Near-Optimal Sample Complexity Bounds for Constrained MDPs

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Abstract

In contrast to the advances in characterizing the sample complexity for solving Markov decision processes (MDPs), the optimal statistical complexity for solving constrained MDPs (CMDPs) remains unknown. We resolve this question by providing *minimax* upper and lower bounds on the sample complexity for learning near-optimal policies in a discounted CMDP with access to a generative model (simulator). In particular, we design a model-based algorithm that addresses two settings: (i) *relaxed feasibility*, where small constraint violations are allowed, and (ii) *strict feasibility*, where the output policy is required to satisfy the constraint. For (i), we prove that our algorithm returns an ε-optimal policy with probability $1-\delta$, by making $\tilde{O}\left(\frac{SA\log(1/\delta)}{(1-\gamma)^3\varepsilon^2}\right)$ queries to the generative model, thus matching the sample-complexity for unconstrained MDPs. For (ii), we show that the algorithm's sample complexity is upper-bounded by $\tilde{O}\left(\frac{SA\ \log(1/\delta)}{(1-\gamma)^5\ \varepsilon^2\zeta^2}\right)$ where ζ is the problem-dependent Slater constant that characterizes the size of the feasible region. Finally, we prove a matching lower-bound for the strict feasibility setting, thus obtaining the first near minimax optimal bounds for discounted CMDPs. Our results show that learning CMDPs is as easy as MDPs when small constraint violations are allowed, but inherently more difficult when we demand zero constraint violation.

1 Introduction

Common reinforcement learning (RL) algorithms focus on optimizing an unconstrained objective, and have found applications in games such as Atari [21] or Go [26], robot manipulation tasks [27, 33] or clinical trials [24]. However, many applications require the planning agent to satisfy constraints – for example, in wireless sensor networks [10] where there is a constraint on average power consumption. More generally, in the constrained Markov decision processes (CMDP) framework, the goal is to find a policy that maximizes the value associated with a reward function subject to the policy achieving a return (for a second reward function) that exceeds an apriori determined threshold [3]. There has been substantial work addressing the planning problem to find a near-optimal policy in a known CMDP [8, 7, 28, 22, 1, 32]. However, since the CMDP is unknown in most practical applications, we consider the problem of finding a near-optimal policy in this more challenging setting.

There have been multiple recent approaches to obtain a near-optimal policy in CMDPs in the regret-minimization or PAC-RL settings [13, 34, 9, 18, 29, 12, 14, 15, 11]. These works tackle the exploration, estimation and planning problems simultaneously. On the other hand, recent works [15, 30, 6] consider an easier, but even more fundamental problem of obtaining a near-optimal policy with access to a simulator or *generative model* [19, 17, 2]. In particular, these works assume that the transition probabilities in the underlying CMDP are unknown, but the planner has access to a sampling oracle (the generative model) that returns a sample of the next state when given any

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state-action pair as input. This is the problem setting we consider and *aim to obtain matching upper and lower bounds on the sample complexity of planning in CMDPs with access to a generative model*.

Given a target error $\varepsilon > 0$, the approximate CMDP objective is to return a policy that achieves a cumulative reward within an ε additive error of the optimal policy in the CMDP. Previous work can be classified into two categories based on how it tackles the constraint – for the easier problem that we term *relaxed feasibility*, the policy returned by an algorithm is allowed to violate the constraint by at most ε. On the other hand, for the more difficult *strict feasibility* problem, the returned policy is required to strictly satisfy the constraint and achieve zero constraint violation. Except for the recent works of Wei et al. [30] and Bai et al. [6], most provably efficient approaches including those in the regret-minimization and PAC-RL settings consider the relaxed feasibility setting. For this problem, the best model-based algorithm requires $\tilde{O}\left(\frac{S^2 A}{(1-\gamma)^3 \varepsilon^2}\right)$ samples to return an ε-optimal policy in an infinite-horizon γ-discounted CMDP with S states and A actions [15], while the best model-free approach requires $\tilde{O}\left(\frac{SA}{(1-\gamma)^5 \varepsilon^2}\right)$ samples for achieving the objective [12]. On the other hand, the best known upper bounds for the strict feasibility problem are achieved by the model-free algorithm of Bai et al. [6]. In particular, this algorithm requires $\tilde{O}\left(\frac{L \cdot SA}{(1-\gamma)^4 \varepsilon^2}\right)$ samples [6, Theorem 2] that depends on a potentially large parameter L that is not explicitly bounded by Bai et al. [6].

Importantly, there are no lower bounds characterizing the difficulty of either the relaxed or strict feasibility problems (except in degenerate cases where the constraint is always satisfied and the CMDP problem reduces to an unconstrained MDP). To get an indication of what the optimal bounds might be, it is instructive to compare these results to the unconstrained MDP setting. For unconstrained MDPs with access to a generative model, both model-based [2, 20] and model-free approaches [25] can return an ε-optimal policy within near-optimal $\tilde{\Theta}\left(\frac{SA}{(1-\gamma)^3 \varepsilon^2}\right)$ sample-complexity [4]. Hence, compared to the sample-complexity for unconstrained MDPs, the best-known upper-bounds for CMDPs are worse for both the relaxed and strict feasibility settings. However, it is unclear whether solving CMDPs is inherently more difficult than unconstrained MDPs. We resolve these questions for both the relaxed and strict feasibility settings, and make the following contributions.

Generic model-based algorithm: In Section 3, we provide a generic model-based primal-dual algorithm (Algorithm 1) that can be used to achieve both the relaxed and strict feasibility objectives (with appropriate parameter settings). The proposed algorithm requires solving a sequence of unconstrained empirical MDPs using any black-box MDP planner.

Upper-bound on sample complexity under relaxed feasibility: In Section 4, we prove that with a specific set of parameters, Algorithm 1 uses no more than $\tilde{O}\left(\frac{SA}{(1-\gamma)^3 \varepsilon^2}\right)$ samples to achieve the relaxed feasibility objective. This improves upon the bounds of HasanzadeZonuzy et al. [15] and matches the lower-bound in the easier unconstrained MDP setting, implying that our bounds are near-optimal. Our result indicates that under relaxed feasibility solving CMDPs is as easy as solving unconstrained MDPs. To the best of our knowledge, these are the first such bounds.

Upper-bound on sample-complexity under strict feasibility: In Section 5, we prove that with a specific set of parameters, Algorithm 1 uses no more than $\tilde{O}\left(\frac{SA}{(1-\gamma)^5 \zeta^2 \varepsilon^2}\right)$ to achieve the strict feasibility objective. Here $\zeta \in (0, 1/1-\gamma]$ is the problem-dependent *Slater constant* that characterizes the size of the feasible region and influences the difficulty of the problem. Unlike Bai et al. [6], our bounds do not depend on additional (potentially large) problem-dependent quantities.

Lower-bound on sample-complexity under strict feasibility: In Section 7, we prove a matching problem-dependent $\Omega\left(\frac{SA}{(1-\gamma)^5\, \zeta^2\, \varepsilon^2}\right)$ lower bound on the sample-complexity in the strict feasibility setting. Our results thus demonstrate that the proposed model-based algorithm is near minimax optimal. Furthermore, our bounds indicate that under strict feasibility (i) solving CMDPs is inherently more difficult than solving unconstrained MDPs, and (ii) the problem hardness (in terms of the sample-complexity) increases as ζ (and hence the size of the feasible region) decreases. To the best of our knowledge, these are first results characterizing the difficulty of solving CMDPs with access to a generative model and demonstrate a separation between the relaxed and strict feasibility settings.

Overview of techniques: For proving the upper bounds, we use a specific primal-dual algorithm that reduces the CMDP planning problem to solving multiple unconstrained MDPs. Specifically, by

using a strong-duality argument, we show that we can obtain an optimal CMDP policy by averaging the optimal policies of a specific sequence of MDPs. For each MDP in this sequence, we use the model-based techniques from Agarwal et al. [2], Li et al. [20] to prove concentration results for data-dependent policies. This allows us to prove concentration for the optimal data-dependent policy in the CMDP, and subsequently bound the sample complexity for both the relaxed and strict feasibility problems. For the lower bound, we modify the MDP hard instances [5, 31] to handle a constraint reward. This makes the resulting gadgets significantly more complex than those required for MDPs, but we show that similar likelihood arguments can be used to prove the lower-bound.

2 Problem Formulation

We consider an infinite-horizon discounted constrained Markov decision process (CMDP) [3] denoted by M, and defined by the tuple $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, r, c, b, \rho, \gamma \rangle$ where $\mathcal{S}$ is the set of states, $\mathcal{A}$ is the action set, $\mathcal{P} : \mathcal{S} \times \mathcal{A} \to \Delta_{\mathcal{S}}$ is the transition probability function, $\rho \in \Delta_{\mathcal{S}}$ is the initial distribution of states and $\gamma \in [0, 1)$ is the discount factor. The primary reward to be maximized is denoted by $r : \mathcal{S} \times \mathcal{A} \to [0, 1]$, whereas the constraint reward is denoted by $c : \mathcal{S} \times \mathcal{A} \to [0, 1]$[1]. If $\Delta_{\mathcal{A}}$ denotes the simplex over the action space, the expected discounted return or *reward value function* of a stationary, stochastic policy[2] $\pi : \mathcal{S} \to \Delta_{\mathcal{A}}$ is defined as $V_r^\pi(\rho) = \mathbb{E}_{s_0, a_0, \ldots}\left[\sum_{t=0}^{\infty} \gamma^t r(s_t, a_t)\right]$, where $s_0 \sim \rho, a_t \sim \pi(\cdot|s_t)$, and $s_{t+1} \sim \mathcal{P}(\cdot|s_t, a_t)$. For each state-action pair (s, a) and policy π, the reward action-value function is defined as $Q_r^\pi : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$, and satisfies the relation: $V_r^\pi(s) = \langle \pi(\cdot|s), Q_r^\pi(s, \cdot) \rangle$, where $V_r^\pi(s)$ is the reward value function when the starting state is equal to s. Analogously, the *constraint value function* and constraint action-value function of policy π is denoted by $V_c^\pi(\rho)$ and Q_c^π respectively. The CMDP objective is to return a policy that maximizes $V_r^\pi(\rho)$, while ensuring that $V_c^\pi(\rho) \geq b$. Formally,

$$\max_\pi V_r^\pi(\rho) \quad \text{s.t.} \quad V_c^\pi(\rho) \geq b. \tag{1}$$

The optimal stochastic policy for the above CMDP is denoted by π^* and the corresponding reward value function is denoted by $V_r^*(\rho)$. We also define $\zeta := \max_\pi V_c^\pi(\rho) - b$ as the problem-dependent quantity referred to as the Slater constant [12, 6]. The Slater constant is a measure of the size of the feasible region and determines the difficulty of solving Eq. (1).

For simplicity of exposition, we assume that the rewards r and constraint rewards c are known, but the transition matrix $\mathcal{P}$ is unknown and needs to be estimated. We note that assuming the knowledge of the rewards does not affect the leading terms of the sample complexity since learning these is an easier problem compared to the transition matrix [5, 25]. We assume access to a *generative model* or simulator that allows the agent to obtain samples from the $\mathcal{P}(\cdot|s, a)$ distribution for any (s, a). Assuming access to such a generative model, our aim is to characterize the sample complexity required to return a near-optimal policy $\hat{\pi}$ in M. Given a target error $\varepsilon > 0$, we can characterize the performance of policy $\hat{\pi}$ in two ways:

Relaxed feasibility: We require $\hat{\pi}$ to achieve an approximately optimal reward value, while allowing it to have a small constraint violation in M. Formally, we require $\hat{\pi}$ s.t.

$$V_r^{\hat{\pi}}(\rho) \geq V_r^*(\rho) - \varepsilon, \text{ and } V_c^{\hat{\pi}}(\rho) \geq b - \varepsilon. \tag{2}$$

Strict feasibility We require $\hat{\pi}$ to achieve an approximately optimal reward value, while simultaneously demanding zero constraint violation in M. Formally, we require $\hat{\pi}$ s.t.

$$V_r^{\hat{\pi}}(\rho) \geq V_r^*(\rho) - \varepsilon, \text{ and } V_c^{\hat{\pi}}(\rho) \geq b \tag{3}$$

Next, we describe a general model-based algorithm to handle both these cases, and subsequently instantiate the algorithm for the relaxed feasibility (Section 4) and strict feasibility (Section 5) settings.

3 Methodology

We will use a model-based approach [2, 20, 15] for achieving the objectives in Eq. (2) and Eq. (3). In particular, for each (s, a) pair, we collect N independent samples from $\mathcal{P}(\cdot|s, a)$ and form an empirical

[1] These ranges for r and c are chosen for simplicity. Our results can be easily extended to handle other ranges.

[2] The performance of an optimal policy in a CMDP can always be achieved by a stationary, stochastic policy [3]. On the other hand, for an MDP, it suffices to only consider stationary, deterministic policies [23].

Algorithm 1: Model-based algorithm for CMDPs with generative model

Input: $\mathcal{S}$ (state space), $\mathcal{A}$ (action space), r (rewards), c (constraint rewards), ζ (Slater constant), N (number of samples), b' (constraint RHS), ω (perturbation magnitude), U (projection upper bound), ε_1 (epsilon-net resolution), T (number of iterations), $\lambda_0 = 0$ (initialization).

For each state-action (s, a) pair, collect N samples from $\mathcal{P}(.|s, a)$ and form $\hat{\mathcal{P}}$.

Perturb the rewards to form vector $r_p(s, a) := r(s, a) + \xi(s, a)$ where $\xi(s, a) \sim \mathcal{U}[0, \omega]$.

Form the empirical CMDP $\hat{M} = \langle \mathcal{S}, \mathcal{A}, \hat{\mathcal{P}}, r_p, c, b', \rho, \gamma \rangle$.

Form the epsilon-net $\Lambda = \{0, \varepsilon_1, 2\varepsilon_1, \ldots, U\}$.

for $t \leftarrow 0$ **to** $T - 1$ **do**

  Update the policy by solving an unconstrained MDP: $\hat{\pi}_t = \arg\max \hat{V}^{\pi}_{r_p + \lambda_t c}$.

  Update the dual-variables: $\lambda_{t+1} = \mathcal{R}_{\Lambda}\left[\mathbb{P}_{[0,U]}\left[\lambda_t - \eta\left(\hat{V}^{\hat{\pi}_t}_c(\rho) - b'\right)\right]\right]$.

end

Output: Mixture policy $\bar{\pi}_T = \frac{1}{T}\sum_{t=0}^{T-1} \hat{\pi}_t$.

transition matrix $\hat{\mathcal{P}}$ such that $\hat{\mathcal{P}}(s'|s, a) = \frac{N(s'|s,a)}{N}$, where $N(s'|s, a)$ is the number of samples that have transitions from (s, a) to s'. These estimated transition probabilities are used to form an empirical CMDP. Due to a technical requirement, (which we will clarify in the Section 6), we require adding a small random perturbation to the rewards in the empirical CMDP[3]. In particular, for each $s \in \mathcal{S}$ and $a \in \mathcal{A}$, we define the perturbed rewards $r_p(s, a) := r(s, a) + \xi(s, a)$ where $\xi(s, a) \sim \mathcal{U}[0, \omega]$ are i.i.d. uniform random variables. Finally, compared to Eq. (1), we will require solving the empirical CMDP with a constraint right-hand side equal to b'. Note that setting $b' < b$ corresponds to loosening the constraint, while $b' > b$ corresponds to tightening the constraint. This completes the specification of the empirical CMDP $\hat{M}$ that is defined by the tuple $\langle \mathcal{S}, \mathcal{A}, \hat{\mathcal{P}}, r_p, c, b', \rho, \gamma \rangle$. For $\hat{M}$, the corresponding reward value function (and constraint value function) for policy π is denoted as $\hat{V}^{\pi}_{r_p}(\rho)$ (and $\hat{V}^{\pi}_c(\rho)$ respectively). In order to fully instantiate $\hat{M}$, we require setting the values of ω (the magnitude of the perturbation) and b' (the constraint right-hand side). This depends on the specific setting (relaxed vs strict feasibility) and we do this in Sections 4 and 5 respectively. We compute the optimal policy for the empirical CMDP $\hat{M}$ as follows:

$$\hat{\pi}^* \in \arg\max \hat{V}^{\pi}_{r_p}(\rho) \text{ s.t. } \hat{V}^{\pi}_c(\rho) \geq b' \tag{4}$$

In contrast to Agarwal et al. [2], Li et al. [20] that consider model-based approaches for unconstrained MDPs and can solve the resulting empirical MDP using any black-box approach, we will require solving Eq. (4) using a specific primal-dual approach that we outline next. Using this algorithm enables us to prove optimal sample complexity bounds under both relaxed and strict feasibility.

First, observe that Eq. (4) can be written as an equivalent saddle-point problem – $\max_{\pi} \min_{\lambda \geq 0} \left[\hat{V}^{\pi}_{r_p}(\rho) + \lambda\left(\hat{V}^{\pi}_c(\rho) - b'\right)\right]$, where $\lambda \in \mathbb{R}$ corresponds to the Lagrange multiplier for the constraint. The solution to this saddle-point problem is $(\hat{\pi}^*, \lambda^*)$ where $\hat{\pi}^*$ is the optimal empirical policy and λ^* is the optimal Lagrange multiplier. We solve the above saddle-point problem iteratively, by alternatively updating the policy (primal variable) and the Lagrange multiplier (dual variable). If T is the total number of iterations of the primal-dual algorithm, we define $\hat{\pi}_t$ and λ_t to be the primal and dual iterates for $t \in [T] := \{1, \ldots, T\}$. The primal update at iteration t is given as:

$$\hat{\pi}_t = \arg\max \left[\hat{V}^{\pi}_{r_p}(\rho) + \lambda_t \hat{V}^{\pi}_c(\rho)\right] = \arg\max \hat{V}^{\pi}_{r_p + \lambda_t c}. \tag{5}$$

Hence, iteration t of the algorithm requires solving an unconstrained MDP with a reward equal to $r_p + \lambda_t c$. This can be done using any black-box MDP solver such as policy iteration. The algorithm updates the Lagrange multipliers using a gradient descent step and requires projecting and rounding the resulting dual variables. In particular, the dual variables are first projected onto the $[0, U]$ interval, where U is chosen to be an upper-bound on $|\lambda^*|$. After the projection, the resulting iterates are rounded to the closest element in the set $\Lambda = \{0, \varepsilon_1, 2\varepsilon_1, \ldots, U\}$, a one-dimensional epsilon-net (with resolution ε_1) over the dual variables. In Section 6, we will see that constructing such an ε_1-net will

[3] Similar to MDPs [20], we can instead perturb the Q function while planning in the empirical CMDP.

enable us to prove concentration results for all $\lambda \in \Lambda$. Formally, the dual update at iteration t is given as:

$$\lambda_{t+1} = \mathcal{R}_\Lambda \left[\mathbb{P}_{[0,U]} \left[\lambda_t - \eta \left(\hat{V}_c^{\hat{\pi}_t}(\rho) - b' \right) \right] \right] , \tag{6}$$

where $\mathbb{P}_{[0,U]}[\lambda] = \arg\min_{p \in [0,U]} |\lambda - p|$ projects λ onto the $[0, U]$ interval and $\mathcal{R}_\Lambda[\lambda] = \arg\min_{p \in \Lambda} |\lambda - p|$ rounds λ to the closest element in Λ. Since Λ is an epsilon-net, for all $\lambda \in [0, U]$, $|\lambda - \mathcal{R}_\Lambda[\lambda]| \leq \varepsilon_{\text{l}}$. Finally, η in Eq. (6) corresponds to the step-size for the gradient descent update. The above primal-dual updates are similar to the dual-descent algorithm proposed in Paternain et al. [22]. The pseudo-code summarizing the entire model-based algorithm is given in Algorithm 1. We note that although Algorithm 1 requires the knowledge of ζ, this is not essential and we can instead use an estimate of ζ. In Appendix F, we show that we can estimate ζ to within a factor of 2 using $\tilde{O}\left(\frac{|\mathcal{S}||\mathcal{A}|}{(1-\gamma)^3 \zeta^2}\right)$ additional queries. Next, we show that the primal-dual updates in Algorithm 1 can be used to solve the empirical CMDP $\hat{M}$. Specifically, we prove the following theorem (proof in Appendix A) that bounds the average optimality gap (in the reward value function) and constraint violation for the mixture policy returned by Algorithm 1.

Theorem 1 (Guarantees for the primal-dual algorithm)**.** For a target error $\varepsilon_{\text{opt}} > 0$ and the primal-dual updates in Eq. (5)-Eq. (6) with $U > |\lambda^*|$, $T = \frac{4U^2}{\varepsilon_{\text{opt}}^2 (1-\gamma)^2} \left[1 + \frac{1}{(U - \lambda^*)^2} \right]$, $\eta = \frac{U(1-\gamma)}{\sqrt{T}}$ and $\varepsilon_{\text{l}} = \frac{\varepsilon_{\text{opt}}^2 (1-\gamma)^2 (U - \lambda^*)}{6U}$, the mixture policy $\bar{\pi}_T := \frac{1}{T} \sum_{t=0}^{T-1} \hat{\pi}_t$ satisfies,

$$\hat{V}_{r_p}^{\bar{\pi}_T}(\rho) \geq \hat{V}_{r_p}^{\hat{\pi}^*}(\rho) - \varepsilon_{\text{opt}} \quad ; \quad \hat{V}_c^{\bar{\pi}_T}(\rho) \geq b' - \varepsilon_{\text{opt}}.$$

Hence, with $T = O(1/\varepsilon_{\text{opt}}^2)$ and $\varepsilon_{\text{l}} = O(\varepsilon_{\text{opt}}^2)$, the algorithm outputs a policy $\bar{\pi}_T$ that achieves a reward ε_{opt} close to that of the optimal empirical policy $\hat{\pi}^*$, while violating the constraint by at most ε_{opt}. Hence, with sufficient number of iterations T and by choosing a sufficiently small resolution ε_{l} for the epsilon-net, we can use the above primal-dual algorithm to approximately solve the problem in Eq. (4). In order to completely instantiate the primal-dual algorithm, we require setting $U > |\lambda^*|$. We will subsequently do this for the the relaxed and strict feasibility settings in Sections 4 and 5 respectively. We note that in contrast to Paternain et al. [22, Theorem 3] that bounds the Lagrangian, Theorem 1 provides explicit bounds on both the reward suboptimality and constraint violation.

We conclude this section by making some observations about the primal-dual algorithm – while the subsequent bounds for both settings heavily depend on using the "best-response" primal update in Eq. (5), the algorithm does not require using the specific form of the dual updates in Eq. (6). Indeed, when used in conjunction with the projection and rounding operations in Eq. (6), we can use any method to update the dual variables (not necessarily gradient descent) provided that it results in an $O\left(T^a + \varepsilon_{\text{l}} T^b\right)$ (for $a < 1$) bound on the dual regret (see the proof of Theorem 1 for the definition). Next, we specify the values of N, b', ω, ε_{l}, T, U in Algorithm 1 to achieve the objective in Eq. (2).

4 Upper-bound under Relaxed Feasibility

In order to achieve the objective in Eq. (2) for a target error $\varepsilon > 0$, we require setting $N = \tilde{O}\left(\frac{\log(1/\delta)}{(1-\gamma)^3 \varepsilon^2}\right)$, $b' = b - \frac{3\varepsilon}{8}$ and $\omega = \frac{\varepsilon(1-\gamma)}{8}$. This completely specifies the empirical CMDP $\hat{M}$ and the problem in Eq. (4). In order to specify the primal-dual algorithm, we set $U = O\left(1/\varepsilon\,(1-\gamma)\right)$, $\varepsilon_{\text{l}} = O\left(\varepsilon^2 (1-\gamma)^2\right)$ and $T = O\left(1/(1-\gamma)^4 \varepsilon^4\right)$. With these choices, we prove the following theorem in Appendix B and provide a proof sketch below.

Theorem 2. For a fixed $\varepsilon \in (0, 1/1-\gamma]$ and $\delta \in (0, 1)$, Algorithm 1 with $N = \tilde{O}\left(\frac{\log(1/\delta)}{(1-\gamma)^3 \varepsilon^2}\right)$ samples, $b' = b - \frac{3\varepsilon}{8}$, $\omega = \frac{\varepsilon(1-\gamma)}{8}$, $U = O\left(1/\varepsilon\,(1-\gamma)\right)$, $\varepsilon_{\text{l}} = O\left(\varepsilon^2 (1-\gamma)^2\right)$ and $T = O\left(1/(1-\gamma)^4 \varepsilon^4\right)$, returns policy $\bar{\pi}_T$ that satisfies the objective in Eq. (2) with probability at least $1 - 4\delta$.

Proof Sketch: We prove the result for a general primal-dual error $\varepsilon_{\text{opt}} < \varepsilon$ and $b' = b - \frac{\varepsilon - \varepsilon_{\text{opt}}}{2}$, and subsequently specify ε_{opt} and hence b'. In Lemma 11 (proved in Appendix B), we show that if the constraint value functions are sufficiently concentrated (the empirical value function is close to the

ground truth value function) for both the optimal policy π^* in M and the mixture policy $\bar{\pi}_T$ returned by Algorithm 1, i.e., if

$$\left|V_c^{\bar{\pi}_T}(\rho) - \hat{V}_c^{\bar{\pi}_T}(\rho)\right| \leq \frac{\varepsilon - \varepsilon_{\text{opt}}}{2} \quad ; \quad \left|V_c^{\pi^*}(\rho) - \hat{V}_c^{\pi^*}(\rho)\right| \leq \frac{\varepsilon - \varepsilon_{\text{opt}}}{2}, \tag{7}$$

then (i) policy $\bar{\pi}_T$ violates the constraint in M by at most ε, i.e., $V_c^{\bar{\pi}_T}(\rho) \geq b - \varepsilon$, and (ii) its suboptimality in M (compared to π^*) can be decomposed as:

$$V_r^{\pi^*}(\rho) - V_r^{\bar{\pi}_T}(\rho) \leq \frac{2\omega}{1-\gamma} + \varepsilon_{\text{opt}} + \left|V_{r_p}^{\pi^*}(\rho) - \hat{V}_{r_p}^{\pi^*}(\rho)\right| + \left|\hat{V}_{r_p}^{\bar{\pi}_T}(\rho) - V_{r_p}^{\bar{\pi}_T}(\rho)\right|. \tag{8}$$

In order to instantiate the primal-dual algorithm, we require a concentration result for policy π_c^* that maximizes the the constraint value function, i.e. if $\pi_c^* := \arg\max V_c^{\pi}(\rho)$, then we require $\left|V_c^{\pi_c^*}(\rho) - \hat{V}_c^{\pi_c^*}(\rho)\right| \leq \varepsilon + \varepsilon_{\text{opt}}$. In Case 1 of Lemma 9 (proved in Appendix A), we show that if this concentration result holds, then we can upper-bound the optimal dual variable $|\lambda^*|$ by $\frac{2(1+\omega)}{(\varepsilon+\varepsilon_{\text{opt}})(1-\gamma)}$. With these results in hand, we can instantiate all the algorithm parameters except N (the number of samples required for each state-action pair). In particular, we set $\varepsilon_{\text{opt}} = \frac{\varepsilon}{4}$ and hence $b' = b - \frac{3\varepsilon}{8}$, and $\omega = \frac{\varepsilon(1-\gamma)}{8} < 1$. Setting $U = \frac{32}{5\varepsilon\,(1-\gamma)}$ ensures that the $U > |\lambda^*|$ condition required by Theorem 1 holds. In order to guarantee that the primal-dual algorithm outputs an $\frac{\varepsilon}{4}$-approximate policy, we use Theorem 1 to set $T = O\left(\frac{1}{(1-\gamma)^4\varepsilon^4}\right)$ iterations and $\varepsilon_1 = O\left(\varepsilon^2(1-\gamma)^2\right)$. Eq. (8) can then be simplified as,

$$V_r^{\pi^*}(\rho) - V_r^{\bar{\pi}_T}(\rho) \leq \frac{\varepsilon}{2} + \left|V_{r_p}^{\pi^*}(\rho) - \hat{V}_{r_p}^{\pi^*}(\rho)\right| + \left|\hat{V}_{r_p}^{\bar{\pi}_T}(\rho) - V_{r_p}^{\bar{\pi}_T}(\rho)\right|.$$

Putting everything together, in order to guarantee an ε-reward suboptimality for $\bar{\pi}_T$, we require the following results:

$$\begin{aligned}
&\left|V_c^{\pi_c^*}(\rho) - \hat{V}_c^{\pi_c^*}(\rho)\right| \leq \frac{5\varepsilon}{4}\,;\ \left|V_c^{\bar{\pi}_T}(\rho) - \hat{V}_c^{\bar{\pi}_T}(\rho)\right| \leq \frac{3\varepsilon}{8}\,;\ \left|V_c^{\pi^*}(\rho) - \hat{V}_c^{\pi^*}(\rho)\right| \leq \frac{3\varepsilon}{8} \\
&\left|V_{r_p}^{\pi^*}(\rho) - \hat{V}_{r_p}^{\pi^*}(\rho)\right| \leq \frac{\varepsilon}{4}\,;\ \left|\hat{V}_{r_p}^{\bar{\pi}_T}(\rho) - V_{r_p}^{\bar{\pi}_T}(\rho)\right| \leq \frac{\varepsilon}{4}.
\end{aligned} \tag{9}$$

We control such concentration terms for both the constraint and reward value functions in Section 6, and bound the terms in Eq. (9). In particular, we prove that for a fixed $\varepsilon \in (0, 1/1-\gamma]$, using $N \geq \tilde{O}\left(\frac{\log(1/\delta)}{(1-\gamma)^3\,\varepsilon^2}\right)$ samples enssures that the statements in Eq. (9) hold with probability $1 - 4\delta$. This guarantees that $V_r^{\pi^*}(\rho) - V_r^{\bar{\pi}_T}(\rho) \leq \varepsilon$ and $V_c^{\bar{\pi}_T}(\rho) \geq b - \varepsilon$. □

Hence, the total sample-complexity of achieving the objective in Eq. (2) is $\tilde{O}\left(\frac{SA\log(1/\delta)}{(1-\gamma)^3\varepsilon^2}\right)$. This result improves over the $\tilde{O}\left(\frac{S^2A\log(1/\delta)}{(1-\gamma)^3\varepsilon^2}\right)$ result in HasanzadeZonuzy et al. [15]. Furthermore, our result matches the lower-bound in the easier unconstrained setting [4], implying that our bounds are near-optimal. We conclude that under relaxed feasibility and with access to a generative model, solving constrained MDPs is as easy as solving MDPs. Algorithmically, we do not require constructing an optimistic CMDP like in HasanzadeZonuzy et al. [15]. Instead, we solve the empirical CMDP in Eq. (4) using specific primal-dual updates Eqs. (5) and (6). In the next section, we instantiate Algorithm 1 for the strict feasibility setting.

5 Upper-bound under Strict Feasibility

In order to achieve the objective in Eq. (3) for a target error $\varepsilon > 0$, we require setting $N = \tilde{O}\left(\frac{\log(1/\delta)}{(1-\gamma)^5\zeta^2\varepsilon^2}\right)$,[4] $b' = b + \frac{\varepsilon(1-\gamma)\zeta}{20}$ and $\omega = \frac{\varepsilon(1-\gamma)}{10}$. This completely specifies the empirical CMDP $\hat{M}$ and the problem in Eq. (4). In order to specify the primal-dual algorithm, we set $U = \frac{4(1+\omega)}{\zeta(1-\gamma)}$, $\varepsilon_1 = O\left(\varepsilon^2(1-\gamma)^4\zeta^2\right)$ and $T = O\left(1/(1-\gamma)^6\zeta^4\varepsilon^2\right)$. With these choices, we prove the following theorem in Appendix C, and provide a proof sketch below.

[4] Again, we do not need to know ζ and it can be replaced by the estimator constructed in Section F.

Theorem 3. For a fixed $\varepsilon \in (0, 1/1-\gamma]$ and $\delta \in (0,1)$, Algorithm 1, with $N = \tilde{O}\left(\frac{\log(1/\delta)}{(1-\gamma)^5 \varepsilon^2 \zeta^2}\right)$ samples, $b' = b + \frac{\varepsilon(1-\gamma)\zeta}{20}$, $\omega = \frac{\varepsilon(1-\gamma)}{10}$, $U = \frac{4(1+\omega)}{\zeta(1-\gamma)}$, $\varepsilon_1 = O\left(\varepsilon^2(1-\gamma)^4\zeta^2\right)$ and $T = O\left(1/(1-\gamma)^6\zeta^4\varepsilon^2\right)$ returns policy $\bar{\pi}_T$ that satisfies the objective in Eq. (3), with probability at least $1-4\delta$.

Proof Sketch: We prove the result for a general $b' = b + \Delta$ for $\Delta > 0$ and primal-dual error $\varepsilon_{\text{opt}} < \Delta$, and subsequently specify Δ (and hence b') and ε_{opt}. In Lemma 12 (proved in Appendix C), we prove that if the constraint value functions are sufficiently concentrated (the empirical value function is close to the ground truth value function) for both the optimal policy π^* in M and the mixture policy $\bar{\pi}_T$ returned by Algorithm 1 i.e. if

$$\left|V_c^{\bar{\pi}_T}(\rho) - \hat{V}_c^{\bar{\pi}_T}(\rho)\right| \leq \Delta - \varepsilon_{\text{opt}} \quad ; \quad \left|V_c^{\pi^*}(\rho) - \hat{V}_c^{\pi^*}(\rho)\right| \leq \Delta \tag{10}$$

then (i) policy $\bar{\pi}_T$ satisfies the constraint in M i.e. $V_c^{\bar{\pi}_T}(\rho) \geq b$, and (ii) its suboptimality in M (compared to π^*) can be decomposed as:

$$V_r^{\pi^*}(\rho) - V_r^{\bar{\pi}_T}(\rho) \leq \frac{2\omega}{1-\gamma} + \varepsilon_{\text{opt}} + 2\Delta|\lambda^*| + \left|V_{r_p}^{\pi^*}(\rho) - \hat{V}_{r_p}^{\pi^*}(\rho)\right| + \left|\hat{V}_{r_p}^{\bar{\pi}_T}(\rho) - V_{r_p}^{\bar{\pi}_T}(\rho)\right| \tag{11}$$

In order to upper-bound $|\lambda^*|$, we require a concentration result for policy $\pi_c^* := \arg\max V_c^{\pi}(\rho)$ that maximizes the the constraint value function. In particular, we require $\Delta \in \left(0, \frac{\zeta}{2}\right)$ and $\left|V_c^{\pi_c^*}(\rho) - \hat{V}_c^{\pi_c^*}(\rho)\right| \leq \frac{\zeta}{2} - \Delta$. In Case 2 of Lemma 9 (proved in Appendix A), we show that if this concentration result holds, then we can upper-bound the optimal dual variable $|\lambda^*|$ by $\frac{2(1+\omega)}{\zeta(1-\gamma)}$. Using the above bounds to simplify Eq. (11),

$$V_r^{\pi^*}(\rho) - V_r^{\bar{\pi}_T}(\rho) \leq \frac{2\omega}{1-\gamma} + \varepsilon_{\text{opt}} + \frac{4\Delta(1+\omega)}{\zeta(1-\gamma)} + \left|V_{r_p}^{\pi^*}(\rho) - \hat{V}_{r_p}^{\pi^*}(\rho)\right| + \left|\hat{V}_{r_p}^{\bar{\pi}_T}(\rho) - V_{r_p}^{\bar{\pi}_T}(\rho)\right|.$$

With these results in hand, we can instantiate all the algorithm parameters except N (the number of samples required for each state-action pair). In particular, we set $\Delta = \frac{\varepsilon\,(1-\gamma)\,\zeta}{40} < \frac{\zeta}{2}$, $\varepsilon_{\text{opt}} = \frac{\Delta}{5} = \frac{\varepsilon\,(1-\gamma)\,\zeta}{200} < \frac{\varepsilon}{5}$, and $\omega = \frac{\varepsilon(1-\gamma)}{10} < 1$. We set $U = \frac{8}{\zeta(1-\gamma)}$ for the primal-dual algorithm, ensuring that the $U > |\lambda^*|$ condition required by Theorem 1 holds. In order to guarantee that the primal-dual algorithm outputs an $\frac{\varepsilon\,(1-\gamma)\,\zeta}{200}$-approximate policy, we use Theorem 1 to set $T = O\left(\frac{1}{(1-\gamma)^6\zeta^4\varepsilon^2}\right)$ iterations and $\varepsilon_1 = O\left(\varepsilon^2(1-\gamma)^4\zeta^2\right)$. With these values, we can further simplify Eq. (11),

$$V_r^{\pi^*}(\rho) - V_r^{\bar{\pi}_T}(\rho) \leq \frac{3\varepsilon}{5} + \left|V_{r_p}^{\pi^*}(\rho) - \hat{V}_{r_p}^{\pi^*}(\rho)\right| + \left|\hat{V}_{r_p}^{\bar{\pi}_T}(\rho) - V_{r_p}^{\bar{\pi}_T}(\rho)\right|.$$

Putting everything together, in order to guarantee an ε-reward suboptimality for $\bar{\pi}_T$, we require the following concentration results to hold for $\Delta = \frac{\varepsilon(1-\gamma)\zeta}{40}$,

$$\begin{aligned}
&\left|V_c^{\bar{\pi}_T}(\rho) - \hat{V}_c^{\bar{\pi}_T}(\rho)\right| \leq \frac{4\Delta}{5}\,;\ \left|V_c^{\pi^*}(\rho) - \hat{V}_c^{\pi^*}(\rho)\right| \leq \Delta\,;\ \left|V_c^{\pi_c^*}(\rho) - \hat{V}_c^{\pi_c^*}(\rho)\right| \leq \frac{19\Delta}{5} \\
&\left|V_{r_p}^{\pi^*}(\rho) - \hat{V}_{r_p}^{\pi^*}(\rho)\right| \leq \frac{\varepsilon}{5}\,;\ \left|\hat{V}_{r_p}^{\bar{\pi}_T}(\rho) - V_{r_p}^{\bar{\pi}_T}(\rho)\right| \leq \frac{\varepsilon}{5}.
\end{aligned} \tag{12}$$

We control such concentration terms for both the constraint and reward value functions in Section 6, and bound the terms in Eq. (12). In particular, we prove that for a fixed $\varepsilon \in (0, 1/1-\gamma]$, using $N \geq \tilde{O}\left(\frac{\log(1/\delta)}{(1-\gamma)^5\,\zeta^2\,\varepsilon^2}\right)$ ensures that the statements in Eq. (12) hold with probability $1-4\delta$. This guarantees that $V_r^{\pi^*}(\rho) - V_r^{\bar{\pi}_T}(\rho) \leq \varepsilon$ and $V_c^{\bar{\pi}_T}(\rho) \geq b$. □

Hence, the total sample-complexity of achieving the objective in Eq. (3) is $\tilde{O}\left(\frac{SA\,\log(1/\delta)}{(1-\gamma)^5\,\zeta^2\varepsilon^2}\right)$. In this setting, we note that Bai et al. [6] propose a model-free algorithm with an $\tilde{O}\left(\frac{SA\,L\,\log(1/\delta)}{(1-\gamma)^4\,\zeta^2\varepsilon^2}\right)$ sample complexity. Here, L is a potentially large problem-dependent parameter that is not explicitly bounded by Bai et al. [6] and depends on the Lipschitz constant of the reward and constraint reward value functions. In Section 7, we prove a matching lower bound showing that Algorithm 1 is minimax optimal in the strict feasibility setting. In the next section, we give more details for the bounding the concentration terms in Theorem 2 and Theorem 3.

6 Bounding the concentration terms

We have seen that proving Theorem 2 and Theorem 3 require bounding the concentration terms in Eq. (9) and Eq. (12) respectively. In this section, we detail the techniques to achieve these bounds.

Our approach requires reasoning about a general unconstrained MDP $M_\alpha = (\mathcal{S}, \mathcal{A}, \mathcal{P}, \gamma, \alpha)$ with the same state-action space, transition probabilities and discount factor as the CMDP in Eq. (1) but with rewards equal to $\alpha \in [0, \alpha_{\max}]$. Analogously, we define the empirical MDP $\hat{M}_\alpha = (\mathcal{S}, \mathcal{A}, \hat{\mathcal{P}}, \gamma, \alpha)$ where the empirical transition matrix $\hat{\mathcal{P}}$ is the same as that of the empirical CMDP in Eq. (4). Similarly, we define MDP (and its empirical counterpart) $M_\beta = (\mathcal{S}, \mathcal{A}, \mathcal{P}, \gamma, \beta)$ (and $\hat{M}_\beta$) where the rewards $\beta \in [0, \beta_{\max}]$. Note that the rewards α and β are independent of the sampling of the transition matrix. The corresponding value functions for policy π in M_α and $\hat{M}_\alpha$ (and M_β and $\hat{M}_\beta$) are denoted as V_α^π and $\hat{V}_\alpha^\pi$ (and V_β^π and $\hat{V}_\beta^\pi$) respectively, with the optimal value functions denoted as V_α^* and $\hat{V}_\alpha^*$ (and V_β^* and $\hat{V}_\beta^*$) respectively. The action-value function in M_α for policy π and state-action pair (s, a) is denoted as $Q_\alpha^\pi(s, a)$ and analogously for $\hat{M}_\alpha$. For the subsequent technical results, we require that $\hat{M}_\alpha$ satisfy the following gap condition [20]:

Definition 4 (ι-Gap Condition)**.** *MDP $\hat{M}_\alpha$ satisfies the ι-gap condition if $\forall s$, $\hat{V}_\alpha^*(s) - \max_{a' : a \neq \hat{\pi}_\alpha^*(s)} \hat{Q}_\alpha^*(s, a') \geq \iota$, where $\hat{\pi}_\alpha^* := \arg\max \hat{V}_\alpha^\pi$ and $\hat{\pi}_\alpha^*(s) = \arg\max_a \hat{Q}_\alpha^*(s, a)$ is the optimal action in state s.*

Intuitively, the gap condition states that there is a unique optimal action at each state and there is a gap between the performance of best action and the second best action. With this gap condition, we use techniques in Li et al. [20] to prove the following lemma in Appendix D.

Lemma 5. *Define $\hat{\pi}_\alpha^* := \arg\max_\pi \hat{V}_\alpha^\pi$. If (i) $\mathcal{E}$ is the event that the ι-gap condition in Definition 4 holds for $\hat{M}_\alpha$ and (ii) for $\delta \in (0, 1)$ and $C(\delta) = 72 \log\left(\frac{16 \alpha_{\max} SA \log(e/1-\gamma)}{(1-\gamma)^2\, \iota\, \delta}\right)$, the number of samples per state-action pair is $N \geq \frac{4\, C(\delta)}{1-\gamma}$, then with probability at least $\Pr[\mathcal{E}] - \delta/10$,*

$$\left\| \hat{V}_\beta^{\hat{\pi}_\alpha^*} - V_\beta^{\hat{\pi}_\alpha^*} \right\|_\infty \leq \sqrt{\frac{C(\delta)}{N \cdot (1-\gamma)^3}}\, \|\beta\|_\infty .$$

Hence, for policy $\hat{\pi}_\alpha^*$, we can obtain a concentration result in another MDP $\hat{M}_\beta$ with an independent reward function β and the same empirical transition matrix $\hat{\mathcal{P}}$.

We wish to use the above lemma for the unconstrained MDP formed at every iteration of the primal update in Eq. (5). In particular, for a given λ_t, we will use Lemma 5 with $\alpha = r_p + \lambda_t c$ and $\beta = r_p$. Doing so will immediately give us a bound on $\left\| V_{r_p}^{\hat{\pi}_t} - \hat{V}_{r_p}^{\hat{\pi}_t} \right\|_\infty$ and hence $\left| V_{r_p}^{\hat{\pi}_t}(\rho) - \hat{V}_{r_p}^{\hat{\pi}_t}(\rho) \right|$. In order to use Lemma 5, we require the unconstrained MDP $\hat{M}_{r_p + \lambda c}$ to satisfy the gap condition in Definition 4 for any $\lambda \in \Lambda$. This is achieved by the perturbation of the rewards in Line 3 of Algorithm 1. Specifically, using Li et al. [20, Lemma 6] with a union-bound over Λ, we prove (in Lemma 16 in Appendix D) that with probability $1 - \delta/10$, $\hat{M}_{r_p + \lambda c}$ satisfies the gap condition in Definition 4 with $\iota = \frac{\omega\, \delta\, (1-\gamma)}{30\, |\Lambda| |S| |A|^2}$ for every $\lambda \in \Lambda$. This allows us to use Lemma 5 with $\alpha = r_p + \lambda_t c$ for all $t \in [T]$, and $\beta = r_p$ and $\beta = c$. In the following theorem, we obtain a concentration result for each $\hat{\pi}_t$ and hence for the mixture policy $\bar{\pi}_T$.

Theorem 6. For $\delta \in (0, 1)$, $\omega \leq 1$ and $C(\delta) = 72 \log\left(\frac{16(1+U+\omega)\, SA \log(e/1-\gamma)}{(1-\gamma)^2\, \iota\, \delta}\right)$ where $\iota = \frac{\omega\, \delta\, (1-\gamma)\, \varepsilon_1}{30\, U |S| |A|^2}$, if $N \geq \frac{4\, C(\delta)}{1-\gamma}$, then for policy $\bar{\pi}_T$ output by Algorithm 1, with probability at least $1 - \delta/5$,

$$\left| V_{r_p}^{\bar{\pi}_T}(\rho) - \hat{V}_{r_p}^{\bar{\pi}_T}(\rho) \right| \leq 2 \sqrt{\frac{C(\delta)}{N \cdot (1-\gamma)^3}} \quad ; \quad \left| V_c^{\bar{\pi}_T}(\rho) - V_c^{\bar{\pi}_T}(\rho) \right| \leq \sqrt{\frac{C(\delta)}{N \cdot (1-\gamma)^3}} .$$

Eqs. (9) and (12) also require proving concentration bounds for fixed (that do not depend on the data) policies π^* and π_c^*. This can be done by directly using Li et al. [20, Lemma 1]. Specifically, we prove the following the lemma in Appendix D.

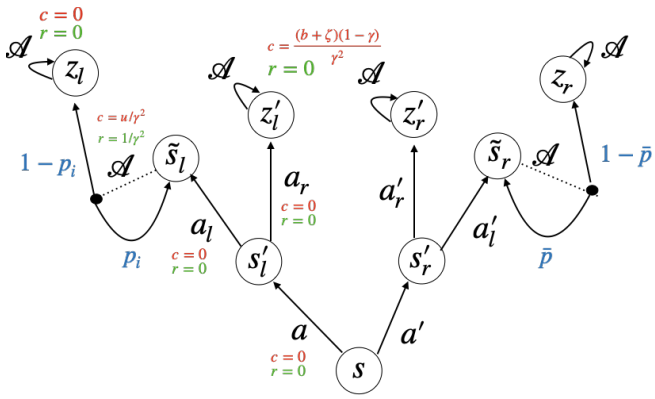

Figure 1: The figure shows the CMDP M_i for $i \in \{0, 1\}$. Except for $\tilde{s}_l$ and $\tilde{s}_r$, the transitions in all states are deterministic. M_0 and M_1 are nearly identical, only differing in the transition probabilities in state $\tilde{s}_l$. The right branch is nearly identical to the left branch, only differing in the transition probability in state $\tilde{s}_r$. The rewards are identical in both the left and right branches and for each M_i.

Lemma 7. *For $\delta \in (0, 1)$, $\omega \leq 1$ and $C'(\delta) = 72 \log\left(\frac{4|S| \log(e/1-\gamma)}{\delta}\right)$, if $N \geq \frac{4\, C'(\delta)}{1-\gamma}$ and $B(\delta, N) := \sqrt{\frac{C'(\delta)}{(1-\gamma)^3 N}}$, then with probability at least $1 - 3\delta$,*

$$\left|V_{r_p}^{\pi^*}(\rho) - \hat{V}_{r_p}^{\pi^*}(\rho)\right| \leq 2B(\delta, N)\,; \left|V_c^{\pi^*}(\rho) - \hat{V}_c^{\pi^*}(\rho)\right| \leq B(\delta, N)\,; \left|V_c^{\pi_c^*}(\rho) - \hat{V}_c^{\pi_c^*}(\rho)\right| \leq B(\delta, N).$$

Using Theorem 6 and Lemma 7, we can bound each term in Eq. (9) and Eq. (12), completing the proof of Theorem 2 and Theorem 3 respectively. In the next section, we prove a lower-bound on the sample-complexity in the strict feasibility setting.

7 Lower-bound under strict feasibility

We define an algorithm to be (ε, δ)-sound if it outputs a policy $\hat{\pi}$ such that with probability $1 - \delta$, $V_r^*(\rho) - V_r^{\hat{\pi}}(\rho) \leq \varepsilon$ and $V_c^{\hat{\pi}}(\rho) \geq b$ i.e. the algorithm achieves the strict feasibility objective in Eq. (3). We prove a lower bound on the number of samples required by any (ε, δ)-sound algorithm on the CMDP instance in Fig. 1. For this instance, with a specific setting of the rewards and probabilities $p_0 < \bar{p} < p_1$, we prove that any (ε, δ)-sound algorithm requires at least $\Omega\left(\frac{\ln(1/4\delta)}{\varepsilon^2 \zeta^2 (1-\gamma)^5}\right)$ samples to distinguish between M_0 and M_1. This enables us to prove the following theorem in Appendix E.

Theorem 8. For constants $\gamma_0 \in (0, 1)$, $c = \frac{1}{2}\left(1 + \frac{1-\gamma_0/2}{1-\gamma_0}\right)$, $\zeta > 0$, any $\gamma \geq \gamma_0$ and $\varepsilon \leq \frac{1}{4c(1-\gamma)} \min\left\{1, \frac{1}{4} \frac{\gamma}{(1-\gamma)\, c\, \zeta}\right\}$, any (ε, δ)-sound algorithm requires $\Omega\left(\frac{SA \ln(1/4\delta)}{\varepsilon^2 \zeta^2 (1-\gamma)^5}\right)$ samples from the generative model in the worst case.

The above lower bound matches the upper bound in Theorem 3 and proves that Algorithm 1 is near minimax optimal in the strict feasibility setting. It also demonstrates that solving CMDPs under strict feasibility is inherently more difficult than solving unconstrained MDPs or CMDPs in the relaxed feasibility setting. Finally, we can conclude that the problem becomes more difficult (requires more samples) as the Slater constant ζ decreases and the feasible region shrinks.

8 Discussion

We proposed a model-based primal-dual algorithm for planning in CMDPs. Via upper and lower bounds, we proved that our algorithm is near minimax optimal for both the relaxed and strict feasibility settings. Our results demonstrate that solving CMDPs is as easy as MDPs when small constraint violations are allowed, but inherently more difficult when we demand zero constraint violation. Algorithmically, we required a specific primal-dual approach that involved solving a sequence of MDPs. In contrast, model-based approaches for MDPs [2, 20] allow the use of any black-box planner. In the future, we aim to extend our sample complexity results to black-box CMDP solvers.

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Checklist

1. For all authors...
   (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [Yes]
   (b) Did you describe the limitations of your work? [Yes]
   (c) Did you discuss any potential negative societal impacts of your work? [N/A]
   (d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [N/A]
2. If you are including theoretical results...
   (a) Did you state the full set of assumptions of all theoretical results? [Yes]
   (b) Did you include complete proofs of all theoretical results? [Yes]
3. If you ran experiments...
   (a) Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? [N/A]
   (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? [N/A]
   (c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? [N/A]
   (d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? [N/A]
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   (c) Did you include the estimated hourly wage paid to participants and the total amount spent on participant compensation? [N/A]