t:t+h}) \tag{5}$$

It is easy to verify that this quantity is 0 given perfect predictions of the model. Note that, because of auto-regressive prediction of a nonlinear model, this is a very difficult object to handle. Nevertheless, since our predictive distribution $p(x|x_t, u_t; \boldsymbol{\vartheta})$ is parametrized by model outputs, we may utilize disagreement in $\boldsymbol{\vartheta}_t$ to approximate $\mathfrak{E}$. To get correct estimations, we need to propagate mean predictions $\bar{x}$ in addition to the particles as illustrated as the yellow lines in Fig. 2. We quantify epistemic uncertainty as ensemble disagreement at time step t:

$$\mathrm{Var}^{\mathfrak{E}}(x_{t+1}) = \mathrm{Var}^e[\mu_\theta^k(\bar{x}_t, u_t)] + \mathrm{Var}^e[\Sigma_\theta^k(\bar{x}_t, u_t)] \tag{6}$$

where Var^e is the empirical variance over the $k = 1 \dots K$ ensembles.

3.5 Probabilistic Safety Constraints

When applying data-driven control algorithms to real systems, safety is of utmost importance. In the realm of zero-order optimization, safety constraints can be easily introduced by putting infinite

cost on constraint-violating trajectories. Nevertheless, we are dealing with erroneous stochastic nonlinear models which lead to nontrivial predictive distributions of future states, based on the control sequence $\boldsymbol{u}$. For this reason, we want to control the risk of violating the safety constraints that we, as practitioners, are willing to tolerate. Given a constraint set $\mathbb{C} \subset \mathbb{X}$, we define the probability of the control sequence $\boldsymbol{u}$ violating the safety constraints as:

$$p(x \in \mathbb{C} | x_t, \boldsymbol{u}) = \int_{x \in \mathbb{C}} \psi_\tau(x | x_t, \boldsymbol{u}).$$

In practice, it is hard to compute this integral efficiently, since our distribution ψ_τ is nontrivial as a result of nonlinear propagation of uncertainty. Furthermore, the constraint set $\mathbb{C}$ might not have the structure necessary to allow an efficient solution to the integral, in which case one needs to resort to Monte Carlo estimation.

To simplify computation and gain speed, we consider constraint sets of the form $[a, b] \in \{a, b \,|\, a, b \in \mathbb{R}^2,\, a < b\}$. Let ψ_τ^t be a time-slice in the predictive trajectory distribution at time step t and x_t the state at time step t of the trajectory. Since we perform moment matching by a Gaussian in this time-slice, the probability of the constraint being violated at time step t is given by $\mathcal{N}(x; \mu_t, \Sigma_t)$, where μ and Σ are estimated by Monte Carlo sampling. If we further assume a diagonal covariance Σ, this integral can be deconstructed into d univariate Gaussian integrals, which can be computed fast. Hence, the probability of a constraint violation happening at time step t is defined by:

$$p(x_{t+1} \in \mathbb{C} | x_t, \boldsymbol{u}) = \prod_{i=0}^{d} \int_{x \in \mathbb{C}} \mathcal{N}(x^i; \mu_t^i, \sigma_t^i) \tag{7}$$

3.6 Implementing Risk-Averse ZERo-Order Trajectory Optimization (RAZER)

We assume the task definition is provided by the cost $c(x, u)$. For trajectory optimization, we start from a state x_t and predict with an action sequence $\boldsymbol{u}$ the future development of the trajectory τ. Along this trajectory, we want to compute a single cost term which is conveniently defined as the expected cost of all particles $\tilde{x}$ summed over the planning horizon H:

$$c(x_t, \boldsymbol{u}) = \sum_{h=1}^{H} \frac{1}{B} \sum_{b=1}^{B} c(\tilde{x}_{t+h}^{(b)}, u_{t+h}). \tag{8}$$

The optimizer, in our case CEM, will optimize the action sequence $\boldsymbol{u}$ to minimize the cost in a probabilistic sense, i.e. $P(\boldsymbol{u} \mid x) \propto \exp(-\beta\, c(x, \boldsymbol{u}))$ where β reflects the strength of the optimizer (the higher the more likely it finds the global optimum). To make the planner uncertainty aware, we need to make sure it avoids unpredictable parts of the state space by making them less likely. Using the aleatoric uncertainty provided by PETSUS Eq. 4, we define the an aleatoric penalty as

$$c_{\mathfrak{A}}(\boldsymbol{x}_t, \boldsymbol{u}) = w_{\mathfrak{A}} \cdot \sum_{h=1}^{H} \sqrt{\mathrm{Var}_{t+h}^{\mathfrak{A}}}, \tag{9}$$

where $w_{\mathfrak{A}} > 0$ is a weighting constant. The larger the aleatoric uncertainty, the higher the cost.

To guide the exploration to states where the model has epistemic uncertainty Eq. 6 (due to lack of data), we use an epistemic bonus:

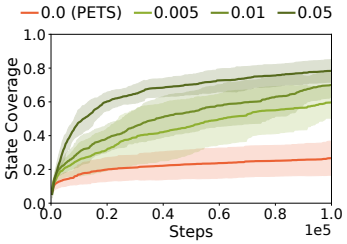
$$c_{\mathfrak{E}}(\boldsymbol{x}_t, \boldsymbol{u}) = -w_{\mathfrak{E}} \cdot \sum_{h=1}^{H} \sqrt{\mathrm{Var}_{t+h}^{\mathfrak{E}}}, \tag{10}$$

where $w_{\mathfrak{E}} > 0$ is a weighting constant.

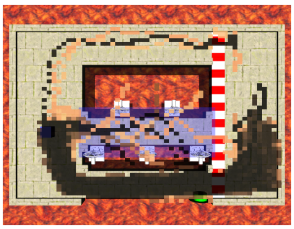
To be able to operate on a real system, the most important part is to adhere to safety constraints. As formulated in Eq. 7, the predicted safety violations need to be uncertainty aware, independent of the source of uncertainty. We integrate this into the planning method by adding:

$$c_{\mathfrak{S}}(\boldsymbol{x}_t, \boldsymbol{u}) = w_{\mathfrak{S}} \cdot \sum_{h=1}^{H} [\![p(\hat{\boldsymbol{x}}_{t+h} \in \mathbb{C}) > \delta]\!] \tag{11}$$

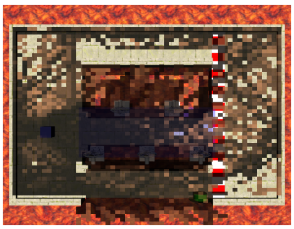
where $[\![\cdot]\!]$ is Iverson bracket and $w_{\mathfrak{S}}$ is either a large penalty $c_{\max}$ or 0 to disable safety. The overall algorithm used in a model-predictive control fashion is outlined in Suppl. B.



(a) State space exploration over time depending on epistemic bonus ($w_{\mathfrak{E}}$).



(b) State-space coverage of PETS.



(c) State-space coverage of RAZER with $w_{\mathfrak{E}} = 0.05$.

Figure 3: Active learning setting: The epistemic bonus let RAZER seek states for which no or only little training data exists (a,c). PETS overfits to a particular solution (b). In (b) and (c), visited states as dots being black for early times and fading into pale orange if visited later.

4 Experiments

We study our uncertainty-aware planner in 4 continuous state-/action-space environments and compare results with PETS. PETS does not use the model's uncertainties explicitly, instead they enter only by taking the mean over the particle costs. We first describe the environments.

BridgeMaze This toy environment (see Fig. 1c), we specifically designed to study the different aspects of uncertainty independently. The agent (blue cube) starts on the left platform and has to reach the goal platform on the right. To reach the goal platform, the agent has to move over one of three bridges without falling into the lava. The upper bridge is safeguarded by walls; hence, it is the safest path to the goal but also the longest. The lower bridge has no walls, and therefore is more dangerous for an unskilled agent to cross but the path is shorter. The middle bridge is the shortest path to the goal. However, randomly appearing strong winds perpendicular to the bridge might cause the agent to fall off the bridge with some probability, rendering this bridge dangerous.

Noisy-HalfCheetah This environment is based on *HalfCheetah-v3* from the OpenAI Gym toolkit. We introduce aleatoric uncertainty to the system by adding Gaussian noise $\xi \sim \mathcal{N}(\mu, \sigma^2)$ to the actions when the velocity is above 6. The action noise translates into a non-Gaussian and potentially very complicated state-space noise distribution that makes the control problem very challenging.

Noisy-FetchPickAndPlace Based on the *FetchPickAndPlace-v1* gym environment. Additive action noise is applied to the gripper, so that its grip on the box might become tighter or looser. The noise is applied for x-positions < 0.8 which is illustrated in Fig. 1a by a blue line causing the agent to drop the box with high probability if it tries to lift the box too early.

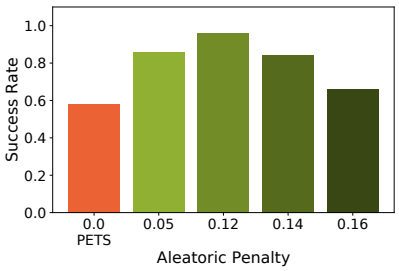
Solo8-LeanOverObject In this robotic environment, the task of a quadrupedal robot [41] is to stand up and learn forward to reach a target position (purple markers need to reach green dots in Fig. 1b) without hitting an object visualized by the red cube representing the unsafe zone. The robots starts in a laying position has shown in the inset of Fig. 1b. As in the *Noisy-HalfCheetah* environment, Gaussian action noise is applied to mimic real-world perturbances.

Additional implementation details of all environments can be found in Suppl. B.1. The code for the modified environments will be published alongside with the main code of this paper.

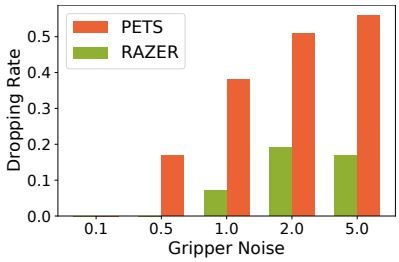
Next, we will present RAZER's exploration and safety in the *BridgeMaze* environment. Afterwards, we are going to discuss planning with external safety constraints in the *Solo8* environment. We complete this section with results on *Noisy-HalfCheetah* and *Noisy-FetchPickAndPlace*.

4.1 Active Learning for Model Improvement

If model uncertainties are used for risk-avers planning, they are only meaningful if the model has the right training data to correctly estimate the parameters of the approximate noise model from it. In case of too little data, the agent might avoid parts of the state space that lead to higher rewards because of an overestimation of the model uncertainties. On the other hand, the agent might enter unsafe regions for which the uncertainties are underestimated. By adding the epistemic bonus to our domain specific cost, the planner can actively seek states with high epistemic uncertainty, i.e. for which no or only little training data exists.



(a) Success rate in BridgeMaze depending on $w_{\mathfrak{A}}$.



(b) Dropping rate in FetchPickAndPlace.

Figure 4: Risk-averse planning in the face of aleatoric uncertainty yields higher success rates in noisy environments. For (b) we use ground truth models and a fixed aleatoric penalty weight $w_{\mathfrak{A}}$.

Figure 4a shows this active data gathering process for the BridgeMaze environment. The PETS model finds one specific solution to the problem, that is, it chooses the somewhat save lower bridge over the dangerous middle and the further away upper bridge (Fig. 3b). Once, this solution is found the model overfits to this particular path without exploring much more other parts of the state space as it can also seen by the red curve in Fig. 3a.

RAZER on the other hand actively explores large parts of the state space with increasing weight of the epistemic bonus (Fig. 3a). RAZER not only finds the one path that happens to minimize the domain specific cost but also explores all the other possible paths (Fig. 3c). One the other hand, since RAZER differentiates between aleatoric and epistemic uncertainties, it does not get stuck on the middle bridge that has high inherent noise, but keeps on exploring yet unexplored parts of the state space. In that way, RAZER can better estimate the noise parameters of larger parts of the state space and adapts better to new situations in which new solutions have to be found.

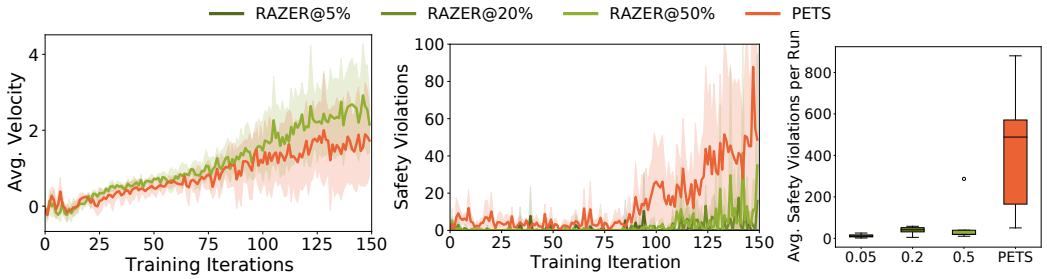
4.2 Risk-Averse Planning

Once a good model is learned, the estimated noise model can be actively used for safe planning. What differentiates RAZER from PETS is that our method makes full usage of the information from the noise model while in PETS the uncertainties only enter planning by taking the mean over the particle costs. First, the PETS approach does not differentiates between aleatoric and epistemic uncertainty, and second, the sample based approach suffers from the typically low (~ 5) sample size.

BridgeMaze:. Figure 4a shows the success rate of PETS and RAZER in the BridgeMaze. In both cases, we use the same model that was trained from data collected during a training run with $w_{\mathfrak{E}} = 0.05$. Hence, the model saw enough training data from all parts of the state space. The noise in the env is tuned such that there is a chance to cross the bridge without falling. While in Fig. 3b PETS avoided this path because of an overestimation of the state due to a lack of enough training data now it sometimes sees a change to cross the bridge. However, these attempts are very likely to fail because of stronger winds that occur randomly, resulting in a success rate of only 58%. RAZER does not rely on sampling for the aleatoric part and can thus avoid risk. With higher penalty constant the success rate increase up to 96% but only as long as the agent is willing to take a risk at all. For large values of $w_{\mathfrak{A}}$ the agent becomes so afraid that it only moves slowly (decreasing reward in Fig. 4a).

Noisy-HalfCheetah:. How does RAZER perform on the Noisy-HalfCheetah environment when learning models from scratch. Without aleatoric penalty, the planner is optimistic. Risky situations are only detected if a failing particle is sampled. The planner is mostly neglecting the noise and thus the robot increases its velocity and gets destabilized and ends up slower than with the aleatoric penalty, as shown in Fig. 5a.

FetchPickAndPlace. In this environment, a 7-DoF robot arm should bring the box to a target position – starting and target position are at the opposite sides of the table. The shortest path is to lift the box and move in a straight line to the target. However, with noise applied to the gripper action, there is a certain probability to drop the box along the way. When penalizing aleatoric uncertainty, this is avoided and also less trajectory samples are "wasted" in high-entropic regions, as presented in Fig. 1a. In Fig. 4b we present the number of times the box is dropped on the table depending on the aleatoric penalty. RAZER adopts a cautious behavior, preferring to slide the box on the table and



(a) With aleatoric penalty. (b) With safety constraints. (c) With safety constraints.

Figure 5: Halfcheetah environment (task lengths 300 steps) with learning models from scratch. At 150 iterations we have seen then only 45k points. (a) Performance under noisy actions. By applying aleatoric penalty, RAZER is able to navigate the uncertainties better – leading to higher returns faster. (b) Safety violations above a certain body height (simulating a low ceiling) for different values of δ. With increasing δ, RAZER is much more seldom violating a constraint in stark contrast to PETS. In (c) the number of violations are averaged over the last 50 iterations (summed over 10 rollouts).

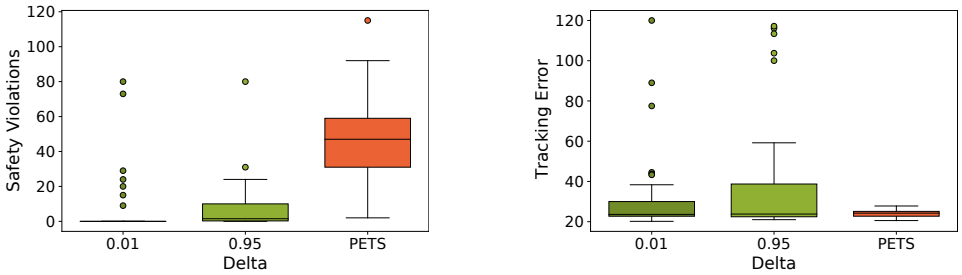


Figure 6: Safe planning vs. task oriented planning in the *Solo8-LeanOverObject* environment with noisy actions. Left: number of safety violations for different values of δ Eq. 11. Right: tracking error.

lifting it only in the area without action noise, keeping below 20%, even when considerable noise is applied.

4.3 Planning with External Safety Constraints

Noisy-HalfCheetah:. We consider a safety constraint on the height of the body above ground simulating a narrow passage. Figure 5b shows the number of safety violations. Note that PETS has the same penalty cost for hard violations.

Solo8-LeanOverObject:. In this experiment, the robot has to move to two target points with its front and rear of the trunk while avoiding to enter a specified rectangular area (fragile object). The front feet are fixed. To track the points, the robot has to lean forward, such that it can loose balance due to noisy actions. RAZER successfully manages to satisfy the safety constraints almost always, which is in contrast to PETS, as shown in Fig. 6. However, satisfying the safety constraint comes with the cost of a reduced tracking accuracy.

5 Discussion

In this work, we have provided a methodology to separate uncertainties in stochastic ensemble models (PETSUS) which can be used as a tool to build risk-averse model-based planners that are also data-efficient and enforce safety through probabilistic safety constraints (RAZER). This type of risk-averseness can be achieved by a simple modification of the cost function in form of uncertainty penalties in zero-order trajectory optimizers.

Furthermore, the separation of uncertainties allows us to do proper exploration via epistemic bonus which benefits generalization of the model and therefore makes it applicable to more settings. As future work, it would be of interest to see this approach applied to a proper transfer learning setting from simulations to real systems, where risk-averseness combined with exploratory behavior is crucial for efficient learning and safe operation.

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